Hindawi Publishing Corporation
Advances in Difference Equations
Volume 2009, Article ID 562329, 10 pages
doi:10.1155/2009/562329

Research Article

Existence of Nonoscillatory Solutions to Second-Order Neutral Delay Dynamic Equations on Time Scales

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Received 5 March 2009; Revised 24 June 2009; Accepted 24 August 2009

Recommended by Alberto Cabada

We employ Kranoselskii's fixed point theorem to establish the existence of nonoscillatory solutions to the second-order neutral delay dynamic equation $[x(t)+p(t)x(\tau_0(t))]^{\Delta\Delta} + q_1(t)x(\tau_1(t)) - q_2(t)x(\tau_2(t)) = e(t)$ on a time scale $\mathbb{T}$. To dwell upon the importance of our results, one interesting example is also included.

1. Introduction

The theory of time scales, which has recently received a lot of attention, was introduced by Hilger in his Ph.D. Thesis in 1988 in order to unify continuous and discrete analysis (see Hilger [1]). Several authors have expounded on various aspects of this new theory; see the survey paper by Agarwal et al. [2] and references cited therein. A book on the subject of time scales, by Bohner and Peterson [3], summarizes and organizes much of the time scale calculus; we refer also to the last book by Bohner and Peterson [4] for advances in dynamic equations on time scales. For the notation used below we refer to the next section that provides some basic facts on time scales extracted from Bohner and Peterson [3].

In recent years, there has been much research activity concerning the oscillation of solutions of various equations on time scales, and we refer the reader to Erbe [5], Saker [6], and Hassan [7]. And there are some results dealing with the oscillation of the solutions of second-order delay dynamic equations on time scales [8–22].

In this work, we will consider the existence of nonoscillatory solutions to the second-order neutral delay dynamic equation of the form

$$\left[x(t)+p(t)x(\tau_0(t))\right]^{\Delta\Delta}+q_1(t)x(\tau_1(t))-q_2(t)x(\tau_2(t))=e(t) \tag{1.1}$$

on a time scale $\mathbb{T}$ (an arbitrary closed subset of the reals).

The motivation originates from Kulenović and Hadžiomerpahić [23] and Zhu and Wang [24]. In [23], the authors established some sufficient conditions for the existence of positive solutions of the delay equation

$$\left[x(t)+p(t)x(t-\tau)\right]''+q_1(t)x(t-\sigma_1)-q_2(t)x(t-\sigma_2)=e(t). \tag{1.2}$$

Recently, [24] established the existence of nonoscillatory solutions to the neutral equation

$$\left[x(t)+p(t)x\left(g(t)\right)\right]^{\Delta}+f(t,x(h(t)))=0 \tag{1.3}$$

on a time scale $\mathbb{T}$.

Neutral equations find numerous applications in natural science and technology. For instance, they are frequently used for the study of distributed networks containing lossless transmission lines. So, we try to establish some sufficient conditions for the existence of equations of (1.1). However, there are few papers to discuss the existence of nonoscillatory solutions for neutral delay dynamic equations on time scales.

Since we are interested in the nonoscillatory behavior of (1.1), we assume throughout that the time scale $\mathbb{T}$ under consideration satisfies $\inf\mathbb{T}=t_0$ and $\sup\mathbb{T}=\infty$.

As usual, by a solution of (1.1) we mean a continuous function $x(t)$ which is defined on $\mathbb{T}$ and satisfies (1.1) for $t\geq t_1\geq t_0$. A solution of (1.1) is said to be eventually positive (or eventually negative) if there exists $c\in\mathbb{T}$ such that $x(t)>0$ (or $x(t)<0$) for all $t\geq c$ in $\mathbb{T}$. A solution of (1.1) is said to be nonoscillatory if it is either eventually positive or eventually negative; otherwise, it is oscillatory.

