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Research Article

Linear Impulsive Periodic System with Time-Varying Generating Operators on Banach Space

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A class of the linear impulsive periodic system with time-varying generating operators on Banach space is considered. By constructing the impulsive evolution operator, the existence of T_0-periodic PC-mild solution for homogeneous linear impulsive periodic system with time-varying generating operators is reduced to the existence of fixed point for a suitable operator. Further the alternative results on T_0-periodic PC-mild solution for nonhomogeneous linear impulsive periodic system with time-varying generating operators are established and the relationship between the boundness of solution and the existence of T_0-periodic PC-mild solution is shown. The impulsive periodic motion controllers that are robust to parameter drift are designed for a given periodic motion. An example is given for demonstration.

1. Introduction

It is well known that periodic motion is a very important and special phenomenon not only in natural science, but also in social science. The periodic solution theory of dynamic equations has been developed over the last decades. We refer the readers to [1–11] for infinite dimensional cases, to [12–15] for finite dimensional cases. Especially, there are many results of periodic solutions (such as existence, the relationship between bounded solutions and periodic solutions, stability, and robustness) for non-autonomous impulsive periodic system on finite dimensional spaces (see [12, 14, 15]). There are also some relative results of periodic solutions for periodic systems with time-varying generating operators on infinite dimensional spaces (see [3, 8, 11, 16, 17]).

On the other hand, in order to describe dynamics of populations subject to abrupt changes as well as other phenomena such as harvesting, diseases, and so forth, some authors have used impulsive differential systems to describe the model since the last century. For the basic theory on impulsive differential equations on finite dimensional spaces, the reader can refer to Yang's book and Lakshmikantham's book (see [15, 18]). For the basic theory on impulsive differential equations on infinite dimensional spaces, the reader can refer to Ahmed's paper, Liu's paper and Xiang's papers (see [4, 8, 11, 19–22]).

Impulsive periodic differential equations serve as basic periodic models to study the dynamics of processes that are subject to sudden changes in their states. To the best of our knowledge, few papers discuss the impulsive periodic systems with time-varying generating operators on infinite dimensional spaces. In this paper, we pay attention to impulsive periodic systems with time-varying generating operators. We consider the following homogeneous linear impulsive periodic system with time-varying generating operators:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + f(t), \quad t \neq \tau_k, \\ \Delta x(\tau_k) &= B_k x(\tau_k) + c_k, \quad t = \tau_k, \end{aligned} \tag{1.1}$$

in the parabolic case on infinite dimensional Banach space X, where $\{A(t),\ t \in [0, T_0]\}$ is a family of closed densely defined linear unbounded operators on X and the resolvent of the unbounded operator $A(t)$ is compact. $0 = \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_k \ldots$, $\lim_{k\to\infty}\tau_k = \infty$, $\tau_{k+\delta} = \tau_k + T_0$, $\widetilde{D} = \{\tau_1, \tau_2, \ldots, \tau_\delta\} \subset (0, T_0)$, $\triangle x(\tau_k) = x(\tau_k^+) - x(\tau_k^-)$, where $k \in \mathbb{Z}_0^+$, T_0 is a fixed positive number. $f(t + T_0) = f(t)$, $B_{k+\delta} = B_k$ and $c_{k+\delta} = c_k$.

First, we construct a new impulsive evolution operator corresponding to the homogeneous linear impulsive periodic system with time-varying generating operators and introduce the suitable definition of T_0-periodic PC-mild solution for homogeneous linear impulsive periodic system with time-varying generating operators. The impulsive evolution operator can be used to reduce the existence of T_0-periodic PC-mild solution for nonhomogeneous linear impulsive periodic system with time-varying generating operators to the existence of fixed points for an operator equation. Using the Fredholm alternative theorem, we exhibit the alternative results on T_0-periodic PC-mild solution for homogeneous linear impulsive periodic system with time-varying generating operators and nonhomogeneous linear impulsive periodic system with time-varying generating operators. At the same time, we show several Massera-type criterias for nonhomogeneous linear impulsive periodic system with time-varying generating operators which conclude the relationship between the boundness of solution and the existence of T_0-periodic PC-mild solution. At last, impulsive periodic motion controllers that are robust to parameter drift are designed for given a periodic motion. This work is fundamental for further discussion about nonlinear impulsive periodic system with time-varying generating operators on infinite dimensional spaces.

This paper is organized as follows. In Section 2, the impulsive evolution operator is constructed and alternative results on T_0-periodic PC-mild solution for homogeneous linear impulsive periodic system with time-varying generating operators are proved. In Section 3, alternative results on T_0-periodic PC-mild solution for nonhomogeneous linear impulsive periodic system with time-varying generating operators are obtained. Massera-type criteria are given to show the relationship between bounded solution and

T_0-periodic PC-mild solution for nonhomogeneous linear impulsive periodic system with time-varying generating operators. In Section 4, impulsive periodic motion controllers that are robust to parameter drift are designed, given T_0-periodic PC-mild solution for nonhomogeneous linear impulsive periodic system with time-varying generating operators. At last, an example is given to demonstrate the applicability of our result.

2. Homogeneous linear impulsive periodic system with time-varying generating operators

Let $L_b(X)$ be the space of bounded linear operators in the Banach space X. Define $PC([0,T_0];X) \equiv \{x : [0,T_0] \to X \mid x$ is continuous at $t \in [0,T_0]\backslash\widetilde{D}$, x is continuous from left and has right hand limits at $t \in \widetilde{D}\}$ and $PC^1([0,T_0];X) \equiv \{x \in PC([0,T_0];X) \mid \dot{x} \in PC([0,T_0];X)\}$. Set

$$\|x\|_{PC} = \max\left\{\sup_{t\in[0,T_0]} \|x(t+0)\|, \sup_{t\in[0,T_0]} \|x(t-0)\|\right\}, \qquad \|x\|_{PC^1} = \|x\|_{PC} + \|\dot{x}\|_{PC}. \tag{2.1}$$

It can be seen that endowed with the norm $\|\cdot\|_{PC}$ ($\|\cdot\|_{PC^1}$), $PC([0,T_0];X)$ ($PC^1([0,T_0];X)$) is a Banach space.

