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Research Article

Oscillation Criteria for Second-Order Delay Dynamic Equations on Time Scales

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By means of Riccati transformation technique, we establish some new oscillation criteria for the second-order nonlinear delay dynamic equations $(p(t)(x^{\Delta}(t))^{\gamma})^{\Delta}+q(t)f(x(\tau(t))) = 0$ on a time scale $\mathbb{T}$, here $\gamma \geq 1$ is a quotient of odd positive integers with p and q real-valued positive rd-continuous functions defined on $\mathbb{T}$.

1. Introduction

The theory of time scales, which has recently received a lot of attention, was introduced by Hilger in his Ph.D. thesis in 1988 in order to unify continuous and discrete analysis (see Hilger [1]). Several authors have expounded on various aspects of this new theory; see the survey paper by Agarwal et al. [2] and references cited therein. A book on the subject of time scales, by Bohner and Peterson [3], summarizes and organizes much of the time scale calculus, we refer also the last book by Bohner and Peterson [4] for advances in dynamic equations on time scales. For the notions used below we refer to the next section that provides some basic facts on time scales extracted from Bohner and Peterson [3].

A time scale $\mathbb{T}$ is an arbitrary closed subset of the reals, and the cases when this time scale is equal to the reals or to the integers represent the classical theories of differential and of difference equations. Many other interesting time scales exist, and they give rise to plenty of applications, among them the study of population dynamic models which are discrete in season (and may follow a difference scheme with variable step-size or often modeled by continuous dynamic systems), die out, say in winter, while their eggs are incubating or dormant, and then in season again, hatching gives rise to a nonoverlapping population (see Bohner and Peterson [3]).

In recent years, there has been much research activity concerning the oscillation and nonoscillation of solutions of various equations on time scales, and we refer the reader to Bohner and Saker [5], Erbe [6], Erbe et al. [7], Saker [8, 9]. However, there are few results dealing with the oscillation of the solutions of delay dynamic equations on time scales [10–17].

To the best of our knowledge, there are no results regarding the oscillation of the solutions of the following second-order nonlinear delay dynamic equations on time scales up to now:

$$\left(p(t)\left(x^{\Delta}(t)\right)^{\gamma}\right)^{\Delta}+q(t)f\left(x(\tau(t))\right)=0 \quad \text{for } t\in\mathbb{T}. \tag{1.1}$$

Zhang and Deng [16] (see also Bohner [12]) considered the first-order delay dynamic equations on time scales

$$x^{\Delta}(t)+p(t)x(\tau(t))=0 \quad \text{for } t\in\mathbb{T}, \tag{1.2}$$

and unified oscillation criteria of the first-order delay differential and difference equations. Agarwal et al. [10] considered the second-order delay dynamic equations on time scales

$$x^{\Delta\Delta}(t)+p(t)x(\tau(t))=0 \quad \text{for } t\in\mathbb{T}, \tag{1.3}$$

and established some sufficient conditions for oscillation of (1.3). Zhang and Zhu [17] considered the second-order nonlinear delay dynamic equations on time scales

$$x^{\Delta\Delta}(t)+p(t)f\left(x(t-\tau)\right)=0 \quad \text{for } t\in\mathbb{T}, \tag{1.4}$$

and the second-order nonlinear dynamic equations on time scales

$$x^{\Delta\Delta}(t)+p(t)f\left(x(\sigma(t))\right)=0 \quad \text{for } t\in\mathbb{T}, \tag{1.5}$$

and established the equivalence of the oscillation of (1.4) and (1.5), from which obtained some oscillation criteria and comparison theorems for (1.4). Sahiner [13] considered the second-order nonlinear delay dynamic equations on time scales

$$x^{\Delta\Delta}(t)+p(t)f\left(x(\tau(t))\right)=0 \quad \text{for } t\in\mathbb{T}, \tag{1.6}$$

and obtained some sufficient conditions for oscillation of (1.6) by means of Riccati transformation technique. Erbe et al. [18] considered the pair of second-order dynamic equations

$$\begin{gathered}\left(r(t)\left(x^{\Delta}\right)^{\gamma}\right)^{\Delta}+p(t)x^{\gamma}(t)=0 \quad \text{for } t\in\mathbb{T},\\ \left(r(t)\left(x^{\Delta}\right)^{\gamma}\right)^{\Delta}+p(t)x^{\gamma}(\sigma(t))=0 \quad \text{for } t\in\mathbb{T},\end{gathered} \tag{1.7}$$

and established some necessary and sufficient conditions for nonoscillation of Hille-Kneser type. Han et al. [19] considered the second-order Emden-Fowler delay dynamic equations on time scales

$$x^{\Delta\Delta}(t)+p(t)x^{\gamma}(\tau(t))=0 \quad \text{for } t\in\mathbb{T}, \tag{1.8}$$

and established some sufficient conditions for oscillation of (1.8). Agarwal et al. [11], Saker [15] considered the second-order nonlinear neutral delay dynamic equations on time scales

$$(r(t)((x(t)+p(t)x(t-\tau))^{\Delta})^{\gamma})^{\Delta}+f(t,x(t-\delta))=0 \quad \text{for } t\in\mathbb{T}, \tag{1.9}$$

and established some oscillation criteria of (1.9). Sahiner [14] considered the second-order neutral delay and mixed-type dynamic equations on time scales

$$(r(t)((x(t)+p(t)x(\tau(t)))^{\Delta})^{\gamma})^{\Delta}+f(t,x(\delta(t)))=0 \quad \text{for } t\in\mathbb{T}, \tag{1.10}$$

and obtained some sufficient conditions for oscillation of (1.10).

Clearly, (1.3), (1.4), and (1.6) are the special cases of (1.1), and (1.9) is different from (1.1). To develop the qualitative theory of delay dynamic equations on time scales, in this paper, we consider the second-order nonlinear delay dynamic equation on time scales (1.1).

As we are interested in oscillatory behavior, we assume throughout this paper that the given time scale $\mathbb{T}$ is unbounded above, that is, it is a time scale interval of the form $[a,\infty)$ with $a\in\mathbb{T}$.

We assume that $\gamma\geq 1$ is a quotient of odd positive integer, p and q are positive, real-valued rd-continuous functions defined on $\mathbb{T}$, $\tau:\mathbb{T}\to\mathbb{T}$ is an rd-continuous function such that $\tau(t)\leq t$ and $\tau(t)\to\infty$ as $t\to\infty$, $f\in C(\mathbb{R},\mathbb{R})$ such that satisfies for some positive constant L, $f(x)/x^{\gamma}\geq L$, for all nonzero x. We will also consider the two cases

$$\int_a^{\infty}\left(\frac{1}{p(t)}\right)^{1/\gamma}\Delta t=\infty, \tag{1.11}$$

$$\int_a^{\infty}\left(\frac{1}{p(t)}\right)^{1/\gamma}\Delta t<\infty. \tag{1.12}$$

By a solution of (1.1), we mean a nontrivial real-valued function $x\in C^1_{\mathrm{rd}}[t_x,\infty)$, $t_x\geq a$, which has the property $p(x^{\Delta})^{\gamma}\in C^1_{\mathrm{rd}}[t_x,\infty)$ and satisfying (1.1) for $t\geq t_x$. A solution x of (1.1) is called oscillatory if it is neither eventually positive nor eventually negative; otherwise it is called nonoscillatory. Equation (1.1) is called oscillatory if all solutions are oscillatory. Our attention is restricted to those solutions x of (1.1) which exist on some half line $[t_x,\infty)$ with $\sup\{|x(t)|:t\geq t_0\}>0$ for any $t_0\geq t_x$.

