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Common fixed point results for three maps in generalized metric space

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Abstract

Mustafa and Sims [Fixed Point Theory Appl. **2009**, Article ID 917175, 10, (2009)] generalized a concept of a metric space and proved fixed point theorems for mappings satisfying different contractive conditions. In this article, we extend and generalize the results obtained by Mustafa and Sims and prove common fixed point theorems for three maps in these spaces. It is worth mentioning that our results do not rely on continuity and commutativity of any mappings involved therein. We also introduce the notation of a generalized probabilistic metric space and obtain common fixed point theorem in the frame work of such spaces.
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1. Introduction and Preliminaries

The study of fixed points of mappings satisfying certain contractive conditions has been at the center of vigorous research activity. Mustafa and Sims [1] generalized the concept of a metric space. Based on the notion of generalized metric spaces, Mustafa et al. [2-5] obtained some fixed point theorems for mappings satisfying different contractive conditions. Abbas and Rhoades [6] motivated the study of a common fixed point theory in generalized metric spaces. Recently, Saadati et al. [7] proved some fixed point results for contractive mappings in partially ordered G-metric spaces.

The purpose of this article is to initiate the study of common fixed point for three mappings in complete G-metric space. It is worth mentioning that our results do not rely on the notion of continuity, weakly commuting, or compatibility of mappings involved therein. We generalize various results of Mustafa et al. [3,5].

Consistent with Mustafa and Sims [1], the following definitions and results will be needed in the sequel.

Definition 1.1. Let X be a nonempty set. Suppose that a mapping $G : X \times X \times X \to R^+$ satisfies:

(a) $G(x, y, z) = 0$ if and only if $x = y = z$,
(b) $0 < G(x, y, z)$ for all $x, y \in X$, with $x \neq y$,
(c) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$, with $z \neq y$,
(d) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \ldots$ (symmetry in all three variables), and
(e) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$.

Then G is called a G-metric on X and (X, G) is called a G-metric space.

Definition 1.2. A G-metric is said to be symmetric if $G(x, y, y) = G(y, x, x)$ for all $x, y \in X$.

Definition 1.3. Let (X, G) be a G-metric space. We say that $\{x_n\}$ is

(i) a *G-Cauchy* sequence if, for any $\varepsilon > 0$, there is an $n_0 \in N$ (the set of all positive integers) such that for all $n, m, l \geq n_0$, $G(x_n, x_m, x_l) < \varepsilon$;
(ii) a *G-Convergent* sequence if, for any $\varepsilon > 0$, there is an $x \in X$ and an $n_0 \in N$, such that for all $n, m \geq n_0$, $G(x, x_n, x_m) < \varepsilon$.

A G-metric space X is said to be complete if every G-Cauchy sequence in X is convergent in X. It is known that $\{x_n\}$ converges to $x \in (X, G)$ if and only if $G(x_m, x_n, x) \to 0$ as $n, m \to \infty$.

Proposition 1.4. *Every G-metric space* (X, G) *will define a metric space* (X, d_G) *by*

$$d_G(x, y) = G(x, y, y) + G(y, x, x), \forall x, y \in X.$$

Definition 1.5. Let (X, G) and (X', G') be G-metric spaces and let $f : (X, G) \to (X', G')$ be a function, then f is said to be G-continuous at a point $a \in X$ if and only if, given $\varepsilon > 0$, there exists $\delta > 0$ such that $x, y \in X$; and $G(a, x, y) < \delta$ implies $G'(f(a), f(x), f(y)) < \varepsilon$. A function f is G-continuous at X if and only if it is G-continuous at all $a \in X$.

2. Common Fixed Point Theorems

In this section, we obtain common fixed point theorems for three mappings defined on a generalized metric space. We begin with the following theorem which generalize [[5], Theorem 1].

Theorem 2.1. *Let f, g, and h be self maps on a complete G-metric space X satisfying*

$$G(fx, gy, hz) \leq kU(x, y, z) \tag{2.1}$$

where $k \in [0, \frac{1}{2})$ *and*

$$\begin{aligned} U(x, y, z) = \max\{&G(x, y, z), G(fx, fx, x), G(y, gy, gy), G(z, hz, hz), \\ &G(x, gy, gy), G(y, hz, hz), G(z, fx, fx)\} \end{aligned}$$

for all $x, y, z \in X$. *Then f, g, and h have a unique common fixed point in X. Moreover, any fixed point of f is a fixed point g and h and conversely.*

