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Research Article

Dynamic Analysis of Stochastic Reaction-Diffusion Cohen-Grossberg Neural Networks with Delays

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Stochastic effects on convergence dynamics of reaction-diffusion Cohen-Grossberg neural networks (CGNNs) with delays are studied. By utilizing Poincaré inequality, constructing suitable Lyapunov functionals, and employing the method of stochastic analysis and nonnegative semimartingale convergence theorem, some sufficient conditions ensuring almost sure exponential stability and mean square exponential stability are derived. Diffusion term has played an important role in the sufficient conditions, which is a preeminent feature that distinguishes the present research from the previous. Two numerical examples and comparison are given to illustrate our results.

1. Introduction

In the recent years, the problems of stability of delayed neural networks have received much attention due to its potential application in associative memories, pattern recognition and optimization. A large number of results have appeared in literature, see, for example, [1–14]. As is well known, a real system is usually affected by external perturbations which in many cases are of great uncertainty and hence may be treated as random [15–17]. As pointed out by Haykin [18] that in real nervous systems synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes, it is of significant importance to consider stochastic effects for neural networks. In recent years, the dynamic behavior of stochastic neural networks, especially the stability of stochastic neural networks, has become a hot study topic. Many interesting results on stochastic effects to the stability of delayed neural networks have been reported (see [16–23]).

In the factual operations, on other hand, diffusion phenomena could not be ignored in neural networks and electric circuits once electrons transport in a nonuniform electromagnetic field. Thus, it is essential to consider state variables varying with time and space variables. The delayed neural networks with diffusion terms can commonly be expressed by partial functional differential equation (PFDE). To study the stability of delayed reaction-diffusion neural networks, for instance, see [24–31], and references therein.

Based on the above discussion, it is significant and of prime importance to consider the stochastic effects on the stability property of the delayed reaction-diffusion networks. Recently, Sun et al. [32, 33] have studied the problem of the almost sure exponential stability and the moment exponential stability of an equilibrium solution for stochastic reaction-diffusion recurrent neural networks with continuously distributed delays and constant delays, respectively. Wan et al. have derived the sufficient condition of exponential stability of stochastic reaction-diffusion CGNNs with delay [34, 35]. In [36], the problem of stochastic exponential stability of the delayed reaction-diffusion recurrent neural networks with Markovian jumping parameters have been investigated. In [32–36], unfortunately, reaction-diffusion terms were omitted in the deductions, which result in that the criteria of obtained stability do not contain the diffusion terms. In other words, the diffusion terms do not take effect in their results. The same cases appear also in other research literatures on the stability of reaction-diffusion neural network [24–31].

Motivated by the above discussions, in this paper, we will further investigate the convergence dynamics of stochastic reaction-diffusion CGNNs with delays, where the activation functions are not necessarily bounded, monotonic, and differentiable. Utilizing Poincaré inequality and constructing appropriate Lyapunov functionals, some sufficient conditions on the almost surely and mean square exponential stability for the equilibrium point are established. The results show that diffusion terms have contributed to the almost surely and mean square exponential stability criteria. Two examples have been provided to illustrate the effectiveness of the obtained results.

The rest of this paper is organized as follows. In Section 2, a stochastic delayed reaction-diffusion CGNNs model is described, and some preliminaries are given. In Section 3, some sufficient conditions to guarantee the mean square and almost surely exponential stability of equilibrium point for the reaction-diffusion delayed CGNNs are derived. Examples and comparisons are given in Section 4. Finally, in Section 5, conclusions are given.

2. Model Description and Preliminaries

To begin with, we introduce some notations and recall some basic definitions and lemmas:

(i) X be an open bounded domain in $\mathbb{R}^m$ with smooth boundary ∂X, and $\mathrm{mes}X > 0$ denotes the measure of X. $\overline{X} = X \cup \partial x$;

(ii) $L^2(X)$ is the space of real Lebesgue measurable functions on X which is a Banach space for the L^2-norm $\|v(x)\|_2 = \left(\int_X |v(x)|^2 dx\right)^{1/2}$, $v \in L^2(X)$;

(iii) $H^1(X) = \{w \in L^2(X),\ D_i w \in L^2(X)\}$, where $D_i w = \partial w/\partial x_i$, $1 \leq i \leq m$. $H_0^1(X) =$ the closure of $C_0^\infty(X)$ in $H^1(X)$;

(iv) $C = C(I \times X, \mathbb{R}^n)$ is the space of continuous functions which map $I \times X$ into $\mathbb{R}^n$ with the norm $\|u(t,x)\|_2 = (\sum_{i=1}^n \|u_i(t,x)\|_2^2)^{1/2}$, for any $u(t,x) = (u_1(t,x),\ldots,u_n(t,x))^T \in C$;

(v) $\zeta = \{(\phi_1(s,x),\ldots,\phi_n(s,x))^T : [-\tau,0]\} \in BC([-\tau,0] \times X, R^n)$ and be an $\mathcal{F}_0$-measurable $\mathbb{R}$-valued random variable, where, for example, $\mathcal{F}_0 = \mathcal{F}_s$ restricted on $[-\tau,0]$, and BC be the Banach space of continuous and bounded functions with the norm $\|\phi\|_\tau = (\Sigma_{i=1}^n \|\phi_i\|_\tau^2)^{1/2}$, where $\|\phi_i\|_\tau = \sup_{-\tau\le s\le 0}\|\phi_i(s,x)\|_2$, for any $\phi(s,x) = (\phi_1(s,x),\ldots,\phi_n(s,x))^T \in BC$, $i = 1,\ldots,n$;

(vi) $\nabla v = (\partial v/\partial x_1,\ldots,\partial v/\partial x_m)$ is the gradient operator, for $v \in C^1(X)$. $|\nabla v|^2 = \sum_{l=1}^m |\partial v/\partial x_m|^2$. $\Delta u = \sum_{l=1}^m (\partial^2 u/\partial x_l^2)$ is the Laplace operator, for $u \in C^2(X)$.

