Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

Finite-time H_∞ control for a class of Markovian jump systems with mode-dependent time-varying delay

Jun Cheng[1*], Guihua Li[2], Hong Zhu[1], Shouming Zhong[3] and Yong Zeng[1]

*Correspondence:
jcheng6819@126.com
[1]School of Automation Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, P.R. China
Full list of author information is available at the end of the article

Abstract

This paper is concerned with the problem of finite-time H_∞ control for a class of Markovian jump system with mode-dependent time-varying delay. By using the new augmented multiple Lyapunov function with more general decomposition approach, a novel sufficient condition for finite-time bounded with an H_∞ performance index is derived. Based on the derived condition, the reliable H_∞ control problem is solved, and an explicit expression of the desired controller is also given, the system trajectory stays within a prescribed bound during a specified time interval. Finally, numerical examples are given to demonstrate that the proposed approach is more effective than some existing ones.

Keywords: H_∞ finite-time stability; time-varying delay; Markov jump system; H_∞ control; delay partition approach

1 Introduction

Markovian jump systems were introduced by Krasovskii and Lidskii [1], which can be described by a set of systems with the transitions in a finite mode set. In the last few decades, there has been increasing interest in Markovian jump systems because this class of systems is appropriate to model many physical systems mainly those random failures, repairs and sudden environment disturbance [2–5]. Such class of systems is a special class of stochastic hybrid systems with finite operation modes, which may switch from one to another at different time. As a crucial factor, it is shown that such jumping can be determined by a Markovian chain [6]. For linear Markovian jumping systems, many important issues have been studied extensively such as stability, stabilization, control synthesis and filter design [6–12]. In finite operation modes, Markovian jump systems is a special class of stochastic systems that can switch from one to another at different time. It is worth pointing out that time delay is one of the instability sources for dynamical systems and is a common phenomenon in many industrial and engineering systems [13–18]. Hence, it is not surprising that much effort has been made to investigate of Markovian jump systems with time delay during the last two decades [19–23]. The exponential stabilization of Markovian jump systems with time delay was first studied in [19] where the decay rate was estimated by solving linear matrix inequalities [20]. However, in the aforementioned works, the network-induced delays have been commonly assumed to be deterministic, which is fairly unrealistic since delays resulting from network transmissions are typically time varying.

Generally speaking, the delay-dependent criterions are less conservative than delay-independent ones, especially when the time delay is small enough in Markovian jump systems. Thus, recent efforts were devoted to the delay-dependent Markovian jump systems stability analysis by employing Lyapunov-Krasovskii functionals [24–32]. However, most efforts have been given on how to construct an appropriate Lyapunov functional by dividing the delay interval $[-\mu_2, -\mu_1]$ into N equal length subintervals [22]. It should be pointed out that the delay decomposition method is not effective when the lower bound of time-varying delay is zero. Furthermore, although the decay rate can be computed, it is a fixed value that one cannot adjust to deduce if a larger decay rate is possible. Therefore, how to obtain the improved results without increasing the computational burden has greatly improved the current study.

Over the years, many research efforts have been devoted to the study of finite-time stability of systems. In finite-time interval, finite-time stability is investigated to address these transient performances of control systems. Recently, the concept of finite-time stability has been revisited in the light of linear matrix inequalities (LMIs) and Lyapunov function theory, some results are obtained to ensure that system is finite-time stable or finite-time bounded [33–49]. It is noted that there are still some related issues to be solved, to the best of our knowledge, the finite-time H_∞ control for a class of Markovian jump systems with time-varying delay has not been fully developed. The analysis method in the existing references seems still conservative to study Markovian jump system. There is room for further investigation.

The main contribution of this paper is as follows: Firstly, we present a new augmented Lyapunov functional by employing the more general decomposition of a delay interval for a class of Markovian jump systems with mode-dependent time-varying delay. Secondly, in order to reduce the possible conservativeness and computational burden, some slack matrices are introduced [18]. Several sufficient conditions are derived to guarantee the finite-time stability and boundedness of the resulting closed-loop system. Last but not the least, it is shown that less conservative and more general results can be derived since the time-varying delays are divided into a more general decomposition. We find that finite-time stability is a concept independent from Lyapunov stability and can always be affected by switching behavior significantly, thus it deserves our investigation. The finite-time boundedness criteria can be tackled in the form of LMIs. Finally, numerical examples illustrate the effectiveness of the developed techniques.

Notations: Throughout this paper, we let $P > 0$ ($P \geq 0$, $P < 0$, $P \leq 0$) denote a symmetric positive definite matrix P (positive semi-definite, negative definite and negative semi-definite). For any symmetric matrix P, $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of matrix P, respectively. $\mathcal{R}^n$ denotes the n-dimensional Euclidean space and $\mathcal{R}^{n\times m}$ refers to the set of all $n \times m$ real matrices and $\mathcal{N} = \{1, 2, \dots, N\}$. The identity matrix of order n is denoted as I_n. $*$ represents the elements below the main diagonal of a symmetric matrix. The superscripts $\top$ and -1 stand for matrix transposition and matrix inverse, respectively.

2 Preliminaries

Given a probability space (Ω, F, P) where Ω, F and P respectively represent the sample space, the algebra of events and the probability measure defined on F. In this paper, we consider the following Markov jump system over the space (Ω, F, P) described by

$$\begin{cases} \dot{x}(t) = A_{r_t}x(t) + A_{\tau r_t}x(t - \tau_{r_t}(t)) + B_{r_t}u(t) + D_{r_t}\omega(t), \\ z(t) = C_{r_t}x(t) + C_{\tau r_t}x(t - \tau_{r_t}(t)) + E_{r_t}u(t) + F_{r_t}\omega(t), \\ x(t) = \varphi(t), \quad t = [-\mu_2, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector of the system, $z(t) \in \mathcal{R}^q$ is the controlled output, $u(t) \in \mathcal{R}^m$ is the control input and $\varphi(t)$, $t = [-\mu_2, 0]$ and $r_0 \in \mathcal{N}$ are initial conditions of continuous state and the mode. $\omega(k) \in \mathcal{R}^q$ is the disturbance input satisfying

$$\int_0^\infty \omega^{\mathsf{T}}(t)\omega(t)\,dt \le d. \tag{2}$$

Let the random form process $\{r_t, t \ge 0\}$ be the Markov stochastic process taking values on a finite set $\mathcal{N} = \{1, 2, \ldots, N\}$ with the transition rate matrix $\Omega = \{\pi_{ij}\}$, $i, j \in \mathcal{N}$, and the transition probabilities described as

$$\Pr(r_{t+\Delta} = j \mid r_t = i) = \begin{cases} \pi_{ij}\Delta + o(\Delta) & \text{if } i \ne j, \\ 1 + \pi_{ii}\Delta + o(\Delta) & \text{if } i = j, \end{cases}$$

where $\Delta > 0$, $\pi_{ij} \ge 0$, for $i \ne j$, is the transition rate from mode i at time t to mode j at time $t + \Delta$ and

$$-\pi_{ii} = \sum_{j=1, j\ne i}^{N} \pi_{ij}$$

for each mode $i \in \mathcal{N}$, $\lim_{\Delta\to 0_+} \frac{o(\Delta)}{\Delta} = 0$. $\tau_i(t)$ denotes the mode dependent time-varying state delay in the system and satisfies the following condition:

$$0 < \mu_{1i} \le \tau_i(t) \le \mu_{2i} < \infty, \tag{3}$$

$$h_{1i} \le \dot{\tau}_i(t) \le h_{2i}, \tag{4}$$

where $\mu_1 = \min\{\mu_{1i}, i \in \mathcal{N}\}$ and $\mu_2 = \max\{\mu_{2i}, i \in \mathcal{N}\}$ are prescribed integers representing the lower and upper bounds of time-varying delay $\tau_i(t)$. Similarly, $h_1 = \min\{h_{1i}, i \in \mathcal{N}\}$ and $h_2 = \max\{h_{2i}, i \in \mathcal{N}\}$ are prescribed integers representing the lower and upper bounds of time-varying delay $\dot{\tau}_i(t)$. $A_{r_t}, A_{\tau r_t}, B_{r_t}, D_{r_t}, C_{r_t}, C_{\tau r_t}, E_{r_t}$ and F_{r_t} are known mode-dependent matrices with appropriate dimension functions of the random jumping process $\{r_t\}$ and represent the nominal systems for each $r_t \in \mathcal{N}$. For notation simplicity, when the system operates in the ith mode ($r_t = i$), A_{r_t}, $A_{\tau r_t}$, B_{r_t}, D_{r_t}, C_{r_t}, $C_{\tau r_t}$, E_{r_t} and F_{r_t} are denoted as A_i, $A_{\tau i}$, B_i, D_i, C_i, $C_{\tau i}$, E_i and F_i, respectively.

Remark 1 In this paper, the lower bound of $\dot{\tau}_i(t)$ is required in order to implement the proposed delay decomposition method. If $h_{1i} = h_{2i} = 0$, then $\tau_i(t)$ corresponds to the constant delay.

Moreover, the transient process of a system can be clearly characterized if its decay rate is available. The objective of this study is to develop a new approach to designing a state feedback controller

$$u(t) = K_i x(t) \tag{5}$$

via a novel Lyapunov functional such that the resulting closed-loop system is finite-time stable, where K_i is the controller gains to be designed.

In this paper, we split the delay interval $[-\mu_2, -\mu_1]$ into two segments: $[-\tau_i(t), -\mu_1] \cup [-\mu_2, -\tau_i(t)]$. Moreover, we further subdivide each interval into l, m equal length subsegments $[-\tau_k, -\tau_{k-1}]$ and $[-\tau_{l+s}, -\tau_{l+s-1}]$, respectively, where

$$\tau_k = \mu_1 + \frac{k}{l}\big(\tau_i(t) - \mu_1\big), \qquad \tau_{l+s} = \tau_i(t) + \frac{s}{m}\big(\mu_2 - \tau_i(t)\big), \quad k = 0, 1, \ldots, l, s = 0, 1, \ldots, m,$$

and l, m are given positive integers.

Remark 2 The delay intervals are divided subsegments dependent on t, thus the proposed delay decomposition method is more general than those in [13–17, 19–24]. The conservatism will be reduced with the partitioning number l and m increase.

In order to more precisely describe the main objective, we introduce the following definitions and lemmas for the underlying system.

Definition 2.1 *System* (1) *is said to be finite-time bounded with respect to* $(c_1, c_2, T, \overline{R}_{r_t}, d)$ *if condition* (2) *and the following inequality hold*:

$$\begin{aligned} &\sup_{-\mu_2 \le \upsilon \le 0} \mathbb{E}\big\{x^{\mathsf T}(\upsilon)\overline{R}_{r_t}x(\upsilon), \dot{x}^{\mathsf T}(\upsilon)\overline{R}_{r_t}\dot{x}(\upsilon)\big\} \le c_1 \\ &\quad\Rightarrow \quad \mathbb{E}\big\{x^{\mathsf T}(t)\overline{R}_{r_t}x(t)\big\} < c_2, \quad \forall t \in [0, T], \end{aligned} \tag{6}$$

where $c_2 > c_1 \ge 0$ *and* $\overline{R}_{r_t} > 0$.

Definition 2.2 [49] *Consider* $V(x_t, r_t)$ *as the stochastic Lyapunov function of the resulting system* (1), *its weak infinitesimal operator is defined as*

$$\begin{aligned} \pounds V(x_t, r_t, t) &= \lim_{\Delta t \to 0} \frac{1}{\Delta t}\big[\mathbb{E}\big\{V(x_{t+\Delta t}, r_{t+\Delta t}, t + \Delta t)\big\} - V(x_t, i, t)\big] \\ &= \frac{\partial}{\partial t}V(x_t, i, t) + \frac{\partial}{\partial x}V(x_t, i, t)\dot{x}(t, i) + \sum_{j=1}^{N} \pi_{ij}V(x_t, j, t). \end{aligned}$$

Definition 2.3 *Given a constant* $T > 0$, *for all admissible* $\omega(t)$ *subject to condition* (2), *under zero initial conditions, if the closed-loop Markovian jump system* (1) *is finite-time bounded and the control outputs satisfy condition* (5) *with attenuation* $\gamma > 0$,

$$\mathbb{E}\left\{\int_0^T z^{\mathsf T}(t)z(t)\,dt\right\} \le \gamma^2 e^{\eta T}\mathbb{E}\left\{\int_0^T \omega^{\mathsf T}(t)\omega(t)\,dt\right\},$$

then the controller system (1) *is called the finite-time bounded with disturbance attenuation* γ.

