

Research Article

Riccati Equations and Delay-Dependent BIBO Stabilization of Stochastic Systems with Mixed Delays and Nonlinear Perturbations

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Received 21 August 2010; Accepted 9 December 2010

Academic Editor: T. Bhaskar

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The mean square BIBO stability is investigated for stochastic control systems with mixed delays and nonlinear perturbations. The system with mixed delays is transformed, then a class of suitable Lyapunov functionals is selected, and some novel delay-dependent BIBO stabilization in mean square criteria for stochastic control systems with mixed delays and nonlinear perturbations are obtained by applying the technique of analyzing controller and the method of existing a positive definite solution to an auxiliary algebraic Riccati matrix equation. A numerical example is given to illustrate the validity of the main results.

1. Introduction

In recent years, Bounded-Input Bounded-Output (BIBO) stabilization has been investigated by many researchers in order to track out the reference input signal in real world, see [1–6] and some references therein. On the other hand, because of the finite switching speed, memory effects, and so on, time delays are unavoidable in technology and nature, commonly exist in various mechanical, chemical engineering, physical, biological, and economic systems. They can make the concerned control system be of poor performance and instable, which cause the hardware implementation of the control system to become difficult. It is necessary to introduce the distributed delay in control systems, which can describe mathematical modeling of many biological phenomena, for instance, in prey-predator systems, see [7–9]. And so, BIBO stabilization analysis for mixed delays and nonlinear systems is of great significance.

In [10, 11], the sufficient condition for BIBO stabilization of the control system with no delays was proposed by the Bihari-type inequality. In [12, 13], employing the parameters

technique and the Gronwall inequality investigated the BIBO stability of the system without distributed time delays. In [14–16], based on Riccati-equations, by constructing appropriate Lyapunov functions, some BIBO stabilization criteria for a class of delayed control systems with nonlinear perturbations were established. In [17], the BIBO stabilization problem of a class of piecewise switched linear system was further investigated. However, up to now, these previous results have been assumed to be in deterministic systems, including continuous time deterministic systems and discrete time deterministic systems, but seldom in stochastic systems (see [18], Fu and Liao get several mean square BIBO stabilization criteria in terms of Razumikhin technique and comparison principle). In practice, stochastic control systems are more applicable to problems that are environmentally noisily in nature or related to biological realities. Thus, the BIBO stabilization analysis problems for stochastic case are necessary.

Up to now, to the best of authors knowledge, the method of existence of a positive definite solution to an auxiliary algebraic Riccati matrix equation is only used to deal with the BIBO stabilization for deterministic differential equations [14–17], not for stochastic differential equations.

Motivated by the above discussions, the main aim of this paper is to study the BIBO stabilization in mean square for the stochastic control system with mixed delays and nonlinear perturbations. Based on the technique of analyzing controller and transforming of the system, various suitable Lyapunov functionals are selected, different Riccati matrix equations are established, and some sufficient conditions guaranteeing BIBO stabilization in mean square are obtained. Finally, a numerical example is provided to demonstrate the effectiveness of the derived results.

2. Problem Formulation and Preliminaries

Consider the stochastic control system described by the following equation:

$$\begin{aligned} dx(t) &= \left[Ax(t) + B_1x(t - h_1) + B_2 \int_{t-h_2}^t x(s)ds + f(t, x(t)) + Du(t) \right] dt \\ &\quad + \left[C_1x(t - \tau_1) + C_2 \int_{t-\tau_2}^t x(s)ds \right] d\mathbf{w}(t), \quad t \geq t_0 \geq 0, \\ y(t) &= Hx(t), \\ x(\theta) &= \varphi(\theta) \in C_{\mathcal{F}_0}^b([t_0 - \tau, t_0]; R^n), \quad \theta \in [t_0 - \tau, t_0], \end{aligned} \quad (2.1)$$

where $x(t)$, $u(t)$, $y(t)$ are the state vector, control input, control output of the system, respectively. $\tau_1 > 0$, $h_1 > 0$ are discrete time delays, and $h_2 > 0$, $\tau_2 > 0$ are distributed time delays, $\tau = \max\{\tau_1, \tau_2, h_1, h_2\}$. $A, B_1, B_2, C_1, C_2, D, H$ are constant matrices with appropriate dimensional, and C_2, D are nonsingular matrices. $\mathbf{w}(t) = (\mathbf{w}_1(t), \mathbf{w}_2(t), \dots, \mathbf{w}_n(t))$ is an n -dimensional standard Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \{F_t\}_{t \geq 0}, P)$ with a natural filtration $\{F_t\}_{t \geq 0}$. $f(t, x(t))$ is the nonlinear vector-valued perturbation bounded in magnitude as

$$\|f(t, x(t))\| \leq \alpha \|x(t)\|, \quad (2.2)$$

where α is known positive constant.