Proof. Suppose x_0 is an arbitrary point in X. Define $\{x_n\}$ by $x_{3n+1} = fx_{3n}$, $x_{3n+2} = gx_{3n+1}$, $x_{3n+3} = hx_{3n+2}$ for $n \geq 0$. We have

$$\begin{aligned} G(x_{3n+1}, x_{3n+2}, x_{3n+3}) &= G(fx_{3n}, gx_{3n+1}, hx_{3n+2}) \\ &\leq kU(x_{3n}, x_{3n+1}, x_{3n+2}) \end{aligned}$$

for $n = 0, 1, 2, \ldots$, where

$$\begin{aligned} &U(x_{3n}, x_{3n+1}, x_{3n+2}) \\ = \;& \max\{G(x_{3n}, x_{3n+1}, x_{3n+2}), G(fx_{3n}, fx_{3n}, x_{3n}), G(x_{3n+1}, gx_{3n+1}, gx_{3n+1}), \\ & G(x_{3n+2}, hx_{3n+2}, hx_{3n+2}), G(x_{3n}, gx_{3n+1}, gx_{3n+1}), \\ & G(x_{3n+1}, hx_{3n+2}, hx_{3n+2}), G(x_{3n+2}, fx_{3n}, fx_{3n})\} \\ = \;& \max\{G(x_{3n}, x_{3n+1}, x_{3n+2}), G(x_{3n+1}, x_{3n+1}, x_{3n}), G(x_{3n+1}, x_{3n+2}, x_{3n+2}), \\ & G(x_{3n+2}, x_{3n+3}, x_{3n+3}), G(x_{3n}, x_{3n+2}, x_{3n+2}), \\ & G(x_{3n+1}, x_{3n+3}, x_{3n+3}), G(x_{3n+2}, x_{3n+1}, x_{3n+1})\} \\ \leq \;& \max\{G(x_{3n}, x_{3n+1}, x_{3n+2}), G(x_{3n}, x_{3n+1}, x_{3n+2}), G(x_{3n}, x_{3n+1}, x_{3n+2}), \\ & G(x_{3n+1}, x_{3n+2}, x_{3n+3}), G(x_{3n}, x_{3n+1}, x_{3n+2}), \\ & G(x_{3n+1}, x_{3n+2}, x_{3n+3}), (x_{3n}, x_{3n+1}, x_{3n+2})\} \\ = \;& \max\{G(x_{3n}, x_{3n+1}, x_{3n+2}), G(x_{3n+1}, x_{3n+2}, x_{3n+3})\} \end{aligned}$$

In case $\max\{G(x_{3n}, x_{3n+1}, x_{3n+2}), G(x_{3n+1}, x_{3n+2}, x_{3n+3})\} = G(x_{3n}, x_{3n+1}, x_{3n+2})$, we obtain that

$$G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \leq kG(x_{3n}, x_{3n+1}, x_{3n+2}).$$

If $\max\{G(x_{3n}, x_{3n+1}, x_{3n+2}), G(x_{3n+1}, x_{3n+2}, x_{3n+3})\} = G(x_{3n+1}, x_{3n+2}, x_{3n+3})$, then

$$G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \leq kG(x_{3n+1}, x_{3n+2}, x_{3n+3}),$$

which implies that $G(x_{3n+1}, x_{3n+2}, x_{3n+3}) = 0$, and $x_{3n+1} = x_{3n+2} = x_{3n+3}$ and the result follows immediately.

Hence,

$$G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \leq kG(x_{3n}, x_{3n+1}, x_{3n+2}).$$

Similarly it can be shown that

$$G(x_{3n+2}, x_{3n+3}, x_{3n+4}) \leq kG(x_{3n+1}, x_{3n+2}, x_{3n+3})$$

and

$$G(x_{3n+3}, x_{3n+4}, x_{3n+5}) \leq kG(x_{3n+2}, x_{3n+3}, x_{3n+4}).$$

Therefore, for all n,

$$\begin{aligned} G(x_{n+1}, x_{n+2}, x_{n+3}) &\leq kG(x_n, x_{n+1}, x_{n+2}) \\ &\leq \cdots \leq k^{n+1}G(x_0, x_1, x_2). \end{aligned}$$

Now, for any l, m, n with $l > m > n$,

$$\begin{aligned} G(x_n, x_m, x_l) &\leq G(x_n, x_{n+1}, x_{n+1}) + G(x_{n+1}, x_{n+1}, x_{n+2}) \\ &\quad + \cdots + G(x_{l-1}, x_{l-1}, x_l) \\ &\leq G(x_n, x_{n+1}, x_{n+2}) + G(x_n, x_{n+1}, x_{n+2}) \\ &\quad + \cdots + G(x_{l-2}, x_{l-1}, x_l) \\ &\leq [k^n + k^{n+1} + \cdots + k^l]G(x_0, x_1, x_2) \\ &\leq \frac{k^n}{1-k}G(x_0, x_1, x_2). \end{aligned}$$