Remark 3 It should be pointed that the assumption of zero initial condition in system (1) is only for the purpose of technical simplification in the derivation, and it does not lose generality. In fact, if this assumption is lost, the same control result can still be got along the same lines, except adding extra manipulations in the derivation and extra terms in the control presentation. However, in real world applications, the initial condition of the underlying system is generally not zero.

Lemma 2.1 [18] *Let $f_i : \mathcal{R}^m \to \mathcal{R}$ $(i = 1, 2, \ldots, N)$ have positive values in an open subset $\mathcal{D}$ of $\mathcal{R}^m$. Then the reciprocally convex combination of f_i over $\mathcal{D}$ satisfies*

$$\min_{\{\beta_i \mid \beta_i > 0, \sum_i \beta_i = 1\}} \sum_i \frac{1}{\beta_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \neq j} g_{i,j}(t)$$

subject to

$$\left\{ g_{i,j} : \mathcal{R}^m \to \mathcal{R}, g_{j,i}(t) = g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \geq 0 \right\}.$$

Lemma 2.2 *For a given function $\mu_{1i} \leq \tau_i(t) \leq \mu_{2i}$, $h_{1i} \leq \dot{\tau}_i(t) \leq h_{2i}$ $(i \in \mathcal{N})$, there exist four functions $\alpha_1(t) \geq 0$, $\alpha_2(t) \geq 0$, $\beta_1(t) \geq 0$ and $\beta_2(t) \geq 0$ satisfying $\alpha_1(t) + \alpha_2(t) = 1$ and $\beta_1(t) + \beta_2(t) = 1$, respectively, such that $\forall i \in \mathcal{N}$, the following equation holds*:

$$\tau_i(t) = \alpha_1(t)\mu_{1i} + \alpha_2(t)\mu_{2i}, \qquad \dot{\tau}_i(t) = \beta_1(t)h_{1i} + \beta_2(t)h_{2i}.$$

Lemma 2.3 [50] *For matrices A, $Q = Q^\top > 0$ and $P > 0$, the following matrix inequality*

$$A^\top PA - Q < 0$$

holds if and only if there exists a matrix G of appropriate dimension such that

$$\begin{bmatrix} -Q & A^\top G \\ * & P - G - G^\top \end{bmatrix} < 0.$$

Lemma 2.4 ([14] Schur complement) *Given constant matrices X, Y, Z, where $X = X^\top$ and $0 < Y = Y^\top$, then $X + Z^\top Y^{-1} Z < 0$ if and only if*

$$\begin{bmatrix} X & Z^\top \\ * & -Y \end{bmatrix} < 0 \quad or \quad \begin{bmatrix} -Y & Z \\ * & X \end{bmatrix} < 0. \tag{7}$$

3 Finite-time H_∞ performance analysis

We first consider the problem of stability analysis for system (1) with $u(t) = 0$. The following results actually present the finite-time stability for the Markov jump system with time-varying delay.

Theorem 3.1 *System* (1) *is finite-time bounded with respect to $(c_1, c_2, d, \overline{R}_i, T)$ if there exist matrices $P_i > 0$, $Q_i^{(r)} > 0$, $Q^{(r)} > 0$ $(r = 1, 2, \ldots, (m + l))$, R_i, $R > 0$, $W > 0$, S_i, $\forall i, j \in \mathcal{N}$, scalars*

$c_1 < c_2$, $T > 0$, $\kappa_1 > 0$, $\kappa_2 > 0$, $\kappa_3 > 0$, $\lambda_s > 0$ $(s = 1, 2, \ldots, 7)$, $\lambda > 0$, $\eta > 0$ *and* $\Lambda > 0$, *such that for all* $i, j \in \mathcal{N}$, $k = 1, 2, \ldots, l$, $s = 1, 2, \ldots, m$, *the following inequalities hold*:

$$\Omega_i(\mu_{qi}, h_{qi}) = \begin{bmatrix} \Omega_{1i}(\mu_{pi}, h_{qi}) & \Upsilon_{1i} & \Upsilon_{2i} \\ * & \Omega_{2i}(\mu_{pi}, h_{qi}) & \Upsilon_{3i} \\ * & * & \Upsilon_{4i} \end{bmatrix} < 0, \quad p = 1, 2, q = 1, 2, \tag{8}$$

$$e^{\frac{\lambda}{l}(\mu_{pi}-\mu_1)} \sum_{j=1}^{N} \pi_{ij} Q_j^{(k)} \le Q^{(k)}, \qquad e^{\frac{\lambda}{m}(\mu_2-\mu_{pi})} \sum_{j=1}^{N} \pi_{ij} Q_j^{(l+s)} \le Q^{(l+s)}, \quad p = 1, 2, \tag{9}$$

$$e^{\lambda\mu_2} \sum_{j=1}^{N} \pi_{ij} R_j \le R, \tag{10}$$

$$c_1 \Lambda + d\lambda\lambda_7 \frac{1}{\eta}\left(1 - e^{-\eta T}\right) < \lambda_1 c_2 e^{-\eta T}, \tag{11}$$

where

$$\Omega_{1i}(\mu_{pi}, h_{qi}) = \begin{bmatrix} \widetilde{\Omega}_{1i}(\mu_{pi}, h_{qi}) & 0 & 0 & 0 & \cdots & 0 \\ * & \widetilde{\Omega}_{2i}(\mu_{pi}, h_{qi}) & -\frac{l}{\mu_2-\mu_1} R_i & 0 & \cdots & 0 \\ * & * & \widetilde{\Omega}_{3i}(\mu_{pi}, h_{qi}) & -\frac{l}{\mu_2-\mu_1} R_i & \cdots & 0 \\ * & * & * & \ddots & \ddots & \vdots \\ * & * & * & * & \widetilde{\Omega}_{li}(\mu_{pi}, h_{qi}) & -\frac{l}{\mu_2-\mu_1} R_i \\ * & * & * & * & * & \widetilde{\Omega}_{(l+1)i}(\mu_{pi}, h_{qi}) \end{bmatrix},$$

$$\Omega_{2i}(\mu_{pi}, h_{qi}) = \begin{bmatrix} \overline{\Omega}_{1i}(\mu_{pi}, h_{qi}) & -\frac{m}{\mu_2-\mu_1} R_i & 0 & \cdots & 0 \\ * & \overline{\Omega}_{2i}(\mu_{pi}, h_{qi}) & -\frac{m}{\mu_2-\mu_1} R_i & \cdots & 0 \\ * & * & \ddots & \ddots & 0 \\ * & * & * & \overline{\Omega}_{mi}(\mu_{pi}, h_{qi}) & -\frac{m}{\mu_2-\mu_1} R_i \\ * & * & * & * & \overline{\Omega}_{(m+1)i}(\mu_{pi}, h_{qi}) \end{bmatrix},$$

$$\Upsilon_{1i} = \begin{bmatrix} P_i A_{\tau i} & 0 & \cdots & 0 & 0 \\ S_i & 2S_i & \cdots & 2S_i & S_i \\ S_i & 2S_i & \cdots & 2S_i & S_i \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -\frac{l}{\mu_2-\mu_1} R_i + S_i & 2S_i & \cdots & 2S_i & S_i \end{bmatrix},$$

$$\Upsilon_{2i} = \begin{bmatrix} P_i D_i & A_i^{\mathsf{T}} R_i & A_i^{\mathsf{T}} R \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix}, \qquad \Upsilon_{3i} = \begin{bmatrix} 0 & A_{\tau i}^{\mathsf{T}} R_i & A_{\tau i}^{\mathsf{T}} R \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Upsilon_{4i} = \begin{bmatrix} -\lambda W & D_i^{\mathsf{T}} R_i & D_i^{\mathsf{T}} R \\ * & -\frac{1}{\kappa_1} R_i & 0 \\ * & * & -\frac{1}{\kappa_2} R \end{bmatrix},$$

$$
\begin{aligned}
\widetilde{\Omega}_{1i}(\mu_{pi},h_{qi}) &= -\lambda P_i + P_iA_i + A_i^{\mathsf{T}}P_i + \sum_{j=1}^{N}\pi_{ij}P_j \\
&\quad + \sum_{k=1}^{l}\frac{e^{\lambda[\mu_1+\frac{k(\mu_{pi}-\mu_1)}{l}]} - e^{\lambda[\mu_1+\frac{(k-1)(\mu_{pi}-\mu_1)}{l}]}}{\lambda}Q^{(k)} \\
&\quad + \sum_{s=1}^{m}\frac{e^{\lambda[\mu_{pi}+\frac{(l+s)(\mu_2-\mu_{pi})}{m}]} - e^{\lambda[\mu_{pi}+\frac{(l+s-1)(\mu_2-\mu_{pi})}{m}]}}{\lambda}Q^{(l+s)}, \\
\widetilde{\Omega}_{2i}(\mu_{pi},h_{qi}) &= e^{\frac{\lambda}{l}(\mu_{pi}-\mu_1)}Q_i^{(1)} + \frac{l}{\mu_2-\mu_1}R_i, \\
\widetilde{\Omega}_{3i}(\mu_{pi},h_{qi}) &= -\left(1-\frac{1}{l}h_{qi}\right)Q_i^{(1)} + \left(1-\frac{1}{l}h_{qi}\right)e^{\frac{\lambda}{l}(\mu_{pi}-\mu_1)}Q_i^{(2)} + \frac{2l}{\mu_2-\mu_1}R_i, \\
\widetilde{\Omega}_{li}(\mu_{pi},h_{qi}) &= -\left(1-\frac{l-2}{l}h_{qi}\right)Q_i^{(l-2)} + \left(1-\frac{l-2}{l}h_{qi}\right)e^{\frac{\lambda}{l}(\mu_{pi}-\mu_1)}Q_i^{(l-1)} + \frac{2l}{\mu_2-\mu_1}R_i, \\
\widetilde{\Omega}_{(l+1)i}(\mu_{pi},h_{qi}) &= -\left(1-\frac{l-1}{l}h_{qi}\right)Q_i^{(l-1)} + \left(1-\frac{l-1}{l}h_{qi}\right)e^{\frac{\lambda}{l}(\mu_{pi}-\mu_1)}Q_i^{(l)} + \frac{2l}{\mu_2-\mu_1}R_i, \\
\overline{\Omega}_{1i}(\mu_{pi},h_{qi}) &= -(1-h_{qi})Q_i^{(l)} + (1-h_{qi})e^{\frac{\lambda}{m}(\mu_2-\mu_{pi})}Q_i^{(l+1)} + \frac{l+m}{\mu_2-\mu_1}R_i, \\
\overline{\Omega}_{2i}(\mu_{pi},h_{qi}) &= -\left(1-\frac{m-1}{m}h_{qi}\right)Q_i^{(l+1)} + \left(1-\frac{m-1}{m}h_{qi}\right)e^{\frac{\lambda}{m}(\mu_2-\mu_{pi})}Q_i^{(l+2)} + \frac{2m}{\mu_2-\mu_1}R_i, \\
\overline{\Omega}_{mi}(\mu_{pi},h_{qi}) &= -\left(1-\frac{1}{m}h_{qi}\right)Q_i^{(l+m-1)} + \left(1-\frac{1}{m}h_{qi}\right)e^{\frac{\lambda}{m}(\mu_2-\mu_{pi})}Q_i^{(l+m)} + \frac{2m}{\mu_2-\mu_1}R_i, \\
\overline{\Omega}_{(m+1)i}(\mu_{pi},h_{qi}) &= -Q_i^{(l+m)} + \frac{m}{\mu_2-\mu_1}R_i, \\
\kappa_1 &= \frac{e^{\lambda\mu_2}-e^{\lambda\mu_1}}{\lambda}, \qquad \kappa_2 = \frac{e^{\lambda\mu_2}-e^{\lambda\mu_1}-\lambda(\mu_2-\mu_1)}{\lambda^2}, \\
\kappa_3 &= \frac{2e^{\lambda\mu_2}-2e^{\lambda\mu_1}-\lambda^2(\mu_2^2-\mu_1^2)-2\lambda(\mu_2-\mu_1)}{2\lambda^3}, \\
\Lambda &= \lambda_2 + \kappa_1\lambda_3 + \kappa_2(\lambda_4+\lambda_5) + \kappa_3\lambda_6, \\
\lambda_1 &= \max_{i\in\mathcal{N}}\lambda_{\max}(P_i), \qquad \lambda_2 = \max_{i\in\mathcal{N}}\lambda_{\max}(\widetilde{P}_i), \qquad \lambda_3 = \max_{i\in\mathcal{N},1\le r\le(m+l)}\lambda_{\max}\big(\widetilde{Q}_i^{(r)}\big), \\
\lambda_4 &= \lambda_{\max}(\widetilde{Q}), \qquad \lambda_5 = \max_{i\in\mathcal{N}}\lambda_{\max}(\widetilde{R}_i), \qquad \lambda_6 = \lambda_{\max}(\widetilde{R}), \qquad \lambda_7 = \lambda_{\max}(W), \\
\widetilde{P}_i &= \overline{R}_i^{-\frac{1}{2}}P_i\overline{R}_i^{-\frac{1}{2}}, \qquad \widetilde{Q}_i^{(r)} = \overline{R}_i^{-\frac{1}{2}}Q_i^{(r)}\overline{R}_i^{-\frac{1}{2}}, \qquad \widetilde{Q} = \overline{R}_i^{-\frac{1}{2}}Q\overline{R}_i^{-\frac{1}{2}}, \\
\widetilde{R}_i &= \overline{R}_i^{-\frac{1}{2}}R_i\overline{R}_i^{-\frac{1}{2}}, \qquad \widetilde{R} = \overline{R}_i^{-\frac{1}{2}}R\overline{R}_i^{-\frac{1}{2}}.
\end{aligned}
$$

