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Research Article

Existence of 2^n Positive Periodic Solutions to n-Species Nonautonomous Food Chains with Harvesting Terms

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By using Mawhin's continuation theorem of coincidence degree theory and some skills of inequalities, we establish the existence of at least 2^n positive periodic solutions for n-species nonautonomous Lotka-Volterra type food chains with harvesting terms. An example is given to illustrate the effectiveness of our results.

1. Introduction

The dynamic relationship between predators and their prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. These problems may appear to be simple mathematically at first; sight, they are, in fact, very challenging and complicated. There are many different kinds of predator-prey models in the literature. For more details, we refer to [1, 2]. Food chain predator-prey system, as one of the most important predator-prey system, has been extensively studied by many scholars, many excellent results concerned with the persistent property and positive periodic solution of the system; see [3–13] and the references cited therein. However, to the best of the authors' knowledge, to this day, still no scholar study the n-species nonautonomous case of Food chain predator-prey system with harvesting terms. Indeed, the exploitation of biological resources and the harvest of population species are commonly practiced in fishery, forestry, and wildlife management; the study of population dynamics with harvesting is an important subject in mathematical bioeconomics, which is related to the optimal management of renewable resources (see [14–16]). This motivates us to consider the following n-species nonautonomous Lotka-Volterra type food chain model with harvesting terms:

$$
\begin{aligned}
\dot{x}_1(t) &= x_1(t)(a_1(t) - b_1(t)x_1(t) - c_{12}(t)x_2(t)) - h_1(t),\\
&\vdots\\
\dot{x}_i(t) &= x_i(t)(-d_i(t) - b_i(t)x_i(t) + c_{i,i-1}(t)x_{i-1}(t) - c_{i,i+1}(t)x_{i+1}(t)) - h_i(t),\\
&\vdots\\
\dot{x}_n(t) &= x_n(t)(-d_n(t) - b_n(t)x_n(t) + c_{n,n-1}(t)x_{n-1}(t)) - h_n(t),
\end{aligned}
\tag{1.1}
$$

where $i = 2,3,\ldots,n-1$, $x_i(t)$ $(i = 1,2,\ldots,n)$ is the ith species population density, $a_1(t)$ is the growth rate of the first species that is the only producer in system (1.1), $b_i(t)$ $(i = 1,2,\ldots,n)$ and $h_i(t)$ $(i = 1,2,\ldots,n)$ stand for the ith species intraspecific competition rate and harvesting rate, respectively, $d_i(t)$ $(i = 2,3,\ldots,n)$ is the death rate of the ith species, $c_{i,i+1}(t)$ $(i = 1,2,\ldots,n-1)$ represents the $(i+1)$th species predation rate on the ith species, and $c_{i,i-1}(t)(i = 2,3,\ldots,n)$ stands for the transformation rate from the $(i-1)$th species to the ith species. In addition, the effects of a periodically varying environment are important for evolutionary theory as the selective forces on systems in a fluctuating environment differ from those in a stable environment. Therefore, the assumptions of periodicity of the parameters are a way of incorporating the periodicity of the environment (e.g, seasonal effects of weather, food supplies, mating habits, etc), which leads us to assume that $a_1(t), b_i(t), d_i(t), c_{ij}(t)$, and $h_i(t)$ $(i,j = 1,2,\ldots,n)$ are all positive continuous ω-periodic functions.

Since a very basic and important problem in the study of a population growth model with a periodic environment is the global existence and stability of a positive periodic solution, which plays a similar role as a globally stable equilibrium does in an autonomous model, this motivates us to investigate the existence of a positive periodic or multiple positive periodic solutions for system (1.1). In fact, it is more likely for some biological species to take on multiple periodic change regulations and have multiple local stable periodic phenomena. Therefore, it is essential for us to investigate the existence of multiple positive periodic solutions for population models. Our main purpose of this paper is by using Mawhin's continuation theorem of coincidence degree theory [17], to establish the existence of 2^n positive periodic solutions for system (1.1). For the work concerning the multiple existence of periodic solutions of periodic population models which was done using coincidence degree theory, we refer to [18–21].

The organization of the rest of this paper is as follows. In Section 2, by employing the continuation theorem of coincidence degree theory and the skills of inequalities, we establish the existence of at least 2^n positive periodic solutions of system (1.1). In Section 3, an example is given to illustrate the effectiveness of our results.

2. Existence of at Least 2^n Positive Periodic Solutions

In this section, by using Mawhin's continuation theorem and the skills of inequalities, we shall show the existence of positive periodic solutions of (1.1). To do so, we need to make some preparations.

Let X and Z be real normed vector spaces. Let $L : \operatorname{Dom} L \subset X \to Z$ be a linear mapping and $N : X \times [0,1] \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim \operatorname{Ker} L = \operatorname{codim} Im\, L < \infty$ and $Im\, L$ is closed in Z.

If L is a Fredholm mapping of index zero, then there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $Im\,P = \operatorname{Ker} L$ and $\operatorname{Ker} Q = Im\,L = Im\,(I - Q)$, and $X = \operatorname{Ker} L \oplus \operatorname{Ker} P$, $Z = Im\,L \oplus Im\,Q$. It follows that $L|_{\operatorname{Dom} L \cap \operatorname{Ker} P} : (I - P)X \to Im\,L$ is invertible and its inverse is denoted by K_P. If Ω is a bounded open subset of X, the mapping N is called L-compact on $\overline{\Omega}\times[0,1]$, if $QN(\overline{\Omega}\times[0,1])$ is bounded and $K_P(I-Q)N : \overline{\Omega}\times[0,1] \to X$ is compact. Because $\operatorname{Im} Q$ is isomorphic to $\operatorname{Ker} L$, there exists an isomorphism $J : Im\,Q \to \operatorname{Ker} L$.

The Mawhin's continuous theorem [17, page 40] is given as follows.

Lemma 2.1 (see [9]). *Let L be a Fredholm mapping of index zero and let N be L-compact on $\overline{\Omega} \times [0,1]$. Assume that*

(a) *for each $\lambda \in (0,1)$, every solution x of $Lx = \lambda N(x,\lambda)$ is such that $x \notin \partial\Omega \cap \operatorname{Dom} L$;*

(b) $QN(x,0)x \neq 0$ *for each $x \in \partial\Omega \cap \operatorname{Ker} L$;*

(c) $\deg(JQN(x,0), \Omega \cap \operatorname{Ker} L, 0) \neq 0$.

Then $Lx = N(x,1)$ has at least one solution in $\overline{\Omega} \cap \operatorname{Dom} L$.