Consider the following stochastic reaction-diffusion CGNNs with constant delays on X:

$$
\begin{aligned}
du_i(t,x) = {} & \Sigma_{l=1}^m \frac{\partial}{\partial x_l}\left(D_{il}\frac{\partial u_i(t,x)}{\partial x_l}\right)dt - a_i(u_i(t,x)) \\
& \times \left[b_i(u_i(t,x)) - \Sigma_{j=1}^n w_{ij} f_j\big(u_j(t,x)\big) - \Sigma_{j=1}^n v_{ij} g_j\big(u_j(t-\tau_j,x)\big) + J_i\right]dt \\
& + \sum_{j=1}^n \sigma_{ij}(u_i(t,x))dw_j(t), \quad (t,x) \in [0,+\infty) \times X,
\end{aligned} \tag{2.1}
$$

$$
B[u_i(t,x)] = 0, \quad (t,x) \in [0,+\infty) \times \partial X,
$$

$$
u_i(t,x) = \phi_i(s,x), \quad (s,x) \in [-\tau,0] \times \overline{X},
$$

where $i = 1,\ldots,n$, $n \ge 2$ corresponds to the number of units in a neural network; $x = (x_1,\ldots,x_m)^T \in X$ is a space variable, $u_i(t,x)$ corresponds to the state of the ith unit at time $t$and in space x; $D_{il} > 0$ corresponds to the transmission diffusion coefficient along the ith neuron; $a_i(u_i(t,x))$ represents an amplification function; $b_i(u_i(t,x))$ is an appropriately behavior function; w_{ij}, v_{ij} denote the connection strengths of the jth neuron on the ith neuron, respectively; $g_j(u_j(t,x))$, $f_j(u_j(t,x))$ denote the activation functions of jth neuron at time t and in space x; τ_j corresponds to the transmission delay and satisfies $0 \le \tau_j \le \tau$ (τ is a positive constant); J_i is the constant input from outside of the network. Moreover, $w(t) = (w_1(t),\ldots,w_n(t))^T$ is an n-dimensional Brownian motion defined on a complete probability space $(\Omega,\mathcal{F},\mathbb{P})$ with the natural filtration $\{\mathcal{F}_t\}_{t\ge0}$ generated by the process $\{w(s) : 0 \le s \le t\}$, where we associate Ω with the canonical space generated by all $\{w_i(t)\}$, and denote by $\mathcal{F}$ the associated σ-algebra generated by $w(t)$ with the probability measure $\mathbb{P}$. The boundary condition is given by $B[u_i(t,x)] = u_i(t,x)$ (Dirichlet type) or $B[u_i(t,x)] = \partial u_i(t,x)/\partial m$ (Neumann type), where $\partial u_i(t,x)/\partial m = \partial u_i(t,x)/\partial x_1,\ldots,\partial u_i(t,x)/\partial x_m)^T$ denotes the outward normal derivative on ∂X.

Model (2.1) includes the following reaction-diffusion recurrent neural networks (RNNs) as a special case:

$$
\begin{aligned}
du_i(t,x) &= \Sigma_{l=1}^{m} \frac{\partial}{\partial x_l}\left(D_{il}\frac{\partial u_i(t,x)}{\partial x_l}\right)dt \\
&\quad + \left[-b_i(u_i(t,x)) + \Sigma_{j=1}^{n} w_{ij} f_j\big(u_j(t,x)\big) + \Sigma_{j=1}^{n} v_{ij} g_j\big(u_j\big(t-\tau_j,x\big)\big) + J_i\right]dt \\
&\quad + \sum_{j=1}^{n} \sigma_{ij}\big(u_j(t,x)\big)dw_j(t), \quad (t,x) \in [0,+\infty)\times X,
\end{aligned}
\tag{2.2}
$$

$$
B[u_i(t,x)] = 0, \quad (t,x) \in [0,+\infty)\times \partial X,
$$

$$
u_i(t,x) = \phi_i(s,x), \quad (s,x) \in [-\tau,0]\times \overline{X},
$$

for $i = 1,\ldots,n$.

When $w_i(t) = 0$ for any $i = 1,\ldots,n$, model (2.1) also comprises the following reaction-diffusion CGNNs with no stochastic effects on space X:

$$
\begin{aligned}
\frac{\partial u_i(t,x)}{\partial t} &= \Sigma_{l=1}^{m} \frac{\partial}{\partial x_l}\left(D_{il}\frac{\partial u_i(t,x)}{\partial x_l}\right) - a_i(u_i(t,x)) \\
&\quad \times \left[b_i(u_i(t,x)) - \Sigma_{j=1}^{n} w_{ij} f_j\big(u_j(t,x)\big) - \Sigma_{j=1}^{n} v_{ij} g_j\big(u_j\big(t-\tau_j,x\big)\big) + J_i\right], \\
&\qquad (t,x) \in [0,+\infty)\times X,
\end{aligned}
\tag{2.3}
$$

$$
B[u_i(t,x)] = 0, \quad (t,x) \in [0,+\infty)\times \partial X,
$$

$$
u_i(t,x) = \phi_i(s,x), \quad (s,x) \in [-\tau,0]\times \overline{X},
$$

for $i = 1,\ldots,n$.

Throughout this paper, we assume that

(H1) each function $a_i(\xi)$ is bounded, positive and continuous, that is, there exist constants $\underline{a}_i$, $\overline{a}_i$ such that $0 < \underline{a}_i \le a_i(\xi) \le \overline{a}_i < \infty$, for $\xi \in R$, $i = 1,\ldots,n$,

(H2) $b_i(\xi) \in C^1(\mathbb{R},\mathbb{R})$ and $b_i = \inf_{\xi\in\mathbb{R}} b_i'(\xi) > 0$, for $i = 1,\ldots,n$,

(H3) f_j, g_j are bounded, and f_j, g_j, σ_{ij} are Lipschitz continuous with Lipschitz constant F_j, G_j, $L_{ij} > 0$, for $i,j = 1,\ldots,n$,

(H4) $\sigma_{ij}(u_i^*) = 0$, for $i,l = 1,\ldots,n$.