Proof First, in order to cast our model into the framework of the Markov processes, we define a new process $\{(x_t, r_t), t \ge 0\}$ by

$$x_t(s) = x(t+s), \quad s \in [-\mu_2, -\mu_1].$$

Now, we consider the following Lyapunov-Krasovskii functional:

$$V(x_t, r_t, t) = \sum_{l=1}^{5} V_l(x_t, r_t, t), \tag{12}$$

where

$$V_1(x_t,r_t,t) = x(t)^{\mathsf T} e^{\lambda t} P_{r_t} x(t),$$

$$\begin{aligned} V_2(x_t,r_t,t) = {} & \sum_{k=1}^{l} \int_{t-\tau_k}^{t-\tau_{k-1}} e^{\lambda(\upsilon+\tau_k)} x^{\mathsf T}(\upsilon) Q_{r_t}^{(k)} x(\upsilon)\,d\upsilon \\ & + \sum_{s=1}^{m} \int_{t-\tau_{l+s}}^{t-\tau_{l+s-1}} e^{\lambda(\upsilon+\tau_s)} x^{\mathsf T}(\upsilon) Q_{r_t}^{(l+s)} x(\upsilon)\,d\upsilon, \end{aligned}$$

$$\begin{aligned} V_3(x_t,r_t,t) = {} & \sum_{k=1}^{l} \int_{-\tau_k}^{-\tau_{k-1}} \int_{t+\theta}^{t} e^{\lambda(\upsilon-\theta)} x^{\mathsf T}(\upsilon) Q^{(k)} x(\upsilon)\,d\upsilon\,d\theta \\ & + \sum_{s=1}^{m} \int_{-\tau_{l+s}}^{-\tau_{l+s-1}} \int_{t+\theta}^{t} e^{\lambda(\upsilon-\theta)} x^{\mathsf T}(\upsilon) Q^{(l+s)} x(\upsilon)\,d\upsilon\,d\theta, \end{aligned}$$

$$V_4(x_t,r_t,t) = \int_{-\mu_2}^{-\mu_1} \int_{t+\theta}^{t} e^{\lambda(\upsilon-\theta)} \dot x^{\mathsf T}(\upsilon) R_{r_t} \dot x(\upsilon)\,d\upsilon\,d\theta,$$

$$V_5(x_t,r_t,t) = \int_{-\mu_2}^{-\mu_1} \int_{\theta}^{0} \int_{t+\kappa}^{t} e^{\lambda(\upsilon-\kappa)} \dot x^{\mathsf T}(\upsilon) R \dot x(\upsilon)\,d\upsilon\,d\kappa\,d\theta.$$

Then, let the mode at time t be i, *i.e.*, $r_t = i \in \mathcal{N}$, we have

$$\begin{aligned} \pounds V_1(x_t,i,t) = {} & \lambda e^{\lambda t} x^{\mathsf T}(t) P_i x(t) + 2e^{\lambda t} x^{\mathsf T}(t) P_i \bigl(A_i x(t) + A_{\tau i} x\bigl(t-\tau_i(t)\bigr) + D_i \omega(t)\bigr) \\ & + e^{\lambda t} x^{\mathsf T}(t) \left(\sum_{j=1}^{N} \pi_{ij} P_j \right) x(t), \end{aligned}$$

$$\begin{aligned} \pounds V_2(x_t,i,t) = {} & \sum_{k=1}^{l} \biggl[\left(1 - \frac{k-1}{l} \dot\tau_i(t)\right) x^{\mathsf T}(t-\tau_{k-1}) e^{\lambda(t+\tau_k-\tau_{k-1})} Q_i^{(k)} x(t-\tau_{k-1}) \\ & - \left(1 - \frac{k}{l} \dot\tau_i(t)\right) x^{\mathsf T}(t-\tau_k) e^{\lambda t} Q_i^{(k)} x(t-\tau_k) \biggr] \\ & + \sum_{k=1}^{l} \int_{t-\tau_k}^{t-\tau_{k-1}} e^{\lambda(\upsilon+\tau_k)} x^{\mathsf T}(\upsilon) \left(\sum_{j=1}^{N} \pi_{ij} Q_j^{(k)} \right) x(\upsilon)\,d\upsilon \\ & + \sum_{s=1}^{m} \biggl[\left(1 - \frac{m-s+1}{m} \dot\tau_i(t)\right) x^{\mathsf T}(t-\tau_{l+s-1}) e^{\lambda(t+\tau_{l+s}-\tau_{l+s-1})} Q_i^{(l+s)} x(t-\tau_{l+s-1}) \\ & - \left(1 - \frac{m-s}{m} \dot\tau_i(t)\right) x^{\mathsf T}(t-\tau_{l+s}) e^{\lambda t} Q_i^{(l+s)} x(t-\tau_{l+s}) \biggr] \\ & + \sum_{s=1}^{m} \int_{t-\tau_{l+s}}^{t-\tau_{l+s-1}} e^{\lambda(\upsilon+\tau_{l+s})} x^{\mathsf T}(\upsilon) \left(\sum_{j=1}^{N} \pi_{ij} Q_j^{(l+s)} \right) x(\upsilon)\,d\upsilon \\ = {} & e^{\lambda t} \sum_{k=1}^{l} \biggl[\left(1 - \frac{k-1}{l} \dot\tau_i(t)\right) x^{\mathsf T}(t-\tau_{k-1}) e^{\frac{\lambda[\tau_i(t)-\mu_1]}{l}} Q_i^{(k)} x(t-\tau_{k-1}) \\ & - \left(1 - \frac{k}{l} \dot\tau_i(t)\right) x^{\mathsf T}(t-\tau_k) Q_i^{(k)} x(t-\tau_k) \biggr] \\ & + e^{\lambda t} \sum_{s=1}^{m} \biggl[\left(1 - \frac{m-s+1}{m} \dot\tau_i(t)\right) x^{\mathsf T}(t-\tau_{l+s-1}) e^{\frac{\lambda}{m}[\mu_2-\tau_i(t)]} Q_i^{(l+s)} x(t-\tau_{l+s-1}) \end{aligned}$$

$$
\begin{aligned}
&- \left(1 - \frac{m-s}{m}\dot{\tau}_i(t)\right)x^{\mathsf{T}}(t-\tau_{l+s})Q_i^{(l+s)}x(t-\tau_{l+s})\Bigg] \\
&+ \sum_{k=1}^{l}\int_{t-\tau_k}^{t-\tau_{k-1}} e^{\lambda(\upsilon+\tau_k)}x^{\mathsf{T}}(\upsilon)\left(\sum_{j=1}^{N}\pi_{ij}Q_j^{(k)}\right)x(\upsilon)\,d\upsilon \\
&+ \sum_{s=1}^{m}\int_{t-\tau_{l+s}}^{t-\tau_{l+s-1}} e^{\lambda(\upsilon+\tau_{l+s})}x^{\mathsf{T}}(\upsilon)\left(\sum_{j=1}^{N}\pi_{ij}Q_j^{(l+s)}\right)x(\upsilon)\,d\upsilon,
\end{aligned}
$$

$$
\begin{aligned}
\pounds V_3(x_t,i,t) ={}& e^{\lambda t}\sum_{k=1}^{l}\frac{e^{\lambda\tau_k}-e^{\lambda\tau_{k-1}}}{\lambda}x^{\mathsf{T}}(t)Q^{(k)}x(t) - e^{\lambda t}\sum_{k=1}^{l}\int_{t-\tau_k}^{t-\tau_{k-1}} x^{\mathsf{T}}(\upsilon)Q^{(k)}x(\upsilon)\,d\upsilon \\
&+ \sum_{k=1}^{l}\dot{\tau}_k\int_{t-\tau_k}^{t} e^{\lambda(\upsilon+\tau_k)}x^{\mathsf{T}}(\upsilon)Q^{(k)}x(\upsilon)\,d\upsilon \\
&- \sum_{k=1}^{l}\dot{\tau}_{k-1}\int_{t-\tau_{k-1}}^{t} e^{\lambda(\upsilon+\tau_{k-1})}x^{\mathsf{T}}(\upsilon)Q^{(k)}x(\upsilon)\,d\upsilon \\
&+ e^{\lambda t}\sum_{s=1}^{m}\frac{e^{\lambda\tau_{l+s}}-e^{\lambda\tau_{l+s-1}}}{\lambda}x^{\mathsf{T}}(t)Q^{(l+s)}x(t) \\
&- e^{\lambda t}\sum_{s=1}^{m}\int_{t-\tau_{l+s}}^{t-\tau_{l+s-1}} x^{\mathsf{T}}(\upsilon)Q^{(l+s)}x(\upsilon)\,d\upsilon \\
&+ \sum_{s=1}^{m}\dot{\tau}_{l+s}\int_{t-\tau_{l+s}}^{t} e^{\lambda(\upsilon+\tau_{l+s})}x^{\mathsf{T}}(\upsilon)Q^{(l+s)}x(\upsilon)\,d\upsilon \\
&- \sum_{s=1}^{m}\dot{\tau}_{l+s-1}\int_{t-\tau_{l+s-1}}^{t} e^{\lambda(\upsilon+\tau_{l+s-1})}x^{\mathsf{T}}(\upsilon)Q^{(l+s)}x(\upsilon)\,d\upsilon \\
\le{}& e^{\lambda t}x^{\mathsf{T}}(t)\left\{\sum_{k=1}^{l}\frac{e^{\lambda[\mu_1+\frac{k(\tau_i(t)-\mu_1)}{l}]}-e^{\lambda[\mu_1+\frac{(k-1)(\tau_i(t)-\mu_1)}{l}]}}{\lambda}Q^{(k)}\right. \\
&\left.+ \sum_{s=1}^{m}\frac{e^{\lambda[\tau_i(t)+\frac{(l+s)(\mu_2-\tau_i(t))}{m}]}-e^{\lambda[\tau_i(t)+\frac{(l+s-1)(\mu_2-\tau_i(t))}{m}]}}{\lambda}Q^{(l+s)}\right\}x(t) \\
&- e^{\lambda t}\sum_{k=1}^{l}\int_{t-\tau_k}^{t-\tau_{k-1}} x^{\mathsf{T}}(\upsilon)Q^{(k)}x(\upsilon)\,d\upsilon \\
&- e^{\lambda t}\sum_{s=1}^{m}\int_{t-\tau_{l+s}}^{t-\tau_{l+s-1}} x^{\mathsf{T}}(\upsilon)Q^{(l+s)}x(\upsilon)\,d\upsilon,
\end{aligned}
$$

$$
\begin{aligned}
\pounds V_4(x_t,i,t) ={}& \dot{x}^{\mathsf{T}}(t)e^{\lambda t}\times\frac{e^{\lambda\mu_2}-e^{\lambda\mu_1}}{\lambda}R_i\dot{x}(t) - e^{\lambda t}\int_{t-\mu_2}^{t-\mu_1}\dot{x}^{\mathsf{T}}(\upsilon)R_i\dot{x}(\upsilon)\,d\upsilon \\
&+ \int_{-\mu_2}^{-\mu_1}\int_{t+\theta}^{t} e^{\lambda(\upsilon-\theta)}\dot{x}^{\mathsf{T}}(\upsilon)\left(\sum_{j=1}^{N}\pi_{ij}R_j\right)\dot{x}(\upsilon)\,d\upsilon
\end{aligned}
$$

$$
\begin{aligned}
\pounds V_5(x_t,i,t) ={}& \frac{e^{\lambda\mu_2}-e^{\lambda\mu_1}-\lambda(\mu_2-\mu_1)}{\lambda^2}e^{\lambda t}\dot{x}^{\mathsf{T}}(t)R\dot{x}(t) \\
&- e^{\lambda t}\int_{-\mu_2}^{-\mu_1}\int_{t+\theta}^{t}\dot{x}^{\mathsf{T}}(s)R\dot{x}(s)\,ds\,d\theta.
\end{aligned}
$$