For the sake of convenience, we denote $f^l = \min_{t\in[0,\omega]} f(t)$, $f^M = \max_{t\in[0,\omega]} f(t)$, $\overline{f} = (1/\omega)\int_0^\omega f(t)\,dt$, respectively; here $f(t)$ is a continuous ω-periodic function.

For simplicity, we need to introduce some notations as follows:

$$
\begin{gathered}
A_1^{\pm} = \frac{\left(a_1^l - c_{12}^M l_2^+\right) \pm \sqrt{\left(a_1^l - c_{12}^M l_2^+\right)^2 - 4b_1^M h_1^M}}{2b_1^M}, \quad l_i^{\pm} = \frac{c_{i,i-1}^M l_{i-1}^+ \pm \sqrt{\left(c_{i,i-1}^M l_{i-1}^+\right)^2 - 4b_i^l h_i^l}}{2b_i^l}, \\
l_1^{\pm} = \frac{a_1^M \pm \sqrt{\left(a_1^M\right)^2 - 4b_n^l h_n^l}}{2b_n^l}, \quad A_n^{\pm} = \frac{\left(c_{n,n-1}^l l_{n-1}^- - d_n^M\right) \pm \sqrt{\left(c_{n,n-1}^l l_{n-1}^- - d_n^M\right)^2 - 4b_n^M h_n^M}}{2b_n^M}, \\
A_i^{\pm} = \frac{\left(c_{i,i-1}^l l_{i-1}^- - c_{i,i+1}^M l_{i+1}^+ - d_i^M\right) \pm \sqrt{\left(c_{i,i-1}^l l_{i-1}^- - c_{i,i+1}^M l_{i+1}^+ - d_i^M\right)^2 - 4b_i^M h_i^M}}{2b_i^M}, \\
B_1^{\pm} = \frac{a_1^l \pm \sqrt{\left(a_1^l\right)^2 - 4b_n^M h_n^M}}{2b_n^M}, \quad B_i^{\pm} = \frac{c_{i,i-1}^l l_{i-1}^- \pm \sqrt{\left(c_{i,i-1}^l l_{i-1}^-\right)^2 - 4b_i^M h_i^M}}{2b_i^M},
\end{gathered}
\tag{2.1}
$$

where $i = 2, 3, \ldots, n$.

Throughout this paper, we need the following assumptions:

(H_1) $a_1^l - c_{12}^M l_2^+ > 2\sqrt{b_1^M h_1^M}$ and $c_{n,n-1}^l l_{n-1}^- - d_n^M > 2\sqrt{b_n^M h_n^M}$;

(H_2) $c_{i,i-1}^l l_{i-1}^- - c_{i,i+1}^M l_{i+1}^+ - d_i^M > 2\sqrt{b_i^M h_i^M}$, $i = 2, 3, \ldots, n-1$.

Lemma 2.2. *Let $x > 0$, $y > 0$, $z > 0$ and $x > 2\sqrt{yz}$, for the functions $f(x,y,z) = (x + \sqrt{x^2 - 4yz})/2z$ and $g(x,y,z) = (x - \sqrt{x^2 - 4yz})/2z$, the following assertions hold:*

(1) *$f(x,y,z)$ and $g(x,y,z)$ are monotonically increasing and monotonically decreasing on the variable $x \in (0,\infty)$, respectively.*

(2) *$f(x,y,z)$ and $g(x,y,z)$ are monotonically decreasing and monotonically increasing on the variable $y \in (0,\infty)$, respectively.*

(3) *$f(x,y,z)$ and $g(x,y,z)$ are monotonically decreasing and monotonically increasing on the variable $z \in (0,\infty)$, respectively.*

Proof. In fact, for all $x > 0, y > 0, z > 0$, we have

$$\frac{\partial f}{\partial x} = \frac{x + \sqrt{x^2 - 4yz}}{2z\sqrt{x^2 - 4yz}} > 0, \qquad \frac{\partial g}{\partial x} = \frac{\sqrt{x^2 - 4yz} - x}{2z\sqrt{x^2 - 4yz}} < 0, \qquad \frac{\partial f}{\partial y} = \frac{-1}{\sqrt{x^2 - 4yz}} < 0,$$
$$\frac{\partial g}{\partial y} = \frac{1}{\sqrt{x^2 - 4yz}} > 0, \qquad \frac{\partial f}{\partial z} = \frac{-x\left(x + \sqrt{x^2 - 4yz}\right)}{2z^2\sqrt{x^2 - 4yz}} < 0, \qquad \frac{\partial g}{\partial z} = \frac{x\left(x - \sqrt{x^2 - 4yz}\right)}{2z^2\sqrt{x^2 - 4yz}} > 0. \tag{2.2}$$

By the relationship of the derivative and the monotonicity, the above assertions obviously hold. The proof of Lemma 2.2 is complete. □

Lemma 2.3. *Assume that (H_1) and (H_2) hold, then we have the following inequalities:*

$$\ln l_i^- < \ln B_i^- < lnA_i^- < lnA_i^+ < lnB_i^+ < lnl_i^+, \quad i = 1,2,\dots,n. \tag{2.3}$$

Proof. Since

$$\begin{gathered}
a_1^M \geq a_1^l > a_1^l - c_{12}^M l_2^+ > 2\sqrt{b_1^M h_1^M} > 0, \quad 0 < b_1^l \leq b_1^M, \quad 0 < h_1^l \leq h_1^M, \\
c_{i,i-1}^M l_{i-1}^+ > c_{i,i-1}^l l_i^- > c_{i,i-1}^l l_{i-1}^- - c_{i,i+1}^M l_{i+1}^+ - d_i^M > 0, \\
0 < b_i^l \leq b_i^M, \quad 0 < h_i^l \leq h_i^M, \ i = 2,3,\dots,n-1, \\
c_{n,n-1}^M l_{n-1}^+ > c_{n,n-1}^l l_{n-1}^- > c_{n,n-1}^l l_{n-1}^- - d_n^M > 2\sqrt{b_n^M h_n^M} > 0, \quad 0 < b_n^l \leq b_n^M, \quad 0 < h_n^l \leq h_n^M.
\end{gathered} \tag{2.4}$$