The same holds if $l = m > n$ and if $l > m = n$ we have

$$G(x_n, x_m, x_l) \leq \frac{k^{n-1}}{1-k}G(x_0, x_1, x_2).$$

Consequently $G(x_n, x_m, x_l) \to 0$ as $n, m, l \to \infty$. Hence $\{x_n\}$ is a G-Cauchy sequence. By G-completeness of X, there exists $u \in X$ such that $\{x_n\}$ converges to u as $n \to \infty$. We claim that $fu = u$. If not, then consider

$$G(fu, x_{3n+2}, x_{3n+3}) = G(fu, gx_{3n+1}, hx_{3n+2}) \leq kU(u, x_{3n+1}, x_{3n+2}),$$

where

$$\begin{aligned} &U(u, x_{3n+1}, x_{3n+2}) \\ = &\max\{G(u, x_{3n+1}, x_{3n+2}), G(fu, fu, u), G(x_{3n+1}, gx_{3n+1}, gx_{3n+1}), \\ &G(x_{3n+2}, hx_{3n+2}, hx_{3n+2}), G(u, gx_{3n+1}, gx_{3n+2}), \\ &G(x_{3n+1}, hx_{3n+2}, hx_{3n+2}), G(x_{3n+2}, fu, fu)\} \\ = &\max\{G(u, x_{3n+1}, x_{3n+2}), G(fu, fu, u), G(x_{3n+1}, x_{3n+2}, x_{3n+2}), \\ &G(x_{3n+2}, x_{3n+3}, x_{3n+3}), G(u, x_{3n+2}, x_{3n+2}), \\ &G(x_{3n+1}, x_{3n+3}, x_{3n+3}), G(x_{3n+2}, fu, fu)\}. \end{aligned}$$

On taking limit $n \to \infty$, we obtain that

$$G(fu, u, u) \leq kU(u, u, u),$$

where

$$\begin{aligned} U(u, u, u) &= \max\{G(u, u, u), G(fu, fu, u), G(u, u, u), G(u, u, u) \\ &\quad G(u, u, u), G(u, u, u), G(u, fu, fu)\} \\ &= G(fu, fu, u). \end{aligned}$$

Thus

$$G(fu, u, u) \leq kG(fu, fu, u) \leq 2kG(fu, u, u),$$

a contradiction. Hence, $fu = u$. Similarly it can be shown that $gu = u$ and $hu = u$. To prove the uniqueness, suppose that if v is another common fixed point of f, g, and h, then

$$G(u, v, v) = G(fu, gv, hv) \leq kU(u, v, v),$$

where

$$\begin{aligned} U(u, v, v) &= \max\{G(u, v, v), G(fu, fu, u), G(v, gv, gv), G(v, hv, hv), \\ &\quad G(u, gv, gv), G(v, hv, hv), G(v, fu, fu)\} \\ &= \max\{G(u, v, v), G(u, u, u), G(v, v, v), G(v, v, v), \\ &\quad G(u, v, v), G(v, v, v), G(v, u, u)\} \\ &= \max\{G(u, v, v), G(v, u, u)\} \end{aligned}$$

If $U(u, v, v) = G(u, v, v)$, then

$$G(u, v, v) \leq kG(u, v, v),$$

which gives that $G(u, v, v) = 0$, and $u = v$. Also for $U(u, v, v) = G(v, u, u)$ we obtain

$$G(u, v, v) \leq kG(v, u, u) \leq 2kG(u, v, v),$$

which gives that $G(u, v, v) = 0$ and $u = v$. Hence, u is a unique common fixed point of f, g, and h.

Now suppose that for some p in X, we have $f(p) = p$. We claim that $p = g(p) = h(p)$, if not then in case when $p \neq g(p)$ and $p \neq h(p)$, we obtain

$$G(p, gp, hp) = G(fp, gp, hp) \leq kU(p, p, p),$$

where

$$\begin{aligned} U(p, p, p) &= \max\{G(p, p, p), G(fp, fp, p), G(p, gp, gp), G(p, hp, hp), \\ &\quad G(p, gp, gp), G(p, hp, hp), G(p, fp, fp)\} \\ &= \max\{G(p, gp, gp), G(p, hp, hp)\}. \end{aligned}$$