In this paper we intend to use the Riccati transformation technique for obtaining several oscillation criteria for (1.1) when (1.11) or (1.12) holds.

The paper is organized as follows: in the next section we present the basic definitions and the theory of calculus on time scales. In Section 3, we apply a simple consequence of Keller's chain rule, and the inequality

$$\lambda AB^{\lambda-1}-A^{\lambda}\leq(\lambda-1)B^{\lambda}, \quad \lambda\geq 1, \tag{1.13}$$

where A and B are nonnegative constants, devoted to the proof of the sufficient conditions for oscillation of all solutions of (1.1). In Section 4, we present some examples to illustrate our main results.

We note that if $\mathbb{T}=\mathbb{R}$, then $\sigma(t)=0$, $\mu(t)=0$, $x^{\Delta}(t)=x'(t)$ and (1.1) becomes the second-order nonlinear delay differential equation

$$(p(t)(x'(t))^{\gamma})'+q(t)f(x(\tau(t)))=0 \quad \text{for } t\in\mathbb{R}. \tag{1.14}$$

If $\mathbb{T}=\mathbb{Z}$, then $\sigma(t)=t+1$, $\mu(t)=1$, $x^{\Delta}(t)=\Delta x(t)=x(t+1)-x(t)$ and (1.1) becomes the second-order nonlinear delay difference equation

$$\Delta(p(t)(\Delta x(t))^{\gamma})+q(t)f(x(\tau(t)))=0 \quad \text{for } t\in\mathbb{Z}. \tag{1.15}$$

Numerous oscillation and nonoscillation criteria have been established for the forms of (1.14) and (1.15); see, for example, [20–26] and references therein.

2. Some preliminaries

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$. On any time scale we define the forward and backward jump operators by

$$\sigma(t):=\inf\{s\in\mathbb{T}\mid s>t\}, \qquad \rho(t):=\sup\{s\in\mathbb{T}\mid s<t\}. \tag{2.1}$$

A point $t\in\mathbb{T}$ is said to be left-dense if $\rho(t)=t$, right-dense if $\sigma(t)=t$, left-scattered if $\rho(t)<t$, and right-scattered if $\sigma(t)>t$. The graininess μ of the time scale is defined by $\mu(t):=\sigma(t)-t$.

For a function $f:\mathbb{T}\to\mathbb{R}$ (the range $\mathbb{R}$ of f may actually be replaced by any Banach space), the (delta) derivative is defined by

$$f^{\Delta}(t)=\frac{f(\sigma(t))-f(t)}{\sigma(t)-t}, \tag{2.2}$$

if f is continuous at t and t is right-scattered. If t is right-dense, then derivative is defined by

$$f^{\Delta}(t)=\lim_{s\to t^+}\frac{f(\sigma(t))-f(s)}{t-s}=\lim_{s\to t^+}\frac{f(t)-f(s)}{t-s}, \tag{2.3}$$

provided this limit exists.

A function $f:\mathbb{T}\to\mathbb{R}$ is said to be rd-continuous if it is continuous at each right-dense point and if there exists a finite left limit at all left-dense points. The set of rd-continuous functions $f:\mathbb{T}\to\mathbb{R}$ is denoted by $C_{\mathrm{rd}}(\mathbb{T},\mathbb{R})$.

f is said to be differentiable if its derivative exists. The set of functions $f:\mathbb{T}\to\mathbb{R}$ that are differentiable and whose derivative is rd-continuous function is denoted by $C^1_{\mathrm{rd}}(\mathbb{T},\mathbb{R})$.

The derivative and the shift operator σ are related by the formula

$$f^{\sigma}=f+\mu f^{\Delta}, \quad \text{where } f^{\sigma}:=f\circ\sigma. \tag{2.4}$$

Let f be a real-valued function defined on an interval $[a,b]$. We say that f is increasing, decreasing, nondecreasing, and nonincreasing on $[a,b]$ if $t_1,t_2\in[a,b]$ and $t_2>t_1$ imply $f(t_2)>f(t_1)$, $f(t_2)<f(t_1)$, $f(t_2)\ge f(t_1)$, and $f(t_2)\le f(t_1)$, respectively. Let f be

a differentiable function on $[a,b]$. Then f is increasing, decreasing, nondecreasing, and nonincreasing on $[a,b]$ if $f^{\Delta}(t) > 0$, $f^{\Delta}(t) < 0$, $f^{\Delta}(t) \geq 0$, and $f^{\Delta}(t) \leq 0$ for all $t \in [a,b)$, respectively.

We will make use of the following product and quotient rules for the derivative of the product fg and the quotient f/g of two differentiable functions f and g:

$$(fg)^{\Delta}(t) = f^{\Delta}(t)g(t) + f(\sigma(t))g^{\Delta}(t) = f(t)g^{\Delta}(t) + f^{\Delta}(t)g(\sigma(t)), \tag{2.5}$$

$$\left(\frac{f}{g}\right)^{\Delta}(t) = \frac{f^{\Delta}(t)g(t) - f(t)g^{\Delta}(t)}{g(t)g(\sigma(t))}. \tag{2.6}$$

For $a,b \in \mathbb{T}$ and a differentiable function f, the Cauchy integral of f^{Δ} is defined by

$$\int_a^b f^{\Delta}(t)\Delta t = f(b) - f(a). \tag{2.7}$$

The integration by parts formula reads

$$\int_a^b f^{\Delta}(t)g(t)\Delta t = f(b)g(b) - f(a)g(a) - \int_a^b f^{\sigma}(t)g^{\Delta}(t)\Delta t, \tag{2.8}$$

and infinite integrals are defined as

$$\int_a^{\infty} f(s)\Delta s = \lim_{t\to\infty}\int_a^t f(s)\Delta s. \tag{2.9}$$

In case $\mathbb{T} = \mathbb{R}$ we have

$$\sigma(t) = \rho(t) = t, \quad \mu(t) \equiv 0, \quad f^{\Delta} = f', \quad \int_a^b f(t)\Delta t = \int_a^b f(t)\mathrm{d}t, \tag{2.10}$$

and in case $\mathbb{T} = \mathbb{Z}$ we have

$$\sigma(t) = t+1, \quad \rho(t) = t-1, \quad \mu(t) \equiv 1, \quad f^{\Delta} = \Delta f, \quad \int_a^b f(t)\Delta t = \sum_{t=a}^{b-1} f(t). \tag{2.11}$$

3. Main results

In this section we give some new oscillation criteria for (1.1). In order to prove our main results, we will use the formula

$$\left((x(t))^{\gamma}\right)^{\Delta} = \gamma\int_0^1 \left[hx^{\sigma} + (1-h)x\right]^{\gamma-1}x^{\Delta}(t)\mathrm{d}h, \tag{3.1}$$

which is a simple consequence of Keller's chain rule (see Bohner and Peterson [3, Theorem 1.90]). Also, we need the following auxiliary result.

