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Global attractivity of a discrete cooperative system incorporating harvesting

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Abstract

A discrete cooperative model incorporating harvesting that takes the form

$$\begin{aligned} x(k+1) &= x(k)\exp\left\{r_1 - Eq - b_1x(k) - \frac{a_1x(k)}{y(k)+k_1}\right\}, \\ y(k+1) &= y(k)\exp\left\{r_2 - b_2y(k) - \frac{a_2y(k)}{x(k)+k_2}\right\} \end{aligned}$$

is proposed and studied in this paper. By using the iterative method and the comparison principle of difference equations, a set of sufficient conditions which ensure the global attractivity of the interior equilibrium of the system is obtained. Numeric simulations show the feasibility of the main result.

MSC: 34D23; 92B05; 34D40

Keywords: global attractivity; cooperation; equilibrium; iterative method

1 Introduction

In [1], Wei and Li proposed and studied the following cooperative system incorporating harvesting:

$$\begin{aligned} \dot{x} &= x\left(r_1 - b_1x - \frac{a_1x}{y+k_1}\right) - Eqx, \\ \dot{y} &= y\left(r_2 - b_2y - \frac{a_2y}{x+k_2}\right), \end{aligned} \tag{1.1}$$

where x and y denote the densities of two populations at time t. The parameters r_1, r_2, a_1, a_2, b_1, b_2, k_1, k_2, E, q are all positive constants. By applying the comparison theorem of differential equations and constructing a suitable Lyapunov function, they obtained sufficient conditions which ensure the persistent and stability of the positive equilibrium, respectively.

Recently, Xie *et al.* [2] revisited the dynamic behaviors of the system (1.1). By using the iterative method, they showed that the condition which ensures the existence of a unique positive equilibrium is enough to ensure the globally attractive of the positive equilibrium. Their result significantly improves the corresponding results of Wei and Li [1].

It is well known that the discrete time models governed by difference equations are more appropriate than the continuous ones when the populations have nonoverlapping generations; corresponding to system (1.1), we propose the following discrete cooperative model incorporating harvesting:

$$\begin{aligned} x(k+1) &= x(k)\exp\left\{r_1 - Eq - b_1 x(k) - \frac{a_1 x(k)}{y(k)+k_1}\right\}, \\ y(k+1) &= y(k)\exp\left\{r_2 - b_2 y(k) - \frac{a_2 y(k)}{x(k)+k_2}\right\}, \end{aligned} \tag{1.2}$$

where $x(k)$, $y(k)$ are the population density of the species x and y at k-generation. Throughout this paper, we assume that the coefficients of the system (1.2) satisfies:

(H_1) r_i, b_i, a_i, E, q, $i = 1, 2$ are all positive constants, $r_1 > Eq$.

We mention here that under the assumption (H_1), system (1.2) admits a unique positive equilibrium (x^*, y^*). Indeed, the positive equilibrium of system (1.2) satisfies

$$\begin{cases} r_1 - Eq - b_1 x - \frac{a_1 x}{y+k_1} = 0, \\ r_2 - b_2 y - \frac{a_2 y}{x+k_2} = 0, \end{cases} \tag{1.3}$$

which is equivalent to

$$\begin{cases} A_1 x^2 + A_2 x + A_3 = 0, \\ B_1 y^2 + B_2 y + B_3 = 0, \end{cases} \tag{1.4}$$

where

$$\begin{aligned} A_1 &= b_1 b_2 k_1 + a_1 b_2 + b_1 r_2, \\ A_2 &= b_1 b_2 k_1 k_2 + a_1 b_2 k_2 + a_2 b_1 k_1 + b_1 k_2 r_2 + a_1 a_2 - (r_1 - qE)(b_2 k_1 + r_2), \\ A_3 &= -(r_1 - Eq)(b_2 k_1 k_2 + a_2 k_1 + k_2 r_2), \\ B_1 &= b_1 b_2 k_2 + (r_1 - Eq) b_2 + a_2 b_1, \\ B_2 &= b_1 b_2 k_1 k_2 + a_1 b_2 k_2 + a_2 b_1 k_1 - b_1 k_2 r_2 + a_1 a_2 + (r_1 - qE)(b_2 k_1 - r_2), \\ B_3 &= -r_2\big(b_1 k_1 k_2 + (r_1 - Eq) k_1 + a_1 k_2\big). \end{aligned} \tag{1.5}$$