Moreover, denote

$$\eta(k) = \int_{t-\tau_k}^{t-\tau_{k-1}} \dot{x}(\upsilon)\,d\upsilon, \qquad \eta(l+s) = \int_{t-\tau_{l+s}}^{t-\tau_{l+s-1}} \dot{x}(\upsilon)\,d\upsilon.$$

By using Lemma 2.1, it yields that

$$\begin{aligned}
&-\int_{t-\mu_2}^{t-\mu_1} \dot{x}^{\mathsf{T}}(\upsilon)R_i\dot{x}(\upsilon) \\
&\quad= -\sum_{k=1}^{l}\int_{t-\tau_k}^{t-\tau_{k-1}} \dot{x}^{\mathsf{T}}(\upsilon)R_i\dot{x}(\upsilon) - \sum_{s=1}^{m}\int_{t-\tau_{l+s}}^{t-\tau_{l+s-1}} \dot{x}^{\mathsf{T}}(\upsilon)R_i\dot{x}(\upsilon) \\
&\quad\le -\frac{\mu_2-\mu_1}{\tau_i(t)-\mu_1}\sum_{k=1}^{l}\eta^{\mathsf{T}}(k)\frac{l}{\mu_2-\mu_1}R_i\eta(k) - \frac{\mu_2-\mu_1}{\mu_2-\tau_i(t)}\sum_{s=1}^{m}\eta^{\mathsf{T}}(l+s)\frac{m}{\mu_2-\mu_1}R_i\eta(l+s) \\
&\quad= -\sum_{k=1}^{l}\eta^{\mathsf{T}}(k)\frac{l}{\mu_2-\mu_1}R_i\eta(k) - \frac{\mu_2-\tau_i(t)}{\tau_i(t)-\mu_1}\sum_{k=1}^{l}\eta^{\mathsf{T}}(k)\frac{l}{\mu_2-\mu_1}R_i\eta(k) \\
&\qquad -\sum_{k=1}^{l}\eta^{\mathsf{T}}(l+s)\frac{m}{\mu_2-\mu_1}R_i\eta(l+s) - \frac{\tau_i(t)-\mu_1}{\mu_2-\tau_i(t)}\sum_{s=1}^{m}\eta^{\mathsf{T}}(l+s)\frac{m}{\mu_2-\mu_1}R_i\eta(l+s) \\
&\quad\le -\begin{bmatrix} \sum_{k=1}^{l}\eta(k) \\ \sum_{s=1}^{m}\eta(l+s)\end{bmatrix}^{\mathsf{T}} \begin{bmatrix} \frac{l}{\mu_2-\mu_1}R_i & S_i \\ * & \frac{m}{\mu_2-\mu_1}R_i\end{bmatrix} \begin{bmatrix} \sum_{k=1}^{l}\eta(k) \\ \sum_{s=1}^{m}\eta(l+s)\end{bmatrix}.
\end{aligned} \tag{13}$$

It follows from (9) and $Q^{(r)} > 0$ $(r = 1, 2, \ldots, (m + l))$ that

$$\begin{aligned}
&\sum_{k=1}^{l}\int_{t-\tau_k}^{t-\tau_{k-1}} e^{\lambda(\upsilon+\tau_k)}x^{\mathsf{T}}(\upsilon)\left(\sum_{j=1}^{N}\pi_{ij}Q_j^{(k)}\right)x(\upsilon)\,d\upsilon \\
&\quad\le e^{\lambda t}\sum_{k=1}^{l}\int_{t-\tau_k}^{t-\tau_{k-1}} x^{\mathsf{T}}(\upsilon)e^{\lambda(\tau_k-\tau_{k-1})}\left(\sum_{j=1}^{N}\pi_{ij}Q_j^{(k)}\right)x(\upsilon)\,d\upsilon \\
&\quad\le e^{\lambda t}\sum_{k=1}^{l}\int_{t-\tau_k}^{t-\tau_{k-1}} x^{\mathsf{T}}(\upsilon)Q^{(k)}x(\upsilon)\,d\upsilon,
\end{aligned} \tag{14}$$

$$\begin{aligned}
&\sum_{s=1}^{m}\int_{t-\tau_{l+s}}^{t-\tau_{l+s-1}} e^{\lambda(\upsilon+\tau_{l+s})}x^{\mathsf{T}}(\upsilon)\left(\sum_{j=1}^{N}\pi_{ij}Q_j^{(l+s)}\right)x(\upsilon)\,d\upsilon \\
&\quad\le e^{\lambda t}\sum_{s=1}^{m}\int_{t-\tau_{l+s}}^{t-\tau_{l+s-1}} x^{\mathsf{T}}(\upsilon)e^{\lambda(\tau_{l+s}-\tau_{l+s-1})}\left(\sum_{j=1}^{N}\pi_{ij}Q_j^{(l+s)}\right)x(\upsilon)\,d\upsilon \\
&\quad\le e^{\lambda t}\sum_{s=1}^{m}\int_{t-\tau_{l+s}}^{t-\tau_{l+s-1}} x^{\mathsf{T}}(\upsilon)Q^{(l+s)}x(\upsilon)\,d\upsilon.
\end{aligned} \tag{15}$$

Similarly, (10) implies

$$\int_{-\mu_2}^{-\mu_1}\int_{t+\theta}^{t} e^{\lambda(\upsilon-\theta)}\dot{x}^{\mathsf{T}}(\upsilon)\left(\sum_{j=1}^{N}\pi_{ij}R_j\right)\dot{x}(\upsilon)\,d\upsilon \le e^{\lambda t}\int_{-\mu_2}^{-\mu_1}\int_{t+\theta}^{t}\dot{x}^{\mathsf{T}}(s)R\dot{x}(s)\,ds\,d\theta. \tag{16}$$

From (12)-(16), we can eventually obtain

$$\pounds V(x_t, r_t, t) - \lambda\omega^{\mathsf{T}}(t)W\omega(t) \le e^{\lambda t}\xi^{\mathsf{T}}(t)\Xi_i\big(\tau_i(t), \dot{\tau}_i(t)\big)\xi(t), \tag{17}$$

where

$$\xi^{\mathsf{T}}(t) = \big[x^{\mathsf{T}}(t), x^{\mathsf{T}}(t-\mu_1), x^{\mathsf{T}}(t-\tau_1), \ldots, x^{\mathsf{T}}(t-\tau_l), x^{\mathsf{T}}(t-\tau_{l+1}), \ldots, x^{\mathsf{T}}(t-\tau_{l+m}), \omega^{\mathsf{T}}(t)\big],$$

and

$$\Xi_i\big(\tau_i(t), \dot{\tau}_i(t)\big) = \begin{bmatrix} \Xi_{1i}(\tau_i(t), \dot{\tau}_i(t)) & \Sigma_{1i} & \Sigma_{2i} \\ * & \Xi_{2i}(\tau_i(t), \dot{\tau}_i(t)) & \Sigma_{3i} \\ * & * & -\lambda W + D_i^{\mathsf{T}}(\kappa_1 R_i + \kappa_2 R)D_i \end{bmatrix},$$

$$\Xi_{1i}\big(\tau_i(t), \dot{\tau}_i(t)\big) = \begin{bmatrix} \widetilde{\Xi}_{1i}(\tau_i(t), \dot{\tau}_i(t)) & 0 & 0 & 0 & \cdots & 0 \\ * & \widetilde{\Xi}_{2i}(\tau_i(t), \dot{\tau}_i(t)) & -\frac{l}{\mu_2-\mu_1}R_i & 0 & \cdots & 0 \\ * & * & \widetilde{\Xi}_{3i}(\tau_i(t), \dot{\tau}_i(t)) & -\frac{l}{\mu_2-\mu_1}R_i & \cdots & 0 \\ * & * & * & \ddots & \ddots & \vdots \\ * & * & * & * & \widetilde{\Xi}_{(l)i}(\tau_i(t), \dot{\tau}_i(t)) & -\frac{l}{\mu_2-\mu_1}R_i \\ * & * & * & * & * & \widetilde{\Xi}_{(l+1)i}(\tau_i(t), \dot{\tau}_i(t)) \end{bmatrix},$$

$$\Xi_{2i}\big(\tau_i(t), \dot{\tau}_i(t)\big) = \begin{bmatrix} \overline{\Xi}_{1i}(\tau_i(t), \dot{\tau}_i(t)) & -\frac{m}{\mu_2-\mu_1}R_i & 0 & \cdots & 0 \\ * & \overline{\Xi}_{2i}(\tau_i(t), \dot{\tau}_i(t)) & -\frac{m}{\mu_2-\mu_1}R_i & \cdots & 0 \\ * & * & \ddots & \ddots & 0 \\ * & * & * & \overline{\Xi}_{mi}(\tau_i(t), \dot{\tau}_i(t)) & -\frac{m}{\mu_2-\mu_1}R_i \\ * & * & * & * & \overline{\Xi}_{(m+1)i}(\tau_i(t), \dot{\tau}_i(t)) \end{bmatrix},$$

$$\Sigma_{1i} = \begin{bmatrix} P_iA_{\tau i} + A_i^{\mathsf{T}}(\kappa_1 R_i + \kappa_2 R)A_{\tau i} & 0 & \cdots & 0 & 0 \\ S_i & 2S_i & \cdots & 2S_i & S_i \\ S_i & 2S_i & \cdots & 2S_i & S_i \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -\frac{l}{\mu_2-\mu_1}R_i + S_i & 2S_i & \cdots & 2S_i & S_i \end{bmatrix},$$

$$\Sigma_{2i} = \begin{bmatrix} P_iD_i + A_i^{\mathsf{T}}(\kappa_1 R_i + \kappa_2 R)D_i \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \qquad \Sigma_{3i} = \begin{bmatrix} A_{\tau i}^{\mathsf{T}}(\kappa_1 R_i + \kappa_2 R)D_i \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\begin{aligned} \widetilde{\Xi}_{1i}\big(\tau_i(t), \dot{\tau}_i(t)\big) = {} & \lambda P_i + P_iA_i + A_i^{\mathsf{T}}P_i + \sum_{j=1}^{N}\pi_{ij}P_j + {+}A_i^{\mathsf{T}}(\kappa_1 R_i + \kappa_2 R)A_i \\ & + \sum_{k=1}^{l}\frac{e^{\lambda[\mu_1 + \frac{k(\tau_i(t)-\mu_1)}{l}]} - e^{\lambda[\mu_1 + \frac{(k-1)(\tau_i(t)-\mu_1)}{l}]}}{\lambda}Q^{(k)} \\ & + \sum_{s=1}^{m}\frac{e^{\lambda[\tau_i(t) + \frac{(l+s)(\mu_2-\tau_i(t))}{m}]} - e^{\lambda[\tau_i(t) + \frac{(l+s-1)(\mu_2-\tau_i(t))}{m}]}}{\lambda}Q^{(l+s)}, \end{aligned}$$