Lemma 3.1 (Sahiner [13]). *Suppose that the following conditions hold:*
(H_1) $u \in C^2_{\mathrm{rd}}(I,\mathbb{R})$ *where* $I = [t_*,\infty) \subset \mathbb{T}$ *for some* $t_* > 0$,
(H_2) $u(t) > 0$, $u^{\Delta}(t) > 0$ *and* $u^{\Delta\Delta}(t) \leq 0$ *for* $t \geq t_*$.

Then, for each $k \in (0,1)$, there exists a constant $t_k \in \mathbb{T}$, $t_k \geq t_*$, such that

$$u(\sigma(t)) \leq \frac{\sigma(t)}{k\tau(t)} u(\tau(t)) \quad \text{for } t \geq t_k. \tag{3.2}$$

Lemma 3.2. *Assume (1.11) holds. Furthermore, assume that $p \in C^1_{\mathrm{rd}}([a,\infty),\mathbb{R})$, $p^\Delta \geq 0$ and x is an eventually positive solution of (1.1). Then, there exists a $t_1 \geq a$ such that*

$$x^\Delta(t) > 0, \quad x^{\Delta\Delta}(t) < 0, \quad \left(p(t)(x^\Delta(t))^\gamma\right)^\Delta < 0 \quad \text{for } t \geq t_1. \tag{3.3}$$

Proof. Since $x(t)$ is an eventually positive solution of (1.1), there exists a number $t_0 \geq a$ such that $x(t) > 0$ and $x(\tau(t)) > 0$ for all $t \geq t_0 > a$. In view of (1.1), we have

$$\left(p(t)(x^\Delta(t))^\gamma\right)^\Delta = -q(t)f(x(\tau(t))) \leq -Lq(t)(x(\tau(t)))^\gamma < 0 \quad \text{for } t \geq t_0, \tag{3.4}$$

and so $p(t)(x^\Delta(t))^\gamma$ is an eventually decreasing function. We first show that $p(t)(x^\Delta(t))^\gamma$ is eventually positive. Indeed, the decreasing function $p(t)(x^\Delta(t))^\gamma$ is either eventually positive or eventually negative. Suppose that there exists an integer $t_1 \geq t_0$ such that $p(t_1)(x^\Delta(t_1))^\gamma = c < 0$, then from (3.4) we have $p(t)(x^\Delta(t))^\gamma \leq p(t_1)(x^\Delta(t_1))^\gamma = c$ for $t \geq t_1$, hence

$$x^\Delta(t) \leq c^{1/\gamma}\left(\frac{1}{p(t)}\right)^{1/\gamma}, \tag{3.5}$$

which implies by (1.11) that

$$x(t) \leq x(t_1) + c^{1/\gamma}\int_{t_1}^{t}\left(\frac{1}{p(s)}\right)^{1/\gamma}\Delta s \longrightarrow -\infty \quad \text{as } t \longrightarrow \infty, \tag{3.6}$$

and this contradicts the fact that $x(t) > 0$ for all $t \geq t_0$. Hence $p(t)(x^\Delta(t))^\gamma$ is eventually positive. So $x^\Delta(t)$ is eventually positive. Then $x(t)$ is eventually increasing.

By (2.5), we get

$$\left(p(t)(x^\Delta(t))^\gamma\right)^\Delta = p^\Delta(t)(x^\Delta(t))^\gamma + p(\sigma(t))\left((x^\Delta(t))^\gamma\right)^\Delta. \tag{3.7}$$

From (3.4), (3.7) and $p^\Delta(t) \geq 0$, we can easily verify that

$$\left((x^\Delta(t))^\gamma\right)^\Delta < 0. \tag{3.8}$$

Using (3.1), we get

$$\left((x^\Delta(t))^\gamma\right)^\Delta = \gamma\int_0^1 \left[h(x^\Delta)^\sigma + (1-h)x^\Delta\right]^{\gamma-1} x^{\Delta\Delta}(t)\mathrm{d}h. \tag{3.9}$$

From (3.8), (3.9), and $\int_0^1[h(x^\Delta)^\sigma + (1-h)x^\Delta]^{\gamma-1}\mathrm{d}h > 0$, we have $x^{\Delta\Delta}(t)$ is eventually negative. Therefore, we see that there is some $t_1 \geq t_0$ such that (3.3) holds. The proof is complete. □

THEOREM 3.3. *Assume (1.11) holds, $p \in C_{rd}^1([a,\infty),\mathbb{R})$, and $p^\Delta \geq 0$. Furthermore, assume that there exists a positive function $\delta \in C_{rd}^1([a,\infty),\mathbb{R})$ such that for some positive constant $k \in (0,1)$,*

$$\limsup_{t\to\infty} \int_a^t \left(Lk^\gamma q(s)\delta(s)\left(\frac{\tau(s)}{\sigma(s)}\right)^\gamma - \frac{p(s)(\delta^\Delta(s))_+^{\gamma+1}}{(\gamma+1)^{\gamma+1}(\delta(s))^\gamma} \right)\Delta s = \infty, \tag{3.10}$$

where $(\delta^\Delta(s))_+ = \max\{0,\delta^\Delta(s)\}$. Then (1.1) is oscillatory on $[a,\infty)$.