Consider the following homogeneous linear impulsive periodic system with time-varying generating operators (THLIPS):

$$\begin{aligned} \dot{x}(t) &= A(t)x(t), \quad t \neq \tau_k, \\ \Delta x(\tau_k) &= B_k x(\tau_k), \quad t = \tau_k, \end{aligned} \tag{2.2}$$

in the Banach space X, $\{A(t),\ t \in [0,T_0]\}$ is a family of closed densely defined linear unbounded operators on X satisfying the following assumption.

Assumption 2.1 (see [23], page 158). For $t \in [0,T_0]$ one has the following.

(P_1) The domain $D(A(t)) = D$ is independent of t and is dense in X.

(P_2) For $t \geq 0$, the resolvent $R(\lambda, A(t)) = (\lambda I - A(t))^{-1}$ exists for all λ with $\operatorname{Re}\lambda \leq 0$, and there is a constant M independent of λ and t such that

$$\|R(\lambda, A(t))\| \leq M(1+|\lambda|)^{-1} \quad \text{for } \operatorname{Re}\lambda \leq 0. \tag{2.3}$$

(P_3) There exist constants $L > 0$ and $0 < \alpha \leq 1$ such that

$$\|(A(t) - A(\theta))A^{-1}(\tau)\| \leq L|t-\theta|^{\alpha} \quad \text{for } t, \theta, \tau \in [0,T_0]. \tag{2.4}$$

Lemma 2.2 (see [23], page 159). *Under Assumption 2.1, the Cauchy problem*

$$\dot{x}(t) + A(t)x(t) = 0, \quad t \in (0,T_0] \text{ with } x(0) = x_0 \tag{2.5}$$

has a unique evolution system $\{U(t,\theta) \mid 0 \leq \theta \leq t \leq T_0\}$ in X satisfying the following properties:

(1) $U(t,\theta) \in L_b(X)$, *for* $0 \leq \theta \leq t \leq T_0$;

(2) $U(t,r)U(r,\theta) = U(t,\theta)$, *for* $0 \le \theta \le r \le t \le T_0$;
(3) $U(\cdot,\cdot)x \in C(\Delta,X)$, *for* $x \in X$, $\Delta = \{(t,\theta) \in [0,T_0] \times [0,T_0] \mid 0 \le \theta \le t \le T_0\}$;
(4) *for* $0 \le \theta < t \le T_0$, $U(t,\theta)\colon X \to D$ *and* $t \to U(t,\theta)$ *is strongly differentiable in* X. *The derivative* $(\partial/\partial t)U(t,\theta) \in L_b(X)$ *and it is strongly continuous on* $0 \le \theta < t \le T_0$; *moreover,*

$$\frac{\partial}{\partial t}U(t,\theta) = -A(t)U(t,\theta) \quad \text{for } 0 \le \theta < t \le T_0,$$
$$\left\|\frac{\partial}{\partial t}U(t,\theta)\right\|_{L_b(X)} = \|A(t)U(t,\theta)\|_{L_b(X)} \le \frac{C}{t-\theta}, \tag{2.6}$$
$$\|A(t)U(t,\theta)A(\theta)^{-1}\|_{L_b(X)} \le C \quad \text{for } 0 \le \theta \le t \le T_0;$$

(5) *for every* $v \in D$ *and* $t \in (0,T_0]$, $U(t,\theta)v$ *is differentiable with respect to* θ *on* $0 \le \theta \le t \le T_0$

$$\frac{\partial}{\partial \theta}U(t,\theta)v = U(t,\theta)A(\theta)v. \tag{2.7}$$

And, for each $x_0 \in X$, *the Cauchy problem (2.5) has a unique classical solution* $x \in C^1([0,T_0];X)$ *given by*

$$x(t) = U(t,0)x_0, \quad t \in [0,T_0]. \tag{2.8}$$

In addition to Assumption 2.1, we introduce the following assumptions.

Assumption 2.3. There exists $T_0 > 0$ such that $A(t+T_0) = A(t)$ for $t \in [0,T_0]$.

Assumption 2.4. For $t \ge 0$, the resolvent $R(\lambda,A(t))$ is compact.

Then we have

Lemma 2.5 (see [5], page 105). *Assumptions 2.1, 2.3, and 2.4 hold. Then evolution system* $\{U(t,\theta) \mid 0 \le \theta \le t \le T_0\}$ *in* X *also satisfying the following two properties:*
(6) $U(t+T_0,\theta+T_0) = U(t,\theta)$ *for* $0 \le \theta \le t \le T_0$;
(7) $U(t,\theta)$ *is compact operator for* $0 \le \theta < t \le T_0$.

In order to construct an impulsive evolution operator and investigate its properties, we need the following assumption.

Assumption 2.6. For each $k \in \mathbb{Z}_0^+$, $B_k \in L_b(X)$, there exists $\delta \in \mathbb{N}$ such that $\tau_{k+\delta} = \tau_k + T_0$ and $B_{k+\delta} = B_k$.