By assumptions (H_1), (H_2), Lemma 2.2 and the expressions of $A_i^{\pm}$, $B_i^{\pm}$, and $l_i^{\pm}$, we have

$$\begin{aligned}
0 &< l_1^- = g\left(a_1^M, b_1^l, h_1^l\right) < g\left(a_1^l, b_1^M, h_1^M\right) = B_1^- < g\left(a_1^l - c_{12}^M l_2^+, b_1^M, h_1^M\right) = A_1^- \\
&< A_1^+ = f\left(a_1^l - c_{12}^M l_2^+, b_1^M, h_1^M\right) < B_1^+ = f\left(a_1^l, b_1^M, h_1^M\right) < f\left(a_1^M, b_1^l, h_1^l\right) = l_1^+, \\
0 &< l_i^- = g\left(c_{i,i-1}^M l_{i-1}^+, b_i^l, h_i^l\right) < g\left(c_{i,i-1}^l l_{i-1}^-, b_i^M, h_i^M\right) = B_i^- \\
&< g\left(c_{i,i-1}^l l_{i-1}^- - c_{i,i+1}^M l_{i+1}^+ - d_i^M, b_i^M, h_i^M\right) = A_i^- \\
&< A_i^+ = f\left(c_{i,i-1}^l l_{i-1}^- - c_{i,i+1}^M l_{i+1}^+ - d_i^M, b_i^M, h_i^M\right) \\
&< B_i^+ = f\left(c_{i,i-1}^l l_{i-1}^-, b_i^M, h_i^M\right) < f\left(c_{i,i-1}^M l_{i-1}^+, b_i^l, h_i^l\right) = l_i^+, \\
0 &< l_n^- = g\left(c_{n,n-1}^M l_{n-1}^+, b_n^l, h_n^l\right) < g\left(c_{n,n-1}^l l_{n-1}^-, b_n^M, h_n^M\right) = B_n^- < g\left(c_{n,n-1}^l l_{n-1}^-, b_n^M, h_n^M\right) = A_n^- \\
&< A_n^+ = f\left(c_{n,n-1}^l l_{n-1}^-, b_n^M, h_n^M\right) < B_n^+ = f\left(c_{n,n-1}^l l_{n-1}^-, b_n^M, h_n^M\right) < f\left(c_{n,n-1}^M l_{n-1}^+, b_n^l, h_n^l\right) = l_n^+,
\end{aligned} \tag{2.5}$$

where $i = 2, 3, \ldots, n-1$, that is $0 < l_i^- < B_i^- < A_i^- < A_i^+ < B_i^+ < l_i^+, i = 1, 2, \ldots, n$. Thus, we have $\ln l_i^- < lnB_i^- < \ln A_i^- < \ln A_i^+ < \ln B_i^+ < \ln l_i^+, i = 1, 2, \ldots, n$. The proof of Lemma 2.3 is complete. □

Theorem 2.4. *Assume that* (H_1) *and* (H_2) *hold. Then system* (1.1) *has at least* 2^n *positive* ω*-periodic solutions.*

Proof. By making the substitution

$$x_i(t) = \exp\{u_i(t)\}, \quad i = 1, 2, \ldots, n, \tag{2.6}$$

system (1.1) can be reformulated as

$$\begin{aligned}
\dot{u}_1(t) &= a_1(t) - b_1(t)e^{u_1(t)} - c_{12}(t)e^{u_2(t)} - h_1(t)e^{-u_1(t)}, \\
&\vdots \\
\dot{u}_i(t) &= -d_i(t) - b_i(t)e^{u_i(t)} + c_{i,i-1}(t)e^{u_{i-1}(t)} - c_{i,i+1}(t)e^{u_{i+1}(t)} - h_i(t)e^{-u_i(t)}, \\
&\vdots \\
\dot{u}_n(t) &= -d_n(t) - b_n(t)e^{u_n(t)} + c_{n,n-1}(t)e^{u_{n-1}(t)} - h_n(t)e^{-u_n(t)},
\end{aligned} \tag{2.7}$$

where $i = 2, 3, \ldots, n-1$.

Let

$$X = Z = \left\{ u = (u_1, u_2, \ldots, u_n)^T \in C(R, R^n) : u(t+\omega) = u(t) \right\} \tag{2.8}$$

and define

$$\|u\| = \sum_{i=1}^{n} \max_{t\in[0,\omega]} |u_i(t)|, \quad u \in X \text{ or } Z. \tag{2.9}$$

Equipped with the above norm $\|\cdot\|$, X and Z are Banach spaces. Let

$$N(u,\lambda) = \begin{pmatrix} a_1(t) - b_1(t)e^{u_1(t)} - \lambda c_{12}(t)e^{u_2(t)} - h_1(t)e^{-u_1(t)} \\ \vdots \\ -\lambda d_i(t) - b_i(t)e^{u_i(t)} + c_{i,i-1}(t)e^{u_{i-1}(t)} - \lambda c_{i,i+1}(t)e^{u_{i+1}(t)} - h_i(t)e^{-u_i(t)} \\ \vdots \\ -\lambda d_n(t) - b_n(t)e^{u_n(t)} + c_{n,n-1}(t)e^{u_{n-1}(t)} - h_n(t)e^{-u_n(t)} \end{pmatrix}_{n\times 1}, \tag{2.10}$$

where $i = 2, 3, \ldots, n-1$, $Lu = \dot{u} = (\mathrm{d}u(t))/\mathrm{d}t$. We put $Pu = (1/\omega)\int_0^\omega u(t)\, \mathrm{d}t$, $u \in X; Qz = (1/\omega)\int_0^\omega z(t)\, \mathrm{d}t$, $z \in Z$. Thus it follows that $\operatorname{Ker} L = R^n$, $ImL = \{z \in Z : \int_0^\omega z(t)\, \mathrm{d}t = 0\}$ is closed in Z, $\dim \operatorname{Ker} L = n = \operatorname{codim} ImL$, and P, Q are continuous projectors such that

$$ImP = \operatorname{Ker} L, \qquad \operatorname{Ker} Q = ImL = Im\ (I - Q). \tag{2.11}$$

Hence, L is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L) $K_P : Im\, L \to \operatorname{Ker} P \bigcap \operatorname{Dom} L$ is given by

$$K_P(z) = \int_0^t z(s)\, \mathrm{d}s - \frac{1}{\omega}\int_0^\omega \int_0^s z(s)\, \mathrm{d}s. \tag{2.12}$$