2. Main Results

In this section, we establish the existence of nonoscillatory solutions to (1.1). For $T_0,T_1\in\mathbb{T}$, let $[T_0,\infty)_{\mathbb{T}}:=\{t\in\mathbb{T}:t\geq T_0\}$ and $[T_0,T_1]_{\mathbb{T}}:=\{t\in\mathbb{T}:T_0\leq t\leq T_1\}$. Further, let $C([T_0,\infty)_{\mathbb{T}},\mathbb{R})$ denote all continuous functions mapping $[T_0,\infty)_{\mathbb{T}}$ into $\mathbb{R}$, and

$$BC[T_0,\infty)_{\mathbb{T}}:=\left\{x:x\in C([T_0,\infty)_{\mathbb{T}},\mathbb{R}),\ \sup_{t\in[T_0,\infty)_{\mathbb{T}}}|x(t)|<\infty\right\}. \tag{2.1}$$

Endowed on $BC[T_0,\infty)_{\mathbb{T}}$ with the norm $\|x\|=\sup_{t\in[T_0,\infty)_{\mathbb{T}}}|x(t)|$, $(BC[T_0,\infty)_{\mathbb{T}},\|\cdot\|)$ is a Banach space (see [24]). Let $X\subseteq BC[T_0,\infty)_{\mathbb{T}}$, we say that X is uniformly Cauchy if for any given $\varepsilon>0$, there exists $T_1\in[T_0,\infty)_{\mathbb{T}}$ such that for any $x\in X$, $|x(t_1)-x(t_2)|<\varepsilon$, for all $t_1,t_2\in[T_1,\infty)_{\mathbb{T}}$.

X is said to be equicontinuous on $[a,b]_{\mathbb{T}}$ if for any given $\varepsilon>0$, there exists $\delta>0$ such that for any $x\in X$, and $t_1,t_2\in[a,b]_{\mathbb{T}}$ with $|t_1-t_2|<\delta$, $|x(t_1)-x(t_2)|<\varepsilon$.

Also, we need the following auxiliary results.

Lemma 2.1 (see [24, Lemma 4]). *Suppose that* $X \subseteq BC[T_0, \infty)_{\mathbb{T}}$ *is bounded and uniformly Cauchy. Further, suppose that* X *is equicontinuous on* $[T_0, T_1]_{\mathbb{T}}$ *for any* $T_1 \in [T_0, \infty)_{\mathbb{T}}$. *Then* X *is relatively compact.*

Lemma 2.2 (see [25, Kranoselskii's fixed point theorem]). *Suppose that* Ω *is a Banach space and* X *is a bounded, convex, and closed subset of* Ω. *Suppose further that there exist two operators* $U, S : X \to \Omega$ *such that*

(i) $Ux + Sy \in X$ *for all* $x, y \in X$;

(ii) U *is a contraction mapping;*

(iii) S *is completely continuous.*

Then $U + S$ *has a fixed point in* X.

Throughout this section, we will assume in (1.1) that

$(H)\tau_i(t) \in C_{rd}(\mathbb{T}, \mathbb{T})$, $\tau_i(t) \le t$, $\lim_{t\to\infty} \tau_i(t) = \infty$, $i = 0, 1, 2$, $p(t)$,$q_j(t) \in C_{rd}(\mathbb{T}, \mathbb{R})$, $q_j(t) > 0$, $\int_{t_0}^{\infty} \sigma(s) q_j(s) \Delta s < \infty$, $j = 1, 2$, and there exists a function $E(t) \in C_{rd}^2(\mathbb{T}, \mathbb{R})$ such that $E^{\Delta\Delta}(t) = e(t)$, $\lim_{t\to\infty} E(t) = e_0 \in \mathbb{R}$.

Theorem 2.3. *Assume that* (H) *holds and* $|p(t)| \le p < 1/3$. *Then* (1.1) *has an eventually positive solution.*

Proof. From the assumption (H), we can choose $T_0 \in \mathbb{T}$ $(T_0 \ge 1)$ large enough and positive constants M_1 and M_2 which satisfy the condition

$$1 < M_2 < \frac{1 - p - 2M_1}{2p}, \tag{2.2}$$

such that

$$\int_{T_0}^{\infty} \sigma(s) q_1(s) \Delta s \le \frac{(1-p)(M_2 - 1)}{M_2}, \tag{2.3}$$

$$\int_{T_0}^{\infty} \sigma(s) q_2(s) \Delta s \le \frac{1 - p(1 + 2M_2) - 2M_1}{2M_2}, \tag{2.4}$$

$$\int_{T_0}^{\infty} \sigma(s) \left[q_1(s) + q_2(s)\right] \Delta s \le \frac{3(1-p)}{4}, \tag{2.5}$$

$$|E(t) - e_0| \le \frac{1-p}{4}, \quad t \ge T_0. \tag{2.6}$$

Furthermore, from (H) we see that there exists $T_1 \in \mathbb{T}$ with $T_1 > T_0$ such that $\tau_i(t) \ge T_0$, $i = 0, 1, 2$, for $t \in [T_1, \infty]_{\mathbb{T}}$.