First consider the following Cauchy problem:

$$\begin{gathered} \dot{x}(t) = A(t)x(t), \quad t \in [0,T_0] \setminus \widetilde{D}, \\ \Delta x(\tau_k) = B_k x(\tau_k), \quad k = 1,2,\ldots,\delta, \\ x(0) = x_0. \end{gathered} \tag{2.9}$$

For every $x_0 \in X$, D is an invariant subspace of B_k, using Lemma 2.2, step by step, one can verify that the Cauchy problem (2.9) has a unique classical solution $x \in PC^1([0,T_0];X)$

represented by $x(t) = \mathcal{S}(t,0)x_0$, where $\mathcal{S}(\cdot,\cdot) : \Delta \to X$ given by

$$\mathcal{S}(t,\theta) = \begin{cases} U(t,\theta), & \tau_{k-1} \le \theta \le t \le \tau_k, \\ U(t,\tau_k^+)(I+B_k)U(\tau_k,\theta), & \tau_{k-1} \le \theta < \tau_k < t \le \tau_{k+1}, \\ U(t,\tau_k^+)\left[\prod_{\theta<\tau_j<t} (I+B_j)U(\tau_j,\tau_{j-1}^+)\right](I+B_i)U(\tau_i,\theta), & \tau_{i-1} \le \theta < \tau_i \le \cdots < \tau_k < t \le \tau_{k+1}. \end{cases} \tag{2.10}$$

The operator $\mathcal{S}(t,\theta)$ $((t,\theta) \in \Delta)$ is called impulsive evolution operator associated with $\{B_k; \tau_k\}_{k=1}^{\infty}$.

The following lemma on the properties of the impulsive evolution operator $\mathcal{S}(t,\theta)$ $((t,\theta) \in \Delta)$ associated with $\{B_k;\tau_k\}_{k=1}^{\infty}$ is widely used in this paper.

Lemma 2.7. *Assumptions 2.1, 2.3, 2.4, and 2.6 hold. The impulsive evolution operator $\mathcal{S}(t,\theta)$ $((t,\theta) \in \Delta)$ has the following properties:*

(1) *$\mathcal{S}(t,\theta) \in L_b(X)$, for $0 \le \theta \le t \le T_0$;*
(2) *for $0 \le \theta \le t \le T_0$, $\mathcal{S}(t+T_0, \theta+T_0) = \mathcal{S}(t,\theta)$;*
(3) *for $0 \le t \le T_0$, $\mathcal{S}(t+T_0,0) = \mathcal{S}(t,0)\mathcal{S}(T_0,0)$;*
(4) *$\mathcal{S}(t,\theta)$ is compact operator, for $0 \le \theta < t \le T_0$.*

Proof. By (1) of Lemma 2.2 and Assumption 2.6, $\mathcal{S}(t,\theta) \in L_b(X)$, for $0 \le \theta \le t \le T_0$. By (6) of Lemma 2.5 and Assumption 2.6, $\mathcal{S}(t+T_0,\theta+T_0) = \mathcal{S}(t,\theta)$, for $0 \le \theta \le t \le T_0$. By (2) of Lemma 2.2, (6) of Lemma 2.5 and Assumption 2.6, $\mathcal{S}(t+T_0,0) = \mathcal{S}(t+T_0,T_0)\mathcal{S}(T_0,0) = \mathcal{S}(t,0)S(T_0,0)$, for $0 \le \theta \le t \le T_0$. By (7) of Lemma 2.5 and Assumption 2.6, one can obtain that $\mathcal{S}(t,\theta)$ is compact operator, for $0 \le \theta < t \le T_0$. □

Now we can introduce the PC-mild solution of Cauchy problem (2.9) and T_0-periodic PC-mild solution of the THLIPS (2.2).

Definition 2.8. For every $x_0 \in X$, the function $x \in PC([0,T_0];X)$ given by $x(t) = \mathcal{S}(t,0)x_0$ is said to be the PC-mild solution of the Cauchy problem (2.9).

Definition 2.9. A function $x \in PC([0,+\infty);X)$ is said to be a T_0-periodic PC-mild solution of THLIPS (2.2) if it is a PC-mild solution of Cauchy problem (2.9) corresponding to some x_0 and $x(t+T_0) = x(t)$, for $t \ge 0$.

The following theorem implies that the existence of periodic solution is equivalent to a fixed point of operator.

Theorem 2.10. *Assumptions 2.1, 2.3, and 2.6 hold. THLIPS (2.2) has a T_0-periodic PC-mild solution x if and only if $\mathcal{S}(T_0,0)$ has a fixed point.*

Proof. If THLIPS (2.2) has a T_0-periodic PC-mild solution x, then we have $x(T_0) = \mathcal{S}(T_0,0)x(0) = x(0)$ where $x(0) = x_0$ is a fixed point of $\mathcal{S}(T_0,0)$. On the other hand, if

$\overline{x}$ is a fixed point of $\mathcal{S}(T_0,0)$, consider the following Cauchy problem:

$$\begin{gathered}\dot{x}(t)=A(t)x(t),\quad t\in[0,T_0]\backslash\widetilde{D},\\ \Delta x(\tau_k)=B_kx(\tau_k),\quad t=\tau_k,\\ x(0)=\overline{x}.\end{gathered}\tag{2.11}$$

Using Lemma 2.2, step by step, one can verify that the above impulsive Cauchy problem has a PC-mild solution given by $x(t)=\mathcal{S}(t,0)\overline{x}$. By (3) of Lemma 2.7, we have

$$x(t+T_0)=\mathcal{S}(t,0)\mathcal{S}(T_0,0)\overline{x}=\mathcal{S}(t,0)\overline{x}=x(t).\tag{2.12}$$

This implies that x is a T_0-periodic PC-mild solution of THLIPS (2.2). □

Further, we can give the following theorem of the alternative result on periodic solution.