$$\begin{aligned}
\widetilde{\Xi}_{2i}\big(\tau_i(t),\dot{\tau}_i(t)\big) &= e^{\frac{\lambda}{l}(\tau_i(t)-\mu_1)}Q_i^{(1)} + \frac{l}{\mu_2-\mu_1}R_i,\\
\widetilde{\Xi}_{3i}\big(\tau_i(t),\dot{\tau}_i(t)\big) &= -\left(1-\frac{1}{l}\dot{\tau}_i(t)\right)Q_i^{(1)} + \left(1-\frac{1}{l}\dot{\tau}_i(t)\right)e^{\frac{\lambda}{l}(\tau_i(t)-\mu_1)}Q_i^{(2)} + \frac{2l}{\mu_2-\mu_1}R_i,\\
\widetilde{\Xi}_{li}\big(\tau_i(t),\dot{\tau}_i(t)\big) &= -\left(1-\frac{l-2}{l}\dot{\tau}_i(t)\right)Q_i^{(l-2)}\\
&\quad + \left(1-\frac{l-2}{l}\dot{\tau}_i(t)\right)e^{\frac{\lambda}{l}(\tau_i(t)-\mu_1)}Q_i^{(l-1)} + \frac{2l}{\mu_2-\mu_1}R_i,\\
\widetilde{\Xi}_{(l+1)i}\big(\tau_i(t),\dot{\tau}_i(t)\big) &= -\left(1-\frac{l-1}{l}\dot{\tau}_i(t)\right)Q_i^{(l-1)}\\
&\quad + \left(1-\frac{l-1}{l}\dot{\tau}_i(t)\right)e^{\frac{\lambda}{l}(\tau_i(t)-\mu_1)}Q_i^{(l)} + \frac{2l}{\mu_2-\mu_1}R_i,\\
\overline{\Xi}_{1i}\big(\tau_i(t),\dot{\tau}_i(t)\big) &= -\big(1-\dot{\tau}_i(t)\big)Q_i^{(l)} + \big(1-\dot{\tau}_i(t)\big)e^{\frac{\lambda}{m}(\mu_2-\tau_i(t))}Q_i^{(l+1)}\\
&\quad + \frac{l+m}{\mu_2-\mu_1}R_i + A_{\tau i}^{\mathsf{T}}(\kappa_1 R_i + \kappa_2 R)A_{\tau i},\\
\overline{\Xi}_{2i}\big(\tau_i(t),\dot{\tau}_i(t)\big) &= -\left(1-\frac{m-1}{m}\dot{\tau}_i(t)\right)Q_i^{(l+1)}\\
&\quad + \left(1-\frac{m-1}{m}\dot{\tau}_i(t)\right)e^{\frac{\lambda}{m}(\mu_2-\tau_i(t))}Q_i^{(l+2)} + \frac{2m}{\mu_2-\mu_1}R_i,\\
\overline{\Xi}_{mi}\big(\tau_i(t),\dot{\tau}_i(t)\big) &= -\left(1-\frac{1}{m}\dot{\tau}_i(t)\right)Q_i^{(l+m-1)}\\
&\quad + \left(1-\frac{1}{m}\dot{\tau}_i(t)\right)e^{\frac{\lambda}{m}(\mu_2-\tau_i(t))}Q_i^{(l+m)} + \frac{2m}{\mu_2-\mu_1}R_i,\\
\overline{\Xi}_{(m+1)i}\big(\tau_i(t),\dot{\tau}_i(t)\big) &= -Q_i^{(l+m)} + \frac{m}{\mu_2-\mu_1}R_i.
\end{aligned}$$

By Lemma 2.2, there exist functions $\alpha_1(t) \ge 0$, $\alpha_2(t) \ge 0$, $\beta_1(t) \ge 0$ and $\beta_2(t) \ge 0$ satisfying $\alpha_1(t) + \alpha_2(t) = 1$ and $\beta_1(t) + \beta_2(t) = 1$, respectively. Using the Schur complement such that

$$\begin{aligned}
\Xi_i\big(\tau_i(t),\dot{\tau}_i(t)\big) &= \alpha_1(t)\beta_1(t)\Omega_i(\mu_{1i},h_{1i}) + \alpha_2(t)\beta_1(t)\Omega_i(\mu_{2i},h_{1i})\\
&\quad + \alpha_1(t)\beta_2(t)\Omega_i(\mu_{1i},h_{2i}) + \alpha_2(t)\beta_2(t)\Omega_i(\mu_{2i},h_{2i}),
\end{aligned} \tag{18}$$

where $\Omega_i(\mu_{1i},h_{1i})$, $\Omega_i(\mu_{2i},h_{1i})$, $\Omega_i(\mu_{1i},h_{2i})$ and $\Omega_i(\mu_{2i},h_{2i})$ are defined in Theorem 3.1. Substituting (18) into (17), then (18) can be rewritten as

$$\pounds V(x_t,r_t,t) - \lambda\omega^{\mathsf{T}}(t)W\omega(t) \le \sum_{p=1}^{2}\sum_{q=1}^{2}\Omega_i(\mu_{pi},h_{qi}). \tag{19}$$

Therefore, the following relation holds by condition (8) and (19):

$$\mathbb{E}\big\{\pounds V(x_t,r_t,t)\big\} \le \mathbb{E}\big[\eta V(x_t,r_t,t)\big] + \lambda\omega^{\mathsf{T}}(t)W\omega(t).$$

Multiplying the aforementioned inequality by $e^{-\eta t}$, we can get

$$\mathbb{E}\big\{\pounds\big[e^{-\eta t}V(x_t,r_t,t)\big]\big\} \le e^{-\eta t}\lambda\omega^{\mathsf{T}}(t)W\omega(t).$$

By integrating the aforementioned inequality between 0 and t, it follows that

$$e^{-\eta t}\mathbb{E}\big[V(x_t,r_t,t)\big]-\mathbb{E}\big[V(x_0,r_0,0)\big]\le\lambda\int_0^t e^{-\eta s}\omega^{\mathsf T}(s)W\omega(s)\,ds.$$

Denote $\widetilde{P}_i=\overline{R}_i^{-\frac12}P_i\overline{R}_i^{-\frac12}$, $\widetilde{Q}_i^{(r)}=\overline{R}_i^{-\frac12}Q_i^{(r)}\overline{R}_i^{-\frac12}$, $\widetilde{Q}=\overline{R}_i^{-\frac12}Q\overline{R}_i^{-\frac12}$, $\widetilde{R}_i=\overline{R}_i^{-\frac12}R_i\overline{R}_i^{-\frac12}$, $\widetilde{R}=\overline{R}_i^{-\frac12}R\overline{R}_i^{-\frac12}$, it yields that

$$\begin{aligned}
&\mathbb{E}\big[V(x_0,r_0,0)\big]\\
&\quad\le\Big\{\max_{i\in\mathcal N}\lambda_{\max}(\widetilde{P}_i)+\kappa_1\max_{i\in\mathcal N,1\le r\le(m+l)}\lambda_{\max}\big(\widetilde{Q}_i^{(r)}\big)+\kappa_2\lambda_{\max}(\widetilde{Q})\\
&\qquad+\kappa_2\max_{i\in\mathcal N}\lambda_{\max}(\widetilde{R}_i)+\kappa_3\lambda_{\max}(\widetilde{R})\Big\}\times\sup_{-\mu_2\le\upsilon\le0}\big\{x^{\mathsf T}(\upsilon)\overline{R}_ix(\upsilon),\dot{x}^{\mathsf T}(\upsilon)\overline{R}_i\dot{x}(\upsilon)\big\}\\
&\quad=\big(\lambda_2+\kappa_1\lambda_3+\kappa_2(\lambda_4+\lambda_5)+\kappa_3\lambda_6\big)\times\sup_{-\mu_2\le\upsilon\le0}\big\{x^{\mathsf T}(\upsilon)\overline{R}_ix(\upsilon),\dot{x}^{\mathsf T}(\upsilon)\overline{R}_i\dot{x}(\upsilon)\big\}\\
&\quad=c_1\Lambda.
\end{aligned}\tag{20}$$

Noting that $\eta>0$ and $0\le t\le T$, we have

$$\begin{aligned}
\mathbb{E}\big[V(x_t,r_t,t)\big]&\le\mathbb{E}\big[e^{\eta t}V(x_0,r_0,0)\big]+e^{\eta t}\lambda\int_0^t e^{-\eta s}\omega^{\mathsf T}(s)W\omega(s)\,ds\\
&\le e^{\eta T}c_1\Lambda+d\lambda e^{\eta T}\lambda_{\max}(W)\int_0^T e^{-\eta s}\,ds\\
&\le e^{\eta T}\Big\{c_1\Lambda+d\lambda\lambda_7\frac{1}{\eta}\big(1-e^{-\eta T}\big)\Big\}.
\end{aligned}\tag{21}$$

On the other hand, it follows from (12) that

$$\begin{aligned}
\mathbb{E}\big[V(x_t,r_t,t)\big]&\ge\mathbb{E}\big[x^{\mathsf T}(t)e^{\lambda t}P_ix(t)\big]\ge\max_{i\in\mathcal N}\lambda_{\min}(P_i)\mathbb{E}\big[x^{\mathsf T}(t)\overline{R}_ix(t)\big]\\
&=\lambda_1\mathbb{E}\big[x^{\mathsf T}(t)\overline{R}_ix(t)\big].
\end{aligned}\tag{22}$$

It can be derived from (21)-(22) that

$$\mathbb{E}\big[x^{\mathsf T}(t)\overline{R}_ix(t)\big]\le\frac{e^{\eta T}}{\lambda_1}\Big\{c_1\Lambda+d\lambda\lambda_7\frac{1}{\eta}\big(1-e^{-\eta T}\big)\Big\}.\tag{23}$$

From (11) and (23), we have

$$\mathbb{E}\big[x^{\mathsf T}(t)\overline{R}_ix(t)\big]<c_2.\tag{24}$$

Then the system is finite-time bounded with respect to $(c_1,c_2,d,\overline{R}_i,T)$. □

Remark 4 It should be mentioned that novel terms $V_2(x_t,i,t)$ and $V_3(x_t,i,t)$ are continuous at $\tau_i(t)=\tau_l$ is included in the Lyapunov-Krasovskii functional (12), which plays an important role in reducing conservativeness of the derived result.

Remark 5 In this paper, $\tau_i(t)$ and $\dot{\tau}_i(t)$ may have different upper bounds in various delay intervals satisfying (3) and (4), respectively. While in previous work such as [16, 17], $\tau_i(t)$ and $\dot{\tau}_i(t)$ are enlarged to $\tau_i(t) \leq \mu_2 = \max\{\mu_{2i}, i \in \mathcal{N}\}$ and $\dot{\tau}_i(t) \leq h_2 = \max\{h_{2i}, i \in \mathcal{N}\}$, respectively, which may lead to conservativeness inevitably. However, the case above can be taken fully into account by employing the Lyapunov-Krasovskii functional (12).

Remark 6 When dealing with term $-\int_{t-\mu_2}^{t-\mu_1} \dot{x}^{\mathrm{T}}(\upsilon)R_i\dot{x}(\upsilon)\,d\upsilon$, the convex combination is not employed, Lemma 2.1 is used in this paper, then the free-weighting matrices-dependent null add items are necessary to be introduced in our proof, which leads to the decrease in the number of LMIs and LMIs scalar decision variables.

Remark 7 The feature of this paper is the way to deal with the integral term. Many researchers have enlarged the derivative of the Lyapunov functional in order to deal with the integral term in mathematical operations. In this paper, we transform different integral intervals with the same integral length into an integral interval. It is worth pointing out that in the proof of the theorem no extra inequality is introduced. We propose a novel delay-dependent sufficient criterion, which ensures that the Markovian jump system with time-varying delays is finite-time stable.

Remark 8 One can clearly see from the proof of Theorem 3.1 that neither free-weighting matrices nor model transformation has been employed to deal with the sum terms, and none of useful items are ignored, resulting in better results with the less number of LMIs scalar decision variables, which deduces some conservatism in some sense.