Define the Banach space $BC[T_0, \infty)_{\mathbb{T}}$ as in (2.1) and let

$$X = \{x \in BC[T_0, \infty)_{\mathbb{T}} : M_1 \le x(t) \le M_2\}. \tag{2.7}$$

It is easy to verify that X is a bounded, convex, and closed subset of $BC[T_0,\infty)_{\mathbb{T}}$. Now we define two operators U and $S : X \to BC[T_0,\infty)_{\mathbb{T}}$ as follows:

$$\begin{aligned}
(Ux)(t) &= \frac{1-p}{4} - p(t)x(\tau_0(T_1)) + E(T_1) - e_0, \quad t \in [T_0,T_1]_{\mathbb{T}},\\
(Ux)(t) &= \frac{1-p}{4} - p(t)x(\tau_0(t)) + E(t) - e_0, \quad t \in [T_1,\infty)_{\mathbb{T}},\\
(Sx)(t) &= \frac{1-p}{2} + T_1\int_{T_1}^{\infty}\left[q_1(s)x(\tau_1(s)) - q_2(s)x(\tau_2(s))\right]\Delta s, \quad t \in [T_0,T_1]_{\mathbb{T}},\\
(Sx)(t) &= \frac{1-p}{2} + t\int_{t}^{\infty}\left[q_1(s)x(\tau_1(s)) - q_2(s)x(\tau_2(s))\right]\Delta s\\
&\quad + \int_{T_1}^{t}\sigma(s)\left[q_1(s)x(\tau_1(s)) - q_2(s)x(\tau_2(s))\right]\Delta s, \quad t \in [T_1,\infty)_{\mathbb{T}}.
\end{aligned} \tag{2.8}$$

Next, we will show that U and S satisfy the conditions in Lemma 2.2.

(i) We first prove that $Ux + Sy \in X$ for any $x, y \in X$. Note that for any $x, y \in X$, $M_1 \le x \le M_2$, $M_1 \le y \le M_2$. For any $x, y \in X$ and $t \in [T_1,\infty)_{\mathbb{T}}$, in view of (2.3), (2.4) and (2.6), we have

$$\begin{aligned}
(Ux)(t) + (Sy)(t) &\ge \frac{3(1-p)}{4} - \frac{1-p}{4} - pM_2 - t\int_{t}^{\infty} q_2(s)x(\tau_2(s))\Delta s - \int_{T_1}^{t}\sigma(s)q_2(s)x(\tau_2(s))\Delta s\\
&\ge \frac{1-p}{2} - pM_2 - M_2\int_{T_1}^{\infty}\sigma(s)q_2(s)\Delta s \ge M_1,\\
(Ux)(t) + (Sy)(t) &\le \frac{3(1-p)}{4} + \frac{1-p}{4} + pM_2 + t\int_{t}^{\infty} q_1(s)x(\tau_1(s))\Delta s + \int_{T_1}^{t}\sigma(s)q_1(s)x(\tau_1(s))\Delta s\\
&\le 1 - p + pM_2 + M_2\int_{T_1}^{\infty}\sigma(s)q_1(s)\Delta s \le M_2.
\end{aligned} \tag{2.9}$$

Similarly, we can prove that $M_1 \le (Ux)(t) + (Sy)(t) \le M_2$ for any $x, y \in X$ and $t \in [T_0,T_1]_{\mathbb{T}}$. Hence, $Ux + Sy \in X$ for any $x, y \in X$.

(ii) We prove that U is a contraction mapping. Indeed, for $x, y \in X$, we have

$$\left|(Ux)(t) - (Uy)(t)\right| = \left|p(t)\left[x(\tau_0(T_1)) - y(\tau_0(T_1))\right]\right| \le p \sup_{t\in[T_0,\infty)_{\mathbb{T}}}\left|x(t) - y(t)\right| \tag{2.10}$$

for $t \in [T_0,T_1]_{\mathbb{T}}$ and

$$\left|(Ux)(t) - (Uy)(t)\right| = \left|p(t)\left[x(\tau_0(t)) - y(\tau_0(t))\right]\right| \le p \sup_{t\in[T_0,\infty)_{\mathbb{T}}}\left|x(t) - y(t)\right| \tag{2.11}$$

for $t \in [T_1, \infty)_{\mathbb{T}}$. Therefore, we have

$$\|Ux - Uy\| \le p\|x - y\| \tag{2.12}$$

for any $x, y \in X$. Hence, U is a contraction mapping.