Theorem 2.11. *Assumptions 2.1, 2.3, 2.4, and 2.6 hold. Then either the THLIPS (2.2) has a unique trivial T_0-periodic PC-mild solution or it has finitely many linearly independent nontrivial T_0-periodic PC-mild solutions in $PC([0,+\infty);X)$.*

Proof. By Assumptions 2.1 and 2.4 and Lemma 2.7(4), $\mathcal{S}(T_0,0)$ is a compact operator. By the Fredholm alternative theorem, either (i) $\mathcal{S}(T_0,0)x_0=x_0$ only has trivial T_0-periodic PC-mild solution and $[I-\mathcal{S}(T_0,0)]^{-1}$ exists or (ii) $\mathcal{S}(T_0,0)x_0=x_0$ has nontrivial T_0-periodic PC-mild solutions which form a finite dimensional subspace of X. In fact, operator equation $[I-\mathcal{S}(T_0,0)]x_0=0$ has m linearly independent nontrivial solutions $x_0^1,x_0^2,\dots,x_0^m$. Thus, $\mathcal{S}(T_0,0)$ has fixed points $x_0^1,x_0^2,\dots,x_0^m$. By Theorem 2.10, we know that the PC-mild solution of Cauchy problem (2.9) corresponding to initial value x_0^i given by $x^i(t)=\mathcal{S}(t,0)x_0^i$, $i=1,2,\dots,m$ is T_0-periodic. Thus THLIPS (2.2) has m linearly independent T_0-periodic PC-mild solutions $x^1,x^2,\dots,x^m$. By linearity of THLIPS (2.2), one can easily verify every T_0-periodic PC-mild solution of THLIPS (2.2) can be written as

$$x(t)=\sum_{i=1}^{m}\alpha_i\mathcal{S}(t,0)x_0^i,\tag{2.13}$$

where m is finite and $\alpha_1,\alpha_2,\dots,\alpha_m$ are constants. □

3. Nonhomogeneous linear impulsive periodic system with time-varying generating operators

Consider the following nonhomogeneous linear impulsive periodic system with time-varying generating operators (TNLIPS)

$$\begin{gathered}\dot{x}(t)=A(t)x(t)+f(t),\quad t\neq\tau_k,\\ \Delta x(\tau_k)=B_kx(\tau_k)+c_k,\quad t=\tau_k,\end{gathered}\tag{3.1}$$

and the Cauchy problem:

$$\begin{gathered}\dot{x}(t)=A(t)x(t)+f(t), \quad t\in[0,T_0]\backslash\widetilde{D},\\ \Delta x(\tau_k)=B_kx(\tau_k)+c_k, \quad k=1,2,\dots,\delta,\\ x(0)=x_0.\end{gathered} \tag{3.2}$$

In addition to Assumptions 2.1, 2.3, 2.4, and 2.6, we make following assumption.

Assumption 3.1. (1) Input $f\in L^1([0,T_0];X)$ and there exists $T_0>0$ such that $f(t+T_0)=f(t)$. (2) For each $k\in\mathbb{Z}_0^+$ and $c_k\in X$, there exists $\delta\in\mathbb{N}$ such that $c_{k+\delta}=c_k$.

Now we can introduce the PC-mild solution of Cauchy problem (3.2) and T_0-periodic PC-mild solution of the TNLIPS (3.1).

Definition 3.2. For every $x_0\in X$, $f\in L^1([0,T_0];X)$, the function $x\in PC([0,T_0];X)$ given by

$$x(t)=\mathcal{S}(t,0)x_0+\int_0^t\mathcal{S}(t,\theta)f(\theta)d\theta+\sum_{0\le\tau_k<t}\mathcal{S}(t,\tau_k^+)c_k, \quad \text{for } t\in[0,T_0], \tag{3.3}$$

is said to be a PC-mild solution of the Cauchy problem (3.2).

Definition 3.3. A function $x\in PC([0,+\infty);X)$ is said to be a T_0-periodic PC-mild solution of TNLIPS (3.1) if it is a PC-mild solution of Cauchy problem (3.2) corresponding to some x_0 and $x(t+T_0)=x(t)$, for $t\ge 0$.

Theorem 3.4. *Assumptions 2.1, 2.3, 2.4, 2.6, and 3.1 hold. If THLIPS (2.2) has no nontrivial T_0-periodic PC-mild solution, then TNLIPS (3.1) has a unique T_0-periodic PC-mild solution given by*

$$x_{T_0}(t)=\mathcal{S}(t,0)[I-\mathcal{S}(T_0,0)]^{-1}z+\int_0^t\mathcal{S}(t,\theta)f(\theta)d\theta+\sum_{0\le\tau_k<t}\mathcal{S}(t,\tau_k^+)c_k, \tag{3.4}$$

where

$$z=\int_0^{T_0}\mathcal{S}(T_0,\theta)f(\theta)d\theta+\sum_{0\le\tau_k<T_0}\mathcal{S}(T_0,\tau_k^+)c_k. \tag{3.5}$$

Further, one has the following estimate:

$$\|x_{T_0}(t)\|_X\le L_1(L_1L_2+1)\Big[\|f\|_{L^1([0,T_0];X)}+\delta\max_{1\le k\le\delta}\|c_k\|_X\Big], \tag{3.6}$$

where $L_1=\sup_{0\le\theta\le t\le T_0}\|\mathcal{S}(t,\theta)\|$ and $L_2=\|[I-\mathcal{S}(T_0,0)]^{-1}\|$.

Proof. By Lemma 2.7, $\mathcal{S}(t,\theta)((t,\theta)\in\Delta)$ is a compact operator. In addition, THLPS (2.2) has no nontrivial T_0-periodic PC-mild solution, by the Fredholm alternative theorem, $[I-\mathcal{S}(T_0,0)]^{-1}$ exists and is bounded. By the operator equation $[I-\mathcal{S}(T_0,0)]\overline{x}=z$ is solvable and has a unique solution $\overline{x} = [I - \mathcal{S}(T_0,0)]^{-1}z$. Consider the following Cauchy problem:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(t), \quad t \in [0,T_0]\backslash\widetilde{D}, \\ \Delta x(\tau_k) &= B_k x(\tau_k) + c_k, \quad t = \tau_k, \\ x(0) &= \overline{x}. \end{aligned} \tag{3.7}$$

It has a PC-mild solution $x_{T_0}(\cdot)$ given by

$$x_{T_0}(t) = \mathcal{S}(t,0)\overline{x} + \int_0^t \mathcal{S}(t,\theta)f(\theta)d\theta + \sum_{0\le\tau_k<t} \mathcal{S}(t,\tau_k^+)c_k. \tag{3.8}$$