To obtain the control law described by (2.1) of tracking out the reference input of the system, we let the controller be in the form of

$$u(t) = u_1(t) + u_2(t) + u_3(t), \quad (2.3)$$

with

$$\begin{aligned} u_1(t) &= K_1(t)x(t) + r(t), \\ u_2(t) &= K_2(t)x(t - h_1) + r(t - h_1), \\ u_3(t) &= K_3(t)x(t - h_2) + r(t - h_2), \end{aligned} \quad (2.4)$$

where $K_1(t)$, $K_2(t)$, $K_3(t)$ are the feedback gain matrices, $r(t)$, $r(t - h_1)$, $r(t - h_2)$ are the reference inputs.

To derive our main results, we need to introduce the following definitions and lemmas.

Definition 2.1 (see [18]). A vector function $r(t) = (r_1(t), r_2(t), \dots, r_n(t))^T$ is said to be an element of L_∞^n if $\|r\|_\infty = \sup_{t \in [t_0, +\infty)} \|r(t)\| < +\infty$, where $\|\cdot\|$ denotes the Euclid norm in R^n or the norm of a matrix.

Definition 2.2 (see [18]). The nonlinear stochastic control system (2.2) is said to be BIBO stabilized in mean square if one can construct a controller (2.5) such that the output $y(t)$ satisfies

$$\mathbb{E}(\|y(t)\|^2) \leq N_1 + N_2 \|r\|_\infty^2, \quad (2.5)$$

where N_1, N_2 are positive constants.

Definition 2.3 (L-operator). Let Lyapunov functional $V : C([- \tau, 0]; R^n) \times R_+ \rightarrow R$; its infinitesimal operator, \mathcal{L} , acting on functional V is defined by

$$\mathcal{L}V(x(t), t) = \lim_{\Delta \rightarrow 0^+} \sup \frac{1}{\Delta} [EV(x(t + \Delta), t + \Delta) - V(x(t), t)]. \quad (2.6)$$

Lemma 2.4 (see [19]). For any constant symmetric matrix $M \in R^{n \times n}$, $M = M^T > 0$, scalar $r > 0$, vector function $g : [0, r] \rightarrow R^n$, such that the integrations in the following are well defined:

$$r \int_0^r g^T(s) M g(s) ds \geq \left[\int_0^r g(s) ds \right]^T M \left[\int_0^r g(s) ds \right]. \quad (2.7)$$

Lemma 2.5 (see [20]). Let $x, y \in R^n$ and any $n \times n$ positive-definite matrix $Q > 0$. Then, one has

$$2x^T y \leq x^T Q^{-1} x + y^T Q y. \quad (2.8)$$

3. BIBO Stabilization for Nonlinear Stochastic Systems

Transform the original system (2.1) to the following system:

$$\begin{aligned}
d \left[x(t) + B_1 \int_{t-h_1}^t x(s) ds \right] &= \left[(A + B_1)x(t) + B_2 \int_{t-h_2}^t x(s) ds + f(t, x(t)) \right] dt \\
&\quad + \left[C_1 x(t - \tau_1) + C_2 \int_{t-\tau_2}^t x(s) ds \right] d\mathbf{w}(t) + Du(t)dt, \quad t \geq t_0 \geq 0, \\
y(t) &= Hx(t), \\
x(\theta) &= \varphi(\theta) \in C_{\mathcal{F}_0}^b([t_0 - \tau, t_0]; R^n), \quad \theta \in [t_0 - \tau, t_0].
\end{aligned} \tag{3.1}$$

Theorem 3.1. *The nonlinear stochastic control system (2.1) or (3.1) with the control law (2.3) is BIBO stabilized in mean square if $\|h_1 B_1\| < 1$ and there exist symmetric positive-definite matrices $R_i > 0$, $i = 1, 2, \dots, 10$, and $Q_1 > 0$ such that*

$$\lambda_{\min}(Q_1) - 2\alpha\|P\| > 0 \tag{3.2}$$

and P is the symmetric positive solution of the Riccati equation

$$P(A + B_1) + (A + B_1)^T P + P\Sigma_1 P + \Xi_1 + \Delta_1 = -Q_1, \tag{3.3}$$

