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Research Article

Oscillation Criteria for Second-Order Quasilinear Neutral Delay Dynamic Equations on Time Scales

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We establish some new oscillation criteria for the second-order quasilinear neutral delay dynamic equations $[r(t)(z^{\Delta}(t))^{\gamma}]^{\Delta} + q_1(t)x^{\alpha}(\tau_1(t)) + q_2(t)x^{\beta}(\tau_2(t)) = 0$ on a time scale $\mathbb{T}$, where $z(t) = x(t) + p(t)x(\tau_0(t))$, $0 < \alpha < \gamma < \beta$. Our results generalize and improve some known results for oscillation of second-order nonlinear delay dynamic equations on time scales. Some examples are considered to illustrate our main results.

1. Introduction

In this paper, we are concerned with oscillation behavior of the second order quasilinear neutral delay dynamic equations

$$\left[r(t)\left(z^{\Delta}(t)\right)^{\gamma}\right]^{\Delta} + q_1(t)x^{\alpha}(\tau_1(t)) + q_2(t)x^{\beta}(\tau_2(t)) = 0, \tag{1.1}$$

on an arbitrary time scale $\mathbb{T}$, where $z(t) = x(t) + p(t)x(\tau_0(t)), \gamma, \alpha$, and β are quotient of odd positive integers such that $0 < \alpha < \gamma < \beta$, r, p, q_1, and q_2 are rd-continuous functions on $\mathbb{T}$, and r, q_1, and q_2 are positive, $-1 < -p_0 \leq p(t) < 1$, $p_0 > 0$; the so-called delay functions $\tau_i : \mathbb{T} \to \mathbb{T}$ satisfy that $\tau_i(t) \leq t$ for $t \in \mathbb{T}$ and $\tau_i(t) \to \infty$ as $t \to \infty$, for $i = 0, 1, 2$, and there exists a function $\tau : \mathbb{T} \to \mathbb{T}$ which satisfies that $\tau(t) \leq \tau_1(t)$, $\tau(t) \leq \tau_2(t)$, and $\tau(t) \to \infty$ as $t \to \infty$.

Since we are interested in the oscillatory and asymptotic behavior of solutions near infinity, we assume that $\sup \mathbb{T} = \infty$ and define the time scale interval $[t_0, \infty)_{\mathbb{T}}$ by $[t_0, \infty)_{\mathbb{T}} := [t_0, \infty) \cap \mathbb{T}$.

We will also consider the two cases

$$\int_{t_0}^{\infty} \frac{\Delta t}{r^{1/\gamma}(t)} = \infty, \tag{1.2}$$

$$\int_{t_0}^{\infty} \frac{\Delta t}{r^{1/\gamma}(t)} < \infty. \tag{1.3}$$

Recently, there has been a large number of papers devoted to the delay dynamic equations on time scales, and we refer the reader to the papers in [1–17].

Agarwal et al. [1], Sahiner [10], Saker [11], Saker et al. [12], and Wu et al. [15] studied the second-order nonlinear neutral delay dynamic equations on time scales

$$\left(r(t)\left(\left(y(t)+p(t)y(\tau(t))\right)^{\Delta}\right)^{\gamma}\right)^{\Delta} + f\left(t, y(\delta(t))\right) = 0, \quad t \in \mathbb{T}, \tag{1.4}$$

where $0 \le p(t) < 1$, and (1.2) holds. By means of Riccati transformation technique, the authors established some sufficient conditions for oscillation of (1.4).

Sun et al. [14] considered (1.1), where $r^{\Delta}(t) \ge 0$, $-1 < -p_0 \le p(t) \le 0$, and (1.2) holds. The authors established some oscillation results of (1.1). To the best of our knowledge, there are no results regarding the oscillation of the solutions of (1.1) when (1.3) holds.

We note that if $\mathbb{T} = \mathbb{R}$, (1.1) becomes the second-order Emden-Fowler neutral delay differential equation

$$\left[r(t)\left(z'(t)\right)^{\gamma}\right]' + q_1(t)x^{\alpha}(\tau_1(t)) + q_2(t)x^{\beta}(\tau_2(t)) = 0, \quad t \ge t_0. \tag{1.5}$$

Chen and Xu [18] as well as Xu and Liu [19] considered (1.5) and obtained some oscillation criteria for (1.5) when $r(t) = 1$. Qin et al. [20] found that some results under the case when $-1 < p_0 \le p(t) \le 0$ in [18, 19] are incorrect.

The paper is organized as follows. In the next section, by developing a Riccati transformation technique some sufficient conditions for oscillation of all solutions of (1.1) on time scales are established. In Section 3, we give some examples to illustrate our main results.

2. Main Results

In this section, by employing the Riccati transformation technique, we establish some new oscillation criteria for (1.1). In order to prove our main results, we will use the formula

$$\left(x^{\gamma}(t)\right)^{\Delta} = \gamma \int_0^1 \left[hx^{\sigma}(t) + (1-h)x(t)\right]^{\gamma-1} x^{\Delta}(t)\mathrm{d}h, \tag{2.1}$$

which is a simple consequence of Keller's chain rule [21, Theorem 1.90]. Also, we need the following lemmas.