It follows from Lemma 2.7 that

$$x_{T_0}(t+T_0) = \mathcal{S}(t,0)(\mathcal{S}(T_0,0)\overline{x}+z) + \int_0^t \mathcal{S}(t,\theta)f(\theta)d\theta + \sum_{0\le\tau_k<t} \mathcal{S}(t,\tau_k^+)c_k = x_{T_0}(t). \tag{3.9}$$

This implies that $x_{T_0}(\cdot)$ is just the unique T_0-periodic PC-mild solution of TNLIPS (3.1). Further

$$\begin{aligned} \|x_{T_0}(t)\| &\le (\|\mathcal{S}(t,0)[I-\mathcal{S}(T_0,0)]^{-1}\|+1)\|z\| \\ &\le \|\mathcal{S}(t,\theta)\|(\|\mathcal{S}(t,0)\|\|[I-\mathcal{S}(T_0,0)]^{-1}\|+1)\left[\int_0^{T_0} \|f(\theta)\|_X d\theta + \sum_{0\le\tau_k<T_0} \|c_k\|_X\right]. \end{aligned} \tag{3.10}$$

The estimation (3.6) is immediately obtained. □

Corollary 3.5. *Assumptions 2.1, 2.3, 2.4, 2.6, and 3.1 hold. If $\|\mathcal{S}(T_0,0)\| < 1$ then THLIPS (2.2) has no nontrivial T_0-periodic PC-mild solution and TNLIPS (3.1) has a unique T_0-periodic PC-mild solution. The unique T_0-periodic PC-mild solution of TNLIPS (3.1) is given by the expression (3.4) which satisfies*

$$\|x_{T_0}(t)\|_X \le \frac{L_1}{1-L_1}\left[\|f\|_{L^1([0,T_0];X)} + \delta \max_{1\le k\le\delta}\|c_k\|_X\right]. \tag{3.11}$$

Suppose that X is a Hilbert space. Consider the following Cauchy problem:

$$\begin{aligned} \dot{y}(t) &= -A^*(t)y(t), \quad t \in [0,T_0]\backslash\widetilde{D}, \\ \Delta y(\tau_k) &= -B_k^* y(\tau_k^+), \quad k = 1,2,\dots,\delta, \\ y(T_0) &= y_0 \in X^*, \end{aligned} \tag{3.12}$$

where $A^*(t)$, B_k^* are the adjoint operators of $A(t)$, B_k, respectively. By Assumptions 2.3 and 2.6, $A^*(t) = A^*(t+T_0)$ and for each $k \in \mathbb{Z}_0^+$, $B_k^* \in L_b(X^*)$ and $B_{k+\delta}^* = B_k^*$. Let $U^*(\cdot,\cdot)$ be the adjoint operator of $U(\cdot,\cdot)$. It is well known that $U^*(\cdot,\cdot)$, due to the convexity of X^*, satisfies some properties similar to $U(\cdot,\cdot)$.

Similar to the discussion on Cauchy problem of homogenous linear impulsive system with time-varying generating operators, the PC-mild solution of Cauchy problem (3.12) can be given by

$$y(\theta) = \mathcal{S}^*(T_0,\theta)\,y_0, \quad \theta < T_0, \tag{3.13}$$

where

$$\mathcal{S}^*(T_0,\theta) = \begin{cases} U^*(T_0,\theta), & \tau_{k-1} < \theta \leq T_0, \\ U^*(\tau_{k-1},\theta)(I+B_k^*)U^*(T_0,\tau_{k-1}), & \tau_{k-2} < \theta \leq \tau_{k-1} < T_0, \\ U^*(\tau_i,\theta)(I+B_i^*)\left[\prod\limits_{\theta<\tau_j<T_0}(I+B_j)U(\tau_j,\tau_{j-1})\right]^* U^*(T_0,\tau_{k-1}), & \tau_{i-1} < \theta \leq \tau_i < \cdots < T_0. \end{cases} \tag{3.14}$$

Theorem 3.6. *Assumptions 2.1, 2.3, 2.4, 2.6, and 3.1 hold. Suppose X be a Hilbert space and $[I - \mathcal{S}(T_0,0)]^{-1}$ does not exist. Then one has that*

(1) *the adjoint equation of THLIPS (2.2) (TAHLIPS)*

$$\begin{aligned} \dot{y}(t) &= -A^*(t)y(t), \quad t \neq \tau_k, \\ \Delta y(\tau_k) &= -B_k^* y(\tau_k^+), \quad t = \tau_k, \end{aligned} \tag{3.15}$$

has m linearly independent T_0-periodic PC-mild solutions $y^1, y^2, \ldots, y^m$;

(2) *the TNLIPS (3.1) has a T_0-periodic PC-mild solution if and only if*

$$\langle y_0^i, z\rangle_{X^*,X} = 0, \quad i = 1,2,\ldots,m, \tag{3.16}$$

which is equivalent to

$$\int_0^{T_0} \langle f(\theta), y^i(\theta)\rangle_{X,X^*}\, d\theta + \sum_{0\leq\tau_k<T_0} \langle c_k, y^i(\tau_k)\rangle_{X,X^*} = 0. \tag{3.17}$$

Otherwise, TNLIPS (3.1) has no T_0-periodic PC-mild solution.

Proof. It comes from the compactness of $\mathcal{S}(T_0,0)$ that $\mathcal{S}^*(T_0,0)$ is compact and $\dim\ker[I - \mathcal{S}^*(T_0,0)] = \dim\ker[I - \mathcal{S}(T_0,0)] = m < +\infty$. The operator equation $[I - \mathcal{S}^*(T_0,0)]y_0 = 0$ has m nontrivial linearly independent solutions $\{y_0^i\}_{i=1}^m$. Let y^i be the PC-mild solution of Cauchy problem (3.2) corresponding to initial value y_0^i $(i = 1,2,\ldots,m)$

$$\begin{aligned} \dot{y}(t) &= -A^* y(t), \quad t \neq \tau_k, \\ -\Delta y(\tau_k) &= B_k^* y(\tau_k^+), \quad t = \tau_k, \\ y(0) &= y_0^i. \end{aligned} \tag{3.18}$$

By Theorem 2.10, the PC-mild solution y^i $(i = 1,2,\ldots,m)$ is just a T_0-periodic PC-mild solution of TAHLIPS (3.15).