Using the similar method of [25], it is easily to prove that under assumptions (H1)–(H3), model (2.3) has a unique equilibrium point $u^* = (u_1^*,\ldots,u_n^*)^T$ which satisfies

$$
b_i(u_i^*) - \Sigma_{j=1}^{n} w_{ij} f_j\left(u_j^*\right) - \Sigma_{j=1}^{n} v_{ij} g_j\left(u_j^*\right) + J_i = 0, \quad i = 1,\ldots,n. \tag{2.4}
$$

Suppose that system (2.1) satisfies assumptions (H1)–(H4), then equilibrium point u^* of model (2.3) is also a unique equilibrium point of system (2.1).

By the theory of stochastic differential equations, see [15, 37], it is known that under the conditions (H1)–(H4), model (2.1) has a global solution denoted by $u(t,0,x;\phi)$ or simply $u(t,\phi)$, $u(t,x)$ or $u(t)$, if no confusion should occur. For the effects of stochastic forces to the stability property of delayed CGNNs model (2.1), we will study the almost sure exponential stability and the mean square exponential stability of their equilibrium solution $u(t) \equiv u^*$ in the following sections. For completeness, we give the following definitions [33], in which $\mathbb{E}$ denotes expectation with respect to $\mathbb{P}$.

Definition 2.1. The equilibrium solution u^* of model (2.1) is said to be almost surely exponentially stable, if there exists a positive constant μ such that for any ϕ there is a finite positive random variable M such that

$$\left\|u(t,\phi) - u^*\right\|_2 \le Me^{-\mu t} \quad \forall t \ge 0. \tag{2.5}$$

Definition 2.2. The equilibrium solution u^* of model (2.1) is said to be pth moment exponentially stable, if there exists a pair of positive constants μ and M such that for any ϕ,

$$\mathbb{E}\left\|u(t,\phi) - u^*\right\|_2^p \le M\mathbb{E}\left\|\phi - u^*\right\|_\tau^p e^{-\mu t} \quad \forall t \ge 0. \tag{2.6}$$

When $p = 1$ and 2, it is usually called the exponential stability in mean value and mean square, respectively.

The following lemmas are important in our approach.

Lemma 2.3 (nonnegative semimartingale convergence theorem [16]). *Suppose $A(t)$ and $U(t)$ are two continuous adapted increasing processes on $t \ge 0$ with $A(0) = U(0) = 0$, a.s. Let $M(t)$ be a real-valued continuous local martingale with $M(0) = 0$, a.s. and let ζ be a nonnegative $\mathcal{F}_0$-measurable random variable with $\mathbb{E}\zeta < \infty$. Define $X(t) = \zeta + A(t) - U(t) + M(t)$ for $t \ge 0$. If $X(t)$ is nonnegative, then*

$$\left\{\lim_{t\to\infty} A(t) < \infty\right\} \subset \left\{\lim_{t\to\infty} X(t) < \infty\right\} \cap \left\{\lim_{t\to\infty} U(t) < \infty\right\} \quad a.s., \tag{2.7}$$

where $B \subset D$ a.s. denotes $\mathbb{P}(B \cup D^c) = 0$. In particular, if $\lim_{t\to\infty} A(t) < \infty$ a.s., then for almost all $w \in \Omega \lim_{t\to\infty} X(t,w) < \infty$ and $\lim_{t\to\infty} U(t,w) < \infty$, that is, both $X(t)$ and $U(t)$ converge to finite random variables.

Lemma 2.4 (Poincaré inequality). *Let X be a bounded domain of R^m with a smooth boundary ∂X of class $\mathcal{C}^2$ by X. $v(x)$ is a real-valued function belonging to $H_0^1(X)$ and satisfies $B[v(x)]|_{\partial X} = 0$. Then*

$$\lambda_1 \int_X |v(x)|^2 dx \le \int_X |\nabla v(x)|^2 dx, \tag{2.8}$$

which λ_1 *is the lowest positive eigenvalue of the boundary value problem*

$$\begin{gathered} -\Delta\psi(x) = \lambda\psi(x), \quad x \in X, \\ B[\psi(x)] = 0, \quad x \in \partial X. \end{gathered} \tag{2.9}$$

Proof. We just give a simple sketch here.

Case 1. Under the Neumann boundary condition, that is, $B[v(x)] = \partial v(x)/\partial m$. According to the eigenvalue theory of elliptic operators, the Laplacian $-\Delta$ on X with the Neumann boundary conditions is a self-adjoint operator with compact inverse, so there exists a sequence of nonnegative eigenvalues (going to $+\infty$) and a sequence of corresponding eigenfunctions, which are denoted by $0 = \lambda_0 < \lambda_1 < \lambda_2 < \cdots$ and $\psi_0(x), \psi_1(x), \psi_2(x), \ldots$, respectively. In other words, we have

$$\begin{gathered} \lambda_0 = 0, \qquad \psi_0(x) = 1, \\ -\Delta\psi_k(x) = \lambda_k\psi_k(x), \quad \text{in } X, \\ \psi_k(x) = 0, \quad \text{on } \partial X, \end{gathered} \tag{2.10}$$

where $k \in \mathbb{N}$. Multiply the second equation of (2.10) by $\psi_k(x)$ and integrate over X. By Green's theorem, we obtain

$$\int_X \left|\nabla\psi_k(x)\right|^2 dx = \lambda_k \int_X \psi_k^2(x)dx, \quad \text{for } k \in \mathbb{N}. \tag{2.11}$$

Clearly, (2.11) can also hold for $k = 0$. The sequence of eigenfunctions $\{\psi_k(x)\}_{k=0}^{\infty}$ defines an orthonormal basis of $L^2(X)$. For any $v(x) \in H_0^1(X)$, we have

$$v(x) = \sum_{k=0}^{\infty} c_k\psi_k. \tag{2.12}$$

From (2.11) and (2.12), we can obtain

$$\int_X |\nabla v(x)|^2 dx \geq \lambda_1 \int_X |v(x)|^2 dx. \tag{2.13}$$

Case 2. Under the Dirichlet boundary condition, that is, $B[v(x)] = v(x)$. By the same may, we can obtained the inequality.