where

$$\begin{aligned}
\Sigma_1 &= B_2 R_1^{-1} B_2^T + 2DR_{10}D^T + R_5^{-1} + R_6^{-1} + DR_{10}D^T P B_1 R_7^{-1} B_1^T P D R_{10} D^T \\
&\quad + B_1 R_8^{-1} B_1^T + B_1 R_9^{-1} B_1^T, \\
\Xi_1 &= h_2^2 (R_1 + R_3) + h_1^2 (R_2 + R_7 + R_8 + R_9) + R_5 + R_6 + \tau_2^2 R_4, \\
\Delta_1 &= (A + B_1)^T P B_1 R_2^{-1} B_1^T P (A + B_1) + C_1^T P C_1 + C_1^T P C_2 R_4^{-1} C_2^T P C_1 \\
&\quad + \tau_2^2 C_2^T P C_2 + h_1^2 B_1^T P B_2 R_3^{-1} B_2^T P B_1, \\
K_1 &= R_{10} D^T P, \quad K_2 = K_3 = D^{-1}.
\end{aligned} \tag{3.4}$$

Proof. We define a Lyapunov functional $V(t, x(t))$ as

$$V(t, x(t)) = V_1(t, x(t)) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t) + V_7(t), \tag{3.5}$$

where

$$\begin{aligned}
V_1(t, x(t)) &= \left(x(t) + B_1 \int_{t-h_1}^t x(s) ds \right)^T P \left(x(t) + B_1 \int_{t-h_1}^t x(s) ds \right), \\
V_2(t) &= \int_{t-h_1}^t x^T(s) \left(R_5 + PB_1 R_8^{-1} B_1^T P \right) x(s) ds, \\
V_3(t) &= \int_{t-h_2}^t x^T(s) \left(R_6 + PB_1 R_9^{-1} B_1^T P \right) x(s) ds, \\
V_4(t) &= \int_{t-\tau_1}^t x^T(s) \left(C_1^T P C_1 + C_1^T P C_2 R_4^{-1} C_2^T P C_1 \right) x(s) ds, \\
V_5(t) &= h_2 \int_{t-h_2}^t (s-t+h_2) x^T(s) (R_1 + R_3) x(s) ds, \\
V_6(t) &= \tau_2 \int_{t-\tau_2}^t (s-t+\tau_2) x^T(s) \left(R_4 + C_2^T P C_2 \right) x(s) ds, \\
V_7(t) &= h_1 \int_{t-h_1}^t (s-t+h_1) x^T(s) \left(R_2 + R_7 + R_8 + R_9 + B_1^T P B_2 R_3^{-1} B_2^T P B_1 \right) x(s) ds.
\end{aligned} \tag{3.6}$$

Taking the operator \mathcal{L} of $V_1(t, x(t))$ along the trajectory of system (3.1), by Lemmas 2.4 and 2.5, we have

$$\begin{aligned}
\mathcal{L}V_1(t, x(t)) &= 2 \left[x(t) + B_1 \int_{t-h_1}^t x(s) ds \right]^T \\
&\quad \times P \left[(A + B_1)x(t) + B_2 \int_{t-h_2}^t x(s) ds + Du(t) + f(t, x(t)) \right] \\
&\quad + \frac{1}{2} \text{trace} \left[C_1 x(t - \tau_1) + C_2 \int_{t-\tau_2}^t x(s) ds \right] 2P \left[C_1 x(t - \tau_1) + C_2 \int_{t-\tau_2}^t x(s) ds \right] \\
&\leq x^T(t) \left[P(A + B_1) + (A + B_1)^T P + PB_2 R_1^{-1} B_2^T P \right. \\
&\quad \left. + (A + B_1)^T P B_1 R_2^{-1} B_1^T P(A + B_1) \right] x(t) + 2x^T P D f(t, x(t)) \\
&\quad + h_2 \int_{t-h_2}^t x^T(s) (R_1 + R_3) x(s) ds + 2 \int_{t-h_1}^t x^T(s) ds B_1^T P D u(t) \\
&\quad + x^T(t - \tau_1) \left(C_1^T P C_1 + C_1^T P C_2 R_4^{-1} C_2^T P C_1 \right) x(t - \tau_1) \\
&\quad + h_1 \int_{t-h_1}^t x^T(s) \left(R_2 + B_1^T P B_2 R_3^{-1} B_2^T P B_1 \right) x(s) ds \\
&\quad + 2 \int_{t-h_1}^t x^T(s) ds B_1^T P f(t, x(t)) + 2x^T P D u(t) + \tau_2 \int_{t-\tau_2}^t x^T(s) \left(R_4 + C_2^T P C_2 \right) ds.
\end{aligned} \tag{3.7}$$