Proof. Suppose that (1.1) has a nonoscillatory solution $x(t)$. We may assume without loss of generality that $x(t) > 0$ and $x(\tau(t)) > 0$ for all $t \geq t_1 > a$. We will consider only this case, since the proof when $x(t)$ is eventually negative is similar. In view of Lemmas 3.1 and 3.2, for each positive constant $k \in (0,1)$, there exists a $t_2 = \max\{t_k, t_1\}$ such that

$$x(t) \leq x(\sigma(t)) \leq \frac{\sigma(t)}{k\tau(t)}x(\tau(t)) \leq \frac{\sigma(t)}{k\tau(t)}x(t) \quad \text{for } t \geq t_2. \tag{3.11}$$

We get (3.3), (3.4), and (3.7). Define the function $\omega(t)$ by

$$\omega(t) = \delta(t)\frac{p(t)(x^\Delta(t))^\gamma}{(x(t))^\gamma} \quad \text{for } t \geq t_2. \tag{3.12}$$

Then $\omega(t) > 0$, and using (2.5) and (2.6) we get

$$\begin{aligned}\omega^\Delta(t) = {} & \frac{\delta(t)}{(x(t))^\gamma}\left(p(t)(x^\Delta(t))^\gamma\right)^\Delta \\ & + p(\sigma(t))(x^\Delta(\sigma(t)))^\gamma \frac{(x(t))^\gamma\delta^\Delta(t) - \delta(t)((x(t))^\gamma)^\Delta}{(x(t))^\gamma(x(\sigma(t)))^\gamma}.\end{aligned} \tag{3.13}$$

In view of (3.4), (3.11), and (3.12), we obtain

$$\begin{aligned}\omega^\Delta(t) \leq {} & -Lk^\gamma q(t)\delta(t)\left(\frac{\tau(t)}{\sigma(t)}\right)^\gamma + \frac{\delta^\Delta(t)}{\delta(\sigma(t))}\omega(\sigma(t)) \\ & - \frac{\delta(t)p(\sigma(t))(x^\Delta(\sigma(t)))^\gamma((x(t))^\gamma)^\Delta}{(x(t))^\gamma(x(\sigma(t)))^\gamma}.\end{aligned} \tag{3.14}$$

Using (3.3) we have $x(\sigma(t)) \geq x(t)$, and then from (3.1) that

$$\begin{aligned}\omega^\Delta(t) \leq {} & -Lk^\gamma q(t)\delta(t)\left(\frac{\tau(t)}{\sigma(t)}\right)^\gamma + \frac{\delta^\Delta(t)}{\delta(\sigma(t))}\omega(\sigma(t)) \\ & - \frac{\gamma\delta(t)p(\sigma(t))(x^\Delta(\sigma(t)))^\gamma(x(t))^{\gamma-1}x^\Delta(t)}{(x(t))^\gamma(x(\sigma(t)))^\gamma}.\end{aligned} \tag{3.15}$$

So,

$$\omega^{\Delta}(t) \le -Lk^{\gamma}q(t)\delta(t)\left(\frac{\tau(t)}{\sigma(t)}\right)^{\gamma} + \frac{\delta^{\Delta}(t)}{\delta(\sigma(t))}\omega(\sigma(t)) - \frac{\gamma\delta(t)p(\sigma(t))(x^{\Delta}(\sigma(t)))^{\gamma}x^{\Delta}(t)}{(x(\sigma(t)))^{\gamma+1}}. \tag{3.16}$$

From (3.3), since $(p(t)(x^{\Delta}(t))^{\gamma})^{\Delta} < 0$, we have

$$x^{\Delta}(t) > \frac{(p(\sigma(t)))^{1/\gamma}}{(p(t))^{1/\gamma}}x^{\Delta}(\sigma(t)). \tag{3.17}$$

Substituting (3.17) in (3.16) we find that

$$\omega^{\Delta}(t) \le -Lk^{\gamma}q(t)\delta(t)\left(\frac{\tau(t)}{\sigma(t)}\right)^{\gamma} + \frac{\delta^{\Delta}(t)}{\delta(\sigma(t))}\omega(\sigma(t)) - \frac{\gamma\delta(t)(p(\sigma(t)))^{(\gamma+1)/\gamma}(x^{\Delta}(\sigma(t)))^{\gamma+1}}{(p(t))^{1/\gamma}(x(\sigma(t)))^{\gamma+1}}. \tag{3.18}$$

So,

$$\omega^{\Delta}(t) \le -Lk^{\gamma}q(t)\delta(t)\left(\frac{\tau(t)}{\sigma(t)}\right)^{\gamma} + \frac{(\delta^{\Delta}(t))_{+}}{\delta(\sigma(t))}\omega(\sigma(t)) - \frac{\gamma\delta(t)}{(p(t))^{\lambda-1}(\delta(\sigma(t)))^{\lambda}}(\omega(\sigma(t)))^{\lambda}, \tag{3.19}$$

where $\lambda = (\gamma+1)/\gamma$, $(\delta^{\Delta}(t))_{+} = \max\{0, \delta^{\Delta}(t)\}$. Set

$$A = \left[\frac{\gamma\delta(t)}{(\delta(\sigma(t)))^{\lambda}(p(t))^{\lambda-1}}\right]^{1/\lambda}\omega(\sigma(t)),$$
$$B = \left[\frac{(\delta^{\Delta}(t))_{+}}{\lambda\delta(\sigma(t))}\left(\frac{\gamma\delta(t)}{(\delta(\sigma(t)))^{\lambda}(p(t))^{\lambda-1}}\right)^{-1/\lambda}\right]^{1/(\lambda-1)}. \tag{3.20}$$

Using the inequality (1.13) we have

$$\frac{(\delta^{\Delta}(t))_{+}}{\delta(\sigma(t))}\omega(\sigma(t)) - \frac{\gamma\delta(t)}{(\delta(\sigma(t)))^{\lambda}(p(t))^{\lambda-1}}(\omega(\sigma(t)))^{\lambda} \le (\lambda-1)\lambda^{-\lambda/(\lambda-1)}\left(\frac{(\delta^{\Delta}(t))_{+}}{\delta(\sigma(t))}\right)^{\lambda/(\lambda-1)}\left(\frac{\gamma\delta(t)}{(\delta(\sigma(t)))^{\lambda}(p(t))^{\lambda-1}}\right)^{-1/(\lambda-1)}, \tag{3.21}$$

then

$$\frac{(\delta^{\Delta}(t))_{+}}{\delta(\sigma(t))}\omega(\sigma(t)) - \frac{\gamma\delta(t)}{(\delta(\sigma(t)))^{\lambda}(p(t))^{\lambda-1}}(\omega(\sigma(t)))^{\lambda} \le C\frac{p(t)(\delta^{\Delta}(t))_{+}^{\gamma+1}}{(\delta(t))^{\gamma}}, \tag{3.22}$$

where $C=(\lambda-1)\lambda^{\lambda/(\lambda-1)}\gamma^{-1/(\lambda-1)}=1/(\gamma+1)^{\gamma+1}$. Thus, from (3.19) and (3.22) we obtain

$$\omega^{\Delta}(t)\leq -Lk^{\gamma}q(t)\delta(t)\left(\frac{\tau(t)}{\sigma(t)}\right)^{\gamma}+\frac{p(t)(\delta^{\Delta}(t))_{+}^{\gamma+1}}{(\gamma+1)^{\gamma+1}(\delta(t))^{\gamma}}. \tag{3.23}$$

Integrating the inequality (3.23) from t_2 to t we obtain

$$-\omega(t_2)\leq\omega(t)-\omega(t_2)\leq-\int_{t_2}^{t}\left(Lk^{\gamma}q(s)\delta(s)\left(\frac{\tau(s)}{\sigma(s)}\right)^{\gamma}-\frac{p(s)(\delta^{\Delta}(s))_{+}^{\gamma+1}}{(\gamma+1)^{\gamma+1}(\delta(s))^{\gamma}}\right)\Delta s, \tag{3.24}$$

which yields

$$\int_{t_2}^{t}\left(Lk^{\gamma}q(s)\delta(s)\left(\frac{\tau(s)}{\sigma(s)}\right)^{\gamma}-\frac{p(s)(\delta^{\Delta}(s))_{+}^{\gamma+1}}{(\gamma+1)^{\gamma+1}(\delta(s))^{\gamma}}\right)\Delta s\leq\omega(t_2) \tag{3.25}$$

for all large t, which contradicts (3.10). The proof is complete. □

From Theorem 3.3, we can obtain different conditions for oscillation of all solutions of (1.1) with different choices of $\delta(t)$.