This completes the proof. □

Remark 2.5. (i) The lowest positive eigenvalue λ_1 in the boundary problem (2.9) is sometimes known as the first eigenvalue. (ii) The magnitude of λ_1 is determined by domain X. For example, let Laplacian on $X = \{(x_1, x_2)^T \in \mathbb{R}^2 \mid 0 < x_1 < a,\ 0 < x_2 < b\}$, if $B[v(x)] = v(x)$ and $B[v(x)] = \partial v(x)/\partial m$, respectively, then $\lambda_1 = (\pi/a)^2 + (\pi/b)^2$ and $\lambda_1 = \min\{(\pi/a)^2, (\pi/b)^2\}$

[38, 39]. (iii) Although the eigenvalue λ_1 of the laplacian with the Dirichlet boundary condition on a generally bounded domain X cannot be determined exactly, a lower bound of it may nevertheless be estimated by $\lambda_1 \geq (m^2/(m+2))((2\pi)^2/\omega_{m-1})(1/V)^{2/m}$, where ω_{m-1} is a surface area of the unit ball in $\mathbb{R}^m$, V is a volume of domain X [40].

In Section 4, we will compare the results between this paper and previous literatures. To this end, we recall some previous results as follows (according to the symbols in this paper).

In [35], Wan and Zhou have studied the problem of convergence dynamics of model (2.1) with the Neumann boundary condition and obtained the following result (see [35, Theorem 3.1]).

Proposition 2.6. *Suppose that system* (2.1) *satisfies the assumptions (H1)–(H4) and*

(A) $C > 0$, $\rho(C^{-1}(A_1W^+F + A_1V^+G)) < 1$, *where* $C = \operatorname{diag}(\delta_1, \ldots, \delta_n)$, $\delta_i = \underline{a}_i b_i - (1/2)\sum_{j=1}^{n} L_{ij}^2$, $i = 1, \ldots, n$, $A_1 = \operatorname{diag}(\overline{a}_1, \ldots, \overline{a}_n)$, $W^+ = (|w_{ij}|)_{n\times n}$, $V^+ = (|v_{ij}|)_{n\times n}$, $F = \operatorname{diag}(F_1, \ldots, F_n)$, $G = \operatorname{diag}(G_1, \ldots, G_n)$. *Also,* $\rho(A)$ *denotes the spectral radius of a square matrix* A.

Then model (2.1) *is mean value exponentially stable.*

Remark 2.7. It should be noted that condition (A) in Proposition 2.6 is equivalent to $C - (A_1W^+F + A_1V^+G)$ is a nonsingular M-matrix, where $C > 0$. Thus, the following result is treated as a special case of Proposition 2.6.

Proposition 2.8 (see [33, Theorem 3.1]). *Suppose that model* (2.2) *satisfies the assumptions (H2)–(H4) and*

(B) $B - \overline{B} - W^+F - V^+G$ *is a nonsingular* M*-matrix, where* $\overline{B} = \operatorname{diag}\{\overline{b}_1, \ldots, \overline{b}_n\}$, $\overline{b}_i := -b_i + \sum_{j=1}^{n} |w_{ij}|F_j + \sum_{j=1}^{n} |V_{ij}|G_j + \sum_{j=1}^{n} L_{ij}^2 \geq 0$, *for* $1 \leq i \leq n$.

Then model (2.2) *is almost surely exponentially stable.*

Remark 2.9. It is obvious that conditions (A) and (B) are irrelevant to the diffusion term. In other words, the diffusion term does not take effect in Propositions 2.6 and 2.8.

3. Main Results

Theorem 3.1. *Under assumptions (H1)–(H4), if the following conditions hold:*

(H5) $a = 2(\lambda_1 D_i + \underline{a}_i b_i) - \sum_{j=1}^{n}(|w_{ij}|\overline{a}_i F_j + |w_{ji}|\overline{a}_j F_i + |v_{ij}|\overline{a}_i G_j + L_{ij}^2) > b = \sum_{j=1}^{n} |v_{ji}|\overline{a}_j G_i$, *for any* $i = 1, \ldots, n$,

where λ_1 *is the lowest positive eigenvalue of problem* (2.9), $D_i = \min_{1\leq l\leq m}\{D_{il}\}$, $i = 1, \ldots, n$.

Then model (2.1) *is almost surely exponentially stable.*

Proof. Let $u(t) = (u_1(t), \ldots, u_n(t))^T$ be an any solution of model (2.1) and $y_i(t) = u_i(t) - u_i^*$. Model (2.1) is equivalent to

$$dy_i(t) = \Sigma_{l=1}^{m} \frac{\partial}{\partial x_l}\left(D_{il}\frac{\partial y_i(t)}{\partial x_l}\right)dt - a_i(u_i(t))$$

$$\times \left[\widetilde{b}_i(y_i(t)) - \Sigma_{j=1}^{n} v_{ij}\widetilde{g}_j(u_j(t-\tau_j)) - \Sigma_{j=1}^{n} w_{ij}\widetilde{f}_j(y_j(t))\right]dt \tag{3.1a}$$

$$+ \sum_{j=1}^{n}\widetilde{\sigma}_{ij}(y_i(t))dw_j(t), \quad (t,x) \in [0,+\infty) \times \mathrm{X},$$

$$B[y_i(t)] = 0, \quad (t,x) \in [0,+\infty) \times \partial\mathrm{X}, \tag{3.1b}$$

$$y_i(s,x) = \phi_i(s,x) - u_i^*, \quad (s,x) \in [-\tau, 0] \times \overline{\mathrm{X}}, \tag{3.1c}$$

where

$$\widetilde{b}_i(y_i(t)) = b_i(y_i(t) + u_i^*) - b_i(u_i^*), \qquad \widetilde{f}_j(y_j(t)) = f_j\left(y_i(t) + u_j^*\right) - f_j\left(u_j^*\right),$$
$$\widetilde{g}_j(y_j(t)) = g_j(y_j(t) + u_i^*) - g_j\left(u_j^*\right), \qquad \widetilde{\sigma}_{ij}(y_j(t)) = \sigma_{ij}\left(y_j(t) + u_j^*\right) - \sigma_{ij}\left(u_j^*\right), \tag{3.2}$$

for $i, j = 1, \ldots, n$.