Then

$$QN(u,\lambda)=\begin{pmatrix}\dfrac{1}{\omega}\displaystyle\int_0^{\omega}F_1(s,\lambda)\mathrm{d}s\\ \vdots\\ \dfrac{1}{\omega}\displaystyle\int_0^{\omega}F_n(s,\lambda)\mathrm{d}s\end{pmatrix}_{n\times1}, \tag{2.13}$$

$$K_P(I-Q)N(u,\lambda)=\begin{pmatrix}\displaystyle\int_0^t F_1(s,\lambda)\,\mathrm{d}s-\frac{1}{\omega}\int_0^{\omega}\int_0^t F_1(s,\lambda)\,\mathrm{d}s\mathrm{d}t+\left(\frac{1}{2}-\frac{t}{\omega}\right)\int_0^{\omega}F_1(s,\lambda)\mathrm{d}s\\ \vdots\\ \displaystyle\int_0^t F_n(s,\lambda)\,\mathrm{d}s-\frac{1}{\omega}\int_0^{\omega}\int_0^t F_n(s,\lambda)\,\mathrm{d}s\mathrm{d}t+\left(\frac{1}{2}-\frac{t}{\omega}\right)\int_0^{\omega}F_n(s,\lambda)\mathrm{d}s\end{pmatrix}_{n\times1}, \tag{2.14}$$

where

$$F(u,\lambda)=\begin{pmatrix}a_1(s)-b_1(s)e^{u_1(s)}-\lambda c_{12}(s)e^{u_2(s)}-h_1(s)e^{-u_1(s)}\\ \vdots\\ -\lambda d_i(s)-b_i(s)e^{u_i(s)}+c_{i,i-1}(s)e^{u_{i-1}(s)}-\lambda c_{i,i+1}(s)e^{u_{i+1}(s)}-h_i(s)e^{-u_i(s)}\\ \vdots\\ -\lambda d_n(s)-b_n(s)e^{u_n(s)}+c_{n,n-1}(s)e^{u_{n-1}(s)}-h_n(s)e^{-u_n(s)}\end{pmatrix}_{n\times1}. \tag{2.15}$$

Obviously, QN and $K_P(I-Q)N$ are continuous. It is not difficult to show that $K_P(I-Q)N(\overline{\Omega})$ is compact for any open bounded set $\Omega\subset X$ by using the Arzela-Ascoli theorem. Moreover, $QN(\overline{\Omega})$ is clearly bounded. Thus, N is L-compact on $\overline{\Omega}$ with any open bounded set $\Omega\subset X$.

In order to use Lemma 2.1, we have to find at least 2^n appropriate open bounded subsets of X. Corresponding to the operator equation $Lu=\lambda N(u,\lambda)$, $\lambda\in(0,1)$, we have

$$\begin{aligned}\dot{u}_1(t)&=\lambda\Big(a_1(t)-b_1(t)e^{u_1(t)}-\lambda c_{12}(t)e^{u_2(t)}-h_1(t)e^{-u_1(t)}\Big),\\ &\vdots\\ \dot{u}_i(t)&=\lambda\Big(-\lambda d_i(t)-b_i(t)e^{u_i(t)}+c_{i,i-1}(t)e^{u_{i-1}(t)}-\lambda c_{i,i+1}(t)e^{u_{i+1}(t)}-h_i(t)e^{-u_i(t)}\Big),\\ &\vdots\\ \dot{u}_n(t)&=\lambda\Big(-\lambda d_n(t)-b_n(t)e^{u_n(t)}+c_{n,n-1}(t)e^{u_{n-1}(t)}-h_n(t)e^{-u_n(t)}\Big),\end{aligned} \tag{2.16}$$

where $i = 2,3,\dots,n-1$. Assume that $u \in X$ is an ω-periodic solution of system (2.16) for some $\lambda \in (0,1)$. Then there exist $\xi_i, \eta_i \in [0,\omega]$ such that $u_i(\xi_i) = \max_{t\in[0,\omega]} u_i(t), u_i(\eta_i) = \min_{t\in[0,\omega]} u_i(t),\ i = 1,2,\dots,n$. It is clear that $\dot{u}_i(\xi_i) = 0, \dot{u}_i(\eta_i) = 0, i = 1,2,\dots,n$. From this and (2.16), we have

$$\begin{gathered}
a_1(\xi_1) - b_1(\xi_1)e^{u_1(\xi_1)} - \lambda c_{12}(\xi_1)e^{u_2(\xi_1)} - h_1(\xi_1)e^{-u_1(\xi_1)} = 0, \\
\vdots \\
-\lambda d_i(\xi_i) - b_i(\xi_i)e^{u_i(\xi_i)} + c_{i,i-1}(\xi_i)e^{u_{i-1}(\xi_i)} - \lambda c_{i,i+1}(\xi_i)e^{u_{i+1}(\xi_i)} - h_i(\xi_i)e^{-u_i(\xi_i)} = 0, \\
\vdots \\
-\lambda d_n(\xi_i) - b_n(\xi_n)e^{u_n(\xi_n)} + c_{n,n-1}(\xi_n)e^{u_{n-1}(\xi_n)} - h_n(\xi_n)e^{-u_n(\xi_n)} = 0
\end{gathered} \tag{2.17}$$

$$\begin{gathered}
a_1(\eta_1) - b_1(\eta_1)e^{u_1(\eta_1)} - \lambda c_{12}(\eta_1)e^{u_2(\eta_1)} - h_1(\eta_1)e^{-u_1(\eta_1)} = 0, \\
\vdots \\
-\lambda d_i(\eta_i) - b_i(\eta_i)e^{u_i(\eta_i)} + c_{i,i-1}(\eta_i)e^{u_{i-1}(\eta_i)} - \lambda c_{i,i+1}(\eta_i)e^{u_{i+1}(\eta_i)} - h_i(\eta_i)e^{-u_i(\eta_i)} = 0, \\
\vdots \\
-\lambda d_n(\eta_i) - b_n(\eta_n)e^{u_n(\eta_n)} + c_{n,n-1}(\eta_n)e^{u_{n-1}(\eta_n)} - h_n(\eta_n)e^{-u_n(\eta_n)} = 0,
\end{gathered} \tag{2.18}$$

where $i = 2,3,\dots,n-1$.