It is well known that the operator equation

$$[I-\mathcal{S}(T_0,0)]\overline{x}=z \tag{3.19}$$

has a solution if and only if

$$\langle y_0^i,z\rangle_{X^*,X}=0,\quad i=1,2,\dots,m, \tag{3.20}$$

which is equivalent to

$$\begin{aligned}0=\langle z,y_0^i\rangle_{X,X^*}&=\int_0^{T_0}\langle\mathcal{S}(T_0,\theta)f(\theta),y_0^i\rangle d\theta+\sum_{0\le\tau_k<T_0}\langle\mathcal{S}(T_0,\tau_k)c_k,y_0^i\rangle\\&=\int_0^{T_0}\langle f(\theta),\mathcal{S}^*(T_0,\theta)y_0^i\rangle_{X,X^*}d\theta+\sum_{0\le\tau_k<T_0}\langle c_k,\mathcal{S}^*(T_0,\tau_k)y_0^i\rangle_{X,X^*}\\&=\int_0^{T_0}\langle f(\theta),y^i(\theta)\rangle_{X,X^*}d\theta+\sum_{0\le\tau_k<T_0}\langle c_k,y^i(\tau_k)\rangle_{X,X^*}.\end{aligned} \tag{3.21}$$

Suppose that $\overline{x}$ is the solution of operator equation (3.19). By Theorem 2.10, one can verify that the PC-mild solution of Cauchy problem (3.2) corresponding to initial value $\overline{x}$

$$\begin{gathered}\dot{x}(t)=A(t)x(t)+f(t),\quad t\in[0,T_0]\backslash\widetilde{D},\\\Delta x(\tau_k)=B_kx(\tau_k)+c_k,\quad k=1,2,\dots,\delta,\\x(0)=\overline{x},\end{gathered} \tag{3.22}$$

is just the T_0-periodic PC-mild solution of TNLIPS (3.1). Furthermore, by linearity of TNLIPS (3.1), one can verify that every T_0-periodic PC-mild solution of TNLIPS (3.1) can be given by

$$x(t)=x_{T_0}(t)+\sum_{i=1}^m\alpha_ix^i(t), \tag{3.23}$$

where $x_{T_0}(\cdot)$ is a T_0-periodic PC-mild solution of TNLIPS (3.1), $x^1,x^2,\dots,x^m$ are m linearly independent T_0-periodic PC-mild solutions of THLIPS (2.2) and $\alpha_1,\dots,\alpha_m$ are constants. □

The following result shows the relationship between bounded solutions and periodic solutions.

Theorem 3.7. *If TNLIPS (3.1) has a bounded solution, then it has at least one T_0-periodic PC-mild solution.*

Proof. By contradiction, we assume TNLIPS (3.1) has no T_0-periodic PC-mild solution. This means the following operator equation

$$[I-\mathcal{S}(T_0,0)]x(0)=z \tag{3.24}$$

has no solution. By the Fredholm alternative theorem, there is a $y \in X^*$ such that

$$[I - \mathcal{S}^*(T_0,0)]y = 0, \qquad \langle y,z\rangle \equiv \gamma \neq 0. \tag{3.25}$$

Further

$$[\mathcal{S}^*(T_0,0)]^i y = y, \quad i = 1,2,\dots,m. \tag{3.26}$$

Hence

$$\begin{aligned}
x(mT_0) &= \mathcal{S}^m(T_0,0)x(0) + \sum_{i=0}^{m-1}\mathcal{S}^i(T_0,0)z,\\
\langle y,x(mT_0)\rangle &= \langle y,\mathcal{S}^m(T_0,0)x(0)\rangle + \left\langle y,\sum_{i=0}^{m-1}\mathcal{S}^i(T_0,0)z\right\rangle\\
&= \langle(\mathcal{S}^*(T_0,0))^m y,x(0)\rangle + \left\langle\sum_{i=0}^{m-1}[\mathcal{S}^*(T_0,0)]^i y,z\right\rangle\\
&= \langle y,x(0)\rangle + m\gamma.
\end{aligned} \tag{3.27}$$

This implies $\lim_{m\to\infty}\langle y,x(mT_0)\rangle = \infty$. This contradicts the boundedness of x. The proof is completed. □

Corollary 3.8. *(1) Suppose that TNLIPS (3.1) has no T_0-periodic PC-mild solution, then all the PC-mild solutions of TNLIPS (3.1) are unbounded for $t \geq 0$. (2) Suppose that TNLIPS (3.1) has a unique bounded PC-mild solution, for $t \geq 0$, the PC-mild solution is just T_0-periodic.*

4. Parameter perturbation methods and robustness varying with time

Define

$$PC_{T_0}([0,\infty);X) = \{x \in PC([0,\infty);X) \mid x(t+T_0) = x(t),\ \text{for } t \in [0,\infty)\}. \tag{4.1}$$

Set

$$\|x\|_{PC_{T_0}} = \max\left\{\sup_{t\in[0,T_0]}\|x(t+0)\|,\ \sup_{t\in[0,T_0]}\|x(t-0)\|\right\}. \tag{4.2}$$

It can be seen that endowed with the norm $\|\cdot\|_{PC_{T_0}}$ $PC_{T_0}([0,T_0];X)$ is a Banach space.