By the Lemma 2.5, (2.3) and (3.5), we conclude

$$\begin{aligned}
\mathcal{L}V_1(t, x(t)) &\leq x^T(t) \left[P(A + B_1) + (A + B_1)^T P + PB_2 R_1^{-1} B_2^T P + PR_5^{-1} P \right. \\
&\quad + (A + B_1)^T P B_1 R_2^{-1} B_1^T P (A + B_1) + 2PDR_{10} D^T P \\
&\quad \left. + PR_6^{-1} P + PDR_{10} D^T P B_1 R_7^{-1} B_1^T PDR_{10} D^T P \right] x(t) \\
&+ x^T(t - \tau_1) \left(C_1^T P C_1 + C_1^T P C_2 R_4^{-1} C_2^T P C_1 \right) x(t - \tau_1) \\
&+ x^T(t - h_1) \left(R_5 + P B_1 R_8^{-1} B_1^T P \right) x(t - h_1) \\
&+ x^T(t - h_2) \left(R_6 + P B_1 R_9^{-1} B_1^T P \right) x(t - h_2) \\
&+ h_1 \int_{t-h_1}^t x^T(s) \left(R_2 + B_1^T P B_2 R_3^{-1} B_2^T P B_1 + R_7 + R_8 + R_9 \right) x(s) ds \\
&+ \tau_2 \int_{t-\tau_2}^t x^T(s) \left(R_4 + C_2^T P C_2 \right) ds + h_2 \int_{t-h_2}^t x^T(s) (R_1 + R_3) x(s) ds \\
&+ 2\alpha \left(\|P\| + h_1 \|B_1^T P\| \right) \|x(t)\|^2 + 6 \left(\|PD\| + h_1 \|B_1^T P\| \right) \|r(t)\|_\infty \|x(t)\|.
\end{aligned} \tag{3.8}$$

Taking the operator \mathcal{L} of $V_i(t)$, $i = 2, 3, \dots, 7$ along the trajectory of system (3.1), we get

$$\begin{aligned}
\mathcal{L}V_2(t) &= x^T(t) \left(R_5 + P B_1 R_8^{-1} B_1^T P \right) x(t) \\
&\quad - x^T(t - h_1) \left(R_5 + P B_1 R_8^{-1} B_1^T P \right) x(t - h_1), \\
\mathcal{L}V_3(t) &= x^T(t) \left(R_6 + P B_1 R_9^{-1} B_1^T P \right) x(t) \\
&\quad - x^T(t - h_2) \left(R_6 + P B_1 R_9^{-1} B_1^T P \right) x(t - h_2), \\
\mathcal{L}V_4(t) &= x^T(t) \left(C_1^T P C_1 + C_1^T P C_2 R_4^{-1} C_2^T P C_1 \right) x(t) \\
&\quad - x^T(t - \tau_1) \left(C_1^T P C_1 + C_1^T P C_2 R_4^{-1} C_2^T P C_1 \right) x(t - \tau_1), \\
\mathcal{L}V_5(t) &= h_2^2 x^T(t) (R_1 + R_3) x(t) - h_2 \int_{t-h_2}^t x^T(s) (R_1 + R_3) x(s) ds, \\
\mathcal{L}V_6(t) &= \tau_2^2 x^T(t) \left(R_4 + C_2^T P C_2 \right) x(t) - \tau_2 \int_{t-\tau_2}^t x^T(s) \left(R_4 + C_2^T P C_2 \right) x(s) ds, \\
\mathcal{L}V_7(t) &= h_1^2 x^T(t) \left(R_2 + R_7 + R_8 + R_9 + B_1^T P B_2 R_3^{-1} B_2^T P B_1 \right) x(t) \\
&\quad - h_1 \int_{t-h_1}^t x^T(s) \left(R_2 + R_7 + R_8 + R_9 + B_1^T P B_2 R_3^{-1} B_2^T P B_1 \right) x(s) ds.
\end{aligned} \tag{3.9}$$