From $A_1 > 0$, $A_3 < 0$, $B_1 > 0$, $B_3 < 0$ one could easily see that system (1.3) admits a unique positive solution

$$x^* = \frac{-A_2 + \sqrt{A_2^2 - 4A_1 A_3}}{2A_1}, \qquad y^* = \frac{-B_2 + \sqrt{B_2^2 - 4B_1 B_3}}{2B_1}. \tag{1.6}$$

The aim of this paper is, by further developing the analysis technique of Xie *et al.* [2], Yang *et al.* [3], and Chen and Teng [4], to obtain a set of sufficient conditions to ensure the global attractivity of the interior equilibrium of system (1.1). More precisely, we will prove the following result.

Theorem 1.1 *In addition to* (H_1), *further assume that*

(H_2) $0 < r_1 - qE \le 1$, $r_2 \le 1$, *hold, then system* (1.2) *admits a unique positive equilibrium* (x^*, y^*) *which is globally attractive*.

The rest of the paper is arranged as follows. With the help of several useful lemmas, we will prove Theorem 1.1 in Section 2. Two examples together with their numeric simulations are presented in Section 3 to show the feasibility of our results. We end this paper by a brief discussion. For more work about cooperative systems, we can refer to [1–30] and the references therein.

2 Global attractivity

We will give a strict proof of Theorem 1.1 in this section. To achieve this objective, we introduce several useful lemmas.

Lemma 2.1 ([4]) *Let* $f(u) = u\exp(\alpha - \beta u)$, *where* α *and* β *are positive constants, then* $f(u)$ *is nondecreasing for* $u \in (0, \frac{1}{\beta}]$.

Lemma 2.2 ([4]) *Assume that the sequence* $\{u(k)\}$ *satisfies*

$$u(k+1) = u(k)\exp\bigl(\alpha - \beta u(k)\bigr), \quad k = 1, 2, \ldots,$$

where α *and* β *are positive constants and* $u(0) > 0$. *Then*:

(i) *If* $\alpha < 2$, *then* $\lim_{k\to+\infty} u(k) = \frac{\alpha}{\beta}$.

(ii) *If* $\alpha \leq 1$, *then* $u(k) \leq \frac{1}{\beta}$, $k = 2, 3, \ldots$.

Lemma 2.3 ([25]) *Suppose that the functions* $f, g : Z_+ \times [0,\infty) \to [0,\infty)$ *satisfy* $f(k,x) \leq g(k,x)$ $(f(k,x) \geq g(k,x))$ *for* $k \in Z_+$ *and* $x \in [0,\infty)$ *and* $g(k,x)$ *is nondecreasing with respect to* x. *If* $\{x(k)\}$ *and* $\{u(k)\}$ *are the nonnegative solutions of the following difference equations*:

$$x(k+1) = f\bigl(k, x(k)\bigr), \qquad u(k+1) = g\bigl(k, u(k)\bigr),$$

respectively, and $x(0) \leq u(0)$ $(x(0) \geq u(0))$, *then*

$$x(k) \leq u(k) \qquad \bigl(x(k) \geq u(k)\bigr) \quad \text{for all } k \geq 0.$$

Proof of Theorem 1.1 Let $(x_1(k), x_2(k))$ be an arbitrary solution of system (1.2) with $x_1(0) > 0$ and $x_2(0) > 0$. Denote

$$U_1 = \limsup_{k\to+\infty} x_1(k), \qquad V_1 = \liminf_{k\to+\infty} x_1(k),$$
$$U_2 = \limsup_{k\to+\infty} x_2(k), \qquad V_2 = \liminf_{k\to+\infty} x_2(k).$$

We claim that $U_1 = V_1 = x^*$ and $U_2 = V_2 = y^*$.