For example, let $\delta(t)=t$, $t\geq a$. Now, Theorem 3.3 yields the following result.

Corollary 3.4. *Assume (1.11) holds and* $p\in C_{\mathrm{rd}}^{1}([a,\infty),\mathbb{R})$, $p^{\Delta}\geq 0$. *Furthermore, assume that for some positive constant* $k\in(0,1)$,

$$\limsup_{t\to\infty}\int_{a}^{t}\left(Lk^{\gamma}sq(s)\left(\frac{\tau(s)}{\sigma(s)}\right)^{\gamma}-\frac{p(s)}{(\gamma+1)^{\gamma+1}s^{\gamma}}\right)\Delta s=\infty, \tag{3.26}$$

then (1.1) is oscillatory on $[a,\infty)$.

Let $\delta(t)=1$, $t\geq a$. Now, Theorem 3.3 yields the following well-known result (Leighton-Wintner theorem).

Corollary 3.5 (Leighton-Wintner). *Assume (1.11) holds and* $p\in C_{\mathrm{rd}}^{1}([a,\infty),\mathbb{R})$, $p^{\Delta}\geq 0$. *If*

$$\limsup_{t\to\infty}\int_{a}^{t}q(s)\left(\frac{\tau(s)}{\sigma(s)}\right)^{\gamma}\Delta s=\infty, \tag{3.27}$$

then (1.1) is oscillatory on $[a,\infty)$.

Let $\gamma=1$ and $p(t)=1$ for $t\geq a$. Now, Theorem 3.3 yields the following result.

Corollary 3.6. *Assume that there exists a positive function* $\delta\in C_{\mathrm{rd}}^{1}([a,\infty),\mathbb{R})$ *such that for some positive constant* $k\in(0,1)$,

$$\limsup_{t\to\infty}\int_{a}^{t}\left(Lkq(s)\delta(s)\frac{\tau(s)}{\sigma(s)}-\frac{(\delta^{\Delta}(s))_{+}^{2}}{4\delta(s)}\right)\Delta s=\infty, \tag{3.28}$$

where $(\delta^{\Delta}(s))_{+}=\max\{0,\delta^{\Delta}(s)\}$. *Then every solution of (1.1) is oscillatory on* $[a,\infty)$.

Remark 3.7. From Theorem 3.3, we can give some special sufficient conditions for oscillation of (1.1) on different type of time scales, for example, we can deduce that if there exists a positive function $\delta \in C^1([a,\infty),\mathbb{R})$ such that for some positive constant $k \in (0,1)$,

$$\int_a^\infty \left(\frac{1}{p(t)}\right)^{1/\gamma} dt = \infty, \qquad \limsup_{t\to\infty} \int_a^t \left(Lk^\gamma q(s)\delta(s)\left(\frac{\tau(s)}{s}\right)^\gamma - \frac{p(s)\left(\delta'(s)\right)_+^{\gamma+1}}{(\gamma+1)^{\gamma+1}\left(\delta(s)\right)^\gamma}\right) ds = \infty,$$
$$p'(t) \geq 0, \tag{3.29}$$

where $(\delta'(s))_+ = \max\{0, \delta'(s)\}$, are sufficient conditions for oscillation of (1.14).

If there exists a positive sequence $\{\delta_n\}$ such that for some positive constant $k \in (0,1)$,

$$\sum_{i=a}^{\infty} \left(\frac{1}{p(i)}\right)^{1/\gamma} = \infty, \qquad \limsup_{t\to\infty} \sum_{i=a}^{n-1} \left(Lk^\gamma q(i)\delta(i)\left(\frac{\tau(i)}{i+1}\right)^\gamma - \frac{p(i)\left(\Delta\delta(i)\right)_+^{\gamma+1}}{(\gamma+1)^{\gamma+1}\left(\delta(i)\right)^\gamma}\right) = \infty,$$
$$\Delta p(n) \geq 0, \tag{3.30}$$

where $(\Delta\delta(i))_+ = \max\{0, \Delta\delta(i)\}$, are sufficient conditions for oscillation of (1.15).

THEOREM 3.8. *Assume (1.11) holds and $p \in C^1_{\mathrm{rd}}([a,\infty),\mathbb{R})$, $p^\Delta \geq 0$. Furthermore, assume that there exists a positive function $\delta \in C^1_{\mathrm{rd}}([a,\infty),\mathbb{R})$ such that for some positive constant $k \in (0,1)$, and $m \geq 1$,*

$$\limsup_{t\to\infty} \frac{1}{t^m} \int_a^t (t-s)^m \left(Lk^\gamma q(s)\delta(s)\left(\frac{\tau(s)}{\sigma(s)}\right)^\gamma - \frac{p(s)\left(\delta^\Delta(s)\right)_+^{\gamma+1}}{(\gamma+1)^{\gamma+1}\left(\delta(s)\right)^\gamma}\right)\Delta s = \infty, \tag{3.31}$$

where $(\delta^\Delta(s))_+ = \max\{0, \delta^\Delta(s)\}$. Then (1.1) is oscillatory on $[a,\infty)$.