Combining (3.8) and (3.9), we have

$$\begin{aligned}
 \mathcal{L}V(t, x(t)) &\leq x^T(t) \left[P(A + B_1) + (A + B_1)^T P + PB_2 R^{-1} B_2^T P + PR_6^{-1} P \right. \\
 &\quad + (A + B_1)^T PB_1 R_2^{-1} B_1^T P(A + B_1) + 2PDR_{10} D^T P + R_5 \\
 &\quad + PDR_{10} D^T PB_1 R_7^{-1} B_1^T PDR_{10} D^T P + h_2^2(R_1 + R_3) + R_6 \\
 &\quad + PB_1 R_8^{-1} B_1^T P + PB_1 R_9^{-1} B_1^T P + C_1^T PC_2 R_4^{-1} C_2^T PC_1 \\
 &\quad + h_1^2 \left(R_2 + B_1^T PB_2 R_3^{-1} B_2^T PB_1 + R_7 + R_8 + R_9 \right) + C_1^T PC_1 \\
 &\quad \left. + \tau_2^2 \left(R_4 + C_2^T PC_2 \right) + PR_5^{-1} P \right] x(t) + 2\alpha h_1 \|B_1^T P\| \|x(t)\|^2 \\
 &\quad + 2\alpha \|P\| \|x(t)\|^2 + 6 \left(\|PD\| + h_1 \|B_1^T P\| \right) \|r(t)\|_\infty \|x(t)\| \\
 &\leq - \left(\lambda_{\min}(Q_1) - 2\alpha \left(\|P\| + h_1 \|B_1^T P\| \right) \right) \|x(t)\|^2 \\
 &\quad + 6 \left(\|PD\| + h_1 \|B_1^T P\| \right) \|r(t)\|_\infty \|x(t)\|.
 \end{aligned} \tag{3.10}$$

Let $\rho_1 = \lambda_{\min}(Q_1) - 2\alpha(\|P\| + h_1\|B_1^T P\|)$, $\rho_2 = 6(\|PD\| + h_1\|B_1^T P\|)\|r(t)\|_\infty$; we have

$$\mathcal{L}V(t, x(t)) \leq -\rho_1 \|x(t)\|^2 + \rho_2 \|x(t)\|. \tag{3.11}$$

Set

$$\begin{aligned}
 \beta_1 &= \lambda_{\max}(P) + h_1 \lambda_{\max}(PB_1) + h_1 \lambda_{\max}(B_1^T P) + h_1^2 \lambda_{\max}(B_1^T PB_1), \\
 \beta_2 &= h_1 \lambda_{\max}(R_5 + PB_1 R_8^{-1} B_1^T P), \quad \beta_3 = h_2 \lambda_{\max}(R_6 + PB_1 R_9^{-1} B_1^T P), \\
 \beta_4 &= \tau_1 \lambda_{\max}(C_1^T PC_1 + C_1^T PC_2 R_4^{-1} C_2^T PC_1), \quad \beta_6 = \tau_2^3 \lambda_{\max}(R_4 + C_2^T PC_2), \\
 \beta_5 &= h_2^3 \lambda_{\max}(R_1 + R_3), \quad \beta_7 = h_1^3 \lambda_{\max}(R_2 + B_1^T PB_2 R_3^{-1} B_2^T PB_1 + R_7 + R_8 + R_9),
 \end{aligned} \tag{3.12}$$

under an assumption that $V(t, x(t)) \leq V(t_0, x(t_0))$ for all $t \geq t_0$, then

$$\lambda_{\min}(P) \mathbb{E} \left\| x(t) + B_1 \int_{t-h_1}^t x(s) ds \right\|^2 \leq V(t, x(t)) \leq V(t_0, x(t_0)) \leq \sum_{i=1}^7 \beta_i \mathbb{E} \|\varphi(\theta)\|^2, \tag{3.13}$$

so

$$\mathbb{E} \left\| x(t) + B_1 \int_{t-h_1}^t x(s) ds \right\|^2 \leq \frac{\sum_{i=1}^7 \beta_i \mathbb{E} \|\varphi(\theta)\|^2}{\lambda_{\min}(P)}. \tag{3.14}$$

Thus, according to [21, Theorem 1.3 page 331], we have

$$\mathbb{E}\|x(t)\|^2 \leq \left(\frac{1 + h_1\|B_1\|}{1 - h_1\|B_1\|} \right)^2 \frac{\sum_{i=1}^7 \beta_i \mathbb{E}\|\varphi(\theta)\|^2}{\lambda_{\min}(P)}. \quad (3.15)$$

If not, there exist $t > t_0$, such that $V(t, x(t)) \geq V(s, x(s))$ for all $s \in [t_0, t)$, and one has

$$D^+ \mathbb{E}V(t, x(t)) \geq 0. \quad (3.16)$$

In view of Ito's formula, we obtain

$$D^+ \mathbb{E}V(t, x(t)) = \mathbb{E}\mathcal{L}V(t, x(t)). \quad (3.17)$$

By (3.16) and (3.17), it is easy to derive that

$$0 \leq D^+ \mathbb{E}V(t, x(t)) = \mathbb{E}\mathcal{L}V(t, x(t)) \leq -\rho_1 \mathbb{E}\|x(t)\|^2 + \rho_2 \mathbb{E}\|x(t)\|, \quad (3.18)$$

so $\mathbb{E}\|x(t)\| \leq \rho_2/\rho_1$. By (3.18), we can conclude that

$$\mathbb{E}\|x(t)\|^2 \leq \frac{\rho_2}{\rho_1} \mathbb{E}\|x(t)\| \leq \left(\frac{\rho_2}{\rho_1} \right)^2. \quad (3.19)$$