From the first equation of system (1.1), we obtain

$$\begin{aligned} x(k+1) &= x(k)\exp\left\{r_1 - Eq - b_1 x(k) - \frac{a_1 x(k)}{y(k) + k_1}\right\} \\ &\leq x(k)\exp\bigl\{r_1 - Eq - b_1 x(k)\bigr\}, \quad k = 0, 1, 2, \ldots. \end{aligned} \tag{2.1}$$

Considering the auxiliary equation as follows:

$$u(k+1) = u(k)\exp\{r_1 - Eq - b_1u(k)\}, \quad k = 0,1,2,\ldots. \tag{2.2}$$

Because of $0 < r_1 - Eq \le 1$, according to (ii) of Lemma 2.2, we can obtain $u(k) \le \frac{1}{b_1}$ for all $k \ge 2$, where $u(k)$ is arbitrary positive solution of (2.2) with initial value $u(0) > 0$. From Lemma 2.1, $f(u) = u\exp(r_1 - Eq - b_1u)$ is nondecreasing for $u \in (0, \frac{1}{b_1}]$. According to Lemma 2.3 we can obtain $x(k) \le u(k)$ for all $k \ge 2$, where $u(k)$ is the solution of (2.2) with the initial value $u(2) = x(2)$. According to (i) of Lemma 2.2, we can obtain

$$U_1 = \limsup_{k\to+\infty} x(k) \le \lim_{k\to+\infty} u(k) = \frac{r_1 - Eq}{b_1}. \tag{2.3}$$

From the second equation of system (1.2), we obtain

$$y(k+1) \le y(k)\exp\{r_2 - b_2y(k)\}, \quad k = 0,1,2,\ldots.$$

Similar to the analysis of (2.1)-(2.3), we have

$$U_2 = \limsup_{k\to+\infty} y(k) \le \frac{r_2}{b_2}. \tag{2.4}$$

Then, for sufficiently small constant $\varepsilon > 0$, without loss of generality, we may assume that $\varepsilon < \frac{1}{2}\min\{\frac{r_1-Eq}{b_1+\frac{a_1}{k_1}}, \frac{r_2}{b_2+\frac{a_2}{k_2}}\}$, it follows from (2.3) and (2.4) that there is an integer $k_1 > 2$ such that

$$x(k) < \frac{r_1 - Eq}{b_1} + \varepsilon \stackrel{\text{def}}{=} M_1^x, \qquad y(k) < \frac{r_2}{b_2} + \varepsilon \stackrel{\text{def}}{=} M_1^y \quad \text{for all } k > k_1. \tag{2.5}$$

Equation (2.5) combined with the first equation of system (1.2) leads to

$$\begin{aligned} x(k+1) &= x(k)\exp\left\{r_1 - Eq - b_1x(k) - \frac{a_1x(k)}{y(k)+k_1}\right\} \\ &\le x(k)\exp\left\{r_1 - Eq - b_1x(k) - \frac{a_1x(k)}{M_1^y+k_1}\right\}. \end{aligned} \tag{2.6}$$

Considering the auxiliary equation as follows:

$$u(k+1) = u(k)\exp\left\{r_1 - Eq - b_1u(k) - \frac{a_1u(k)}{M_1^y+k_1}\right\}, \quad k = 0,1,2,\ldots. \tag{2.7}$$

Because of $0 < r_1 - Eq \le 1$, according to (ii) of Lemma 2.2, we can obtain

$$u(k) \le \frac{1}{b_1 + \frac{a_1}{M_1^y+k_1}}$$

for all $k \ge k_1$, where $u(k)$ is arbitrary positive solution of (2.7) with initial value $u(k_1) > 0$. From Lemma 2.1,

$$f(u) = u\exp\left(r_1 - Eq - b_1u - \frac{a_1u(k)}{M_1^y+k_1}\right)$$

is nondecreasing for

$$u \in \left(0, \frac{1}{b_1 + \frac{a_1}{M_1^y + k_1}}\right].$$

According to Lemma 2.3 we can obtain $x(k) \le u(k)$ for all $k \ge k_1 + 1$, where $u(k)$ is the solution of (2.7) with the initial value $u(k_1 + 1) = x(k_1 + 1)$. According to (i) of Lemma 2.2, we can obtain

$$U_1 = \limsup_{k \to +\infty} x(k) \le \lim_{k \to +\infty} u(k) = \frac{r_1 - Eq}{b_1 + \frac{a_1}{M_1^y + k_1}}. \tag{2.8}$$