On one hand, according to (2.17), we have

$$b_1^l e^{2u_1(\xi_1)} - a_1^M e^{u_1(\xi_1)} + h_1^l \le b_1(\xi_1)e^{2u_1(\xi_1)} - a_1(\xi_1)e^{u_1(\xi_1)} + h_1(\xi_1) = -\lambda c_{12}(\xi_1)e^{u_1(\xi_1)+u_2(\xi_1)} < 0, \tag{2.19}$$

namely,

$$b_1^l e^{2u_1(\xi_1)} - a_1^M e^{u_1(\xi_1)} + h_1^l < 0, \tag{2.20}$$

which implies that

$$\ln l_1^- < u_1(\xi_1) < \ln l_1^+, \tag{2.21}$$

$$\begin{aligned}
b_2^l e^{2u_2(\xi_2)} + h_2^l &< b_2(\xi_2)e^{2u_2(\xi_2)} + \lambda c_{23}(\xi_2)e^{u_2(\xi_2)+u_3(\xi_2)} + \lambda d_2(\xi_2)e^{u_2(\xi_2)} + h_2(\xi_2) \\
&= c_{21}(\xi_2)e^{u_2(\xi_2)+u_1(\xi_2)} < c_{21}^M l_1^+ e^{u_2(\xi_2)};
\end{aligned} \tag{2.22}$$

that is,

$$b_2^l e^{2u_2(\xi_2)} - c_{21}^M l_1^+ e^{u_2(\xi_2)} + h_2^l < 0, \tag{2.23}$$

which implies that

$$\ln l_2^- < u_2(\xi_2) < \ln l_2^+. \tag{2.24}$$

By deducing for $i = 3, 4, \ldots, n-1$, we obtain

$$\begin{aligned} b_i^l e^{2u_i(\xi_i)} + h_i^l &< b_i(\xi_i) e^{2u_i(\xi_i)} + \lambda c_{i,i+1}(\xi_i) e^{u_i(\xi_i)+u_{i+1}(\xi_i)} + \lambda d_i(\xi_i) e^{u_i(\xi_i)} + h_i(\xi_i) \\ &= c_{i,i-1}(\xi_i) e^{u_i(\xi_i)+u_{i-1}(\xi_i)} < c_{i,i-1}^M l_{i-1}^+ e^{u_i(\xi_i)}, \end{aligned} \tag{2.25}$$

namely,

$$b_i^l e^{2u_i(\xi_i)} - c_{i,i-1}^M l_{i-1}^+ e^{u_i(\xi_i)} + h_i^l < 0, \tag{2.26}$$

which implies that

$$\ln l_i^- < u_i(\xi_i) < \ln l_i^+, \quad i = 3, 4, \ldots, n-1, \tag{2.27}$$

$$\begin{aligned} b_n^l e^{2u_n(\xi_i)} + h_n^l &< b_n(\xi_n) e^{2u_n(\xi_n)} + \lambda d_n(\xi_n) e^{u_n(\xi_n)} + h_n(\xi_n) \\ &= c_{n,n-1}(\xi_n) e^{u_n(\xi_n)+u_{n-1}(\xi_n)} < c_{n,n-1}^M l_{n-1}^+ e^{u_n(\xi_n)}, \end{aligned} \tag{2.28}$$

namely,

$$b_n^l e^{2u_n(\xi_n)} - c_{n,n-1}^M l_{n-1}^+ e^{u_n(\xi_n)} + h_n^l < 0, \tag{2.29}$$

which implies that

$$\ln l_n^- < u_n(\xi_n) < \ln l_n^+. \tag{2.30}$$

In view of (2.20), (2.23), (2.26), and (2.29), we have

$$\ln l_i^- < u_i(\xi_i) < \ln l_i^+, \quad i = 1, 2, \ldots, n. \tag{2.31}$$

From (2.18), one can analogously obtain

$$\ln l_i^- < u_i(\eta_i) < \ln l_i^+, \quad i = 1, 2, \ldots, n. \tag{2.32}$$

By (2.30) and (2.31), we get

$$\ln l_i^- < u_i(\eta_i) < u_i(\xi_i) < \ln l_i^+, \quad i = 1, 2, \ldots, n. \tag{2.33}$$

On the other hand, in view of (2.17), we have

$$-b_1^M e^{2u_1(\xi_1)} + a_1^l e^{u_1(\xi_1)} - h_1^M \le -b_1(\xi_1)e^{2u_1(\xi_1)} + a_1(\xi_1)e^{u_1(\xi_1)} - h_1(\xi_1) \\ = \lambda c_{12}(\xi_1)e^{u_1(\xi_1)+u_2(\xi_1)} < c_{12}^M l_2^+ e^{u_1(\xi_1)}, \tag{2.34}$$

namely,

$$b_1^M e^{2u_1(\xi_1)} - \left(a_1^l - c_{12}^M l_2^+\right)e^{u_1(\xi_1)} + h_1^M > 0, \tag{2.35}$$

which implies that

$$\ln A_1^+ < u_1(\xi_1) \quad \text{or} \quad u_1(\xi_1) < \ln A_1^-, \tag{2.36}$$

$$c_{21}^l l_1^- < c_{21}(\xi_2)e^{u_1(\xi_2)} = b_2(\xi_2)e^{u_2(\xi_2)} + h_2(\xi_2)e^{-u_2(\xi_2)} + \lambda d_2(\xi_2) + \lambda c_{23}(\xi_2)e^{u_3(\xi_2)} \\ < b_2^M e^{u_2(\xi_2)} + h_2^M e^{-u_2(\xi_2)} + d_2^M + c_{23}^M l_3^+, \tag{2.37}$$

that is,

$$b_2^M e^{2u_2(\xi_2)} - \left(c_{21}^l l_1^- - c_{23}^M l_3^+ - d_2^M\right)e^{u_2(\xi_2)} + h_2^M > 0, \tag{2.38}$$

which implies that

$$\ln A_2^+ < u_2(\xi_2) \text{ 2003or } u_2(\xi_2) < \ln A_2^-. \tag{2.39}$$

By deducing for $i = 3, 4, \ldots, n-1$, we obtain

$$c_{i,i-1}^l l_{i-1}^- < c_{i,i-1}(\xi_i)e^{u_{i-1}(\xi_i)} = b_i(\xi_i)e^{u_i(\xi_i)} + h_i(\xi_i)e^{-u_i(\xi_i)} + \lambda d_i(\xi_i) + \lambda c_{i,i+1}(\xi_i)e^{u_{i+1}(\xi_i)} \\ < b_i^M e^{u_i(\xi_i)} + h_i^M \mathrm{e}^{-u_i(\xi_i)} + d_i^M + c_{i,i+1}^M l_{i+1}^+, \tag{2.40}$$

that is,

$$b_i^M e^{2u_i(\xi_i)} - \left(c_{i,i-1}^l l_{i-1}^- - c_{i,i+1}^M l_{i+1}^+ - d_i^M\right)e^{u_i(\xi_i)} + h_i^M > 0, \tag{2.41}$$

which implies that

$$\ln A_i^+ < u_i(\xi_i) \quad \text{or} \quad u_i(\xi_i) < \ln A_i^-, \quad i = 3, 4, \ldots, n-1, \tag{2.42}$$

$$c_{n,n-1}^l l_{n-1}^- < c_{n,n-1}(\xi_n)e^{u_{n-1}(\xi_n)} = b_n(\xi_n)e^{u_n(\xi_n)} + h_n(\xi_n)e^{-u_n(\xi_n)} + \lambda d_n(\xi_n) \\ < b_n^M e^{u_n(\xi_n)} + h_n^M e^{-u_n(\xi_n)} + d_n^M, \tag{2.43}$$

namely,

$$b_n^M e^{2u_n(\xi_n)} - \left(c_{n,n-1}^l l_{n-1}^- - d_n^M\right) e^{u_n(\xi_n)} + h_n^M > 0, \tag{2.44}$$

which implies that

$$\ln A_n^+ < u_n(\xi_n) \quad \text{or} \quad u_n(\xi_n) < \ln A_n^-. \tag{2.45}$$