It will be convenient to make the following notations:

$$d_+(t) := \max\{0, d(t)\}, \qquad \theta(a,b;u) := \frac{\int_u^a \Delta s / r^{1/\gamma}(s)}{\int_u^b \Delta s / r^{1/\gamma}(s)},$$

$$\alpha(t,u) := \theta(\tau(t),\sigma(t);u), \qquad \beta(t,u) := \theta(t,\sigma(t);u), \qquad \nu := \min\left\{\frac{\beta-\alpha}{\beta-\gamma}, \frac{\beta-\alpha}{\gamma-\alpha}\right\},$$

$$Q_1(t) := \nu\left(q_1(t)(1-p(\tau_1(t)))^{\alpha}\right)^{(\beta-\gamma)/(\beta-\alpha)}\left(q_2(t)(1-p(\tau_2(t)))^{\beta}\right)^{(\gamma-\alpha)/(\beta-\alpha)}(\alpha(t,T))^{\gamma},$$

$$Q_2(t) := \nu(q_1(t))^{(\beta-\gamma)/(\beta-\alpha)}(q_2(t))^{(\gamma-\alpha)/(\beta-\alpha)}(\alpha(t,T))^{\gamma},$$

$$Q_{1*}(t) = Q_1(t) - \eta^{\Delta}(t), \qquad Q_{2*}(t) = Q_2(t) - \eta^{\Delta}(t). \tag{2.2}$$

Lemma 2.1 (see [3, Lemma 2.4]). *Assume that there exists $T \ge t_0$, sufficiently large, such that*

$$x(t) > 0, \quad x^{\Delta}(t) > 0, \quad \left(r(t)\left(x^{\Delta}(t)\right)^{\gamma}\right)^{\Delta} < 0, \quad t \ge T. \tag{2.3}$$

Then

$$x(\tau(t)) \ge \alpha(t,T)x^{\sigma}(t), \qquad x(t) \ge \beta(t,T)x^{\sigma}(t), \quad \text{for } t \ge T_1 \ge T. \tag{2.4}$$

Lemma 2.2. *Assume that* (1.2) *holds;* $0 \le p(t) < 1$. *Furthermore, x is an eventually positive solution of* (1.1). *Then there exists $t_1 \ge t_0$ such that*

$$z(t) > 0, \quad z^{\Delta}(t) > 0, \quad \left(r(t)\left(z^{\Delta}(t)\right)^{\gamma}\right)^{\Delta} < 0, \quad \text{for } t \ge t_1. \tag{2.5}$$

Proof. Let x be an eventually positive solution of (1.1). Then there exists $t_1 \ge t_0$ such that $x(t) > 0$, and $x(\tau_i(t)) > 0$ for $t \ge t_1$, $i = 0,1,2$. From (1.1), we have

$$\left[r(t)\left(z^{\Delta}(t)\right)^{\gamma}\right]^{\Delta} = -q_1(t)x^{\alpha}(\tau_1(t)) - q_2(t)x^{\beta}(\tau_2(t)) < 0 \tag{2.6}$$

for all $t \ge t_1$, and so $r(t)(z^{\Delta}(t))^{\gamma}$ is an eventually decreasing function.

We first show that $r(t)(z^{\Delta}(t))^{\gamma}$ is eventually positive. Otherwise, there exists $t_2 \ge t_1$ such that $r(t_2)(z^{\Delta}(t_2))^{\gamma} = c < 0$; then from (2.6) we have $r(t)(z^{\Delta}(t))^{\gamma} \le r(t_2)(z^{\Delta}(t_2))^{\gamma} = c$ for $t \ge t_2$, and so

$$z^{\Delta}(t) \le c^{1/\gamma}\left(\frac{1}{r(t)}\right)^{1/\gamma}, \tag{2.7}$$

which implies by (1.2) that

$$z(t) \le z(t_2) + c^{1/\gamma} \int_{t_2}^{t} \left(\frac{1}{r(s)} \right)^{1/\gamma} \Delta s \longrightarrow -\infty \quad \text{as } t \longrightarrow \infty, \tag{2.8}$$

and this contradicts the fact that $z(t) \ge x(t) > 0$ for all $t \ge t_1$. Hence, we have that (2.5) holds and completes the proof. □

Lemma 2.3. *Assume that* (1.2) *holds,* $-1 < -p_0 \le p(t) \le 0$, *and* $\lim_{t\to\infty} p(t) = p > -1$. *Furthermore, assume that there exists* $\{c_k\}_{k\ge 0}$ *such that* $\lim_{k\to\infty} c_k = \infty$ *and* $\tau_0(c_{k+1}) = c_k$. *Then an eventually positive solution* x *of* (1.1) *satisfies eventually* (2.5) *or* $\lim_{t\to\infty} x(t) = 0$.

Proof. Suppose that x is an eventually positive solution of (1.1). Then there exists $t_1 \ge t_0$ such that $x(t) > 0$, and $x(\tau_i(t)) > 0$ for $t \ge t_1$, $i = 0, 1, 2$. From (1.1), we have that (2.6) holds for all $t \ge t_1$, and so $r(t)(z^{\Delta}(t))^{\gamma}$ is an eventually decreasing function.

We first show that $r(t)(z^{\Delta}(t))^{\gamma}$ is eventually positive. Otherwise, there exists $t_2 \ge t_1$ such that $r(t_2)(z^{\Delta}(t_2))^{\gamma} = c < 0$; then from (2.6) we have $r(t)(z^{\Delta}(t))^{\gamma} \le r(t_2)(z^{\Delta}(t_2))^{\gamma} = c$ for $t \ge t_2$, and so

$$z^{\Delta}(t) \le c^{1/\gamma} \left(\frac{1}{r(t)} \right)^{1/\gamma}, \tag{2.9}$$

which implies by (1.2) that

$$z(t) \le z(t_2) + c^{1/\gamma} \int_{t_2}^{t} \left(\frac{1}{r(s)} \right)^{1/\gamma} \Delta s \longrightarrow -\infty \quad \text{as } t \longrightarrow \infty. \tag{2.10}$$

Therefore, there exist $d > 0$ and $t_3 \ge t_2$ such that

$$x(t) \le -d - p(t)x(\tau_0(t)) \le -d + p_0 x(\tau_0(t)), \quad t \ge t_3. \tag{2.11}$$

Thus, we can choose some positive integer k_0 such that $c_k \ge t_3$ for $k \ge k_0$, and

$$\begin{aligned} x(c_k) &\le -d + p_0 x(\tau_0(c_k)) = -d + p_0 x(c_{k-1}) \le -d - p_0 d + p_0^2 x(\tau_0(c_{k-1})) \\ &= -d - p_0 d + p_0^2 x(c_{k-2}) \le \cdots \le -d - p_0 d - \cdots - p_0^{k-k_0-1} d + p_0^{k-k_0} x(\tau_0(c_{k_0+1})) \\ &= -d - p_0 d - \cdots - p_0^{k-k_0-1} d + p_0^{k-k_0} x(c_{k_0}). \end{aligned} \tag{2.12}$$

The above inequality implies that $x(c_k) < 0$ for sufficiently large k, which contradicts the fact that $x(t)$ is eventually positive. Hence $z^{\Delta}(t)$ is eventually positive. Consequently, there are two possible cases:

(i) $z(t)$ is eventually positive, or

(ii) $z(t)$ is eventually negative.