By using the novel Lyapunov functionals with the more general decomposition of delay interval, a state feedback controller (3) can be designed such that the resulting closed-loop system is finite-time bounded with H_∞ performance. When $r_t = i$, the closed-loop system is expressed by

$$\begin{cases} \dot{x}(t) = \overline{A}_i x(t) + A_i x(t - \tau_{r_t}(t)) + D_i \omega(t), \\ z(t) = \overline{C}_i x(t) + C_{\tau i} x(t - \tau_i(t)) + F_i \omega(t), \end{cases} \tag{25}$$

where

$$\overline{A}_i = A_i + B_i K_i, \qquad \overline{C}_i = C_i + E_i K_i.$$

Theorem 3.2 *System* (25) *is finite-time bounded with respect to* $(c_1, c_2, d, \overline{R}_i, T)$ *if there exist matrices* $P_i > 0$, $Q_i^{(r)} > 0$, $Q^{(r)} > 0$ $(r = 1, 2, \ldots, (m + l))$, R_i, $R > 0$, S_i, $\forall i, j \in \mathcal{N}$, *scalars* $\gamma > 0$, $c_1 < c_2$, $T > 0$, $\kappa_1 > 0$, $\kappa_2 > 0$, $\kappa_3 > 0$, $\lambda_s > 0$ $(s = 1, 2, \ldots, 6)$, $\lambda > 0$, $\eta > 0$ *and* $\Lambda > 0$, *such that for all* $i, j \in \mathcal{N}$, $k = 1, 2, \ldots, l$, $s = 1, 2, \ldots, m$, *the following inequalities hold*:

$$\Theta_i(\mu_{qi}, h_{qi}) = \begin{bmatrix} \Theta_{1i}(\mu_{pi}, h_{qi}) & \Upsilon_{1i} & \widetilde{\Upsilon}_{2i} \\ * & \Omega_{2i}(\mu_{pi}, h_{qi}) & \widetilde{\Upsilon}_{3i} \\ * & * & \widetilde{\Upsilon}_{4i} \end{bmatrix} < 0, \quad p = 1, 2, q = 1, 2, \tag{26}$$

$$c_1 \Lambda + d\lambda\gamma^2 \frac{1}{\eta}\left(1 - e^{-\eta T}\right) < \lambda_1 c_2 e^{-\eta T}, \tag{27}$$

where

$$\Theta_{1i}(\mu_{pi},h_{qi}) = \begin{bmatrix} \widetilde{\Theta}_{1i}(\mu_{pi},h_{qi}) & 0 & 0 & 0 & \cdots & 0 \\ * & \widetilde{\Omega}_{2i}(\mu_{pi},h_{qi}) & -\frac{l}{\mu_2-\mu_1}R_i & 0 & \cdots & 0 \\ * & * & \widetilde{\Omega}_{3i}(\mu_{pi},h_{qi}) & -\frac{l}{\mu_2-\mu_1}R_i & \cdots & 0 \\ * & * & * & \ddots & \ddots & \vdots \\ * & * & * & * & \widetilde{\Omega}_{li}(\mu_{pi},h_{qi}) & -\frac{l}{\mu_2-\mu_1}R_i \\ * & * & * & * & * & \widetilde{\Omega}_{(l+1)i}(\mu_{pi},h_{qi}) \end{bmatrix},$$

$$\widetilde{\Upsilon}_{2i} = \begin{bmatrix} P_iD_i & \overline{C}_i & \overline{A}_i^{\mathsf{T}}R_i & \overline{A}_i^{\mathsf{T}}R \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{\Upsilon}_{3i} = \begin{bmatrix} 0 & C_{\tau i} & A_{\tau i}^{\mathsf{T}}R_i & A_{\tau i}^{\mathsf{T}}R \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{\Upsilon}_{4i} = \begin{bmatrix} -\gamma^2 I & F_i & D_i^{\mathsf{T}}R_i & D_i^{\mathsf{T}}R \\ * & -I & 0 & 0 \\ * & * & -\frac{1}{\kappa_1}R_i & 0 \\ * & * & * & -\frac{1}{\kappa_2}R \end{bmatrix},$$

$$\begin{aligned} \widetilde{\Theta}_{1i}(\mu_{pi},h_{qi}) &= -\lambda P_i + P_i\overline{A}_i + \overline{A}_i^{\mathsf{T}}P_i + \sum_{j=1}^{N}\pi_{ij}P_j \\ &\quad + \sum_{k=1}^{l}\frac{e^{\lambda[\mu_1+\frac{k(\mu_{pi}-\mu_1)}{l}]} - e^{\lambda[\mu_1+\frac{(k-1)(\mu_{pi}-\mu_1)}{l}]}}{\lambda}Q^{(k)} \\ &\quad + \sum_{s=1}^{m}\frac{e^{\lambda[\mu_{pi}+\frac{(l+s)(\mu_2-\mu_{pi})}{m}]} - e^{\lambda[\mu_{pi}+\frac{(l+s-1)(\mu_2-\mu_{pi})}{m}]}}{\lambda}Q^{(l+s)}. \end{aligned}$$

Proof We now consider the H_∞ performance of system (25). Select the same Lyapunov-Krasovskii functional as Theorem 3.1 and the Schur complement, it yields that

$$£V(x_t,i,t) + z^{\mathsf{T}}(t)z(t) - \gamma^2\omega^{\mathsf{T}}(t)\omega(t) \le \xi^{\mathsf{T}}(t)\Theta_i(\mu_{pi},h_{qi})\xi(t). \tag{28}$$

It follows from (26) that

$$\mathbb{E}\{£V(x_t,i,t)\} \le \mathbb{E}[\eta V(x_t,i,t)] + \gamma^2\omega^{\mathsf{T}}(t)\omega(t) - \mathbb{E}[z^{\mathsf{T}}(t)z(t)]. \tag{29}$$

Multiplying the aforementioned inequality by $e^{-\eta t}$, one has

$$\mathbb{E}\{£[e^{-\eta t V(x_t,i,t)}]\} \le e^{-\eta t}[\gamma^2\omega^{\mathsf{T}}(t)\omega(t) - z^{\mathsf{T}}(t)z(t)]. \tag{30}$$

Under zero initial condition and $\mathbb{E}[V(x_t, i, t)] > 0$, by integrating the aforementioned inequality between 0 and T, we can get

$$\mathbb{E}\left[\int_0^T e^{-\eta\upsilon} z^\mathsf{T}(\upsilon) z(\upsilon)\, d\upsilon\right] \le \gamma^2 \mathbb{E}\left[\int_0^T e^{-\eta\upsilon} \omega^\mathsf{T}(\upsilon)\omega(\upsilon)\, d\upsilon\right]. \tag{31}$$

Then it yields

$$\mathbb{E}\left[\int_0^T z^\mathsf{T}(\upsilon) z(\upsilon)\, d\upsilon\right] \le \gamma^2 e^{\eta T} \mathbb{E}\left[\int_0^T \omega^\mathsf{T}(\upsilon)\omega(\upsilon)\, d\upsilon\right]. \tag{32}$$

Thus it is concluded by Definition 2.3 that system (25) is finite-time bounded with an H_∞ performance γ. The proof is completed. □

Remark 9 From the proof process of Theorem 3.1 and Theorem 3.2, it is easy to see that neither bounding technique for cross terms nor model transformation is involved. In other words, the obtained result is expected to be less conservative.

Remark 10 Lyapunov asymptotic stability and finite-time stability of a class of systems are independent concepts. Lyapunov asymptotically stable system may not be finite-time stable. Moreover, finite-time stable system may also not be Lyapunov asymptotically stable. There exist some results on Lyapunov stability, while finite-time stability also needs our full investigation, which was neglected by most previous work.

4 Finite-time H_∞ control

Theorem 4.1 *System* (25) *is finite-time bounded with respect to* $(c_1, c_2, d, \overline{R}_i, T)$ *if there exist matrices* $\widehat{P}_i > 0$, $\widehat{Q}_i^{(r)} > 0$, $\widehat{Q}^{(r)} > 0$ $(r = 1, 2, \ldots, (m+l))$, $\widehat{R}_i$, $\widehat{R} > 0$, $\widehat{S}_i$, $\forall i, j \in \mathcal{N}$, *scalars* $c_1 < c_2$, $T > 0$, $\kappa_1 > 0$, $\kappa_2 > 0$, $\kappa_3 > 0$, $\sigma_s > 0$ $(s = 1, 2, \ldots, 5)$, $\lambda > 0$, $\eta > 0$ *and* $\overline{\Lambda} > 0$, *such that for all* $i, j \in \mathcal{N}$, $k = 1, 2, \ldots, l$, $s = 1, 2, \ldots, m$, *the following inequalities hold*:

$$\Phi_i(\mu_{qi}, h_{qi}) = \begin{bmatrix} \Phi_{1i}(\mu_{pi}, h_{qi}) & \widetilde{\Psi}_{1i} & \widetilde{\Psi}_{2i} & \Pi_{1i} \\ * & \Phi_{2i}(\mu_{pi}, h_{qi}) & \widetilde{\Psi}_{3i} & 0 \\ * & * & \widetilde{\Psi}_{4i} & 0 \\ * & * & * & \Pi_{2i} \end{bmatrix} < 0, \quad p = 1, 2, q = 1, 2, \tag{33}$$

$$\begin{aligned} &e^{\frac{\lambda}{l}(\mu_{pi}-\mu_1)} \sum_{j=1}^N \pi_{ij} \widehat{Q}_j^{(k)} \le \widehat{Q}^{(k)}, \\ &e^{\frac{\lambda}{m}(\mu_2-\mu_{pi})} \sum_{j=1}^N \pi_{ij} \widehat{Q}_j^{(l+s)} \le \widehat{Q}^{(l+s)}, \quad p = 1, 2, \end{aligned} \tag{34}$$

$$e^{\lambda\mu_2} \sum_{j=1}^N \pi_{ij} \widehat{R}_{ij} \le \widehat{R}_i, \tag{35}$$

$$\begin{aligned} &\sigma_1 \overline{R}_i^{-1} < X_i < \overline{R}_i^{-1}, \qquad 0 < \widehat{Q}_{ii} < \sigma_2 \overline{R}_i, \qquad 0 < \widehat{Q}_i < \sigma_3 \overline{R}_i, \\ &0 < \widehat{R}_{ii} < \sigma_4 \overline{R}_i, \qquad 0 < \widehat{R}_i < \sigma_5 \overline{R}_i, \end{aligned} \tag{36}$$

$$\begin{bmatrix} c_1 \overline{\Lambda} + d\lambda\gamma^2 \frac{1}{\eta}(1 - e^{-\eta T}) - c_2 e^{-\eta T} & \sqrt{c_1} \\ \sqrt{c_1} & \sigma_1 \end{bmatrix} < 0, \tag{37}$$

where

$$\Phi_{1i}(\mu_{pi},h_{qi}) = \begin{bmatrix} \widetilde{\Phi}_{1i}(\mu_{pi},h_{qi}) & 0 & 0 & 0 & \cdots & 0 \\ * & \widetilde{\Phi}_{2i}(\mu_{pi},h_{qi}) & -\frac{l}{\mu_2-\mu_1}\widehat{R}_{ii} & 0 & \cdots & 0 \\ * & * & \widetilde{\Phi}_{3i}(\mu_{pi},h_{qi}) & -\frac{l}{\mu_2-\mu_1}\widehat{R}_{ii} & \cdots & 0 \\ * & * & * & \ddots & \ddots & \vdots \\ * & * & * & * & \widetilde{\Phi}_{li}(\mu_{pi},h_{qi}) & -\frac{l}{\mu_2-\mu_1}\widehat{R}_{ii} \\ * & * & * & * & * & \widetilde{\Phi}_{(l+1)i}(\mu_{pi},h_{qi}) \end{bmatrix},$$

$$\Phi_{2i}(\mu_{pi},h_{qi}) = \begin{bmatrix} \overline{\Phi}_{1i}(\mu_{pi},h_{qi}) & -\frac{m}{\mu_2-\mu_1}\widehat{R}_{ii} & 0 & \cdots & 0 \\ * & \overline{\Phi}_{2i}(\mu_{pi},h_{qi}) & -\frac{m}{\mu_2-\mu_1}\widehat{R}_{ii} & \cdots & 0 \\ * & * & \ddots & \ddots & 0 \\ * & * & * & \overline{\Phi}_{mi}(\mu_{pi},h_{qi}) & -\frac{m}{\mu_2-\mu_1}\widehat{R}_{ii} \\ * & * & * & * & \overline{\Phi}_{(m+1)i}(\mu_{pi},h_{qi}) \end{bmatrix},$$

$$\widetilde{\Psi}_{1i} = \begin{bmatrix} A_{\tau i}X_i & 0 & \cdots & 0 & 0 \\ \widehat{S}_{ii} & 2\widehat{S}_{ii} & \cdots & 2\widehat{S}_{ii} & \widehat{S}_{ii} \\ \widehat{S}_{ii} & 2\widehat{S}_{ii} & \cdots & 2\widehat{S}_{ii} & \widehat{S}_{ii} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -\frac{l}{\mu_2-\mu_1}\widehat{R}_{ii} + \widehat{S}_{ii} & 2\widehat{S}_{ii} & \cdots & 2\widehat{S}_{ii} & \widehat{S}_{ii} \end{bmatrix},$$