It follows from (H5) that there exists a sufficiently small constant $\mu > 0$ such that

$$2(\lambda_1 D_i + \underline{a}_i b_i) - \mu - \sum_{j=1}^{n}\left(|w_{ij}|\overline{a}_i F_j + |w_{ji}|\overline{a}_j F_i + |v_{ij}|\overline{a}_i G_j + L_{ij}^2\right)$$
$$- \sum_{j=1}^{n}|v_{ji}|\overline{a}_j G_i e^{\mu\tau} > 0, \quad i = 1, \ldots, n. \tag{3.3}$$

To derive the almost surely exponential stability result, we construct the following Lyapunov functional:

$$V(z(t),t) = \sum_{i=1}^{n}\int_{\Omega} e^{\mu t}\left[y_i^2(t) + \overline{a}_i\sum_{j=1}^{n}|v_{ij}|G_j\int_{t-\tau_j}^{t} e^{\mu(s+\tau_j)}y_j^2(s)ds\right]dx. \tag{3.4}$$

By Itô's formula to $V(z(t),t)$ along (3.1a), we obtain

$$
\begin{aligned}
V(z(t),t) &= V(z(0),0) \\
&\quad + \int_0^t e^{\mu s}\sum_{i=1}^{n}\int_\Omega \Bigg\{ \mu y_i^2(s) + 2y_i(s)\frac{\partial}{\partial x_l}\left(D_{il}\frac{\partial y_i(s)}{\partial x_l}\right) - 2y_i(s)a_i(u_i(s)) \\
&\qquad \times \left[\widetilde{b}_i(y_i(s)) + \sum_{j=1}^{n} w_{ij}\widetilde{f}_j(y_j(s)) + \Sigma_{j=1}^{n} v_{ij}\widetilde{g}_j(y_j(s-\tau_j))\right] \\
&\qquad + \overline{a}_i\sum_{j=1}^{n}|v_{ij}|G_j e^{\mu\tau_j}y_j^2(s) - \overline{a}_i\sum_{j=1}^{n}|v_{ij}|G_j y_j^2(s-\tau_j)\Bigg\}\,ds\,dx \\
&\quad + \int_0^t\int_\Omega \sum_{i=1}^{n}\sum_{j=1}^{n} e^{\mu s}\sigma_{ij}^2(y_i(s))\,ds\,dx \\
&\quad + 2\sum_{i=1}^{n}\int_0^t\int_\Omega \Sigma_{j=1}^{n} y_i(s)\sigma_{ij}(y_j(s))\,dw_j(s)\,dx,
\end{aligned}
\tag{3.5}
$$

for $t \ge 0$.

By the boundary condition, it is easy to calculate that

$$
\begin{aligned}
&\int_\Omega y_i(s)\Sigma_{l=1}^{m}\frac{\partial}{\partial x_l}\left(D_{il}\frac{\partial y_i(s)}{\partial x_l}\right)dx \\
&\quad = -\sum_{l=1}^{m}\int_\Omega D_{il}\left(\frac{\partial y_i(s)}{\partial x_l}\right)^2 dx \le -D_i\int_\Omega\sum_{l=1}^{m}\left(\frac{\partial y_i(s)}{\partial x_l}\right)^2 dx \\
&\quad = -D_i\int_\Omega |\nabla y_i(s)|^2 dx \\
&\quad \le -\lambda_1 D_i\int_\Omega y_i^2(s)\,dx = -\lambda_1 D_i\|y_i(s)\|_2^2.
\end{aligned}
\tag{3.6}
$$

From assumptions (H1) and (H2), we have

$$
\int_\Omega y_i(s)\widetilde{a}_i(y_i(s))\widetilde{b}_i(y_i(s))\,dx \ge \underline{a}_i b_i\int_\Omega y_i^2(t)\,dx = \underline{a}_i b_i\|y_i(s)\|_2^2. \tag{3.7}
$$

From assumptions (H1) and (H3), we have

$$
\begin{aligned}
&2\int_{\Omega} y_i(s)\widetilde{a}_i(y_i(s))\sum_{j=1}^{n} w_{ij}\widetilde{f}_i(y_j(s))dx \\
&\quad\le 2\int_{\Omega}\sum_{j=1}^{n}\overline{a}_i|w_{ij}||y_i(s)|\left|\widetilde{f}_i(y_j(s))\right|dx \\
&\quad\le 2\int_{\Omega}\sum_{j=1}^{n}\overline{a}_i|w_{ij}||y_i(s)|F_j|y_j(s)|dx \\
&\quad\le \overline{a}_i\int_{\Omega}\sum_{j=1}^{n}|w_{ij}|F_j y_i^2(t)dx + \overline{a}_i\int_{\Omega}\sum_{j=1}^{n}|w_{ij}|F_j|y_j(s)|^2dx \\
&\quad\le \overline{a}_i\sum_{j=1}^{n}|w_{ij}|F_j\|y_i(s)\|_2^2 + \overline{a}_i\sum_{j=1}^{n}|w_{ij}|F_j\|y_j(s)\|_2^2.
\end{aligned}
\tag{3.8}
$$

By the same way, we can obtain

$$
\begin{aligned}
&2\int_{\Omega} y_i(s)\widetilde{a}_i(y_i(s))\Sigma_{j=1}^{n} v_{ij}\widetilde{g}_i(y_j(s-\tau_j))dx \\
&\quad\le \overline{a}_i\sum_{j=1}^{n}|v_{ij}|G_j\|y_i(s)\|_2^2 + \overline{a}_i\sum_{j=1}^{n}|v_{ij}|G_j\|y_j(s-\tau_j)\|_2^2.
\end{aligned}
\tag{3.9}
$$