(iii) We will prove that S is a completely continuous mapping. First, by (i) we know that S maps X into X.

Second, we consider the continuity of S. Let $x_n \in X$ and $\|x_n - x\| \longrightarrow 0$ as $n \longrightarrow \infty$, then $x \in X$ and $|x_n(t) - x(t)| \longrightarrow 0$ as $n \longrightarrow \infty$ for any $t \in [T_0, \infty)_{\mathbb{T}}$. Consequently, by (2.5) we have

$$\begin{aligned}
&|(Sx_n)(t) - (Sx)(t)| \\
&\quad \le t\left[\int_t^\infty q_1(s)|x_n(\tau_1(s)) - x(\tau_1(s))|\Delta s + \int_t^\infty q_2(s)|x_n(\tau_2(s)) - x(\tau_2(s))|\Delta s\right] \\
&\qquad + \int_{T_1}^t \sigma(s)q_1(s)|x_n(\tau_1(s)) - x(\tau_1(s))|\Delta s \\
&\qquad + \int_{T_1}^t \sigma(s)q_2(s)|x_n(\tau_2(s)) - x(\tau_2(s))|\Delta s \\
&\quad \le \|x_n - x\|\left[\int_t^\infty \sigma(s)\left[q_1(s) + q_2(s)\right]\Delta s + \int_{T_1}^t \sigma(s)\left[q_1(s) + q_2(s)\right]\Delta s\right] \\
&\quad = \|x_n - x\|\int_{T_1}^\infty \sigma(s)\left[q_1(s) + q_2(s)\right]\Delta s \le \frac{3(1-p)}{4}\|x_n - x\|
\end{aligned} \tag{2.13}$$

for $t \in [T_0, \infty)_{\mathbb{T}}$. So, we obtain

$$\|Sx_n - Sx\| \le \frac{3(1-p)}{4}\|x_n - x\| \longrightarrow 0, \quad n \longrightarrow \infty, \tag{2.14}$$

which proves that S is continuous on X.

Finally, we prove that SX is relatively compact. It is sufficient to verify that SX satisfies all conditions in Lemma 2.1. By the definition of X, we see that SX is bounded. For any $\varepsilon > 0$, take $T_2 \in [T_1, \infty)_{\mathbb{T}}$ so that

$$\int_{T_2}^\infty \sigma(s)\left[q_1(s) + q_2(s)\right]\Delta s < \varepsilon. \tag{2.15}$$

For any $x \in X$ and $t_1, t_2 \in [T_2, \infty)_{\mathbb{T}}$, we have

$$\begin{aligned}
&|(Sx)(t_1) - (Sx)(t_2)| \\
&\quad \le \left| t_1 \int_{t_1}^{\infty} \left[q_1(s)x(\tau_1(s)) - q_2(s)x(\tau_2(s))\right]\Delta s - t_2 \int_{t_2}^{\infty} \left[q_1(s)x(\tau_1(s)) - q_2(s)x(\tau_2(s))\right]\Delta s \right| \\
&\qquad + \left| \int_{T_1}^{t_1} \sigma(s)\left[q_1(s)x(\tau_1(s)) - q_2(s)x(\tau_2(s))\right]\Delta s - \int_{T_1}^{t_2} \sigma(s)\left[q_1(s)x(\tau_1(s)) - q_2(s)x(\tau_2(s))\right]\Delta s \right| \\
&\quad \le M_2 \int_{t_1}^{\infty} \sigma(s)\left[q_1(s) + q_2(s)\right]\Delta s + M_2 \int_{t_2}^{\infty} \sigma(s)\left[q_1(s) + q_2(s)\right]\Delta s \\
&\qquad + M_2 \left| \int_{t_1}^{t_2} \sigma(s)\left[q_1(s) + q_2(s)\right]\Delta s \right| < 3M_2\varepsilon.
\end{aligned} \tag{2.16}$$

Thus, SX is uniformly Cauchy.