If there exists a $t_4 \geq t_1$ such that case (ii) holds, then $\lim_{t\to\infty} z(t)$ exists, and $\lim_{t\to\infty} z(t) = l \leq 0$; we claim that $\lim_{t\to\infty} z(t) = 0$. Otherwise, $\lim_{t\to\infty} z(t) < 0$. We can choose some positive integer k_0 such that $c_k \geq t_4$ for $k \geq k_0$, and we obtain

$$\begin{aligned} x(c_k) &\leq p_0 x(\tau_0(c_k)) = p_0 x(c_{k-1}) \leq p_0^2 x(\tau_0(c_{k-1})) \\ &= p_0^2 x(c_{k-2}) \leq \cdots \leq p_0^{k-k_0} x(\tau_0(c_{k_0+1})) = p_0^{k-k_0} x(c_{k_0}), \end{aligned} \tag{2.13}$$

which implies that $\lim_{k\to\infty} x(c_k) = 0$, and so $\lim_{k\to\infty} z(c_k) = 0$, which contradicts $\lim_{t\to\infty} z(t) = l < 0$. Now, we assert that $x(t)$ is bounded. If it is not true, then there exists $\{t_k\}$ with $t_k \to \infty$ as $k \to \infty$ such that

$$x(t_k) = \max_{t_0 \leq s \leq t_k} x(s), \qquad \lim_{k\to\infty} x(t_k) = \infty. \tag{2.14}$$

From $\tau_0(t) \leq t$, we obtain

$$z(t_k) = x(t_k) + p(t_k) x(\tau_0(t_k)) \geq (1 - p_0) x(t_k), \tag{2.15}$$

which implies that $\lim_{k\to\infty} z(t_k) = \infty$; it contradicts $\lim_{t\to\infty} z(t) = 0$. Therefore, we can assume that

$$\limsup_{t\to\infty} x(t) = x_1, \qquad \liminf_{t\to\infty} x(t) = x_2. \tag{2.16}$$

By $-1 < p \leq 0$, we get

$$x_1 + p x_1 \leq 0 \leq x_2 + p x_2, \tag{2.17}$$

which implies that $x_1 \leq x_2$, so $x_1 = x_2$. Hence, $\lim_{t\to\infty} x(t) = 0$. The proof is complete. □

Theorem 2.4. *Assume that* (1.2) *holds,* $0 \leq p(t) < 1$, *and* $\gamma \geq 1$. *Furthermore, assume that there exist positive rd-continuous* Δ*-differentiable functions* δ *and* η *such that, for all sufficiently large* T, *for* $T_1 > T$

$$\limsup_{t\to\infty} \int_{T_1}^{t} \left[\delta^{\sigma}(s) Q_{1*}(s) - \delta^{\Delta}(s)\eta(s) - \frac{r(s)}{(\gamma+1)^{\gamma+1}} \frac{\left((\delta^{\Delta}(s))_+\right)^{\gamma+1}}{(\delta^{\sigma}(s))^{\gamma}} (\beta(s,T))^{-\gamma^2} \right] \Delta s = \infty. \tag{2.18}$$

Then every solution of (1.1) *is oscillatory.*

Proof. Suppose that (1.1) has a nonoscillatory solution x. We may assume without loss of generality that $x(\tau_i(t)) > 0$, $i = 0, 1, 2$, for all $t \geq t_0$. By Lemma 2.2, there exists $T \geq t_0$ such that (2.5) holds. Define the function ω by

$$\omega(t) = \delta(t) \left[\frac{r(t)(z^{\Delta}(t))^{\gamma}}{z^{\gamma}(t)} + \eta(t) \right], \quad t \geq T. \tag{2.19}$$

Then $\omega(t) > 0$. By the product rule and the quotient rule, noteing (2.19), we have

$$\omega^{\Delta}(t) = \frac{\delta^{\Delta}(t)}{\delta(t)}\omega(t) + \delta^{\sigma}(t)\left[\frac{\left(r(t)(z^{\Delta}(t))^{\gamma}\right)^{\Delta}}{(z^{\sigma}(t))^{\gamma}} - \frac{r(t)(z^{\Delta}(t))^{\gamma}(z^{\gamma}(t))^{\Delta}}{z^{\gamma}(t)(z^{\sigma}(t))^{\gamma}} + \eta^{\Delta}(t)\right]. \tag{2.20}$$

By (1.1) and (2.5), we obtain

$$\left(r(t)\left(z^{\Delta}(t)\right)^{\gamma}\right)^{\Delta} \le -q_1(t)((1-p(\tau_1(t)))z(\tau_1(t)))^{\alpha} - q_2(t)((1-p(\tau_2(t)))z(\tau_2(t)))^{\beta} < 0. \tag{2.21}$$

In view of $\gamma \ge 1$, from (2.1), we have $(z^{\gamma}(t))^{\Delta} \ge \gamma(z(t))^{\gamma-1}z^{\Delta}(t)$. By (2.20), we obtain

$$\begin{aligned}\omega^{\Delta}(t) \le{}& \frac{\delta^{\Delta}(t)}{\delta(t)}\omega(t) - \delta^{\sigma}(t)q_1(t)(1-p(\tau_1(t)))^{\alpha}\frac{(z(\tau_1(t)))^{\alpha}}{(z^{\sigma}(t))^{\gamma}} \\ &- \delta^{\sigma}(t)q_2(t)(1-p(\tau_2(t)))^{\beta}\frac{(z(\tau_2(t)))^{\beta}}{(z^{\sigma}(t))^{\gamma}} - \gamma\delta^{\sigma}(t)\frac{r(t)(z^{\Delta}(t))^{\gamma+1}}{z(t)(z^{\sigma}(t))^{\gamma}} + \delta^{\sigma}(t)\eta^{\Delta}(t).\end{aligned} \tag{2.22}$$