By (3.15) and (3.19), we get

$$\mathbb{E}\|x(t)\|^2 \leq \frac{\rho_2^2}{\rho_1^2} + \left(\frac{1 + h_1\|B_1\|}{1 - h_1\|B_1\|} \right)^2 \frac{\sum_{i=1}^7 \beta_i \mathbb{E}\|\varphi(\theta)\|^2}{\lambda_{\min}(P)}, \quad (3.20)$$

Thus

$$\begin{aligned} \mathbb{E}\|y(t)\|^2 &\leq \|H\|^2 \mathbb{E}\|x(t)\|^2 \\ &\leq \frac{\|H\|^2 \rho_2^2}{\rho_1^2} + \left(\frac{1 + h_1\|B_1\|}{1 - h_1\|B_1\|} \right)^2 \frac{\|H\|^2 \sum_{i=1}^7 \beta_i \mathbb{E}\|\varphi(\theta)\|^2}{\lambda_{\min}(P)} \\ &\leq N_1 + N_2 \|r\|_{\infty}^2, \end{aligned} \quad (3.21)$$

where

$$N_1 = \left(\frac{1 + h_1\|B_1\|}{1 - h_1\|B_1\|} \right)^2 \frac{\sum_{i=1}^7 \beta_i \|H\|^2}{\lambda_{\min}(P)} \mathbb{E}\|\varphi(\theta)\|^2, \quad N_2 = \frac{36\|H\|^2 (\|PD\| + h_1\|B_1^T P\|)}{\lambda_{\min}(Q_1) - 2\alpha(\|P\| - h_1\|B_1^T P\|)}. \quad (3.22)$$

By Definition 2.2, the nonlinear stochastic control system (3.1) with the control law (2.3) is said to be BIBO stabilized in mean square. This completes the proof. \square

If we transform the original system (2.1) to the following system

$$\begin{aligned}
 d \left[x(t) + B_2 \int_{t-h_2}^t (s-t+h_2)x(s)ds \right] &= [(A+h_2B_2)x(t) + B_1x(t-h_1) + f(t, x(t))]dt + Du(t)dt \\
 &\quad + \left[C_1x(t-\tau_1) + C_2 \int_{t-\tau_2}^t x(s)ds \right]d\mathbf{w}(t), \quad t \geq t_0 \geq 0 \\
 y(t) &= Hx(t) \\
 x(\theta) &= \varphi(\theta) \in C_{\mathcal{F}_0}^b([t_0-\tau, t_0]; \mathbb{R}^n), \quad \theta \in [t_0-\tau, t_0],
 \end{aligned} \tag{3.23}$$

we can get the following result.

Theorem 3.2. *The nonlinear stochastic control system (3.23) with the control law (2.3) is BIBO stabilized in mean square if there exist symmetric positive-definite matrices $S_i > 0, i = 1, 2, \dots, 10$, and $Q_2 > 0$ such that*

$$\lambda_{\min}(Q_2) - 2\alpha\|\hat{P}\| > 0 \tag{3.24}$$

and \hat{P} is the symmetric positive solution of the Riccati equation

$$\hat{P}(A+h_2B_2) + (A+h_2B_2)^T\hat{P} + \hat{P}\Sigma_2\hat{P} + \Delta_2 + \Xi_2 = -Q_2, \tag{3.25}$$

where

$$\begin{aligned}
 \Sigma_2 &= B_1S_2^{-1}B_1^T + 2DS_{10}D^T + S_5^{-1} + S_6^{-1}, \\
 \Delta_2 &= (A+h_2B_2)^T\hat{P}B_2S_1^{-1}B_2^T\hat{P}(A+h_2B_2) + D^T\hat{P}S_7^{-1}\hat{P}D + \tau_2^2C_2^T\hat{P}C_2 \\
 &\quad + D^T\hat{P}S_8^{-1}\hat{P}D + C_1^T\hat{P}C_1 + C_1^T\hat{P}C_2S_4^{-1}C_2^T\hat{P}C_1 + D^T\hat{P}S_9^{-1}\hat{P}D \\
 &\quad + B_1^T\hat{P}B_2S_3^{-1}B_2^T\hat{P}B_1, \\
 \Xi_2 &= S_2 + S_5 + S_6 + \frac{1}{3}h_2^4(S_1 + S_3 + S_7 + S_8 + S_9) + \tau_2^2S_4, \\
 K_1 &= S_{10}D^T\hat{P}, \quad K_2 = K_3 = D^{-1}.
 \end{aligned} \tag{3.26}$$