Combining (3.6)–(3.9) into (3.5), we get

$$
\begin{aligned}
V(z(t),t) \le V(z(0),0) + \int_0^t e^{\mu s}\Bigg\{\sum_{i=1}^{n}\Bigg[&-2(\lambda_1 D_i + \underline{a}_i b_i) + \mu + \sum_{j=1}^{n}|w_{ij}|\overline{a}_i F_j \\
&+\sum_{j=1}^{n}|w_{ji}|\overline{a}_j F_i + \overline{a}_i\sum_{j=1}^{n}|v_{ij}|G_j\Bigg]\|y_i(s)\|_2^2 \\
&+\overline{a}_i\sum_{j=1}^{n}|v_{ij}|G_j e^{\mu\tau_j}\|y_j(s)\|_2^2\Bigg\}ds \\
+\int_0^t\int_{\Omega}\sum_{i=1}^{n}\sum_{j=1}^{n}&e^{\mu s}\widetilde{\sigma}_{ij}^2(y_i(s))dx\,ds \\
+2\int_0^t\sum_{i=1}^{n}\int_{\Omega}&\Sigma_{j=1}^{n} y_i(s)\widetilde{\sigma}_{ij}(y_j(s))dw_j(s)dx
\end{aligned}
$$

$$
\begin{aligned}
&\leq V(z(0),0) - \int_0^t e^{\mu s}\Bigg\{\sum_{i=1}^n \Bigg[2(\lambda_1 D_i + \underline{a}_i b_i) - \mu \\
&\qquad\qquad - \sum_{j=1}^n \left(|w_{ij}|\overline{a}_i F_j + |w_{ji}|\overline{a}_j F_i + |v_{ij}|\overline{a}_i G_j + L_{ij}^2\right) \\
&\qquad\qquad - \sum_{j=1}^n |v_{ji}|\overline{a}_j G_i e^{\mu\tau}\Bigg] \|y_i(s)\|_2^2\Bigg\} ds \\
&\quad + 2\sum_{i=1}^n \int_\Omega \int_0^t \Sigma_{j=1}^n y_i(s)\widetilde{\sigma}_{ij}(y_j(s))dw_j(s)dx \\
&\leq V(z(0),0) + 2\sum_{i=1}^n \int_\Omega \int_0^t \Sigma_{j=1}^n y_i(s)\widetilde{\sigma}_{ij}(y_j(s))dw_j(s)dx, \quad \text{for } t \geq 0.
\end{aligned}
\tag{3.10}
$$

That is,

$$
V(z(t),t) \leq V(z(0),0) + 2\sum_{i=1}^n \int_0^t \int_\Omega \Sigma_{j=1}^n y_i(s)\sigma_{ij}(u_i(s))dw_j(s)dx, \quad \text{for } t \geq 0. \tag{3.11}
$$

It is obvious that the right-hand side of (3.6) is a nonnegative semimartingale. From Lemma 2.3, it is easy to see that its limit is finite almost surely as $t \to \infty$, which shows that

$$
\limsup_{t\to\infty} V(y(t),t) < \infty, \quad \mathbb{P}\text{-a.s.} \tag{3.12}
$$

That is,

$$
\limsup_{t\to\infty}\left(e^{\mu t}\sum_{i=1}^n \|y_i(t,x)\|_2^2\right) < \infty, \quad \mathbb{P}\text{-a.s.}, \tag{3.13}
$$

which implies

$$
\limsup_{t\to\infty} \frac{1}{t}\ln\left(\sum_{i=1}^n \|y_i(t,x)\|_2^2\right) < -\mu, \quad \mathbb{P}\text{-a.s.}, \tag{3.14}
$$

that is,

$$
\limsup_{t\to\infty} \frac{1}{t}\ln\left(\|y(t,x)\|_2^2\right) < -\mu, \quad \mathbb{P}\text{-a.s.} \tag{3.15}
$$

The proof is complete. □

Theorem 3.2. *Under the conditions of Theorem 3.1, model* (2.1) *is mean square exponentially stable.*

Proof. Taking expectations on both sides of (3.11) and noticing that

$$\mathbb{E}\sum_{i=1}^{n}\int_{0}^{t}\int_{\Omega}\Sigma_{j=1}^{n}y_i(s)\sigma_{ij}(u_j(s))dw_j(s)dx = 0, \tag{3.16}$$

we get

$$\mathbb{E}V(z(t),t) \leq \mathbb{E}V(z(0),0). \tag{3.17}$$

Since

$$\begin{aligned}
V(z(0),0) &= \sum_{i=1}^{n}\int_{\Omega}\left[y_i^2(0) + \overline{a}_i\sum_{j=1}^{n}|v_{ij}|G_j\int_{-\tau_j}^{0}e^{\mu(s+\tau_j)}y_j^2(s)ds\right]dx \\
&= \sum_{i=1}^{n}\int_{\Omega}\left[|\phi_i(0) - u_i^*|^2 + \overline{a}_i\sum_{j=1}^{n}|v_{ij}|G_j^2\left(\int_{-\tau_j}^{0}e^{\mu(s+\tau_j)}\left|\phi_j(s) - u_j^*\right|^2 ds\right)\right]dx \\
&\leq \sum_{i=1}^{n}\|\phi_i - u_i^*\|_\tau^2 + \frac{1}{\mu}\sum_{i=1}^{n}\overline{a}_i\sum_{j=1}^{n}|v_{ij}|(e^{\mu\tau}-1)G_j\left\|\phi_j(s) - u_j^*\right\|_2^2 \\
&\leq \sum_{i=1}^{n}\left(1 + \frac{e^{\mu\tau}-1}{\mu}\sum_{j=1}^{n}\overline{a}_j|v_{ji}|G_i^2\right)\|\phi_i - u_i^*\|_\tau^2 \\
&\leq \max_{i\leq i\leq n}\left\{1 + \frac{e^{\mu\tau}-1}{\mu}\sum_{j=1}^{n}\overline{a}_j|v_{ji}|G_i\right\}\|\phi - u^*\|_\tau^2,
\end{aligned} \tag{3.18}$$

we have

$$\mathbb{E}V(z(0),0) \leq \max_{i\leq i\leq n}\left\{1 + \frac{e^{\mu\tau}-1}{\mu}\sum_{j=1}^{n}\overline{a}_j|v_{ji}|G_i\right\}\mathbb{E}\|\phi - u^*\|_\tau^2. \tag{3.19}$$

Also

$$V(z(t),t) \geq \sum_{i=1}^{n}\int_{\Omega}e^{\mu t}y_i^2(t)dx \geq e^{\mu t}\|y(t)\|_2^2. \tag{3.20}$$

By (3.17)–(3.20), we have

$$e^{\mu t}\mathbb{E}\|y\|_2^2 \leq \max_{i\leq i\leq n}\left\{1 + \frac{e^{\mu\tau}-1}{\mu}\sum_{j=1}^{n}\overline{a}_j|v_{ji}|G_i\right\}\mathbb{E}\|\phi - u^*\|_\tau^2, \quad \forall t \geq 0. \tag{3.21}$$

Let $M = \max_{i\leq i\leq n}\{1 + ((e^{\mu\tau}-1)/\mu)\sum_{j=1}^{n}\overline{a}_j|v_{ji}|G_i\}$.