By Young's inequality

$$|ab| \le \frac{1}{p}|a|^{p} + \frac{1}{q}|b|^{q}, \quad a,b \in \mathbb{R},\ p>1,\ q>1,\ \frac{1}{p}+\frac{1}{q}=1, \tag{2.23}$$

we have

$$\begin{aligned}&\frac{\beta-\gamma}{\beta-\alpha}q_1(t)(1-p(\tau_1(t)))^{\alpha}\frac{(z(\tau_1(t)))^{\alpha}}{(z^{\sigma}(t))^{\gamma}} + \frac{\gamma-\alpha}{\beta-\alpha}q_2(t)(1-p(\tau_2(t)))^{\beta}\frac{(z(\tau_2(t)))^{\beta}}{(z^{\sigma}(t))^{\gamma}} \\ &\quad\ge \left[q_1(t)(1-p(\tau_1(t)))^{\alpha}\frac{(z(\tau_1(t)))^{\alpha}}{(z^{\sigma}(t))^{\gamma}}\right]^{(\beta-\gamma)/(\beta-\alpha)}\left[q_2(t)(1-p(\tau_2(t)))^{\beta}\frac{(z(\tau_2(t)))^{\beta}}{(z^{\sigma}(t))^{\gamma}}\right]^{(\gamma-\alpha)/(\beta-\alpha)} \\ &\quad= \left(q_1(t)(1-p(\tau_1(t)))^{\alpha}\right)^{(\beta-\gamma)/(\beta-\alpha)}\left(q_2(t)(1-p(\tau_2(t)))^{\beta}\right)^{(\gamma-\alpha)/(\beta-\alpha)}\left(\frac{(z(\tau_1(t)))^{\alpha}}{(z^{\sigma}(t))^{\gamma}}\right)^{(\beta-\gamma)/(\beta-\alpha)} \\ &\qquad\times\left(\frac{(z(\tau_2(t)))^{\beta}}{(z^{\sigma}(t))^{\gamma}}\right)^{(\gamma-\alpha)/(\beta-\alpha)} \\ &\quad\ge \left(q_1(t)(1-p(\tau_1(t)))^{\alpha}\right)^{(\beta-\gamma)/(\beta-\alpha)}\left(q_2(t)(1-p(\tau_2(t)))^{\beta}\right)^{(\gamma-\alpha)/(\beta-\alpha)}\left(\frac{z(\tau(t))}{z^{\sigma}(t)}\right)^{\gamma}.\end{aligned} \tag{2.24}$$

By Lemma 2.1, we have

$$\frac{z(\tau(t))}{z^{\sigma}(t)} \geq \alpha(t,T), \qquad \frac{z(t)}{z^{\sigma}(t)} \geq \beta(t,T). \tag{2.25}$$

Hence, by (2.19) and (2.22), we obtain

$$\begin{aligned}\omega^{\Delta}(t) \leq{}& \frac{\delta^{\Delta}(t)}{\delta(t)}\omega(t) - \nu\delta^{\sigma}(t)\left(q_1(t)\left(1-p(\tau_1(t))\right)^{\alpha}\right)^{(\beta-\gamma)/(\beta-\alpha)} \\ &\times \left(q_2(t)\left(1-p(\tau_2(t))\right)^{\beta}\right)^{(\gamma-\alpha)/(\beta-\alpha)}(\alpha(t,T))^{\gamma} \\ &- \gamma\delta^{\sigma}(t)\frac{1}{(r(t))^{1/\gamma}}(\beta(t,T))^{\gamma}\left(\frac{\omega(t)}{\delta(t)}-\eta(t)\right)^{(\gamma+1)/\gamma} + \delta^{\sigma}(t)\eta^{\Delta}(t).\end{aligned} \tag{2.26}$$

Thus

$$\begin{aligned}\omega^{\Delta}(t) \leq{}& -\delta^{\sigma}(t)\left[Q_1(t)-\eta^{\Delta}(t)\right] + \delta^{\Delta}(t)\eta(t) + \left(\delta^{\Delta}(t)\right)_{+}\left|\frac{\omega(t)}{\delta(t)}-\eta(t)\right| \\ &- \gamma\delta^{\sigma}(t)\frac{1}{(r(t))^{1/\gamma}}(\beta(t,T))^{\gamma}\left(\frac{\omega(t)}{\delta(t)}-\eta(t)\right)^{(\gamma+1)/\gamma}.\end{aligned} \tag{2.27}$$

Set

$$\begin{aligned}\lambda &= \frac{\gamma+1}{\gamma}, \quad A = \gamma^{1/\lambda}(\delta^{\sigma}(t))^{1/\lambda}\frac{1}{(r(t))^{1/(\gamma+1)}}(\beta(t,T))^{\gamma^2/(\gamma+1)}\left|\frac{\omega(t)}{\delta(t)}-\eta(t)\right|, \\ B &= \left(\left(\delta^{\Delta}(t)\right)_{+}\right)^{\gamma}\left(\frac{\gamma}{\gamma+1}\right)^{\gamma}\frac{(r(t))^{\gamma/(\gamma+1)}}{\gamma^{\gamma^2/(\gamma+1)}(\delta^{\sigma}(t))^{\gamma^2/(\gamma+1)}}\left(\frac{1}{\beta(t,T)}\right)^{\gamma^3/(\gamma+1)}.\end{aligned} \tag{2.28}$$