Denote

$$\begin{aligned}
S_\rho &= \{x \in PC([0,+\infty);X) \mid \|x\|_{PC} < \rho\}\\
\mathcal{B}(x_{T_0},\rho_1) &= \{x \in PC_{T_0}([0,\infty);X) \mid \|x - x_{T_0}\|_{PC_{T_0}} \leq \rho_1\},
\end{aligned} \tag{4.3}$$

where

$$\begin{aligned}\rho &= L_1(L_1L_2+1)\left[\|f\|_{L^1([0,T_0];X)} + 2(T_0+\delta)\sup_{|\xi|\le\tilde{\xi}}\chi(\xi) + \delta\max_{1\le k\le\delta}\|c_k\|_X\right]+2,\\ \rho_1 &= 2L_1(L_1L_2+1)(T_0+\delta)\sup_{|\xi|\le\tilde{\xi}}\chi(\xi),\end{aligned} \tag{4.4}$$

and χ is a nonnegative function.

Consider the following impulsive control system with parameter perturbations (TPNLIPS)

$$\begin{aligned}\dot{x}(t) &= A(t)x(t)+f(t)+p(t,x,\xi), \quad t\neq\tau_k,\\ \Delta x(\tau_k) &= B_kx(\tau_k)+c_k+q_k(x,\xi), \quad t=\tau_k,\end{aligned} \tag{4.5}$$

and the Cauchy problem:

$$\begin{aligned}\dot{x}(t) &= A(t)x(t)+f(t)+p(t,x,\xi), \quad t\in[0,T_0]\setminus\widetilde{D},\\ \Delta x(\tau_k) &= B_kx(\tau_k)+c_k+q_k(x,\xi), \quad t=\tau_k,\\ x(0) &= x_0,\end{aligned} \tag{4.6}$$

where $x\in S_\rho$, $\xi\in\Lambda\equiv(-\tilde{\xi},\tilde{\xi})(\tilde{\xi}>0)$ is a small parameter perturbation that may be caused by some adaptive impulsive control algorithms or parameter drift.

In addition to Assumptions 2.1, 2.3, 2.4, 2.6, and 3.1, we introduce the following assumption.

Assumption 4.1. (1) $p:[0,+\infty)\times S_\rho\times\Lambda\to X$ is measurable for t and $p(t+T_0,x,\xi)=p(t,x,\xi)$.

(2) $q_k:S_\rho\times\Lambda\to X$ and $q_{k+\delta}(x,\xi)=q_k(x,\xi)$.

(3) There exists a nonnegative function ϖ such that $\lim_{\xi\to0}\varpi(\xi)=\varpi(0)=0$ and for any $t\ge0$, x, $y\in S_\rho$ and $\xi\in\Lambda$ such that

$$\|p(t,x,\xi)-p(t,y,\xi)\|\le\varpi(\xi)\|x-y\|, \quad \|q_k(x,\xi)-q_k(y,\xi)\|\le\varpi(\xi)\|x-y\|. \tag{4.7}$$

(4) There exists a nonnegative function χ such that $\lim_{\xi\to0}\chi(\xi)=\chi(0)=0$ and for any $t\ge0$, $x\in S_\rho$, and $\xi\in\Lambda$ such that

$$\|p(t,x,\xi)\|\le\chi(\xi), \quad \|q_k(x,\xi)\|\le\chi(\xi). \tag{4.8}$$

We introduce PC-mild solution of Cauchy problem (4.6) and T_0-periodic PC-mild solution of TPNLIPS (4.5).

Definition 4.2. For every $x_0\in X$, the function $x\in PC([0,T_0];X)$ is said to be the PC-mild solution of the Cauchy problem (4.6) if x satisfies the following integral equation:

$$x(t)=\mathcal{S}(t,0)x_0+\int_0^t\mathcal{S}(t,\theta)[f(\theta)+p(\theta,x(\theta),\xi)]d\theta+\sum_{0\le\tau_k<t}\mathcal{S}(t,\tau_k^+)[c_k+q_k(x(\tau_k^+),\xi)]. \tag{4.9}$$

Definition 4.3. A function $x \in PC([0,+\infty);X)$ is said to be a T_0-periodic PC-mild solution of TPNLIPS (4.5) if it is a PC-mild solution of Cauchy problem (4.6) corresponding to some x_0 and $x(t+T_0) = x(t)$, for $t \geq 0$.

The following result shows that given a periodic motion we can design impulsive periodic motion controllers that are robust to parameter drift.

THEOREM 4.4. *Assumptions 2.1, 2.3, 2.4, 2.6, 3.1, and 4.1 hold and THLIPS (2.2) has no trivial T_0-periodic PC-mild solution. Then there is a $\xi_0 \in (0,\tilde{\xi})$ such that for $|\xi| \leq \xi_0$, TPNLIPS (4.5) has a unique T_0-periodic PC-mild solution $x^{\xi}_{T_0}$ satisfying*

$$\begin{aligned} &||x^{\xi}_{T_0} - x_{T_0}||_{PC_{T_0}} \leq \rho_1, \\ &\lim_{\xi \to 0} x^{\xi}_{T_0}(t) = x_{T_0}(t) \end{aligned} \tag{4.10}$$

uniformly on $t \in [0,+\infty)$ where x_{T_0} is the T_0-periodic PC-mild solution of TNLIPS (3.1).

Proof. Let

$$x_0 = [I - \mathcal{S}(T_0,0)]^{-1}\left[z + \int_0^{T_0} \mathcal{S}(T_0,\theta)p(\theta,x(\theta),\xi)\,d\theta + \sum_{0\leq\tau_k<T_0} \mathcal{S}(T_0,\tau_k^+)q_k(x(\tau_k^+),\xi)\right] \in X \tag{4.11}$$

be fixed. Define the map $\mathbb{O}$ on $\mathcal{B}(x_{T_0},\rho_1)$ which is given by

$$\begin{aligned} (\mathbb{O}x)(t) = {}& \mathcal{S}(t,0)x_0 + \int_0^t \mathcal{S}(t,\theta)[f(\theta) + p(\theta,x(\theta),\xi)]\,d\theta \\ &+ \sum_{0\leq\tau_k<t} \mathcal{S}(t,\tau_k^+)[c_k + q_k(x(\tau_k^+),\xi)]. \end{aligned} \tag{4.12}$$