Equation (2.5) combined with the second equation of system (1.2) leads to

$$\begin{aligned} y(k+1) &= y(k) \exp\left\{ r_2 - b_2 y(k) - \frac{a_2 y(k)}{x(k) + k_2} \right\} \\ &\le y(k) \exp\left\{ r_2 - b_2 y(k) - \frac{a_2 y(k)}{M_1^x + k_2} \right\}, \quad k > k_1. \end{aligned} \tag{2.9}$$

Similar to the analysis of (2.6)-(2.8), we can obtain

$$U_2 = \limsup_{k \to +\infty} y(k) \le \frac{r_2}{b_2 + \frac{a_2}{M_1^x + k_2}}. \tag{2.10}$$

Then, for sufficiently small constant $\varepsilon > 0$, it follows from (2.8) and (2.10) that there is an integer $k_2 > k_1$ such that, for all $k > k_2$,

$$\begin{aligned} x(k) &< \frac{r_1 - Eq}{b_1 + \frac{a_1}{M_1^y + k_1}} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_2^x, \\ y(k) &< \frac{r_2}{b_2 + \frac{a_2}{M_1^x + k_2}} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_2^y. \end{aligned} \tag{2.11}$$

Obviously,

$$M_2^x < M_1^x, \qquad M_2^x < M_1^x. \tag{2.12}$$

According to the first equation of system (1.2) and the positivity of $y(k)$, we can obtain

$$\begin{aligned} x(k+1) &= x(k) \exp\left\{ r_1 - Eq - b_1 x(k) - \frac{a_1 x(k)}{y(k) + k_1} \right\} \\ &\ge x(k) \exp\left\{ r_1 - Eq - b_1 x(k) - \frac{a_1 x(k)}{k_1} \right\}. \end{aligned} \tag{2.13}$$

Considering the auxiliary equation as follows:

$$u(k+1) = u(k) \exp\left\{ r_1 - Eq - b_1 u(k) - \frac{a_1 u(k)}{k_1} \right\}. \tag{2.14}$$

Since $0 < r_1 - Eq \leq 1$, according to (ii) of Lemma 2.2, we can obtain $u(k) \leq \frac{1}{b_1+\frac{a_1}{k_1}}$ for all $k \geq k_2$, where $u(k)$ is arbitrary positive solution of (2.14) with initial value $u(k_2) > 0$. From Lemma 2.1, $f(u) = u\exp\{r_1 - Eq - b_1 u - \frac{a_1 u(k)}{k_1}\}$ is nondecreasing for $u \in (0, \frac{1}{b_1+\frac{a_1}{k_1}}]$. According to Lemma 2.3 we can obtain $x(k) \geq u(k)$ for all $k \geq k_2$, where $u(k)$ is the solution of (2.14) with the initial value $u(k_2) = x(k_2)$. According to (i) of Lemma 2.2, we have

$$V_1 = \liminf_{k\to+\infty} x(k) \geq \lim_{k\to+\infty} u(k) = \frac{r_1 - Eq}{b_1 + \frac{a_1}{k_1}}. \tag{2.15}$$

From the second equation of system (1.2) and the positivity of $x(k)$, we can obtain

$$y(k+1) \geq y(k)\exp\left\{r_2 - b_2 y(k) - \frac{a_2 y(k)}{k_2}\right\}. \tag{2.16}$$

Similar to the analysis of (2.13)-(2.15), we have

$$V_2 = \liminf_{k\to+\infty} y(k) \geq \frac{r_2}{b_2 + \frac{a_2}{k_2}}.$$

Then, for the above $\varepsilon > 0$, there is an integer $k_3 > k_2$ such that, for all $k > k_3$,

$$\begin{aligned} x(k) &> \frac{r_1 - Eq}{b_1 + \frac{a_1}{k_1}} - \varepsilon \stackrel{\text{def}}{=} m_1^x, \\ y(k) &> \frac{r_2}{b_2 + \frac{a_2}{k_2}} - \varepsilon \stackrel{\text{def}}{=} m_1^y. \end{aligned} \tag{2.17}$$