Using the inequality

$$\lambda A B^{\lambda-1} - A^{\lambda} \leq (\lambda-1)B^{\lambda}, \quad \lambda \geq 1,\ A \geq 0,\ B \geq 0, \tag{2.29}$$

we obtain

$$\omega^{\Delta}(t) \leq -\delta^{\sigma}(t)\left[Q_1(t)-\eta^{\Delta}(t)\right] + \delta^{\Delta}(t)\eta(t) + \frac{r(t)}{(\gamma+1)^{\gamma+1}}\frac{\left(\left(\delta^{\Delta}(t)\right)_{+}\right)^{\gamma+1}}{(\delta^{\sigma}(t))^{\gamma}}(\beta(t,T))^{-\gamma^2}. \tag{2.30}$$

Integrating the last inequality from $T_1 > T$ to $t > T_1$, we obtain

$$-\omega(T_1) < \omega(t) - \omega(T_1)$$
$$\leq -\int_{T_1}^{t} \left[\delta^{\sigma}(s)\left(Q_1(s) - \eta^{\Delta}(s)\right) - \delta^{\Delta}(s)\eta(s) - \frac{r(s)}{(\gamma+1)^{\gamma+1}} \frac{\left((\delta^{\Delta}(s))_+\right)^{\gamma+1}}{(\delta^{\sigma}(s))^{\gamma}} (\beta(s,T))^{-\gamma^2} \right] \Delta s, \tag{2.31}$$

which yields

$$\int_{T_1}^{t} \left[\delta^{\sigma}(s)\left(Q_1(s) - \eta^{\Delta}(s)\right) - \delta^{\Delta}(s)\eta(s) - \frac{r(s)}{(\gamma+1)^{\gamma+1}} \frac{\left((\delta^{\Delta}(s))_+\right)^{\gamma+1}}{(\delta^{\sigma}(s))^{\gamma}} (\beta(s,T))^{-\gamma^2} \right] \Delta s \leq \omega(T_1), \tag{2.32}$$

which leads to a contradiction to (2.18). The proof is complete. □

Theorem 2.5. *Assume that* (1.2) *holds,* $0 \leq p(t) < 1$, *and* $\gamma \leq 1$. *Furthermore, assume that there exist positive rd-continuous* Δ*-differentiable functions* δ *and* η *such that, for all sufficiently large* T, *for* $T_1 > T$

$$\limsup_{t\to\infty} \int_{T_1}^{t} \left[\delta^{\sigma}(s)Q_{1*}(s) - \delta^{\Delta}(s)\eta(s) - \frac{r(s)}{(\gamma+1)^{\gamma+1}} \frac{\left((\delta^{\Delta}(s))_+\right)^{\gamma+1}}{(\delta^{\sigma}(s))^{\gamma}} (\beta(s,T))^{-\gamma} \right] \Delta s = \infty. \tag{2.33}$$

Then every solution of (1.1) *is oscillatory.*

Proof. Suppose that (1.1) has a nonoscillatory solution x. We may assume without loss of generality that $x(\tau_i(t)) > 0$, $i = 0, 1, 2$, for all $t \geq t_0$.

By Lemma 2.2, there exists $T \geq t_0$ such that (2.5) holds. Defining the function ω as (2.19), we proceed as in the proof of Theorem 2.4, and we get (2.20). In view of $\gamma \leq 1$, using (2.1), we have $(z^{\gamma}(t))^{\Delta} \geq \gamma(z^{\sigma}(t))^{\gamma-1} z^{\Delta}(t)$. From (2.20) we obtain

$$\begin{aligned} \omega^{\Delta}(t) \leq{}& \frac{\delta^{\Delta}(t)}{\delta(t)}\omega(t) - \delta^{\sigma}(t)q_1(t)\left(1 - p(\tau_1(t))\right)^{\alpha} \frac{(z(\tau_1(t)))^{\alpha}}{(z^{\sigma}(t))^{\gamma}} \\ &- \delta^{\sigma}(t)q_2(t)\left(1 - p(\tau_2(t))\right)^{\beta} \frac{(z(\tau_2(t)))^{\beta}}{(z^{\sigma}(t))^{\gamma}} - \gamma\delta^{\sigma}(t)\frac{r(t)(z^{\Delta}(t))^{\gamma+1}}{z^{\gamma}(t)z^{\sigma}(t)} + \delta^{\sigma}(t)\eta^{\Delta}(t). \end{aligned} \tag{2.34}$$

The remainder of the proof is similar to that of Theorem 2.4, and hence it is omitted. □

Theorem 2.6. *Assume that* (1.3) *holds,* $0 \le p(t) < 1$, $\lim_{t\to\infty} p(t) = p_1 < 1$, *and* $\gamma \ge 1$. *Furthermore, assume that there exist positive rd-continuous* Δ*-differentiable functions* δ, η, *and* ϕ *such that* $\phi^{\Delta}(t) \ge 0$, *then for all sufficiently large* T, *for* $T_1 > T$, *one has that* (2.18) *holds, and*

$$\int_{t_0}^{\infty} \left(\frac{1}{\phi(s)r(s)} \int_{t_0}^{s} \phi^{\sigma}(\tau)\left[q_1(\tau) + q_2(\tau)\right]\Delta\tau \right)^{1/\gamma} \Delta s = \infty. \tag{2.35}$$

Then every solution of (1.1) *is either oscillatory or converges to zero.*

Proof. We proceed as in Theorem 2.4, and we assume that $x(\tau_i(t)) > 0$, $i = 0, 1, 2$, for all $t \ge t_0$. From the proof of Lemma 2.2, we see that there exist two possible cases for the sign of $z^{\Delta}(t)$.

If $z^{\Delta}(t)$ is eventually positive, we are then back to the proof of Theorem 2.4 and we obtain a contradiction with (2.18).

If $z^{\Delta}(t) < 0$, $t \ge t_1 \ge t_0$, then there exist constants $c > 0$, $a > 0$ such that $z(t) \le c$, $x(t) \le z(t) \le c$, $t \ge t_1$, and $\lim_{t\to\infty} z(t) = a \ge 0$. Since x is bounded, we let $\limsup_{t\to\infty} x(t) = x_1$, $\liminf_{t\to\infty} x(t) = x_2$. From definition of $z(t)$, noting $0 \le p_1 < 1$, we have $x_1 + p_1 x_2 \le a \le x_2 + p_1 x_1$; hence, we have $x_1 \le x_2$.