Proof. We define a Lyapunov functional $V(t, x(t))$ as

$$V(t, x(t)) = V_1(t, x(t)) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t), \tag{3.27}$$

where

$$\begin{aligned}
V_1(t, x(t)) &= \left(x(t) + B_2 \int_{t-h_2}^t (s-t+h_2)x(s)ds \right)^T \widehat{P} \left(x(t) + B_2 \int_{t-h_2}^t (s-t+h_2)x(s)ds \right), \\
V_2(t) &= \int_{t-h_1}^t x^T(s) \left(S_2 + S_5 + B_1^T \widehat{P} B_2 S_3^{-1} B_2^T \widehat{P} B_1 + D^T \widehat{P} S_8^{-1} \widehat{P} D \right) x(s) ds, \\
V_3(t) &= \int_{t-h_2}^t x^T(s) \left(S_6 + D^T \widehat{P} S_9^{-1} \widehat{P} D \right) x(s) ds, \\
V_4(t) &= \int_{t-\tau_1}^t x^T(s) \left(C_1^T \widehat{P} C_1 + C_1^T \widehat{P} C_2 S_4^{-1} C_2^T \widehat{P} C_1 \right) x(s) ds, \\
V_5(t) &= \tau_2 \int_{t-\tau_2}^t (s-t+\tau_2) x^T(s) \left(S_4 + C_2^T \widehat{P} C_2 \right) x(s) ds, \\
V_6(t) &= \frac{1}{3} \int_{t-h_2}^t (s-t+h_2)^3 x^T(s) (S_1 + S_3 + S_7 + S_8 + S_9) x(s) ds.
\end{aligned} \tag{3.28}$$

Let $\rho_1 = \lambda_{\min}(Q_2) - 2\alpha(1+h_2^2)\|\widehat{P}\|$, $\rho_2 = 6(1+h_2^2)\|\widehat{P}D\|\|r(t)\|_{\infty}$. The rest of the proof is essentially as that of Theorem 3.1, and hence is omitted. This completes the proof. \square

Remark 3.3. If we transform the original system (2.1) to the following system

$$\begin{aligned}
& d \left[x(t) + B_1 \int_{t-h_1}^t x(s)ds + B_2 \int_{t-h_2}^t (s-t+h_2)x(s)ds \right] \\
&= [Ax(t) + f(t, x(t)) + Du(t)] dt \\
&+ \left[C_1 x(t-\tau_1) + C_2 \int_{t-\tau_2}^t x(s)ds \right] d\mathbf{w}(t), \quad t \geq t_0 \geq 0 \\
&y(t) = Hx(t) \\
&x(\theta) = \varphi(\theta) \in C_{\tau_0}^b([t_0 - \tau, t_0]; \mathbb{R}^n), \quad \theta \in [t_0 - \tau, t_0],
\end{aligned} \tag{3.29}$$

using the same process of Theorem 3.1, we can get the corresponding BIBO stability in mean square results. Here we omitted it.

Theorem 3.4. *The nonlinear stochastic control system (2.1) with the control law (2.3) is BIBO stabilized in mean square if there exist symmetric positive-definite matrices $\Omega_i > 0$, $i = 1, 2, \dots, 6$, and $Q_3 > 0$ such that*

$$\lambda_{\min}(Q_3) - 2\alpha\|\widetilde{P}\| > 0 \tag{3.30}$$

and \tilde{P} is the symmetric positive solution of the Riccati equation

$$\tilde{P}A + A^T\tilde{P} + \tilde{P}\Sigma_3\tilde{P} + \Delta_3 + \Xi_3 = -Q_3, \tag{3.31}$$

where

$$\begin{aligned} \Sigma_3 &= B_1\Omega_1^{-1}B_1^T + h_2B_2\Omega_2^{-1}B_2^T + 2D\Omega_6D^T + \Omega_4^{-1} + \Omega_5^{-1}, \\ \Delta_3 &= C_1^T\tilde{P}C_1 + \tau_2C_2^T\tilde{P}C_1\Omega_3^{-1}C_1^T\tilde{P}C_2 + \tau_2^2C_2^T\tilde{P}C_2, \\ \Xi_3 &= \Omega_1 + h_2\Omega_2 + \tau_2\Omega_3 + \Omega_4 + \Omega_5, \\ K_1 &= \Omega_6D^T\tilde{P}, \quad K_2 = K_3 = D^{-1}. \end{aligned} \tag{3.32}$$