Equation (2.17) combined with the first equation of system (1.2) leads to

$$x(k+1) \geq x(k)x(k)\exp\left\{r_1 - Eq - b_1 x(k) - \frac{a_1 x(k)}{m_1^y + k_1}\right\}, \quad k > k_3. \tag{2.18}$$

Similar to the analysis of (2.13)-(2.15), we have

$$V_1 = \liminf_{k\to+\infty} x(k) \geq \frac{r_1 - Eq}{b_1 + \frac{a_1}{m_1^y + k_1}}. \tag{2.19}$$

Equation (2.17) combined with the second equation of system (1.2) leads to

$$y(k+1) \geq y(k)\exp\left\{r_2 - b_2 y(k) - \frac{a_2 y(k)}{m_1^x + k_2}\right\}, \quad k > k_3. \tag{2.20}$$

Similar to the analysis of (2.13)-(2.15), we can obtain

$$V_2 = \liminf_{k\to+\infty} y(k) \geq \frac{r_2}{b_2 + \frac{a_2}{m_1^x + k_2}}. \tag{2.21}$$

Then, for the above $\varepsilon > 0$, it follows from (2.19) and (2.21) that there is an integer $k_4 > k_3$ such that, for all $k > k_4$,

$$\begin{aligned} x(k) &> \frac{r_1 - Eq}{b_1 + \frac{a_1}{m_1^y + k_1}} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_2^x, \\ y(k) &> \frac{r_2}{b_2 + \frac{a_2}{m_1^x + k_2}} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_2^y. \end{aligned} \tag{2.22}$$

Obviously

$$m_1^x < m_2^x, \qquad m_1^y < m_2^y. \tag{2.23}$$

Continuing the above steps, we can get four sequences $\{M_k^x\}$, $\{M_k^y\}$, $\{m_k^x\}$, and $\{m_k^y\}$ such that

$$\begin{aligned} M_k^x &= \frac{r_1 - Eq}{b_1 + \frac{a_1}{M_{k-1}^y + k_1}} + \frac{\varepsilon}{k}, \\ M_k^y &= \frac{r_2}{b_2 + \frac{a_2}{M_{k-1}^x + k_2}} + \frac{\varepsilon}{k}; \end{aligned} \tag{2.24}$$

and

$$\begin{aligned} m_k^x &= \frac{r_1 - Eq}{b_1 + \frac{a_1}{m_{k-1}^y + k_1}} - \frac{\varepsilon}{k}, \\ m_k^y &= \frac{r_2}{b_2 + \frac{a_2}{m_{k-1}^x + k_2}} - \frac{\varepsilon}{k}. \end{aligned} \tag{2.25}$$

Clearly, we have

$$m_k^x \le V_1 \le U_1 \le M_k^x, \qquad m_k^y \le V_2 \le U_2 \le M_k^y, \quad k = 0, 1, 2, \ldots. \tag{2.26}$$

Now, we will prove $\{M_k^x\}$, $\{M_k^y\}$ is monotonically decreasing, $\{m_k^x\}$, $\{m_k^y\}$ is monotonically increasing by means of inductive method.

First of all, from (2.12) and (2.23) it is clear that $M_2^x < M_1^x$, $M_2^y < M_1^y$, $m_2^x > m_1^x$, $m_2^y > m_1^y$. Now we assume that $M_i^x < M_{i-1}^x$, $M_i^y < M_{i-1}^y$ and $m_i^x > m_{i-1}^x$, $m_i^y > m_{i-1}^y$ hold, then

$$\begin{aligned} b_1 + \frac{a_1}{M_{i-1}^y + k_1} &< b_1 + \frac{a_1}{M_i^y + k_1}, \\ b_2 + \frac{a_2}{M_{i-1}^x + k_2} &< b_2 + \frac{a_2}{M_i^x + k_2}. \end{aligned} \tag{2.27}$$

From (2.27) and the expression of M_i^x, M_i^y, it immediately follows that

$$M_{i+1}^x = \frac{r_1 - Eq}{b_1 + \frac{a_1}{M_i^y + k_1}} + \frac{\varepsilon}{i+1} < \frac{r_1 - Eq}{b_1 + \frac{a_1}{M_{i-1}^y + k_1}} + \frac{\varepsilon}{i} = M_i^x, \tag{2.28}$$

$$M_{i+1}^y = \frac{r_2}{b_2 + \frac{a_2}{M_i^x + k_2}} + \frac{\varepsilon}{i+1} < \frac{r_2}{b_2 + \frac{a_2}{M_{i-1}^x + k_2}} + \frac{\varepsilon}{i} = M_i^y. \tag{2.29}$$