The remainder is to consider the equicontinuous on $[T_0, T_2]_{\mathbb{T}}$ for any $T_2 \in [T_0, \infty)_{\mathbb{T}}$. Without loss of generality, we set $T_1 < T_2$. For any $x \in X$, we have $|(Sx)(t_1) - (Sx)(t_2)| \equiv 0$ for $t_1, t_2 \in [T_0, T_1]_{\mathbb{T}}$ and

$$\begin{aligned}
&|(Sx)(t_1) - (Sx)(t_2)| \\
&\quad \le \left| t_1 \int_{t_1}^{\infty} \left[q_1(s)x(\tau_1(s)) - q_2(s)x(\tau_2(s))\right]\Delta s - t_2 \int_{t_2}^{\infty} \left[q_1(s)x(\tau_1(s)) - q_2(s)x(\tau_2(s))\right]\Delta s \right| \\
&\qquad + \left| \int_{T_1}^{t_1} \sigma(s)\left[q_1(s)x(\tau_1(s)) - q_2(s)x(\tau_2(s))\right]\Delta s - \int_{T_1}^{t_2} \sigma(s)\left[q_1(s)x(\tau_1(s)) - q_2(s)x(\tau_2(s))\right]\Delta s \right| \\
&\quad \le M_2 \left| \int_{t_1}^{t_2} \sigma(s)\left[q_1(s) + q_2(s)\right]\Delta s \right| \\
&\qquad + \left| (t_1 - t_2) \int_{t_1}^{\infty} \left[q_1(s)x(\tau_1(s)) - q_2(s)x(\tau_2(s))\right]\Delta s \right| \\
&\qquad + \left| t_2 \int_{t_1}^{\infty} \left[q_1(s)x(\tau_1(s)) - q_2(s)x(\tau_2(s))\right]\Delta s - t_2 \int_{t_2}^{\infty} \left[q_1(s)x(\tau_1(s)) - q_2(s)x(\tau_2(s))\right]\Delta s \right| \\
&\quad \le (M_2 t_2 + M_2) \left| \int_{t_1}^{t_2} \sigma(s)\left[q_1(s) + q_2(s)\right]\Delta s \right| \\
&\qquad + M_2 |t_1 - t_2| \int_{t_1}^{\infty} \sigma(s)\left[q_1(s) + q_2(s)\right]\Delta s
\end{aligned} \tag{2.17}$$

for $t_1, t_2 \in [T_1, T_2]_{\mathbb{T}}$.

Now, we see that for any $\varepsilon > 0$, there exists $\delta > 0$ such that when $t_1, t_2 \in [T_1, T_2]_{\mathbb{T}}$ with $|t_1 - t_2| < \delta$,

$$|(Sx)(t_1) - (Sx)(t_2)| < \varepsilon \tag{2.18}$$

for all $x \in X$. This means that SX is equicontinuous on $[T_0, T_2]_{\mathbb{T}}$ for any $T_2 \in [T_0, \infty)_{\mathbb{T}}$.

By means of Lemma 2.1, SX is relatively compact. From the above, we have proved that S is a completely continuous mapping.

By Lemma 2.2, there exists $x \in X$ such that $(U + S)x = x$. Therefore, we have

$$\begin{aligned} x(t) = {}& \frac{3(1-p)}{4} - p(t)x(\tau_0(t)) + t\int_t^{\infty} [q_1(s)x(\tau_1(s)) - q_2(s)x(\tau_2(s))]\,\Delta s \\ &+ \int_{T_1}^{t} \sigma(s)\,[q_1(s)x(\tau_1(s)) - q_2(s)x(\tau_2(s))]\,\Delta s + E(t) - e_0, \quad t \in [T_1, \infty)_{\mathbb{T}}, \end{aligned} \tag{2.19}$$

which implies that $x(t)$ is an eventually positive solution of (1.1). The proof is complete. □

Theorem 2.4. *Assume that* (H) *holds and* $0 \le p(t) \le p_1 < 1$. *Then* (1.1) *has an eventually positive solution.*

Proof. From the assumption (H), we can choose $T_0 \in \mathbb{T}$ ($T_0 \ge 1$) large enough and positive constants M_3 and M_4 which satisfy the condition

$$1 - M_4 < p_1 < \frac{1 - 2M_3}{1 + 2M_4}, \tag{2.20}$$

such that

$$\begin{gathered} \int_{T_0}^{\infty} \sigma(s) q_1(s)\,\Delta s \le \frac{p_1 + M_4 - 1}{M_4}, \\ \int_{T_0}^{\infty} \sigma(s) q_2(s)\,\Delta s \le \frac{1 - p_1(1 + 2M_4) - 2M_3}{2M_4}, \\ \int_{T_0}^{\infty} \sigma(s)\,[q_1(s) + q_2(s)]\,\Delta s \le \frac{3(1-p_1)}{4}, \\ |E(t) - e_0| \le \frac{1 - p_1}{4}, \quad t \ge T_0. \end{gathered} \tag{2.21}$$

Furthermore, from (H) we see that there exists $T_1 \in \mathbb{T}$ with $T_1 > T_0$ such that $\tau_i(t) \ge T_0, i = 0, 1, 2$, for $t \in [T_1, \infty]_{\mathbb{T}}$.