It follows from (2.35), (2.38), (2.41), and (2.44) that

$$\ln A_i^+ < u_i(\xi_i) \quad \text{or} \quad u_i(\xi_i) < \ln A_i^-, \quad i = 1, 2, \ldots, n. \tag{2.46}$$

From (2.18), one can analogously obtain

$$\ln A_i^+ < u_i(\eta_i) \quad \text{or} \quad u_i(\eta_i) < \ln A_i^-, \quad i = 1, 2, \ldots, n. \tag{2.47}$$

By (2.32), (2.45), (2.46), and Lemma 2.3, we get

$$\ln A_i^+ < u_i(\eta_i) < u_i(\xi_i) < \ln l_i^+ \quad \text{or} \quad \ln l_i^- < u_i(\eta_i) < u_i(\xi_i) < \ln A_i^-, \quad i = 1, 2, \ldots, n, \tag{2.48}$$

which implies that, for all $t \in R$,

$$\ln A_i^+ < u_i(t) < \ln l_i^+ \quad \text{or} \quad \ln l_i^- < u_i(t) < \ln A_i^-, \quad i = 1, 2, \ldots, n. \tag{2.49}$$

For convenience, we denote

$$G_i = \left(\ln l_i^-, \ln A_i^-\right), \quad H_i = \left(\ln A_i^+, \ln l_i^+\right), \quad i = 1, 2, \ldots, n. \tag{2.50}$$

Clearly, $l_i^\pm$ $(i = 1, 2, \ldots, n)$ and $A_i^\pm$ $(i = 1, 2, \ldots, n)$ are independent of λ. For each $i = 1, 2, \ldots, n$, we choose an interval between two intervals G_i and H_i, and denote it as Δ_i, then define the set

$$\left\{u = (u_1, u_2, \ldots, u_n)^T \in X : u_i(t) \in \Delta_i,\ t \in R,\ i = 1, 2, \ldots, n\right\}. \tag{2.51}$$

Obviously, the number of the above sets is 2^n. We denote these sets as Ω_k, $k = 1, 2, \ldots, 2^n$. $\Omega_k, k = 1, 2, \ldots, 2^n$ are bounded open subsets of $X, \Omega_i \cap \Omega_j = \phi$, $i \neq j$. Thus $\Omega_k (k = 1, 2, \ldots, 2^n)$ satisfies the requirement (a) in Lemma 2.1.

Now we show that (b) of Lemma 2.1 holds; that is, we prove when $u \in \partial\Omega_k \cap \operatorname{Ker} L = \partial\Omega_k \cap R^n,\ QN(u,0) \neq (0,0,\ldots,0)^T,\ k = 1,2,\ldots,2^n$. If it is not true, then when $u \in \partial\Omega_k \cap \operatorname{Ker} L = \partial\Omega_k \cap R^n,\ i = 1,2,\ldots,2^n$, constant vector $u = (u_1,u_2,\ldots,u_n)^T$, with $u \in \partial\Omega_k,\ k = 1,2,\ldots,2^n$, satisfies

$$
\begin{gathered}
\int_0^{\omega}\left(a_1(t) - b_1(t)e^{u_1} - h_1(t)e^{-u_1}\right)\mathrm{d}t = 0,\\
\vdots\\
\int_0^{\omega}\left(-b_i(t)e^{u_i} + c_{i,i-1}(t)e^{u_{i-1}} - h_i(t)e^{-u_i}\right)\mathrm{d}s = 0,\\
\vdots\\
\int_0^{\omega}\left(-b_n(t)e^{u_n} + c_{n,n-1}(t)e^{u_{n-1}} - h_n(t)e^{-u_n}\right)\mathrm{d}s = 0,
\end{gathered}
\tag{2.52}
$$

where $i = 2,3,\ldots,n-1$. In view of the mean value theorem of calculous, there exist n points $t_i\ (i = 1,2,\ldots,n)$ such that

$$
\begin{gathered}
a_1(t_1) - b_1(t_1)e^{u_1} - h_1(t_1)e^{-u_1} = 0,\\
\vdots\\
-b_i(t_i)e^{u_i} + c_{i,i-1}(t_i)e^{u_{i-1}} - h_i(t_i)e^{-u_i} = 0,\\
\vdots\\
-b_n(t_n)e^{u_n} + c_{n,n-1}(t_n)e^{u_{n-1}} - h_n(t_n)e^{-u_n} = 0,
\end{gathered}
\tag{2.53}
$$

where $i = 2,3,\ldots,n-1$. Following the argument of (2.20)–(2.47), from (2.52), we obtain

$$
\ln l_i^- < u_i < \ln B_i^- < \ln A_i^- \quad \text{or} \quad \ln A_i^+ < \ln B_i^+ < u_i < lnl_i^+, \quad i = 1,2,\ldots,n. \tag{2.54}
$$

Then u belongs to one of $\Omega_k \cap R^n, k = 1,2,\ldots,2^n$. This contradicts the fact that $u \in \partial\Omega_k \cap R^n, k = 1,2,\ldots,2^n$. This proves that (b) in Lemma 2.1 holds.