It is not difficult to verify that $(\mathbb{O}x)(t+T_0) = (\mathbb{O}x)(t)$, for $t > 0$ and $\mathbb{O}x \in PC_{T_0}([0,\infty);X)$.
By Assumption 4.1, we can choose a $\xi_0 \in (0,\tilde{\xi})$ such that

$$2L_1(L_1L_2+1)(T_0+\delta)\sup_{|\xi|\leq\xi_0}\chi(\xi) \leq \rho_1, \qquad \eta = L_1(L_1L_2+1)(T_0+\delta)\sup_{|\xi|\leq\xi_0}\varpi(\xi) < 1. \tag{4.13}$$

For $\xi \in (-\xi_0,\xi_0)$ and provided x, $y \in \mathcal{B}(x_{T_0},\rho_1)$, one can verify that

$$||\mathbb{O}x - x_{T_0}||_{PC_{T_0}} \leq 2L_1(L_1L_2+1)(T_0+\delta)\sup_{|\xi|\leq\xi_0}\chi(\xi) \leq \rho_1, \tag{4.14}$$

$$||\mathbb{O}x - \mathbb{O}y||_{PC_{T_0}} \leq \eta\|x-y\|_{PC_{T_0}}. \tag{4.15}$$

This implies that $\mathbb{O}$ is a contraction mapping on $\mathcal{B}(x_{T_0},\rho_1)$. By Banach's fixed point theorem, operator $\mathbb{O}$ has a unique fixed point $x_{T_0}^{\xi} \in \mathcal{B}(x_{T_0},\rho_1)$ given by

$$\begin{aligned} x_{T_0}^{\xi}(t) = \mathcal{S}(t,0)x_0 + \int_0^t \mathcal{S}(t,\theta)[f(\theta)+p(\theta,x_{T_0}^{\xi}(\theta),\xi)]d\theta \\ + \sum_{0\le\tau_k<t} \mathcal{S}(t,\tau_k^+)[c_k+q_k(x_{T_0}^{\xi}(\tau_k^+),\xi)] \end{aligned} \tag{4.16}$$

which is just the unique T_0-periodic PC-mild solution of TPNLIS (4.5).

For any $t \ge 0$, $x_{T_0}^{\xi} \in \mathcal{B}(x_{T_0},\rho_1) \subset S_\rho$ and $\xi \in (-\xi_0,\xi_0) \subset \Lambda$, it comes from (4) of Assumption 4.1 and

$$\begin{aligned} &\|x_{T_0}^{\xi}(t) - x_{T_0}(t)\| \\ &\quad\le (\|\mathcal{S}(t,0)\|\|[I-\mathcal{S}(T_0,0)]^{-1}\|+1)\Bigg[\int_0^{T_0}\|\mathcal{S}(T_0,\theta)p(\theta,x_{T_0}^{\xi}(\theta),\xi)\|d\theta \\ &\qquad\qquad + \sum_{0\le\tau_k<T_0}\|\mathcal{S}(T_0,\tau_k^+)q_k(x_{T_0}^{\xi}(\tau_k^+),\xi)\|\Bigg] \\ &\quad\le 2L_1(L_1L_2+1)(T_0+\delta)\sup_{|\xi|\le\xi_0}\chi(\xi) \end{aligned} \tag{4.17}$$

that $\lim_{\xi\to 0} x_{T_0}^{\xi}(t) = x_{T_0}(t)$ uniformly on $t \in [0,+\infty)$. □

An example is given to illustrate our theory. Consider the following problem:

$$\begin{aligned} \frac{\partial}{\partial t}x(t,y) &= \operatorname{Sin} t\left(\frac{\partial^2 x}{\partial y_1^2}+\frac{\partial^2 x}{\partial y_2^2}+\frac{\partial^2 x}{\partial y_3^2}\right)x(t,y) \\ &\qquad + \operatorname{Cos}(t,y)+\xi\operatorname{Sin}(t,y), \quad y\in\Omega, t\in(0,2\pi]\setminus\left\{\frac{1}{2}\pi,\pi,\frac{3}{2}\pi\right\}, \\ x(t_i+0,y)-x(t_i-0,y) &= x(t_i,y)+\xi x(t_i,y), \quad y\in\Omega,\ t_i=\frac{i}{2}\pi,\ i=1,2,3, \\ x(0,y) &= x(2\pi,y) = 1, \end{aligned} \tag{4.18}$$

where $\xi \in (-1,1)$, $\Omega \subset R^3$ is bounded domain and $\partial\Omega \in C^3$.

Define $X = L_2(\Omega)$, $D(A) = H^2(\Omega)\bigcap H_0^1(\Omega)$, and $A(t)x = \operatorname{Sin} t((\partial^2 x/\partial y_1^2)+(\partial^2 x/\partial y_2^2)+(\partial^2 x/\partial y_3^2))$, for $x \in D(A)$. Define $x(\cdot)(y) = x(\cdot,y)$, $\operatorname{Cos}(\cdot)(y) = \operatorname{Cos}(\cdot,y)$, $\xi\operatorname{Sin}(\cdot)(y) = \xi\operatorname{Sin}(\cdot,y)$. Thus problem (4.18) can be rewritten as

$$\begin{aligned} \dot{x}(t) = A(t)x(t)+\operatorname{Cos} t+\xi\operatorname{Sin} t, \quad t\in(0,2\pi]\setminus\left\{\frac{1}{2}\pi,\pi,\frac{3}{2}\pi\right\}, \\ x(0)=x(2\pi)=1, \quad \Delta x\left(\frac{i}{2}\pi\right) = x\left(\frac{i}{2}\pi\right)+\xi x\left(\frac{i}{2}\pi\right), \quad i=1,2,3. \end{aligned} \tag{4.19}$$

It satisfies all the assumptions given in Theorem 4.4, thus our results can be applied to problem (4.18).

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