We also have

$$\begin{aligned} b_1 + \frac{a_1}{m_{i-1}^y + k_1} &> b_1 + \frac{a_1}{m_i^y + k_1}, \\ b_2 + \frac{a_2}{m_{i-1}^x + k_2} &> b_2 + \frac{a_2}{m_i^x + k_2}. \end{aligned} \tag{2.30}$$

From (2.30) and the expression of m_i^x, m_i^y, it immediately follows that

$$m_{i+1}^x = \frac{r_1 - Eq}{b_1 + \frac{a_1}{m_i^y + k_1}} - \frac{\varepsilon}{i+1} > \frac{r_1 - Eq}{b_1 + \frac{a_1}{m_{i-1}^y + k_1}} - \frac{\varepsilon}{i} = M_i^x, \tag{2.31}$$

$$m_{i+1}^y = \frac{r_2}{b_2 + \frac{a_2}{m_i^x + k_2}} - \frac{\varepsilon}{i+1} > \frac{r_2}{b_2 + \frac{a_2}{m_{i-1}^x + k_2}} - \frac{\varepsilon}{i} = m_i^y. \tag{2.32}$$

Equations (2.27)-(2.32) show that $\{M_k^x\}$ and $\{M_k^y\}$ are monotonically decreasing, $\{m_k^x\}$ and $\{m_k^y\}$ are monotonically increasing. Consequently, $\lim_{k\to+\infty}\{M_k^x\}$, $\lim_{k\to+\infty}\{M_k^y\}$, and $\lim_{k\to+\infty}\{m_k^x\}$, $\lim_{k\to+\infty}\{m_k^y\}$ both exist. Let

$$\lim_{k\to+\infty} M_k^x = \overline{X}, \qquad \lim_{k\to+\infty} m_k^x = \underline{X}, \tag{2.33}$$

$$\lim_{k\to+\infty} M_k^y = \overline{Y}, \qquad \lim_{k\to+\infty} m_k^x = \underline{Y}. \tag{2.34}$$

From (2.24) and (2.25), we have

$$\overline{X} = \frac{r_1 - Eq}{b_1 + \frac{a_1}{\overline{Y} + k_1}}, \qquad \overline{Y} = \frac{r_2}{b_2 + \frac{a_2}{\overline{X} + k_2}}, \tag{2.35}$$

$$\underline{X} = \frac{r_1 - Eq}{b_1 + \frac{a_1}{\underline{Y} + k_1}}, \qquad \underline{Y} = \frac{r_2}{b_2 + \frac{a_2}{\underline{X} + k_2}}. \tag{2.36}$$

Equations (2.35) and (2.36) are equivalent to

$$b_1\overline{X} + \frac{a_1\overline{X}}{\overline{Y} + k_1} = r_1 - Eq, \qquad b_2\overline{Y} + \frac{a_2\overline{Y}}{\overline{X} + k_2} = r_2, \tag{2.37}$$

$$b_1\underline{X} + \frac{a_1\underline{X}}{\underline{Y} + k_1} = r_1 - Eq, \qquad b_2\underline{Y} + \frac{a_2\underline{Y}}{\underline{X} + k_2} = r_2. \tag{2.38}$$

Equations (2.37) and (2.38) show that $(\overline{X}, \overline{Y})$ and $(\underline{X}, \underline{Y})$ are all solutions of system (1.3). however, under the assumption of Theorem 1.1, system (1.3) has unique positive solution (x^*, y^*). Therefore

$$U_1 = V_1 = \lim_{k\to+\infty} x(k) = x^*, \qquad U_2 = V_2 = \lim_{k\to+\infty} y(k) = y^*, \tag{2.39}$$

that is, $E_+(x^*, y^*)$ is globally attractive. This ends the proof of Theorem 1.1. □

3 Examples

In this section, we shall give two examples to illustrate the feasibility of the main result.