On the other hand, $x_1 \ge x_2$; hence, $\lim_{t\to\infty} x(t) = a/(1+p_1)$. Assume that $a > 0$. Then there exist a constant $b > 0$ and $t_2 \ge t_1$ such that $x^{\alpha}(\tau_1(t)) \ge b$, $x^{\beta}(\tau_2(t)) \ge b$ for $t \ge t_2$. Define the function

$$u(t) = \phi(t)r(t)\left(z^{\Delta}(t)\right)^{\gamma}. \tag{2.36}$$

Then $u(t) < 0$ for $t \ge t_2$. From (1.1) we have

$$\begin{aligned} u^{\Delta}(t) &= \phi^{\Delta}(t)r(t)\left(z^{\Delta}(t)\right)^{\gamma} + \phi^{\sigma}(t)\left[r(t)\left(z^{\Delta}(t)\right)^{\gamma}\right]^{\Delta} \le \phi^{\sigma}(t)\left[r(t)\left(z^{\Delta}(t)\right)^{\gamma}\right]^{\Delta} \\ &= -\phi^{\sigma}(t)\left[q_1(t)x^{\alpha}(\tau_1(t)) + q_2(t)x^{\beta}(\tau_2(t))\right] \le -b\phi^{\sigma}(t)\left[q_1(t) + q_2(t)\right]. \end{aligned} \tag{2.37}$$

Integrating the above inequality from t_2 to t, we obtain

$$u(t) \le u(t_2) - b\int_{t_2}^{t} \phi^{\sigma}(s)\left[q_1(s) + q_2(s)\right]\Delta s \le -b\int_{t_2}^{t} \phi^{\sigma}(s)\left[q_1(s) + q_2(s)\right]\Delta s, \tag{2.38}$$

that is,

$$z^{\Delta}(t) \le -b^{1/\gamma}\left(\frac{1}{\phi(t)r(t)} \int_{t_2}^{t} \phi^{\sigma}(s)\left[q_1(s) + q_2(s)\right]\Delta s \right)^{1/\gamma}. \tag{2.39}$$

Integrating the last inequality from t_2 to t, we get

$$z(t) \le z(t_2) - b^{1/\gamma} \int_{t_2}^{t} \left(\frac{1}{\phi(s) r(s)} \int_{t_2}^{s} \phi^{\sigma}(\tau) \left[q_1(\tau) + q_2(\tau) \right] \Delta\tau \right)^{1/\gamma} \Delta s. \tag{2.40}$$

We can easily obtain a contradiction with (2.35). Hence, $\lim_{t\to\infty} x(t) = 0$. This completes the proof. □

From Theorem 2.6, we have the following result.

Theorem 2.7. *Assume that* (1.3) *holds,* $0 \le p(t) < 1$, $\lim_{t\to\infty} p(t) = p_1 < 1$, *and* $\gamma \le 1$. *Furthermore, assume that there exist positive rd-continuous* Δ*-differentiable functions* δ, η, *and* ϕ *such that, for all sufficiently large* T, *for* $T_1 > T$, *one has that* (2.33) *and* (2.35) *hold. Then every solution of* (1.1) *is either oscillatory or converges to zero.*

The proof is similar to that of the proof of Theorem 2.6; hence, we omit the details.

In the following, we give some new oscillation results of (1.1) when $p(t) < 0$.

Theorem 2.8. *Assume that* (1.2) *holds,* $-1 < -p_0 \le p(t) \le 0$, $\lim_{t\to\infty} p(t) = p_2 > -1$, *and* $\gamma \ge 1$. *Furthermore, there exists* $\{c_k\}_{k\ge 0}$ *such that* $\lim_{k\to\infty} c_k = \infty$ *and* $\tau_0(c_{k+1}) = c_k$. *If there exist positive rd-continuous* Δ*-differentiable functions* δ *and* η *such that, for all sufficiently large* T, *for* $T_1 > T$,

$$\limsup_{t\to\infty} \int_{T_1}^{t} \left[\delta^{\sigma}(s) Q_{2*}(s) - \delta^{\Delta}(s)\eta(s) - \frac{r(s)}{(\gamma+1)^{\gamma+1}} \frac{\left(\left(\delta^{\Delta}(s) \right)_+ \right)^{\gamma+1}}{\left(\delta^{\sigma}(s) \right)^{\gamma}} \left(\beta(s,T) \right)^{-\gamma^2} \right] \Delta s = \infty, \tag{2.41}$$

then every solution of (1.1) *is oscillatory or tends to zero.*

Proof. Suppose that (1.1) has a nonoscillatory solution x. We may assume without loss of generality that $x(\tau_i(t)) > 0$, $i = 0, 1, 2$, for all $t \ge t_0$. By Lemma 2.3, there exists $T \ge t_0$ such that (2.5) holds, or $\lim_{t\to\infty} x(t) = 0$. Assume that (2.5) holds. Define the function ω as (2.19), and then we get (2.20). By (1.1), we obtain

$$\left(r(t) \left(z^{\Delta}(t) \right)^{\gamma} \right)^{\Delta} \le -q_1(t) (z(\tau_1(t)))^{\alpha} - q_2(t) (z(\tau_2(t)))^{\beta} < 0. \tag{2.42}$$

In view of $\gamma \ge 1$, from (2.1), we have $(z^{\gamma}(t))^{\Delta} \ge \gamma (z(t))^{\gamma-1} z^{\Delta}(t)$. By (2.20), we obtain

$$\begin{aligned} \omega^{\Delta}(t) \le{}& \frac{\delta^{\Delta}(t)}{\delta(t)} \omega(t) - \delta^{\sigma}(t) q_1(t) \frac{(z(\tau_1(t)))^{\alpha}}{(z^{\sigma}(t))^{\gamma}} - \delta^{\sigma}(t) q_2(t) \frac{(z(\tau_2(t)))^{\beta}}{(z^{\sigma}(t))^{\gamma}} \\ & - \gamma \delta^{\sigma}(t) \frac{r(t) \left(z^{\Delta}(t) \right)^{\gamma+1}}{z(t) (z^{\sigma}(t))^{\gamma}} + \delta^{\sigma}(t) \eta^{\Delta}(t). \end{aligned} \tag{2.43}$$