Finally, in order to show that (c) in Lemma 2.1 holds, we only prove that for $u \in \partial\Omega_k \cap \operatorname{Ker} L = \partial\Omega_k \cap R^n, k = 1,2,\ldots,2^n$, then it holds that $\deg\{JQN(u,0), \Omega_k \cap \operatorname{Ker} L, (0,0,\ldots,0)^T\} \neq 0$. To this end, we define the mapping $\phi : \operatorname{Dom}\ L \times [0,1] \to X$ by

$$
\phi(u,\mu) = \mu QN(u,0) + (1-\mu)G(u); \tag{2.55}
$$

here $\mu \in [0,1]$ is a parameter and $G(u)$ is defined by

$$G(u) = \begin{pmatrix} \int_0^\omega \left(a_1(s) - b_1(s)e^{u_1(s)} - h_1(s)e^{-u_1(s)}\right)\mathrm{d}s \\ \vdots \\ \int_0^\omega \left(c_{i,i-1}^M l_{i-1}^+ - b_i(s)e^{u_i(s)} - h_i(s)e^{-u_i(s)}\right)\mathrm{d}s \\ \vdots \\ \int_0^\omega \left(c_{n,n-1}^M l_{n-1}^+ - b_n(s)e^{u_n(s)} - h_n(s)e^{-u_n(s)}\right)\mathrm{d}s \end{pmatrix}_{n\times 1}, \tag{2.56}$$

where $i = 2,3,\ldots,n-1$. We show that for $u \in \partial\Omega_k \cap \operatorname{Ker} L = \partial\Omega_k \cap R^n,\ k = 1,2,\ldots,2^n,\ \mu \in [0,1]$, then it holds that $\phi(u,\mu) \neq (0,0,\ldots,0)^T$. Otherwise, parameter μ and constant vector $u = (u_1,u_2,\ldots,u_n)^T \in R^n$ satisfy $\phi(u,\mu) = (0,0\ldots,0)^T$, that is,

$$\begin{aligned} 0 &= \mu\int_0^\omega \left(a_1(s) - b_1(s)e^{u_1} - h_1(s)e^{-u_1}\right)\mathrm{d}s + (1-\mu)\int_0^\omega \left(a_1(s) - b_1(s)e^{u_1} - h_1(s)e^{-u_1}\right)\mathrm{d}s, \\ &\vdots \\ 0 &= \mu\int_0^\omega \left(c_{i,i-1}(s)e^{u_{i-1}} - b_i(s)e^{u_i} - h_i(s)e^{-u_i}\right)\mathrm{d}s + (1-\mu)\int_0^\omega \left(c_{i,i-1}^M l_{i-1}^+ - b_i(s)e^{u_i} - h_i(s)e^{-u_i}\right)\mathrm{d}s, \\ &\vdots \\ 0 &= \mu\int_0^\omega \left(c_{n,n-1}(s)e^{u_{n-1}} - b_n(s)e^{u_n} - h_n(s)e^{-u_n}\right)\mathrm{d}s \\ &\quad + (1-\mu)\int_0^\omega \left(c_{n,n-1}^M l_{n-1}^+ - b_n(s)e^{u_n} - h_n(s)e^{-u_n}\right)\mathrm{d}s, \end{aligned} \tag{2.57}$$

where $i = 2,3,\ldots,n-1$. In view of the mean value theorem of calculous, there exist n points $\bar{t}_i \in [0,\omega]$ $(i = 1,2,\ldots,n)$ such that

$$\begin{aligned} a_1\left(\bar{t}_1\right) - b_1\left(\bar{t}_1\right)e^{u_1} - h_1\left(\bar{t}_1\right)e^{-u_1} &= 0, \\ &\vdots \\ c_{i,i-1}^M l_{i-1}^+ - b_i\left(\bar{t}_i\right)e^{u_i} - h_i\left(\bar{t}_i\right)e^{-u_i} &= \mu\left(c_{i,i-1}^M l_{i-1}^+ - c_{i,i-1}\left(\bar{t}_i\right)e^{u_{i-1}}\right), \\ &\vdots \\ c_{n,n-1}^M l_{n-1}^+ - b_n\left(\bar{t}_n\right)e^{u_n} - h_n\left(\bar{t}_n\right)e^{-u_n} &= \mu\left(c_{n,n-1}^M l_{n-1}^+ - c_{i,n-1}\left(\bar{t}_n\right)e^{u_{n-1}}\right), \end{aligned} \tag{2.58}$$

where $i = 2, 3, \ldots, n-1$. Following the argument of (2.20)–(2.47), from (2.57), we obtain

$$\ln l_i^- < u_i < \ln B_i^- < \ln A_i^- \quad \text{or} \quad \ln A_i^+ < \ln B_i^+ < u_i < \ln l_i^+, \quad i = 1, 2, \ldots, n. \tag{2.59}$$

given that u belongs to one of $\Omega_k \cap R^n$, $k = 1, 2, \ldots, 2^n$. This contradicts the fact that $u \in \partial\Omega_k \cap R^n$, $k = 1, 2, \ldots, 2^n$. This proves $\phi(u, \mu) \neq (0, 0, \ldots, 0)^T$ holds. Note that the system of the following algebraic equations:

$$\begin{gathered} a_1\left(\bar{t}_1\right) - b_1\left(\bar{t}_1\right)e^{x_1} - h_1\left(\bar{t}_1\right)e^{-x_1} = 0, \\ \vdots \\ c_{i,i-1}^M l_{i-1}^+ - b_i\left(\bar{t}_i\right)e^{x_i} - h_i\left(\bar{t}_i\right)e^{-x_i} = 0, \\ \vdots \\ c_{n,n-1}^M l_{n-1}^+ - b_n\left(\bar{t}_n\right)e^{x_n} - h_n\left(\bar{t}_n\right)e^{-x_n} = 0 \end{gathered} \tag{2.60}$$

has 2^n distinct solutions since (H_1) and (H_2) hold, $(x_1^*, x_2^*, \ldots, x_n^*) = (\ln \hat{x}_1, \ln \hat{x}_2, \ldots, \ln \hat{x}_n)$, where $x_1^{\pm} = (a_1(\bar{t}_1) \pm \sqrt{(a_1(\bar{t}_1))^2 - 4b_1(\bar{t}_1)h_1(\bar{t}_1)})/(2b_1(\bar{t}_1))$, $x_k^{\pm} = ((c_{k,k-1}^M l_{k-1}^+ \pm \sqrt{(c_{k,k-1}^M l_{k-1}^+)^2 - 4b_k(\bar{t}_k)h_k(\bar{t}_k)})/(2b_k(\bar{t}_k)))$ $(k = 2, 3, \ldots, n)$, $\hat{x}_i = x_i^-$ or $\hat{x}_i = x_i^+$, $i = 1, 2, \ldots, n$. Similar to the proof of Lemma 2.3, it is easy to verify that

$$\ln l_i^- < \ln x_i^- < \ln B_i^- < \ln A_i^- < \ln A_i^+ < \ln B_i^+ < \ln x_i^+ < \ln l_i^+, \quad i = 1, 2, \ldots, n. \tag{2.61}$$