Define the Banach space $BC[T_0, \infty)_{\mathbb{T}}$ as in (2.1) and let

$$X = \{x \in BC[T_0, \infty)_{\mathbb{T}} : M_3 \le x(t) \le M_4\}. \tag{2.22}$$

It is easy to verify that X is a bounded, convex, and closed subset of $BC[T_0, \infty)_{\mathbb{T}}$.

Now we define two operators U and S as in Theorem 2.3 with p replaced by p_1. The rest of the proof is similar to that of Theorem 2.3 and hence omitted. The proof is complete. □

Theorem 2.5. *Assume that* (H) *holds and* $-1 < -p_2 \le p(t) \le 0$. *Then* (1.1) *has an eventually positive solution.*

Proof. From the assumption (H), we can choose $T_0 \in \mathbb{T}$ ($T_0 \ge 1$) large enough and positive constants M_5 and M_6 which satisfy the condition

$$2M_5 + p_2 < 1 < M_6, \tag{2.23}$$

such that

$$\begin{gathered}
\int_{T_0}^{\infty} \sigma(s) q_1(s) \Delta s \le \frac{(1-p_2)(M_6-1)}{M_6}, \\
\int_{T_0}^{\infty} \sigma(s) q_2(s) \Delta s \le \frac{1-p_2-2M_5}{2M_6}, \\
\int_{T_0}^{\infty} \sigma(s)\left[q_1(s)+q_2(s)\right] \Delta s \le \frac{3(1-p_2)}{4}, \\
|E(t)-e_0| \le \frac{1-p_2}{4}, \quad t \ge T_0.
\end{gathered} \tag{2.24}$$

Furthermore, from (H) we see that there exists $T_1 \in \mathbb{T}$ with $T_1 > T_0$ such that $\tau_i(t) \ge T_0, i = 0,1,2$, for $t \in [T_1, \infty]_{\mathbb{T}}$.

Define the Banach space $BC[T_0, \infty)_{\mathbb{T}}$ as in (2.1) and let

$$X = \{x \in BC[T_0,\infty)_{\mathbb{T}} : M_5 \le x(t) \le M_6\}. \tag{2.25}$$

It is easy to verify that X is a bounded, convex, and closed subset of $BC[T_0, \infty)_{\mathbb{T}}$.

Now we define two operators U and S as in Theorem 2.3 with p replaced by p_2. The rest of the proof is similar to that of Theorem 2.3 and hence omitted. The proof is complete. □

We will give the following example to illustrate our main results.

Example 2.6. Consider the second-order delay dynamic equations on time scales

$$\left[x(t)+p(t)x(\tau_0(t))\right]^{\Delta\Delta} + \frac{1}{t^{\alpha}\sigma(t)} x(\tau_1(t)) - \frac{1}{t^{\beta}\sigma(t)} x(\tau_2(t)) = \frac{-(t+\sigma(t))}{t^2\sigma^2(t)}, \quad t \in [t_0,\infty)_{\mathbb{T}}, \tag{2.26}$$

where $t_0 > 0$, $\alpha > 1$, $\beta > 1$, $\tau_i(t) \in C_{rd}(\mathbb{T},\mathbb{T})$, $\tau_i(t) \le t$, $\lim_{t\to\infty}\tau_i(t) = \infty$, $i = 0,1,2$, $|p(t)| \le p < 1/3$. Then $q_1(t) = 1/(t^{\alpha}\sigma(t))$, $q_2(t) = 1/(t^{\beta}\sigma(t))$, $e(t) = -(t+\sigma(t))/(t^2\sigma^2(t))$. Let $E(t) = \int_{t_0}^{t}(1/s^2)\Delta s$. It is easy to see that the assumption (H) holds. By Theorem 2.3, (2.26) has an eventually positive solution.

Acknowledgments

The authors sincerely thank the reviewers for their valuable suggestions and useful comments that have lead to the present improved version of the original manuscript. This research is supported by the Natural Science Foundation of China (60774004), China Postdoctoral Science Foundation Funded Project (20080441126), Shandong Postdoctoral Funded Project (200802018), Shandong Research Funds (Y2008A28, Y2007A27), and also supported by the University of Jinan Research Funds for Doctors (B0621, XBS0843).

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