Then, we easily get

$$\mathbb{E}\|u(t) - u^*\|_2^2 \leq M\mathbb{E}\|\phi - u^*\|_\tau^2 e^{-\mu t}, \quad \forall t \geq 0. \tag{3.22}$$

The proof is completed. □

By the similar way of the proof of Theorem 3.1, it is easy to prove the following results.

Theorem 3.3. *Under assumptions (H2)–(H4), if the following conditions hold:*

(H6) $2(\lambda_1 D_i + b_i) > \sum_{j=1}^{n}(|w_{ij}|F_j + |w_{ji}|F_i + |v_{ij}|G_j + |v_{ji}|G_i + L_{ij}^2)$, $i = 1, \ldots, n$.

Then model (2.2) *is almost surely exponentially stable and mean square exponentially stable.*

Remark 3.4. In the proof of Theorem 3.1, by Poincaré inequality, we have obtained $-D_i\int_\Omega|\nabla y_i|^2 dx \leq -\lambda_1 D_i\|y_i(t)\|_2^2$ (see (3.6)). This is an important step that results in the condition of Theorem 3.1 including the diffusion terms.

Remark 3.5. It should be noted that assumptions (H5) and (H6) allow

$$2\underline{a}_i b_i - \sum_{j=1}^{n}\left(|w_{ij}|\overline{a}_i F_j + |w_{ji}|\overline{a}_j F_i + |v_{ij}|\overline{a}_i G_j + L_{ij}^2\right) \leq \sum_{j=1}^{n}|v_{ji}|\overline{a}_j G_i, \quad i = 1, \ldots, n \tag{3.23}$$

$$2b_i < \sum_{j=1}^{n}\left(|w_{ij}|F_j + |w_{ji}|F_i + |v_{ij}|G_j + |v_{ji}|G_i + L_{ij}^2\right), \quad i = 1, \ldots, n \tag{3.24}$$

respectively, which cannot guarantee the mean square exponential stability of the equilibrium solution of models (2.1) and (2.2). Thus, as we can see form Theorems 3.1, 3.2, and 3.3, reaction-diffusion terms do contribute the almost surely exponential stability and the mean square exponential stability of models (2.1) and (2.2), respectively. However, as we can see from Propositions 2.6 and 2.8, the diffusion term do not take effect in the convergence dynamics of delayed stochastic reaction-diffusion neural networks. Thus, the criteria what we proposed are less conservative and restrictive than Propositions 2.6 and 2.8.

Theorem 3.6. *Under assumptions (H1)–(H3), if*

(H7) $a = 2(\lambda_1 D_i + \underline{a}_i b_i) - \sum_{j=1}^{n}(|w_{ij}|\overline{a}_i F_j + |w_{ji}|\overline{a}_j F_i + |v_{ij}|\overline{a}_i G_j) > b = \sum_{j=1}^{n}|v_{ji}|\overline{a}_j G_i$, *for any* $i = 1, \ldots, n$,

holds, the equilibrium point of system (2.2) *is globally exponentially stable.*

Remark 3.7. Theorem 3.6 shows that the globally exponential stability criteria on reaction-diffusion CGNNs with delays depend on the diffusion term. In exact words, diffusion terms have contributed to exponentially stabilization of reaction-diffusion CGNNs with delays. It should be noted that the authors in [24–28] have studied reaction-diffusion neural networks (including CGNNs and RNNs) with delays and obtained the sufficient condition of exponential stability. However, those sufficient condition are independent of the diffusion term. Obviously, the criteria what we proposed are less conservative and restrictive than those in [24–28].

4. Examples and Comparison

In order to illustrate the feasibility of our above established criteria in the preceding sections, we provide two concrete examples. Although the selection of the coefficients and functions in the examples is somewhat artificial, the possible application of our theoretical theory is clearly expressed.

Example 4.1. Consider the following stochastic reaction-diffusion neural networks model on $X = \{(x_1, x_2)^T \in \mathbb{R}^2 \mid 0 < x_1, x_2 < \sqrt{2/3}\pi\}$

$$
\begin{aligned}
d\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} &= \begin{pmatrix} 0.4\dfrac{\partial u_1(t)}{\partial x_1} & 0.52\dfrac{\partial u_1(t)}{\partial x_2} \\[2ex] 0.42\dfrac{\partial u_2(t)}{\partial x_1} & 0.4\dfrac{\partial u_2(t)}{\partial x_2} \end{pmatrix} \begin{pmatrix} \dfrac{\partial}{\partial x_1} \\[2ex] \dfrac{\partial}{\partial x_2} \end{pmatrix} dt \\
&\quad - \left[\begin{pmatrix} 0.4 & 0 \\ 0 & 0.4 \end{pmatrix} \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} - \begin{pmatrix} 0.1 & 0.2 \\ 0.3 & 0.1 \end{pmatrix} \begin{pmatrix} \sin u_1(t) \\ \sin u_2(t) \end{pmatrix} \right. \\
&\quad \left. - \begin{pmatrix} -0.3 & 0.1 \\ 0.2 & -0.2 \end{pmatrix} \begin{pmatrix} \tanh(u_1(t-1)) \\ \tanh(u_2(t-2)) \end{pmatrix} \right] dt \\
&\quad + \begin{pmatrix} L_{11}(u_1(t)) & L_{12}(u_2(t)) \\ L_{21}(u_2(t)) & L_{22}(u_2(t)) \end{pmatrix} dw(t), \quad (t, x) \in [0, +\infty) \times X, \\
\frac{\partial u_i(t)}{\partial m} &= 0, \quad (t, x) \in [0, +\infty) \times \partial X, \; i = 1, 2, \\
u_i(s) &= \phi_i(s), \quad (s, x) \in [-2, 0] \times \overline{X}, \; i = 1, 2,
\end{aligned}
\tag{4.1}
$$

where $\tanh(x) = (e^x - e^{-x})/(e^x + e^{-x})$. It is clear that $D_i = 0.4$, $b_i = 0.4$, $F_j = G_j = 1$, $i, j = 1, 2$. According to Remark 2.5, we can get $\lambda_1 = 1.5$. Taking

$$
L = \begin{pmatrix} \sqrt{0.1} & 0 \\ 0 & \sqrt{0.2} \end{pmatrix},
\tag{4.2}
$$

we have

$$
2(D_i\lambda_1 + b_i) - \sum_{j=1}^{2} \left(|w_{ij}| F_j + |w_{ji}| F_i + |v_{ij}| G_j + |v_{ji}| G_i + L_{ij}^2 \right) = \begin{cases} 0.8, & i = 1, \\ 1, & i = 2. \end{cases}
\tag{4.3}
$$