Therefore, $(x_1^*, x_2^*, \ldots, x_n^*)$ uniquely belongs to the corresponding Ω_k. Since $\operatorname{Ker} L = ImQ$, we can take $J = I$. A direct computation gives, for $k = 1, 2, \ldots, 2^n$,

$$\begin{aligned} &\deg\left\{JQN(u,0), \Omega_k \cap \operatorname{Ker} L, (0,0,\ldots,0)^T\right\} \\ &\quad = \deg\left\{\phi(u,1), \Omega_k \cap \operatorname{Ker} L, (0,0,\ldots,0)^T\right\} \\ &\quad = \deg\left\{\phi(u,0), \Omega_k \cap \operatorname{Ker} L, (0,0,\ldots,0)^T\right\} \\ &\quad = \operatorname{sign}\left[\prod_{i=1}^{n}\left(-b_i\left(\bar{t}_i\right)x_i^* + \frac{h_i\left(\bar{t}_i\right)}{x_i^*}\right)\right]. \end{aligned} \tag{2.62}$$

Since $a_1(\bar{t}_1) - b_1(\bar{t}_1)x_1^* - (h_1(\bar{t}_1))/(x_1^*) = 0$, $c_{i,i-1}^M l_{i-1}^+ - b_i(\bar{t}_i)x_i^* - (h_i(\bar{t}_i))/(x_i^*) = 0$ $(i = 2,3,\ldots,n)$, then

$$\begin{aligned}
&\deg\left\{JQN(u,0), \Omega_k \cap \operatorname{Ker} L, (0,0,\ldots,0)^T\right\} \\
&\quad = \operatorname{sign}\left[\prod_{i=2}^{n}\left(a_1\left(\bar{t}_1 - 2b_1\left(\bar{t}_1\right)x_1^*\right)\right)\left(c_{i,i-1}^M l_{i-1}^+ - 2b_i^M x_i^*\right)\right] = \pm 1, \; k = 1,2,\ldots,2^n.
\end{aligned} \tag{2.63}$$

So far, we have proved that $\Omega_k (k = 1,2,\ldots,2^n)$ satisfies all the assumptions in Lemma 2.1. Hence, system (2.7) has at least 2^n different ω-periodic solutions. Thus, by (2.6) system (1.1) has at least 2^n different positive ω-periodic solutions. This completes the proof of Theorem 2.4. □

In system (1.1), if $c_{i,i-1}(t) \geq 0$ $(i = 2,3,\ldots,n)$, $c_{j,j+1}(t) \geq 0$ $(j = 1,2,\ldots,n-1)$, and $a_1(t) > 0$, $d_i(t) > 0$, $b_i(t) > 0$, $h_i(t) > 0$ are continuous periodic functions, then similar to the proof of Theorem 2.4, one can prove the following

Theorem 2.5. *Assume that* (H_1) *and* (H_2) *hold. Then system* (1.1) *has at least* 2^n *positive* ω*-periodic solutions.*

Remark 2.6. In Theorem 2.5, $c_{i-1,i}(t) = 0$ means that the ith species does not prey the $(i-1)$th species, thus $c_{i,i-1}(t) = 0$. That is to say, there is no relationship between the ith species and the $(i-1)$th species.

3. Illustrative Examples

Example 3.1. Consider the following three-species food chain with harvesting terms:

$$\begin{aligned}
\dot{x}(t) &= x(t)\left(3 + \sin t - \frac{4+\sin t}{10}x(t) - c_{12}(t)y(t)\right) - \frac{9+\cos t}{20}, \\
\dot{y}(t) &= y(t)\left(-\frac{3+\cos t}{10} - \frac{5+\cos t}{10}y(t) + c_{21}(t)x(t) - c_{23}(t)z(t)\right) - \frac{2+\cos t}{5}, \\
\dot{z}(t) &= z(t)\left(-\frac{3+\sin 2t}{10} - \frac{8+\sin 2t}{10}z(t) + c_{32}(t)y(t)\right) - \frac{8+\cos 2t}{10}.
\end{aligned} \tag{3.1}$$

In this case, $a_1(t) = 3 + \sin t$, $b_1(t) = (4 + \sin t)/10$, $h_1(t) = (9 + \cos t)/20$, $d_2(t) = (3 + \cos t)/10$, $b_2(t) = (5 + \cos t)/10$, $h_2(t) = (2 + \cos t)/5$, $d_3(t) = (3 + \sin 2t)/10$, $b_3(t) = (8 + \sin 2t)/10$, and $h_3(t) = (8 + \cos 2t)/10$. Since

$$l_1^{\pm} = \frac{a_1^M \pm \sqrt{\left(a_1^M\right)^2 - 4b_1^l h_1^l}}{2b_1^l} = \frac{20 \pm 2\sqrt{97}}{3}, \tag{3.2}$$

taking $c_{21}(t) \equiv 14/(5l_1^-)$, then we have

$$l_2^{\pm} = \frac{c_{21}^M l_1^+ \pm \sqrt{\left(c_{21}^M l_1^+\right)^2 - 4b_2^l h_2^l}}{2b_2^l} = \frac{7l_1^+ \pm \sqrt{49\left(l_1^+\right)^2 - 2\left(l_1^-\right)^2}}{2l_1^-}. \quad (3.3)$$

Take $c_{32}(t) \equiv 12/(5l_2^-)$, then

$$l_3^{\pm} = \frac{c_{32}^M l_2^+ \pm \sqrt{\left(c_{32}^M l_2^+\right)^2 - 4b_3^l h_3^l}}{2b_3^l} = \frac{12l_2^+ \pm \sqrt{\left(12l_2^+\right)^2 - \left(7l_2^-\right)^2}}{7l_2^-}. \quad (3.4)$$

Take $c_{12}(t) \equiv 1/(2l_2^+), c_{23}(t) \equiv 1/(l_3^+)$, then

$$\begin{gathered} a_1^l - c_{12}^M l_2^+ = \frac{3}{2} > 1 = 2\sqrt{b_1^M h_1^M}, \quad c_{32}^l l_2^- - d_3^M = 2 > \frac{9}{5} = 2\sqrt{b_3^M h_3^M}, \\ c_{21}^l l_1^- - c_{23}^M l_3^+ - d_2^M = \frac{7}{5} > \frac{6}{5} = 2\sqrt{b_2^M h_2^M}. \end{gathered} \quad (3.5)$$

Therefore, all conditions of Theorem 2.4 are satisfied. By Theorem 2.4, system (3.1) has at least eight positive 2π-periodic solutions.

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