Proof. Suppose that (1.1) has a nonoscillatory solution $x(t)$. We may assume without loss of generality that $x(t) > 0$ and $x(\tau(t)) > 0$ for all $t \geq t_1 > a$. We proceed as in the proof of Theorem 3.3 and we get (3.23). Then from (3.23) we have

$$Lk^\gamma q(t)\delta(t)\left(\frac{\tau(t)}{\sigma(t)}\right)^\gamma - \frac{p(t)\left(\delta^\Delta(t)\right)_+^{\gamma+1}}{(\gamma+1)^{\gamma+1}\delta^\gamma(t)} \leq -\omega^\Delta(t). \tag{3.32}$$

Therefore,

$$\int_{t_2}^t (t-s)^m \left[Lk^\gamma q(s)\delta(s)\left(\frac{\tau(s)}{\sigma(s)}\right)^\gamma - \frac{p(s)\left(\delta^\Delta(s)\right)_+^{\gamma+1}}{(\gamma+1)^{\gamma+1}\delta^\gamma(s)}\right]\Delta s \leq -\int_{t_2}^t (t-s)^m \omega^\Delta(s)\Delta s. \tag{3.33}$$

An integration by parts formula (2.8) the right-hand side leads to

$$\int_{t_2}^t (t-s)^m \omega^\Delta(s)\Delta s = (t-s)^m \omega(s)\Big|_{t_2}^t - \int_{t_2}^t \left((t-s)^m\right)^{\Delta_s} \omega(\sigma(s))\Delta s. \tag{3.34}$$

Note that since $((t-s)^m)^{\Delta_s} \le -m(t-\sigma(s))^{m-1} \le 0$ for $t \ge \sigma(s)$, $m \ge 1$ (see Saker [15]). Then from (3.33) we have

$$\int_{t_2}^{t}(t-s)^m\left[Lk^\gamma q(s)\delta(\sigma(s))\left(\frac{\tau(s)}{\sigma(s)}\right)^\gamma - \frac{p(s)(\delta^\Delta(s))_+^{\gamma+1}}{(\gamma+1)^{\gamma+1}\delta^\gamma(s)}\right]\Delta s \le (t-t_2)^m\omega(t_2). \tag{3.35}$$

Then

$$\frac{1}{t^m}\int_{t_2}^{t}(t-s)^m\left[Lk^\gamma q(s)\delta(\sigma(s))\left(\frac{\tau(s)}{\sigma(s)}\right)^\gamma - \frac{p(s)(\delta^\Delta(s))_+^{\gamma+1}}{(\gamma+1)^{\gamma+1}\delta^\gamma(s)}\right]\Delta s \le \left(\frac{t-t_2}{t}\right)^m\omega(t_2), \tag{3.36}$$

which contradicts (3.31). The proof is complete. □

From Theorem 3.8, we have the following oscillation criteria for (1.14) and (1.15).

Corollary 3.9. *If there exists a positive function $\delta \in C^1([a,\infty),\mathbb{R})$ such that for some positive constant $k \in (0,1)$, $m \ge 1$,*

$$\begin{gathered}\int_a^\infty\left(\frac{1}{p(t)}\right)^{1/\gamma}\mathrm{d}t = \infty,\\ \limsup_{t\to\infty}\frac{1}{t^m}\int_a^t(t-s)^m\left(Lk^\gamma q(s)\delta(s)\left(\frac{\tau(s)}{s}\right)^\gamma - \frac{p(s)(\delta'(s))_+^{\gamma+1}}{(\gamma+1)^{\gamma+1}(\delta(s))^\gamma}\right)\mathrm{d}s = \infty,\\ p'(t) \ge 0,\end{gathered} \tag{3.37}$$

where $(\delta'(s))_+ = \max\{0,\delta'(s)\}$, then (1.14) is oscillatory.

Corollary 3.10. *If there exists a positive sequence $\{\delta(n)\}$ such that for some positive constant $k \in (0,1)$, $m \ge 1$,*

$$\begin{gathered}\sum_{i=a}^{\infty}\left(\frac{1}{p(i)}\right)^{1/\gamma} = \infty,\\ \limsup_{n\to\infty}\frac{1}{n^m}\sum_{i=a}^{n-1}(n-i)^m\left(Lk^\gamma q(i)\delta(i)\left(\frac{\tau(i)}{i+1}\right)^\gamma - \frac{p(i)(\Delta\delta(i))_+^{\gamma+1}}{(\gamma+1)^{\gamma+1}(\delta(i))^\gamma}\right) = \infty,\\ \Delta p(n) \ge 0,\end{gathered} \tag{3.38}$$

where $(\Delta\delta(i))_+ = \max\{0,\Delta\delta(i)\}$, then (1.15) is oscillatory.

Now, we give some sufficient conditions when (1.12) holds, which guarantee that every solution of (1.1) oscillates or converges to zero in $[a,\infty)$.

Theorem 3.11. *Assume (1.12) holds and $p \in C^1_{\mathrm{rd}}([a,\infty),\mathbb{R})$. Furthermore, assume that there exists a positive function $\delta \in C^1_{\mathrm{rd}}([a,\infty),\mathbb{R})$ such that for some positive constant $k \in (0,1)$, (3.10) holds. If*

$$\int_a^\infty\left[\frac{1}{p(t)}\int_a^t q(s)\Delta s\right]^{1/\gamma}\Delta t = \infty, \tag{3.39}$$

then every solution of (1.1) is either oscillatory or converging to zero on $[a,\infty)$.

Proof. We proceed as in Theorem 3.3, we assume that (1.1) has a nonoscillatory solution such that $x(t) > 0$, and $x(\tau(t)) > 0$, for all $t \geq t_1 > a$.

From the proof of Lemma 3.2, we see that there exist two possible cases for the sign of $x^{\Delta}(t)$. The proof when $x^{\Delta}(t)$ is an eventually positive is similar to that of the proof of Theorem 3.3 and hence it is omitted.

Next, suppose that $x^{\Delta}(t) < 0$ for $t \geq t_1 > a$. Then $x(t)$ is decreasing and $\lim_{t\to\infty} x(t) = b \geq 0$. We assert that $b = 0$. If not, then $x(\tau(t)) > x(t) > x(\sigma(t)) > b > 0$ for $t \geq t_2 > t_1$. Since $f(x(\tau(t))) \geq Lb^{\gamma}$, there exists a number $t_3 > t_2$ such that $f(x(\tau(t))) \geq L(x(\tau(t)))^{\gamma}$ for $t \geq t_3$. Defining the function

$$u(t) = p(t)(x^{\Delta}(t))^{\gamma}, \tag{3.40}$$

we obtain from (1.1)

$$u^{\Delta}(t) = -q(t)f(x(\tau(t))) \leq -Lq(t)(x(\tau(t)))^{\gamma} \leq -Lb^{\gamma}q(t), \quad \text{for } t \geq t_3. \tag{3.41}$$

Hence, for $t \geq t_3$, we have

$$u(t) \leq u(t_3) - Lb^{\gamma}\int_a^t q(s)\Delta s \leq -Lb^{\gamma}\int_a^t q(s)\Delta s, \tag{3.42}$$

because of $u(t_3) = p(t_3)(x^{\Delta}(t_3))^{\gamma} < 0$. So, we have

$$\int_{t_3}^t x^{\Delta}(s)\Delta s \leq -L^{1/\gamma}b\int_{t_3}^t \left[\frac{1}{p(s)}\int_{t_3}^s q(\tau)\Delta\tau\right]^{1/\gamma}\Delta s. \tag{3.43}$$

By condition (3.39) we get $x(t) \to -\infty$ as $t \to \infty$, and this is a contradiction to the fact that $x(t) > 0$ for $t \geq t_1$. Thus $b = 0$ and then $x(t) \to 0$ as $t \to \infty$. The proof is complete. □

Similar to that of the proof of Theorem 3.11, we omit the proof of the following theorem.