It follows from Theorem 3.3 that the equilibrium solution of such system is almost surely exponentially stable and mean square exponential stable.

Remark 4.2. It should be noted that

$$2\underline{a}_i b_i - \sum_{j=1}^{2}\left(\left|w_{ij}\right|\overline{a}_i F_j + \left|w_{ji}\right|\overline{a}_j F_i + \left|v_{ij}\right|\overline{a}_i G_j + \left|v_{ji}\right|\overline{a}_j G_i + L_{ij}^2\right) = \begin{cases} -1, & i = 1, \\ -0.8, & i = 2, \end{cases} \tag{4.4}$$

it is well known, which cannot guarantee the mean square exponential stability of the equilibrium solution of model (4.1). Thus, as we can see in Example 4.1, the reaction-diffusion terms have contributed to the almost surely and mean square exponential stability of this model.

Example 4.3. For the model (4.1), the diffusion operator, space X, and the Neumann boundary conditions are replaced by,

$$\begin{gathered}
\begin{pmatrix} 2\dfrac{\partial u_1(t)}{\partial x_1} & 1.2\dfrac{\partial u_1(t)}{\partial x_2} & 1.2\dfrac{\partial u_1(t)}{\partial x_3} \\ 1.2\dfrac{\partial u_2(t)}{\partial x_1} & 2\dfrac{\partial u_2(t)}{\partial x_2} & 2\dfrac{\partial u_2(t)}{\partial x_3} \end{pmatrix} \begin{pmatrix} \dfrac{\partial}{\partial x_1} \\ \dfrac{\partial}{\partial x_2} \\ \dfrac{\partial}{\partial x_3} \end{pmatrix}, \\
\mathrm{X} = \left\{ (x_1, x_2, x_3)^T \in \mathbb{R}^3 \mid |x_i| < 1,\ i = 1, 2, 3 \right\},
\end{gathered} \tag{4.5}$$

and the Dirichlet boundary condition

$$u_i(t) = 0, \quad (t, x) \in [0, +\infty) \times \partial \mathrm{X},\ i = 1, 2, \tag{4.6}$$

respectively. The remainder parameters and functions unchanged. According to Remark 2.5, we see that $\lambda_1 \geq 0.5387$. By the same way of Example 4.1, equilibrium solution of model (4.5) is almost surely exponentially stable and mean square.

Now, we compare the results in this paper with Propositions 2.6 and 2.8.

The authors in [33, 35] have considered the stochastic delayed reaction-diffusion neural networks with Neumann boundary condition and obtained the sufficient conditions to guarantee the almost surely or mean value exponential stability. We notice that the conditions of Propositions 2.6 and 2.8 do not include the diffusion terms, hence, in principal, Propositions 2.6 and 2.8 could be applied to analyze the exponential stability of stochastic system (4.1), but could not be model (4.5) for its the Dirichlet boundary condition. Unfortunately, Propositions 2.6 and 2.8 are not applicable to ascertain the exponential stability of model (4.1).

In fact, it is easy to calculate that

$$\underline{a}_i = \overline{a}_i = 1, \quad i = 1,2, \quad A_1 = \operatorname{diag}(1,1), \quad C = \operatorname{diag}(0.35, 0.35) > 0,$$

$$C^{-1}(A_1 W^+ F + A_1 V^+ G) = \begin{pmatrix} \frac{8}{7} & \frac{6}{7} \\ \frac{10}{7} & \frac{6}{7} \end{pmatrix}, \tag{4.7}$$

$$\rho\Big(C^{-1}(A_1 W^+ F + A_1 V^+ G)\Big) = 1.8690 > 1.$$

That is, condition (A) of Proposition 2.6 does not hold.

Next, we explain that Proposition 2.8 is not applicable to ascertain the almost surely exponential stability of system (4.1):

$$\overline{b}_i = -b_i + \sum_{j=1}^{n} |w_{ij}| F_j + \sum_{j=1}^{n} |V_{ij}| G_j + \sum_{j=1}^{n} L_{ij}^2 = \begin{cases} 0.4, & i = 1, \\ 0.6, & i = 2. \end{cases} \tag{4.8}$$

However,

$$B - \overline{B} - W^+ F - V^+ G = \begin{pmatrix} -0.4 & -0.3 \\ -0.5 & -0.5 \end{pmatrix} \tag{4.9}$$

is not a nonsingular M-matrix. This implies that condition (A) of Proposition 2.6 is not satisfied.

Remark 4.4. The above comparison shows that reaction-diffusion term contributes to the exponentially stabilization of a stochastic reaction-diffusion neural network and the previous results have been improved.

5. Conclusion

The problem of the convergence dynamics for the stochastic reaction-diffusion CGNNs with delays has been studied in this paper. This neural networks is quite general, and can be used to describe some well-known neural networks, including Hopfield neural networks, cellular neural networks, and generalized CGNNs. By Poincaré inequality and constructing suitable Lyapunov functional, we obtain some sufficient condition to ensure the almost sure and mean square exponential stability of the system. It is worth noting that the diffusion term has played an important role in the obtained conditions, a significant feature that distinguishes the results in this paper from the previous. Two examples are given to show the effectiveness of the results. Moreover, the methods in this paper can been used to consider other stochastic delayed reaction-diffusion neural network model with the Neumann or Dirichlet boundary condition.

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