Example 3.1 Consider the following cooperative system:

$$\begin{aligned} x(k+1) &= x(k)\exp\left\{0.5 - 0.1\times 1 - 0.3x(k) - \frac{0.1x(k)}{y(k)+1}\right\}, \\ y(k+1) &= y(k)\exp\left\{0.5 - 0.2y(k) - \frac{0.1y(k)}{x(k)+0.2}\right\}. \end{aligned} \tag{3.1}$$

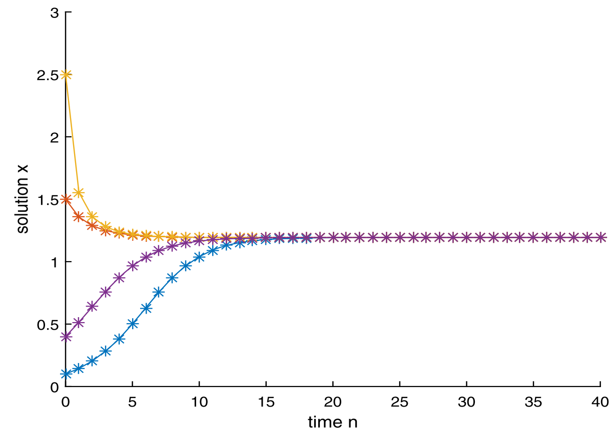

Figure 1 Dynamic behaviors of the first component of the solution $(x(k), y(k))$ of system (3.1) with the initial conditions $(x(0), y(0)) = (0.1, 3), (1.5, 2), (2.5, 1)$, and $(0.4, 0.5)$, respectively.

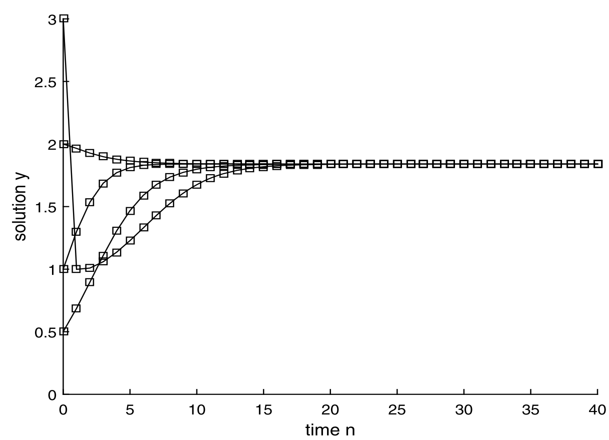

Figure 2 Dynamic behaviors of the second component of the solution $(x(k), y(k))$ of system (3.1) with the initial conditions $(x(0), y(0)) = (0.1, 3), (1.5, 2), (2.5, 1)$, and $(0.4, 0.5)$, respectively.

Corresponding to system (1.2), we have $r_1 = 0.5$; $r_2 = 0.5$; $b_1 = 0.3$; $b_2 = 0.2$; $a_1 = 0.1$; $a_2 = 0.1$; $E = 1$; $q = 0.1$; $k_1 = 1$; $k_2 = 0.2$; by calculating, we see that the positive equilibrium $E_+(x_1^*, x_2^*) \approx (1.193266964, 1.839765589)$, $0 < r_1 - qE = 0.5 - 0.1 = 0.4 < 1$, $r_2 = 0.5 < 1$, thus the coefficients of system (3.1) satisfy (H_1) and (H_2) in Theorem 1.1. From Theorem 1.1, the unique positive equilibrium $E_+(x_1^*, x_2^*)$ is globally attractive. Numeric simulations also support our finding (see Figures 1 and 2).