Proof. We define a Lyapunov functional $V(t, x(t))$ as

$$V(t, x(t)) = V_1(t, x(t)) + V_2(t) + V_3(t) + V_4(t) + V_5(t), \tag{3.33}$$

where

$$\begin{aligned} V_1(t, x(t)) &= x(t)^T\tilde{P}x(t), \\ V_2(t) &= \int_{t-h_1}^t x^T(s)(\Omega_1 + \Omega_4)x(s)ds + \int_{t-h_2}^t x^T(s)\Omega_5x(s)ds, \\ V_3(t) &= \int_{t-h_2}^t (s-t+h_2)x^T(s)\Omega_2x(s)ds, \\ V_4(t) &= \int_{t-\tau_1}^t x^T(s)\left(C_1^T\tilde{P}C_1 + \tau_2C_2^T\tilde{P}C_1\Omega_3^{-1}C_1^T\tilde{P}C_2\right)x(s)ds, \\ V_5(t) &= \int_{t-\tau_2}^t (s-t+\tau_2)x^T(s)\left(\Omega_3 + \tau_2C_2^T\tilde{P}C_2\right)x(s)ds. \end{aligned} \tag{3.34}$$

Let $\rho_1 = \lambda_{\min}(Q_3) - 2\alpha\|\tilde{P}\|$, $\rho_2 = 6\|\tilde{P}D\|\|r(t)\|_{\infty}$. The following proof runs as that of Theorem 3.1., and hence is omitted. This completes the proof. \square

Remark 3.5. The systems (2.1), (3.1), and (3.23) are also asymptotically stable in mean square when all the conditions in Theorems 3.1–3.4 are satisfied, if $r(t) = 0$, $r(t - h_1) = 0$, $r(t - h_2) = 0$ in (2.3).

Remark 3.6. In [18], the authors studied the BIBO stabilization of stochastic delayed systems with uncertainty in terms of Razumikhin technique and comparison principle. In the present paper, we are the first to introduce a new way in the study of BIBO stabilization for stochastic delayed systems by using algebraic Riccati matrix equation, which makes the stability conditions be morefeasible.

Remark 3.7. In [18], researchers investigate the BIBO stabilization of stochastic delayed systems with uncertainty without distributed time delays and nonlinear perturbations. In [15], the authors discussed the BIBO stabilization of mixed time-delayed systems with nonlinear perturbations, but the environmental noise is not taken into account in the models. Therefore, compared with (see [14–18]), the systems reported in this paper are more general.

Remark 3.8. The criteria given in Theorem 3.1–3.4 are delay dependent with respect to delays. Generally speaking, the delay-dependent stability criterion is less conservative than delay-independent stability when the time delay is small.

4. Example

In this section, a numerical example will be presented to show the effectiveness of the main results derived in this paper.

Example 4.1. As a simple application of Theorem 3.4, consider the stochastic control system (2.1) with control law (2.3); the parameters are given by

$$A = \begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix}, \quad B_1 = \begin{pmatrix} -1 & 1 \\ 1 & -2 \end{pmatrix}, \quad B_2 = C_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (4.1)$$

$$C_2 = 0, \quad f = [0.25 \sin(t), 0.25 \cos(t)]^T.$$

For

$$\Omega_i = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad i = 1, 2, \dots, 6, \quad Q_3 = \begin{pmatrix} 11 & 7 \\ 7 & 14 \end{pmatrix}, \quad (4.2)$$

solving for \tilde{P} in the Riccati matrix (3.31) gives us

$$P = \begin{pmatrix} 1 & 1 \\ 1 & 4 \end{pmatrix}, \quad (4.3)$$

therefore, the stabilizing feedback gain matrix is given by

$$K_1 = \begin{pmatrix} -11 & -7 \\ -7 & -14 \end{pmatrix}, \quad K_2 = K_3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (4.4)$$

Meanwhile, we obtain the maximum value $\tau_{\max} = 0.6667$.

5. Conclusions

The problem of delay-dependent BIBO stabilization in mean square for the stochastic control systems with mixed delays and nonlinear perturbations was investigated. A suitable class of

Lyapunov functional combined with the descriptor model transformation and decomposition technique of controller were constructed to derive some novel mean square BIBO stability criteria. This paper was the first to successfully introduce the method of Riccati matrix equation to stochastic BIBO stabilization. A numerical example was given to illustrate the validity of the main results.

Acknowledgments

The authors would like to thank the editor and the reviewers for their detailed comments and valuable suggestions which have led to a much improved paper. This work was supported by the National Natural Science Foundation of China (Grant no. 60736029) and the National Basic Research Program of China (2010CB732501).

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