Theorem 3.12. *Assume (1.12) holds and $p \in C^1_{\mathrm{rd}}([a,\infty),\mathbb{R})$. Furthermore, assume that there exists a positive function $\delta \in C^1_{\mathrm{rd}}([a,\infty),\mathbb{R})$ such that for some positive constant $k \in (0,1)$, (3.31), and (3.39) hold. Then every solution of (1.1) is either oscillatory or converging to zero on $[a,\infty)$.*

From Theorems 3.11 and 3.12, we have the following results for (1.14) and (1.15).

Corollary 3.13. *If there exists a positive function $\delta \in C^1([a,\infty),\mathbb{R})$ such that for some positive constant $k \in (0,1)$,*

$$\begin{gathered} p'(t) \geq 0, \quad \int_a^{\infty}\left(\frac{1}{p(t)}\right)^{1/\gamma}\mathrm{d}t < \infty, \quad \int_a^{\infty}\left[\frac{1}{p(t)}\int_a^t q(s)\mathrm{d}s\right]^{1/\gamma}\mathrm{d}t = \infty, \\ \limsup_{t\to\infty}\int_a^t\left(Lk^{\gamma}q(s)\delta(s)\left(\frac{\tau(s)}{s}\right)^{\gamma} - \frac{p(s)(\delta'(s))_+^{\gamma+1}}{(\gamma+1)^{\gamma+1}(\delta(s))^{\gamma}}\right)\mathrm{d}s = \infty, \end{gathered} \tag{3.44}$$

where $(\delta'(s))_+ = \max\{0,\delta'(s)\}$, then every solution of (1.14) is either oscillatory or converging to zero on $[a,\infty)$.

Corollary 3.14. *If there exists a positive sequence* $\{\delta(n)\}$ *such that for some positive constant* $k \in (0,1)$,

$$\Delta p(n) \geq 0, \quad \sum_{i=a}^{\infty}\left(\frac{1}{p(i)}\right)^{1/\gamma} < \infty, \quad \sum_{i=a}^{\infty}\left(\frac{1}{p(i)}\sum_{j=a}^{i-1} q(j)\right)^{1/\gamma} = \infty,$$
$$\limsup_{n\to\infty}\sum_{i=a}^{n-1}\left(Lk^{\gamma}q(i)\delta(i)\left(\frac{\tau(i)}{i+1}\right)^{\gamma} - \frac{p(i)(\Delta\delta(i))_+^{\gamma+1}}{(\gamma+1)^{\gamma+1}(\delta(i))^{\gamma}}\right) = \infty, \tag{3.45}$$

where $(\Delta\delta(i))_+ = \max\{0,\Delta\delta(i)\}$, *then every solution of (1.15) is either oscillatory or converging to zero on* $[a,\infty)$.

Corollary 3.15. *If there exists a positive function* $\delta \in C^1([a,\infty),\mathbb{R})$ *such that for some positive constant* $k \in (0,1)$, $m \geq 1$,

$$p'(t) \geq 0, \quad \int_a^{\infty}\left(\frac{1}{p(t)}\right)^{1/\gamma} \mathrm{d}t < \infty, \quad \int_a^{\infty}\left[\frac{1}{p(t)}\int_a^t q(s)\mathrm{d}s\right]^{1/\gamma}\mathrm{d}t = \infty,$$
$$\limsup_{t\to\infty}\frac{1}{t^m}\int_a^t (t-s)^m\left(Lk^{\gamma}q(s)\delta(s)\left(\frac{\tau(s)}{s}\right)^{\gamma} - \frac{p(s)(\delta'(s))_+^{\gamma+1}}{(\gamma+1)^{\gamma+1}(\delta(s))^{\gamma}}\right)\mathrm{d}s = \infty, \tag{3.46}$$

where $(\delta'(s))_+ = \max\{0,\delta'(s)\}$, *then every solution of (1.14) is either oscillatory or converging to zero on* $[a,\infty)$.

Corollary 3.16. *If there exists a positive sequence* $\{\delta(n)\}$ *such that for some positive constant* $k \in (0,1)$, $m \geq 1$,

$$\Delta p(n) \geq 0, \quad \sum_{i=a}^{\infty}\left(\frac{1}{p(i)}\right)^{1/\gamma} < \infty, \quad \sum_{i=a}^{\infty}\left(\frac{1}{p(i)}\sum_{j=a}^{i-1} q(j)\right)^{1/\gamma} = \infty,$$
$$\limsup_{n\to\infty}\frac{1}{n^m}\sum_{i=a}^{n-1}(n-i)^m\left(Lk^{\gamma}q(i)\delta(i)\left(\frac{\tau(i)}{i+1}\right)^{\gamma} - \frac{p(i)(\Delta\delta(i))_+^{\gamma+1}}{(\gamma+1)^{\gamma+1}(\delta(i))^{\gamma}}\right) = \infty, \tag{3.47}$$

where $(\Delta\delta(i))_+ = \max\{0,\Delta\delta(i)\}$, *then every solution of (1.15) is either oscillatory or converges to zero on* $[a,\infty)$.

In [8], Saker considered the second-order half-linear dynamic equations on time scales

$$\left(p(t)(x^{\Delta}(t))^{\gamma}\right)^{\Delta} + q(t)x^{\gamma}(t) = 0 \quad \text{for } t \in [a,b] \tag{3.48}$$

and established following main oscillation criteria of (3.48).

Theorem A (Theorem 3.3, Saker [8]). *Assume that*

(H) *p and q are positive, real-valued rd-continuous functions, and* $\gamma > 1$ *is an odd positive integer, and*

$$\int_a^{\infty}\left(\frac{1}{p(t)}\right)^{1/\gamma}\Delta t = \infty \tag{3.49}$$

hold. Furthermore, assume that there exists a positive Δ*-differentiable function* δ *such that*

$$\limsup_{t\to\infty}\int_a^t\left(\delta(s)q(s)-\frac{p(s)(\delta^{\Delta}(s))_+^{\gamma+1}}{(\gamma+1)^{\gamma+1}\delta^{\gamma}(s)}\right)\Delta s=\infty, \tag{3.50}$$

where $(\delta^{\Delta}(t))_+=\max\{0,\delta^{\Delta}(t)\}$. *Then every solution of (3.48) is oscillatory on* $[a,\infty)$.

One can easily see that the result obtained in [8] cannot be applied for (1.1), so our results are new ones.

4. Applications

In this section, we give some examples to illustrate our main results. To obtain the conditions for oscillation, we will use the fact

$$\int_a^{\infty}\frac{\Delta t}{t^p}=\infty \quad \text{if } 0\le p\le 1. \tag{4.1}$$

For more details we refer the reader to [4, Theorem 5.68].