Example 3.2 Consider the following competition system:

$$\begin{aligned} x(k+1) &= x(k)\exp\left\{1.5 - 0.1 \times 1 - 0.3x(k) - \frac{0.1x(k)}{y(k)+1}\right\}, \\ y(k+1) &= y(k)\exp\left\{1.5 - 0.2y(k) - \frac{0.1y(k)}{x(k)+0.2}\right\}. \end{aligned} \tag{3.2}$$

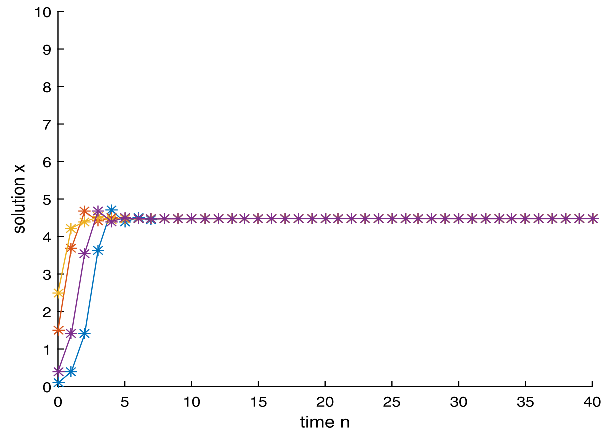

Figure 3 Dynamic behaviors of the first component of the solution $(x(k), y(k))$ of system (3.2) with the initial conditions $(x(0), y(0)) = (0.1, 3), (1.5, 2), (2.5, 1)$, and $(0.4, 0.5)$, respectively.

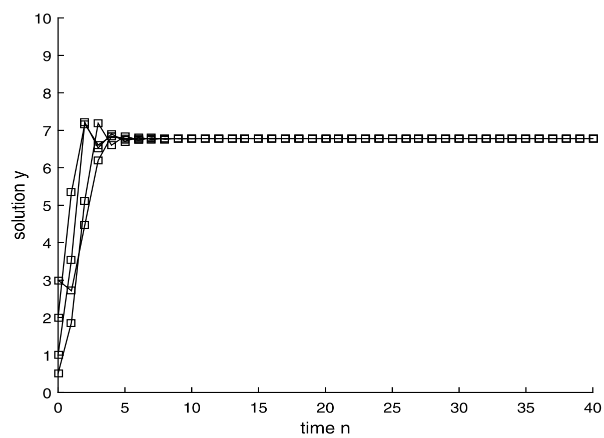

Figure 4 Dynamic behaviors of the second component of the solution $(x(k), y(k))$ of system (3.2) with the initial conditions $(x(0), y(0)) = (0.1, 3), (1.5, 2), (2.5, 1)$, and $(0.4, 0.5)$, respectively.

Here all the other coefficients are as that of Example 3.1, only we change $r_i = 0.5$ to $r_i = 1.5$, $i = 1, 2$. By calculating, we see that the positive equilibrium $E_+(x_1^*, x_2^*) \approx (4.474828147, 6.775338254)$, and $r_1 - qE = 1.5 - 0.1 = 1.4 > 1$, $r_2 = 1.5 > 1$, thus the coefficients of system (3.2) do not satisfy (H_2) in Theorem 1.1, and the stability property of this positive equilibrium could not be judged by Theorem 1.1. However, numeric simulations (see Figures 3 and 4) show that in this case, the positive equilibrium still is globally attractive.

4 Discussion

In [2], Xie *et al.* studied the stability property of the system (1.1), their result shows that once the system (1.1) admits a unique positive equilibrium, it is globally attractive. In this paper, we try to consider the discrete type of system (1.1), we first establish the corresponding system (1.2), then, by developing the analysis technique of [2–4], we also obtain a set of sufficient conditions which ensure the global attractivity of the positive equilibrium. Our result shows that the intrinsic growth rate plays an important role in the stability property of the system.

It brings to our attention that conditions for the continuous system are very simple (one only requires $r_1 > qE$), while conditions for the discrete one is very strong, since one requires $r_1 - qE \leq 1$ and $r_2 \leq 1$. This motivated us to study the case $r_i > 1$, numeric simulation (Example 3.2) shows that in this case, the system still possible admits a unique globally attractive positive equilibrium, and we conjecture that Theorem 1.1 still holds under the condition $r_1 - Eq < 2$, $r_2 < 2$; we leave this for future study.

At the end of the paper, we would like to point out that one of the reviewers of this paper said 'Population models with stochastic noises may also be important and interesting. In fact, many authors have studied stochastic population models with stochastic noises, for example, Beddington and May [31], Liu and Bai [32, 33]. I suggest the authors take stochastic noises into account in their future study.' We do agree with the opinion of the reviewers, and we hope we could do some relevant work in the future.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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