$$\widetilde{\Psi}_{2i} = \begin{bmatrix} D_iX_i & X_iC_i^{\mathsf{T}} + Y_iE_i^{\mathsf{T}} & X_iA_i^{\mathsf{T}} + Y_iB_i^{\mathsf{T}} & X_iA_i^{\mathsf{T}} + Y_iB_i^{\mathsf{T}} \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{\Psi}_{3i} = \begin{bmatrix} 0 & X_iC_{\tau i}^{\mathsf{T}} & X_iA_{\tau i}^{\mathsf{T}} & X_iA_{\tau i}^{\mathsf{T}} \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{\Psi}_{4i} = \begin{bmatrix} -\gamma^2(2X_i - I) & X_iF_i & X_iD_i^{\mathsf{T}} & X_iD_i^{\mathsf{T}} \\ * & -I & 0 & 0 \\ * & * & -\frac{1}{\kappa_1}(2X_i - \widehat{R}_{ii}) & 0 \\ * & * & * & -\frac{1}{\kappa_2}(2X_i - \widehat{R}_i) \end{bmatrix},$$

$$\Pi_{1i} = [\sqrt{\pi_{i1}}X_i, \ldots, \sqrt{\pi_{i(i-1)}}X_i, \sqrt{\pi_{i(i+1)}}X_i, \ldots, \sqrt{\pi_{iN}}X_i],$$

$$\Pi_{2i} = \operatorname{diag}\{X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_N\},$$

$$\begin{aligned} \widetilde{\Phi}_{1i}(\mu_{pi},h_{qi}) = {} & -\lambda X_i + \overline{A}_iX_i + X_i\overline{A}_i^{\mathsf{T}} + \pi_{ii}X_i \\ & + \sum_{k=1}^{l} \frac{e^{\lambda[\mu_1 + \frac{k(\mu_{pi}-\mu_1)}{l}]} - e^{\lambda[\mu_1 + \frac{(k-1)(\mu_{pi}-\mu_{1i})}{l}]}}{\lambda}\widehat{Q}^{(k)} \\ & + \sum_{s=1}^{m} \frac{e^{\lambda[\mu_{pi} + \frac{(l+s)(\mu_2-\mu_{pi})}{m}]} - e^{\lambda[\mu_{pi} + \frac{(l+s-1)(\mu_2-\mu_{pi})}{m}]}}{\lambda}\widehat{Q}^{(l+s)}, \end{aligned}$$

$$
\begin{aligned}
&\widetilde{\Phi}_{2i}(\mu_{pi},h_{qi}) = e^{\frac{\lambda}{l}(\mu_{pi}-\mu_1)}\widehat{Q}_{ii}^{(1)} + \frac{l}{\mu_2-\mu_1}\widehat{R}_{ii},\\
&\widetilde{\Phi}_{3i}(\mu_{pi},h_{qi}) = -\left(1-\frac{1}{l}h_{qi}\right)\widehat{Q}_{ii}^{(1)} + \left(1-\frac{1}{l}h_{qi}\right)e^{\frac{\lambda}{l}(\mu_{pi}-\mu_1)}\widehat{Q}_{ii}^{(2)} + \frac{2l}{\mu_2-\mu_1}\widehat{R}_{ii},\\
&\widetilde{\Phi}_{li}(\mu_{pi},h_{qi}) = -\left(1-\frac{l-2}{l}h_{qi}\right)\widehat{Q}_{ii}^{(l-2)} + \left(1-\frac{l-2}{l}h_{qi}\right)e^{\frac{\lambda}{l}(\mu_{pi}-\mu_1)}\widehat{Q}_{ii}^{(l-1)} + \frac{2l}{\mu_2-\mu_1}\widehat{R}_{ii},\\
&\widetilde{\Phi}_{(l+1)i}(\mu_{pi},h_{qi}) = -\left(1-\frac{l-1}{l}h_{qi}\right)\widehat{Q}_{ii}^{(l-1)} + \left(1-\frac{l-1}{l}h_{qi}\right)e^{\frac{\lambda}{l}(\mu_{pi}-\mu_1)}\widehat{Q}_{ii}^{(l)} + \frac{2l}{\mu_2-\mu_1}\widehat{R}_{ii},\\
&\overline{\Phi}_{1i}(\mu_{pi},h_{qi}) = -(1-h_{qi})\widehat{Q}_{ii}^{(l)} + (1-h_{qi})e^{\frac{\lambda}{m}(\mu_2-\mu_{pi})}\widehat{Q}_{ii}^{(l+1)} + \frac{l+m}{\mu_2-\mu_1}\widehat{R}_{ii},\\
&\overline{\Phi}_{2i}(\mu_{pi},h_{qi}) = -\left(1-\frac{m-1}{m}h_{qi}\right)\widehat{Q}_{ii}^{(l+1)} + \left(1-\frac{m-1}{m}h_{qi}\right)e^{\frac{\lambda}{m}(\mu_2-\mu_{pi})}\widehat{Q}_{ii}^{(l+2)} + \frac{2m}{\mu_2-\mu_1}\widehat{R}_{ii},\\
&\overline{\Phi}_{mi}(\mu_{pi},h_{qi}) = -\left(1-\frac{1}{m}h_{qi}\right)\widehat{Q}_{ii}^{(l+m-1)} + \left(1-\frac{1}{m}h_{qi}\right)e^{\frac{\lambda}{m}(\mu_2-\mu_{pi})}\widehat{Q}_{ii}^{(l+m)} + \frac{2m}{\mu_2-\mu_1}\widehat{R}_{ii},\\
&\overline{\Phi}_{(m+1)i}(\mu_{pi},h_{qi}) = -\widehat{Q}_{ii}^{(l+m)} + \frac{m}{\mu_2-\mu_1}\widehat{R}_{ii},\\
&\overline{\Lambda} = \kappa_1\sigma_2 + \kappa_2(\sigma_3+\sigma_4) + \kappa_3\sigma_5.
\end{aligned}
$$

Moreover, the state feedback gain matrices can be designed as

$$K_i = Y_i X_i^{-1}, \quad \forall i = 1,2,\dots,N.$$

Proof Consider Theorem 3.2 and the overall closed-loop Markov jump system (25). Pre- and post-multiplying inequality (26) by block-diagonal matrix $\operatorname{diag}\{P_i^{-1},\dots,P_i^{-1},I,R_i^{-1},R^{-1}\}$ and its transpose, respectively. Letting

$$
\begin{aligned}
&X_i = P_i^{-1}, \qquad Y_i = K_i X_i, \qquad \widehat{Q}_{ij}^{(r)} = X_i Q_j^{(r)} X_i, \qquad \widehat{Q}_i^{(r)} = X_i Q^{(r)} X_i,\\
&\widehat{R}_{ij} = X_i R_j X_i, \qquad \widehat{R}_i = X_i R X_i, \qquad \widehat{S}_{ij} = X_i S_j X_i.
\end{aligned}
\tag{38}
$$

It can be easily obtained that

$$
\Gamma_i(\mu_{qi},h_{qi}) = \begin{bmatrix} \Phi_{1i}(\mu_{pi},h_{qi}) & \widetilde{\Psi}_{1i} & \widetilde{\Psi}_{2i} & \Pi_{1i}\\ * & \Phi_{2i}(\mu_{pi},h_{qi}) & \widetilde{\Psi}_{3i} & 0\\ * & * & \widetilde{\Gamma}_i & 0\\ * & * & * & \Pi_{2i} \end{bmatrix} < 0, \quad p=1,2, q=1,2, \tag{39}
$$

where

$$
\widetilde{\Gamma}_i = \begin{bmatrix} -\gamma^2 X_i X_i & X_i F_i & X_i D_i^{\mathsf{T}} & X_i D_i^{\mathsf{T}}\\ * & -I & 0 & 0\\ * & * & -\frac{1}{\kappa_1}X_i\widehat{R}_{ii}^{-1}X_i & 0\\ * & * & * & -\frac{1}{\kappa_2}X_i\widehat{R}_i^{-1}X_i \end{bmatrix}.
$$

From Lemma 2.3, for any $X_i > 0$, $\widehat{R}_{ii} > 0$ and $\widehat{R}_i > 0$, one can obtain $-X_iX_i \le 2X_i - I$, $-X_i\widehat{R}_{ii}X_i \le 2X_i - \widehat{R}_{ii}$, $-X_i\widehat{R}_iX_i \le 2X_i - \widehat{R}_i$. Then (33) is equivalent to (39). Therefore, if (33) holds, system (1) is finite-time bounded with a prescribed H_∞ performance index γ. The proof is completed. □

Remark 11 By solving the Markovian jumping system with finite-time observer-based controller, the mode-dependent positive-definite weighting matrices $\overline{R}_i$ in inequalities (33)-(37) should be known first. For convenience, we always choose the initial value for $\overline{R}_i = I$.

Remark 12 In many actual applications, the minimum value of $\gamma^2_{\min}$ is of interest. In Theorem 3.2, with a fixed λ, $\gamma_{\min}$ can be obtained through the following optimization procedure:

$$\min \gamma^2$$
$$\text{s.t. (26)-(27).}$$

In Theorem 4.1, as for finite-time stability and boundedness, once the state bound c_2 is not ascertained, the minimum value $c_{2\min}$ is of interest. With a fixed λ, and define $\lambda_1 = 1$, $\lambda_2 = \sigma_1$, then the following optimization problem can be formulated to get the minimum value $c_{2\min}$

$$\min \varsigma\gamma^2 + (1-\varsigma)c_2$$
$$\text{s.t. (33)-(37),}$$

where ς is weighted factor, and $\varsigma \in [0,1]$.

5 Illustrative example

Example 1 Consider the Markovian jump system (1) with two operation modes and the following data:

$$A_1 = \begin{bmatrix} -0.9 & 0.5 \\ -0.32 & -0.8 \end{bmatrix}, \qquad A_{\tau 1} = \begin{bmatrix} -0.5 & -0.3 \\ 0.3 & -0.2 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} -1.05 & 0.8 \\ -0.15 & -1.3 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 0.6 & -0.4 \\ 0.35 & -0.41 \end{bmatrix},$$

and the transition probability matrix is

$$\Omega = \begin{bmatrix} -0.2 & 0.2 \\ 0.8 & -0.8 \end{bmatrix}.$$

Under different levels of the upper bound μ_2 and λ, Table 1 and Table 2 list the results of the maximum allowable upper bound, the decay rate λ for different time delays and

Table 1 Comparison of the upper bounds of the decay rate for different delays

	$\mu_2 = 0.2$	$\mu_2 = 0.5$	$\mu_2 = 0.8$	$\mu_2 = 1$	$\mu_2 = 1.2$
[22] ($m = 2$)	1.2718	1.0223	0.8234	0.7145	0.6209
[22] ($m = 4$)	1.3648	1.1245	0.9515	0.9980	0.8241
[21] ($m = 2$)	1.3641	1.1972	1.0035	0.8398	0.6934
[21] ($m = 4$)	-	-	-	-	-
Theorem 3.1 ($m = 1, l = 1$)	1.3663	1.2019	1.1012	0.9920	0.7125
Theorem 3.1 ($m = 2, l = 2$)	1.4834	1.3132	1.2563	1.0197	0.9582

Table 2 Comparison of the allowable values of time delay μ_2 for different decay rates

	λ = 0.6	λ = 0.8	λ = 1	λ = 1.2	λ = 1.4
[22] ($m = 2$)	1.2496	0.8045	0.5304	0.2801	0.0662
[22] ($m = 4$)	1.3562	0.9123	0.6294	0.3883	0.1716
[21] ($m = 2$)	1.3525	1.0512	0.8044	0.4953	0.1318
[21] ($m = 4$)	-	-	-	-	-
Theorem 3.1 ($m = 1, l = 1$)	1.4681	1.2003	0.9943	0.6015	0.2726
Theorem 3.1 ($m = 2, l = 2$)	1.5722	1.3620	1.0342	0.7110	0.3675

maximum values of μ_2 derived from various methods including the one proposed in this paper, respectively. One can see from Table 1 and Table 2 that the same results are obtained in [21, 22]. It is clear from Table 1 and Table 2 that the performance achieved by our method is much better than those by [21, 22]. Therefore, our results not only are less conservative, but also require the less number of scalar decision variables.