Example 4.1. Consider the second-order delay dynamic equations on time scales

$$\left(t^{\gamma-1}(x^{\Delta}(t))^{\gamma}\right)^{\Delta}+\frac{\beta}{t^2}\left(\frac{\sigma(t)}{\tau(t)}\right)^{\gamma}f(x(\tau(t)))=0, \quad t\in[1,\infty), \tag{4.2}$$

where $p(t)=t^{\gamma-1}$, $q(t)=(\beta/t^2)(\sigma(t)/\tau(t))^{\gamma}$, $f(x)=x^{\gamma}$, $\gamma>1$, $\beta>1$, $k=1/(\gamma+1)^{(\gamma+1)/\gamma}$. By Corollary 3.4, we have

$$\int_1^{\infty}\left(\frac{1}{s}\right)^{(\gamma-1)/\gamma}\Delta s=\infty,$$

$$\limsup_{t\to\infty}\int_1^t\left(k^{\gamma}s\frac{\beta}{s^2}\left(\frac{\sigma(s)}{\tau(s)}\right)^{\gamma}\left(\frac{\tau(s)}{\sigma(s)}\right)^{\gamma}-\frac{s^{\gamma-1}}{(\gamma+1)^{\gamma+1}s^{\gamma}}\right)\Delta s=\limsup_{t\to\infty}\int_1^t\frac{\beta-1}{(\gamma+1)^{\gamma+1}s}\Delta s=\infty. \tag{4.3}$$

Then (4.2) is oscillatory on $[1,\infty)$.

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References

[1] S. Hilger, "Analysis on measure chains—a unified approach to continuous and discrete calculus," *Results in Mathematics*, vol. 18, no. 1-2, pp. 18–56, 1990.

[2] R. P. Agarwal, M. Bohner, D. O'Regan, and A. Peterson, "Dynamic equations on time scales: a survey," *Journal of Computational and Applied Mathematics*, vol. 141, no. 1-2, pp. 1–26, 2002.

[3] M. Bohner and A. Peterson, *Dynamic Equations on Time Scales. An Introduction with Applications*, Birkhäuser, Boston, Mass, USA, 2001.

[4] M. Bohner and A. Peterson, Eds., *Advances in Dynamic Equations on Time Scales*, Birkhäuser, Boston, Mass, USA, 2003.

[5] M. Bohner and S. H. Saker, "Oscillation of second order nonlinear dynamic equations on time scales," *The Rocky Mountain Journal of Mathematics*, vol. 34, no. 4, pp. 1239–1254, 2004.

[6] L. Erbe, "Oscillation results for second-order linear equations on a time scale," *Journal of Difference Equations and Applications*, vol. 8, no. 11, pp. 1061–1071, 2002.

[7] L. Erbe, A. Peterson, and S. H. Saker, "Oscillation criteria for second-order nonlinear dynamic equations on time scales," *Journal of the London Mathematical Society. Second Series*, vol. 67, no. 3, pp. 701–714, 2003.

[8] S. H. Saker, "Oscillation criteria of second-order half-linear dynamic equations on time scales," *Journal of Computational and Applied Mathematics*, vol. 177, no. 2, pp. 375–387, 2005.

[9] S. H. Saker, "Oscillation of nonlinear dynamic equations on time scales," *Applied Mathematics and Computation*, vol. 148, no. 1, pp. 81–91, 2004.

[10] R. P. Agarwal, M. Bohner, and S. H. Saker, "Oscillation of second order delay dynamic equations," *The Canadian Applied Mathematics Quarterly*, vol. 13, no. 1, pp. 1–17, 2005.

[11] R. P. Agarwal, D. O'Regan, and S. H. Saker, "Oscillation criteria for second-order nonlinear neutral delay dynamic equations," *Journal of Mathematical Analysis and Applications*, vol. 300, no. 1, pp. 203–217, 2004.

[12] M. Bohner, "Some oscillation criteria for first order delay dynamic equations," *Far East Journal of Applied Mathematics*, vol. 18, no. 3, pp. 289–304, 2005.

[13] Y. Sahiner, "Oscillation of second-order delay differential equations on time scales," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 63, no. 5–7, pp. e1073–e1080, 2005.

[14] Y. Sahiner, "Oscillation of second-order neutral delay and mixed-type dynamic equations on time scales," *Advances in Difference Equations*, vol. 2006, Article ID 65626, 9 pages, 2006.

[15] S. H. Saker, "Oscillation of second-order nonlinear neutral delay dynamic equations on time scales," *Journal of Computational and Applied Mathematics*, vol. 187, no. 2, pp. 123–141, 2006.

[16] B. G. Zhang and X. Deng, "Oscillation of delay differential equations on time scales," *Mathematical and Computer Modelling*, vol. 36, no. 11–13, pp. 1307–1318, 2002.

[17] B. G. Zhang and S. Zhu, "Oscillation of second-order nonlinear delay dynamic equations on time scales," *Computers & Mathematics with Applications*, vol. 49, no. 4, pp. 599–609, 2005.

[18] L. Erbe, A. Peterson, and S. H. Saker, "Hille-Kneser-type criteria for second-order dynamic equations on time scales," *Advances in Difference Equations*, vol. 2006, Article ID 51401, 18 pages, 2006.

[19] Z. Han, S. Sun, and B. Shi, "Oscillation criteria for a class of second-order Emden-Fowler delay dynamic equations on time scales," to appear in *Journal of Mathematical Analysis and Applications*.

[20] R. P. Agarwal, S.-L. Shieh, and C.-C. Yeh, "Oscillation criteria for second-order retarded differential equations," *Mathematical and Computer Modelling*, vol. 26, no. 4, pp. 1–11, 1997.

[21] S. Z. Chen and L. H. Erbe, "Riccati techniques and discrete oscillations," *Journal of Mathematical Analysis and Applications*, vol. 142, no. 2, pp. 468–487, 1989.

[22] S. Z. Chen and L. H. Erbe, "Oscillation and nonoscillation for systems of self-adjoint second-order difference equations," *SIAM Journal on Mathematical Analysis*, vol. 20, no. 4, pp. 939–949, 1989.

[23] L. Erbe, "Oscillation criteria for second order nonlinear delay equations," *Canadian Mathematical Bulletin*, vol. 16, pp. 49–56, 1973.
[24] J. Ohriska, "Oscillation of second order delay and ordinary differential equation," *Czechoslovak Mathematical Journal*, vol. 34(109), no. 1, pp. 107–112, 1984.
[25] E. Thandapani, K. Ravi, and J. R. Graef, "Oscillation and comparison theorems for half-linear second-order difference equations," *Computers & Mathematics with Applications*, vol. 42, no. 6-7, pp. 953–960, 2001.
[26] Z. Zhang, J. Chen, and C. Zhang, "Oscillation of solutions for second-order nonlinear difference equations with nonlinear neutral term," *Computers & Mathematics with Applications*, vol. 41, no. 12, pp. 1487–1494, 2001.

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