By Young's inequality (2.23), we have

$$
\begin{aligned}
&\frac{\beta-\gamma}{\beta-\alpha}q_1(t)\frac{(z(\tau_1(t)))^{\alpha}}{(z^{\sigma}(t))^{\gamma}}+\frac{\gamma-\alpha}{\beta-\alpha}q_2(t)\frac{(z(\tau_2(t)))^{\beta}}{(z^{\sigma}(t))^{\gamma}}\\
&\quad\geq\left[q_1(t)\frac{(z(\tau_1(t)))^{\alpha}}{(z^{\sigma}(t))^{\gamma}}\right]^{(\beta-\gamma)/(\beta-\alpha)}\left[q_2(t)\frac{(z(\tau_2(t)))^{\beta}}{(z^{\sigma}(t))^{\gamma}}\right]^{(\gamma-\alpha)/(\beta-\alpha)}\\
&\quad=(q_1(t))^{(\beta-\gamma)/(\beta-\alpha)}(q_2(t))^{(\gamma-\alpha)/(\beta-\alpha)}\left(\frac{(z(\tau_1(t)))^{\alpha}}{(z^{\sigma}(t))^{\gamma}}\right)^{(\beta-\gamma)/(\beta-\alpha)}\left(\frac{(z(\tau_2(t)))^{\beta}}{(z^{\sigma}(t))^{\gamma}}\right)^{(\gamma-\alpha)/(\beta-\alpha)}\\
&\quad\geq(q_1(t))^{(\beta-\gamma)/(\beta-\alpha)}(q_2(t))^{(\gamma-\alpha)/(\beta-\alpha)}\left(\frac{z(\tau(t))}{z^{\sigma}(t)}\right)^{\gamma}.
\end{aligned}
\tag{2.44}
$$

By Lemma 2.1, we have

$$
\frac{z(\tau(t))}{z^{\sigma}(t)}\geq\alpha(t,T),\qquad\frac{z(t)}{z^{\sigma}(t)}\geq\beta(t,T).
\tag{2.45}
$$

Hence, by (2.19) and (2.43), we obtain

$$
\begin{aligned}
\omega^{\Delta}(t)\leq{}&\frac{\delta^{\Delta}(t)}{\delta(t)}\omega(t)-\nu\delta^{\sigma}(t)(q_1(t))^{(\beta-\gamma)/(\beta-\alpha)}(q_2(t))^{(\gamma-\alpha)/(\beta-\alpha)}(\alpha(t,T))^{\gamma}\\
&-\gamma\delta^{\sigma}(t)\frac{1}{(r(t))^{1/\gamma}}(\beta(t,T))^{\gamma}\left(\frac{\omega(t)}{\delta(t)}-\eta(t)\right)^{(\gamma+1)/\gamma}+\delta^{\sigma}(t)\eta^{\Delta}(t).
\end{aligned}
\tag{2.46}
$$

Thus

$$
\begin{aligned}
\omega^{\Delta}(t)\leq{}&-\delta^{\sigma}(t)\left[Q_2(t)-\eta^{\Delta}(t)\right]+\delta^{\Delta}(t)\eta(t)+\left(\delta^{\Delta}(t)\right)_{+}\left|\frac{\omega(t)}{\delta(t)}-\eta(t)\right|\\
&-\gamma\delta^{\sigma}(t)\frac{1}{(r(t))^{1/\gamma}}(\beta(t,T))^{\gamma}\left(\frac{\omega(t)}{\delta(t)}-\eta(t)\right)^{(\gamma+1)/\gamma}.
\end{aligned}
\tag{2.47}
$$

Set

$$
\begin{aligned}
&\lambda=\frac{\gamma+1}{\gamma},\quad A=\gamma^{1/\lambda}(\delta^{\sigma}(t))^{1/\lambda}\frac{1}{(r(t))^{1/\gamma+1}}(\beta(t,T))^{\gamma^2/(\gamma+1)}\left|\frac{\omega(t)}{\delta(t)}-\eta(t)\right|,\\
&B=\left(\left(\delta^{\Delta}(t)\right)_{+}\right)^{\gamma}\left(\frac{\gamma}{\gamma+1}\right)^{\gamma}\frac{(r(t))^{\gamma/(\gamma+1)}}{\gamma^{\gamma^2/(\gamma+1)}(\delta^{\sigma}(t))^{\gamma^2/(\gamma+1)}}\left(\frac{1}{\beta(t,T)}\right)^{\gamma^3/(\gamma+1)}.
\end{aligned}
\tag{2.48}
$$

Using the inequality (2.29), we obtain

$$\omega^{\Delta}(t) \leq -\delta^{\sigma}(t)\left[Q_2(t)-\eta^{\Delta}(t)\right]+\delta^{\Delta}(t)\eta(t)+\frac{r(t)}{(\gamma+1)^{\gamma+1}}\frac{\left(\left(\delta^{\Delta}(t)\right)_+\right)^{\gamma+1}}{\left(\delta^{\sigma}(t)\right)^{\gamma}}(\beta(t,T))^{-\gamma^2}. \tag{2.49}$$