Example 2 Consider a two-mode Markovian jump system (1) with

$$A_1 = \begin{bmatrix} -0.8 & 1.5 \\ 2 & 3 \end{bmatrix}, \qquad A_{\tau 1} = \begin{bmatrix} -0.45 & 1 \\ -0.5 & 2 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -1 & 0.2 \\ 0.5 & -0.1 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad C_{\tau 1} = \begin{bmatrix} 0.03 & 0 \\ 0.01 & 0.02 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.02 & 0 \\ 0.01 & 0.01 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} 0.01 \\ 0.001 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -2 & 1.2 \\ 1 & 4 \end{bmatrix}, \qquad A_{\tau 2} = \begin{bmatrix} -1 & 1.2 \\ 0 & -0.5 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -1 & 1 \\ 0.5 & -2 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.1 & 0.02 \\ 0 & 0.1 \end{bmatrix}, \qquad C_{\tau 2} = \begin{bmatrix} 0.02 & 0 \\ 0.1 & 0.02 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0.04 & 0 \\ 0.1 & 0.01 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 0.04 \\ 0.01 \end{bmatrix}.$$

In addition, the transition rate matrix is given by

$$\Omega = \begin{bmatrix} -1.2 & 1.2 \\ 1 & -1 \end{bmatrix}.$$

Then we choose $\overline{R}_1 = \overline{R}_1 = I$, $T = 2$, $c_1 = 1$, $d = 0.01$, through Theorem 4.1, it yields that $c_2 = 152.4231$. Moreover, we also can obtain the following controller gains:

$$K_1 = \begin{bmatrix} -1.1562 & 2.5461 \\ -0.2712 & 3.2523 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 0.9613 & -1.7163 \\ -2.2864 & 4.3842 \end{bmatrix}.$$

It confirms the effectiveness of Theorem 4.1 for the state feedback controller design to finite-time Markovian jump systems with time-varying delay.

6 Conclusions

In this paper, we have examined the problems of finite-time H_∞ control for a class of Markovian jump systems with mode-dependent time-varying delay. Based on a novel approach, a sufficient condition is derived such that the closed-loop Markovian jump system is finite-time bounded and satisfies the prescribed level of H_∞ disturbance attenuation in a finite time interval. The controller and observer gains can be solved directly by using the existing LMIs optimization techniques. Finally, numerical examples are also given to illustrate the effectiveness of the proposed design approach.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors have equal contributions.

Author details
[1]School of Automation Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, P.R. China. [2]School of Computer Science, Civil Aviation Flight University of China, Guanghan, Sichuan 618307, P.R. China. [3]School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, P.R. China.

Acknowledgements
The authors would like to thank the associate editor and anonymous reviewers for their detailed comments and suggestions. This work was supported by the National Basic Research Program of China (2010CB732501).

Received: 18 March 2013 Accepted: 1 July 2013 Published: 15 July 2013

References

1. Krasovskii, N, Lidskii, E: Analysis and design of controllers in systems with random attributes. Autom. Remote Control **22**, 1021-1025 (1961)
2. Cheng, J, Zhu, H, Zhong, S, Zhang, Y: Robust stability of switched delay systems with average dwell time under asynchronous switching. J. Appl. Math. (2012). doi:10.1155/2012/956370
3. Wu, L, Su, X, Shi, P: Sliding mode control with bounded $\mathcal{L}_2$ gain performance of Markovian jump singular time-delay systems. Automatica **48**(8), 1929-1933 (2012)
4. Gao, H, Fei, Z, Lam, J, Du, B: Further results on exponential estimates of Markovian jump systems with mode-dependent time-varying delays. IEEE Trans. Autom. Control **56**(1), 223-229 (2011)
5. Wu, Z, Su, H, Chu, J: H_∞ filtering for singular Markovian jump systems with time delay. Int. J. Robust Nonlinear Control **20**(8), 939-957 (2010)
6. Tino, P, Cernansky, M, Beunskova, L: Markovian architectural bias of recurrent neural networks. IEEE Trans. Neural Netw. **15**, 6-15 (2004)
7. Dong, H, Wang, Z, Ho, D, Gao, H: Robust H_∞ filtering for Markovian jump systems with randomly occurred nonlinearities and sensor saturation: the finite-horizon case. IEEE Trans. Signal Process. **59**(7), 3048-3057 (2011)
8. Yao, X, Wu, L, Zheng, W, Wang, C: Robust H_∞ filtering of Markovian jump stochastic systems with uncertain transition probabilities. Int. J. Syst. Sci. **42**(7), 1219-1230 (2011)
9. Wu, L, Shi, P, Gao, H: State estimation and sliding-mode control of Markovian jump singular systems. IEEE Trans. Autom. Control **55**(5), 1213-1219 (2010)
10. Wu, L, Shi, P, Gao, H, Wang, C: H_∞ filtering for 2D Markovian jump systems. Automatica **44**(7), 1849-1858 (2008)
11. Wu, Z, Su, H, Chu, J: State estimation for discrete Markovian jumping neural networks with time delay. Neurocomputing **73**(10-12), 2247-2254 (2010)
12. Zhu, Q, Cao, J: Robust exponential stability of Markovian jump impulsive stochastic Cohen-Grossberg neural networks with mixed time delays. IEEE Trans. Neural Netw. **21**, 1314-1325 (2010)
13. Mou, S, Gao, H, Lam, J, Qiang, W, Chen, K: New delay-dependent exponential stability for neural networks with time delay. IEEE Trans. Syst. Man Cybern., Part B, Cybern. **38**, 571-576 (2008)
14. Shao, H: Improved delay-dependent globally asymptotic stability criteria for neural networks with a constant delay. IEEE Trans. Circuits Syst. **55**, 1071-1075 (2008)
15. Hu, L, Gao, H, Zheng, W: Novel stability of cellular neural networks with interval time-varying delay. Neural Netw. **21**(10), 1458-1463 (2008)
16. Zhang, H, Liu, Z, Huang, G-B, Wang, Z: Novel weighting-delay-based stability criteria for recurrent neural networks with time-varying delay. IEEE Trans. Neural Netw. **21**(1), 91-106 (2010)
17. Zuo, Z, Yang, C, Wang, Y: A new method for stability analysis of recurrent neural networks with interval time-varying delay. IEEE Trans. Neural Netw. **21**(2), 339-344 (2010)

18. Park, P, Ko, J, Jeong, C: Reciprocally convex approach to stability of systems with time-varying delays. Automatica **47**, 235-238 (2011)
19. Shu, Z, Lam, J, Xu, S: Robust stabilization of Markovian delay systems with delay-dependent exponential estimates. Automatica **42**, 2001-2008 (2006)
20. Boyd, S, El Ghaoui, L, Feron, E, Balakrishnan, V: Linear Matrix Inequalities in System and Control Theory. SIAM, Philadelphia (1994)
21. Huang, H, Feng, G, Chen, X: Stability and stabilization of Markovian jump systems with time delay via new Lyapunov functionals. IEEE Trans. Circuits Syst. **59**(10), 2413-2421 (2012)
22. Gao, H, Fei, Z, Lam, J, Du, B: Further results on exponential estimates of Markovian jump systems with mode-dependent time-varying delays. IEEE Trans. Autom. Control **56**(1), 223-229 (2011)
23. Sun, M, Lam, J, Xu, S, Zou, Y: Robust exponential stabilization for Markovian jump systems with mode-dependent input delay. Automatica **43**(10), 1799-1807 (2007)
24. Fei, Z, Gao, H, Shi, P: New results on stabilization of Markovian jump systems with time delay. Automatica **45**(10), 2300-2306 (2009)
25. Karimi, HR: Robust delay-dependent control of uncertain time delay systems with mixed neutral, discrete, and distributed time-delays and Markovian switching parameters. IEEE Trans. Circuits Syst. **58**(8), 1910-1923 (2011)
26. Xu, S, Lam, J, Mao, X: Delay-dependent control and filtering for uncertain Markovian jump systems with time-varying delays. IEEE Trans. Circuits Syst. **54**(9), 2070-2077 (2007)
27. Liu, Y, Wang, Z, Liu, X: State estimation for discrete-time Markovian jumping neural networks with mixed mode-dependent delays. Phys. Lett. A **372**(48), 7147-7155 (2008)
28. Zhang, L, Boukas, EL: H_∞ control of a class of extended Markov jump linear systems. IET Control Theory Appl. **3**(7), 834-842 (2009)
29. Zhang, L, Boukas, EL, Lam, J: Analysis and synthesis of Markov jump linear systems with time-varying delays and partially known transition probabilities. IEEE Trans. Autom. Control **53**(10), 2458-2464 (2008)
30. Ma, S, Boukas, EK: Robust H_∞ filtering for uncertain discrete Markov jump singular systems with mode-dependent time delay. IET Control Theory Appl. **3**(3), 351-361 (2009)
31. Chen, Y, Bi, W, Li, W: Stability analysis for neural networks with time-varying delay: a more general delay decomposition approach. Neurocomputing **73**, 853-857 (2010)
32. Cheng, J, Zhu, H, Zhong, S, Li, G: Novel delay-dependent robust stability criteria for neutral systems with mixed time-varying delays and nonlinear perturbations. Appl. Math. Comput. **219**, 7741-7753 (2013)
33. Huang, X, Lin, W, Yang, B: Global finite-time stabilization of a class of uncertain nonlinear systems. Automatica **41**(5), 881-888 (2005)
34. Qian, C, Li, J: Global finite-time stabilization by output feedback for planar systems without observable linearization. IEEE Trans. Autom. Control **50**(6), 549-564 (2005)
35. Amato, F, Ambrosino, R, Cosentino, C, De Tommasi, G: Input-output finite-time stabilization of linear systems. Automatica **46**(9), 1558-1562 (2010)
36. Hong, Y: Finite-time stabilization and stability of a class of controllable systems. Syst. Control Lett. **48**(4), 231-236 (2002)
37. Amato, F, Ariola, M: Finite-time control of discrete-time linear systems. IEEE Trans. Autom. Control **50**(5), 724-729 (2005)
38. Cheng, J, Zhu, H, Zhong, S, Zhang, Y: Finite-time boundness of H_∞ filtering for switched discrete-time systems. Int. J. Control. Autom. Syst. **10**(6), 1129-1135 (2012)
39. Luan, X, Liu, F, Shi, P: Finite-time filtering for non-linear stochastic systems with partially known transition jump rates. IET Control Theory Appl. **4**(5), 735-745 (2010)
40. Liu, H, Zhao, X: Asynchronous finite-time H_∞ control for switched linear systems via mode-dependent dynamic state-feedback. Nonlinear Anal. Hybrid Syst. **8**, 109-120 (2013)
41. Lin, X, Du, H, Li, S: Finite-time boundedness and L_2-gain analysis for switched delay systems with norm-bounded disturbance. Appl. Math. Comput. **217**(12), 982-993 (2011)
42. Lin, J, Fei, S, Gao, Z: Stabilization of discrete-time switched singular time-delay systems under asynchronous switching. J. Franklin Inst. (2012). doi:10.1016/j.jfranklin.2012.02.009
43. Song, H, Yu, L, Zhang, D, Zhang, W: Finite-time H_∞ control for a class of discrete-time switching time-delay systems with quantized feedback. Commun. Nonlinear Sci. Numer. Simul. **17**(12), 4802-4814 (2012)
44. Zuo, Z, Li, H, Liu, Y, Wang, Y: On finite-time stochastic stability and stabilization of Markovian jump systems subject to partial information on transition probabilities. Circuits Syst. Signal Process. (2012). doi:10.1007/s00034-012-9420-3
45. Liu, H, Shen, Y, Zhao, X: Delay-dependent observer-based H_∞ finite-time control for switched systems with time-varying delay. Nonlinear Anal. Hybrid Syst. **6**, 885-898 (2012)
46. Zhang, Y, Liu, C, Mu, X: Robust finite-time H_∞ control of singular stochastic systems via static output feedback. Appl. Math. Comput. **218**, 5629-5640 (2012)
47. Xu, J, Sun, J: Finite-time filtering for discrete-time linear impulsive systems. Signal Process. **92**, 2718-2722 (2012)
48. Cheng, J, Zhu, H, Zhong, S, Zhang, Y, Li, Y: Finite-time H_∞ control for a class of discrete-time Markov jump systems with partly unknown time-varying transition probabilities subject to average dwell time switching. Int. J. Syst. Sci. (2013). doi:10.1080/00207721.2013.808716
49. He, S, Liu, F: Finite-time H_∞ fuzzy control of nonlinear jump systems with time delays via dynamic observer-based state feedback. IEEE Trans. Fuzzy Syst. **20**(4), 605-614 (2012)
50. Zhang, D, Yu, L, Zhang, W: Delay-dependent fault detection for switched linear systems with time-varying delays - the average dwell time approach. Signal Process. **91**, 832-840 (2011)

doi:10.1186/1687-1847-2013-214
Cite this article as: Cheng et al.: **Finite-time H_∞ control for a class of Markovian jump systems with mode-dependent time-varying delay.** *Advances in Difference Equations* 2013 **2013**:214.