Integrating the last inequality from $T_1 > T$ to $t > T_1$, we obtain

$$\begin{aligned}-\omega(T_1) &< \omega(t)-\omega(T_1)\\ &\leq -\int_{T_1}^{t}\left[\delta^{\sigma}(s)\left(Q_2(s)-\eta^{\Delta}(s)\right)-\delta^{\Delta}(s)\eta(s)-\frac{r(s)}{(\gamma+1)^{\gamma+1}}\frac{\left(\left(\delta^{\Delta}(s)\right)_+\right)^{\gamma+1}}{\left(\delta^{\sigma}(s)\right)^{\gamma}}(\beta(s,T))^{-\gamma^2}\right]\Delta s,\end{aligned} \tag{2.50}$$

which yields

$$\int_{T_1}^{t}\left[\delta^{\sigma}(s)\left(Q_2(s)-\eta^{\Delta}(s)\right)-\delta^{\Delta}(s)\eta(s)-\frac{r(s)}{(\gamma+1)^{\gamma+1}}\frac{\left(\left(\delta^{\Delta}(s)\right)_+\right)^{\gamma+1}}{\left(\delta^{\sigma}(s)\right)^{\gamma}}(\beta(s,T))^{-\gamma^2}\right]\Delta s \leq \omega(T_1), \tag{2.51}$$

which leads to a contradiction with (2.41). The proof is complete. □

Theorem 2.9. *Assume that* (1.2) *holds,* $-1 < -p_0 \leq p(t) \leq 0$, $\lim_{t\to\infty}p(t) = p_2 > -1$, *and* $\gamma \leq 1$. *Furthermore, there exists* $\{c_k\}_{k\geq 0}$ *such that* $\lim_{k\to\infty}c_k = \infty$ *and* $\tau_0(c_{k+1}) = c_k$. *If there exist positive rd-continuous* Δ*-differentiable functions* δ *and* η *such that, for all sufficiently large* T, *for* $T_1 > T$,

$$\limsup_{t\to\infty}\int_{T_1}^{t}\left[\delta^{\sigma}(s)Q_{2*}(s)-\delta^{\Delta}(s)\eta(s)-\frac{r(s)}{(\gamma+1)^{\gamma+1}}\frac{\left(\left(\delta^{\Delta}(s)\right)_+\right)^{\gamma+1}}{\left(\delta^{\sigma}(s)\right)^{\gamma}}(\beta(s,T))^{-\gamma}\right]\Delta s = \infty, \tag{2.52}$$

then every solution of (1.1) *is oscillatory or tends to zero.*

Proof. Suppose that (1.1) has a nonoscillatory solution x. We may assume without loss of generality that $x(\tau_i(t)) > 0$, $i = 0, 1, 2$, for all $t \geq t_0$. By Lemma 2.3, there exists $T \geq t_0$ such that (2.5) holds, or $\lim_{t\to\infty}x(t) = 0$. Assume that (2.5) holds.

Define the function ω as (2.19), and then we get (2.20). In view of $\gamma \leq 1$, using (2.1), we have $(z^{\gamma}(t))^{\Delta} \geq \gamma(z^{\sigma}(t))^{\gamma-1}z^{\Delta}(t)$. From (2.20) we obtain

$$\begin{aligned}\omega^{\Delta}(t) \leq{}& \frac{\delta^{\Delta}(t)}{\delta(t)}\omega(t)-\delta^{\sigma}(t)q_1(t)\frac{(z(\tau_1(t)))^{\alpha}}{(z^{\sigma}(t))^{\gamma}}-\delta^{\sigma}(t)q_2(t)\frac{(z(\tau_2(t)))^{\beta}}{(z^{\sigma}(t))^{\gamma}}\\ &-\gamma\delta^{\sigma}(t)\frac{r(t)\left(z^{\Delta}(t)\right)^{\gamma+1}}{z^{\gamma}(t)z^{\sigma}(t)}+\delta^{\sigma}(t)\eta^{\Delta}(t).\end{aligned} \tag{2.53}$$

The remainder of the proof is similar to that of Theorem 2.8, and hence it is omitted. □

Remark 2.10. One can easily see that the results obtained in [1, 10–12, 15] cannot be applied in (1.1), so our results are new.

3. Examples

In this section, we will give some examples to illustrate our main results.

Example 3.1. Consider the second-order quasilinear neutral delay dynamic equations on time scales

$$\left(t\sigma(t)\left(x(t)+\frac{1}{2}x(\tau_0(t))\right)^{\Delta}\right)^{\Delta}+\frac{\sigma(t)}{\tau(t)}x^{1/3}(\tau(t))+\frac{\sigma(t)}{\tau(t)}x^{5/3}(\tau(t))=0, \tag{3.1}$$

where $t\in[t_0,\infty)_{\mathbb{T}}$, and we assume that $\int_{t_0}^{\infty}\Delta t/t\sigma(t)=\infty$.

Let $r(t)=t\sigma(t)$, $p(t)=1/2$, $q_1(t)=q_2(t)=\sigma(t)/\tau(t)$, $\gamma=1$, $\alpha=1/3$, $\beta=5/3$, and $\tau_1(t)=\tau_2(t)=\tau(t)$. Take $\delta(t)=\eta(t)=\phi(t)=1$. It is easy to show that (2.18) and (2.35) hold. Hence, by Theorem 2.6, every solution of (3.1) oscillates or tends to zero.

Example 3.2. Consider the second-order quasilinear neutral delay dynamic equations on time scales

$$\left(\left(x(t)-\frac{1}{2}x(\tau_0(t))\right)^{\Delta}\right)^{\Delta}+\frac{\sigma(t)}{t\tau(t)}x^{1/3}(\tau(t))+\frac{\sigma(t)}{t\tau(t)}x^{5/3}(\tau(t))=0, \tag{3.2}$$

where $t\in[t_0,\infty)_{\mathbb{T}}$, and we assume there exists $\{c_k\}_{k\geq 0}$ such that $\lim_{k\to\infty}c_k=\infty$ and $\tau_0(c_{k+1})=c_k$.

Let $r(t)=1$, $p(t)=-1/2$, $q_1(t)=q_2(t)=\sigma(t)/t\tau(t)$, $\gamma=1$, $\alpha=1/3$, $\beta=5/3$, $\tau_1(t)=\tau_2(t)=\tau(t)$. Take $\delta(t)=\eta(t)=1$. It is easy to show that (2.41) holds. Hence, by Theorem 2.8, every solution of (3.2) oscillates or tends to zero.

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