Now $U(p, p, p) = G(p, gp, gp)$ gives

$$G(p, gp, hp) \leq kG(p, gp, gp) \leq kG(p, gp, hp),$$

a contradiction. For $U(p, p, p) = G(p, hp, hp)$, we obtain

$$G(p, gp, hp) \leq kG(p, hp, hp) \leq kG(p, gp, hp),$$

a contradiction. Similarly when $p \neq g(p)$ and $p = h(p)$ or when $p \neq h(p)$ and $p = g(p)$, we arrive at a contradiction following the similar arguments to those given above. Hence, in all cases, we conclude that $p = gp = hp$. The same conclusion holds if $p = gp$ or $p = hp$. □

Example 2.2. Let $X = \{0, 1, 2, 3\}$ be a set equipped with G-metric defined by

(x, y, z)	$G(x, y, z)$
(0, 0, 0), (1, 1, 1), (2, 2, 2), (3, 3, 3),	0
(0, 0, 2), (0, 2, 0), (2, 0, 0), (0, 2, 2), (2, 0, 2), (2, 2, 0),	1
(0, 0, 1), (0, 1, 0), (1, 0, 0), (0, 1, 1), (1, 0, 1), (1, 1, 0), (0, 0, 3), (0, 3, 0), (3, 0, 0), (0, 3, 3), (3, 0, 3), (3, 3, 0), (1, 1, 2), (1, 2, 1), (2, 1, 1), (1, 2, 2), (2, 1, 2), (2, 2, 1), (1, 1, 3), (1, 3, 1), (3, 1, 1), (1, 3, 3), (3, 1, 3), (3, 3, 1), (2, 2, 3), (2, 3, 2), (3, 2, 2), (2, 3, 3), (3, 2, 3), (3, 3, 2),	3
(0, 1, 2), (0, 1, 3), (0, 2, 1), (0, 2, 3), (0, 3, 1), (0, 3, 2), (1, 0, 2), (1, 0, 3), (1, 2, 0), (1, 2, 3), (1, 3, 0), (1, 3, 2), (2, 0, 1), (2, 0, 3), (2, 1, 0), (2, 1, 3), (2, 3, 0), (2, 3, 1), (3, 0, 1), (3, 0, 2), (3, 1, 0), (3, 1, 2), (3, 2, 0), (3, 2, 1),	3

and $f, g, h : X \to X$ be defined by

x	$f(x)$	$g(x)$	$h(x)$
0	0	0	0
1	0	2	2
2	0	0	0
3	2	0	2

It may be verified that the mappings satisfy contractive condition (2.1) with contractivity factor equal to $\frac{1}{3}$. Moreover, 0 is a common fixed point of mappings f, g, and h.

Corollary 2.3. *Let f, g, and h be self maps on a complete G-metric space X satisfying*

$$\begin{aligned} G(f^m x, g^m y, h^m z) \quad \leq \quad & k \max\{G(x, y, z), G(f^m x, f^m x, x), G(y, g^m y, g^m y), \\ & G(z, h^m z, h^m z), G(x, g^m y, g^m y), \\ & G(y, h^m z, h^m z), G(z, f^m x, f^m x)\} \end{aligned} \tag{2.2}$$

for all x, y, z ∈ X, where $k \in [0, \frac{1}{2})$*. Then f, g, and h have a unique common fixed point in X. Moreover, any fixed point of f is a fixed point g and h and conversely.*

Proof. It follows from Theorem 2.1, that f^m, g^m and h^m have a unique common fixed point p. Now $f(p) = f(f^m(p)) = f^{m+1}(p) = f^m(f(p))$, $g(p) = g(g^m(p)) = g^{m+1}(p) = g^m(g(p))$ and $h(p) = h(h^m(p)) = h^{m+1}(p) = h^m(h(p))$ implies that $f(p)$, $g(p)$ and $h(p)$ are also fixed points for f^m, g^m and h^m. Now we claim that $p = g(p) = h(p)$, if not then in case when $p \neq g(p)$ and $p \neq h(p)$, we obtain

$$\begin{aligned} G(p, gp, hp) &= G(f^m p, g^m(gp), h^m(hp)) \\ &\leq k \max\{G(p, gp, hp), G(f^m p, f^m p, p), G(gp, g^m(gp), g^m(gp)), \\ &\quad G(hp, h^m(hp), h^m(hp)), G(p, g^m(gp), g^m(gp)), \\ &\quad G(gp, h^m(hp), h^m(hp)), G(hp, f^m p, f^m p)\} \\ &= k \max\{G(p, gp, hp), G(p, p, p), G(gp, gp, gp), G(hp, hp, hp), \\ &\quad G(p, gp, gp), G(gp, hp, hp), G(hp, p, p)\} \\ &= k \max\{G(p, gp, hp), G(gp, hp, hp), G(hp, p, p)\} \\ &\leq kG(p, gp, hp), \end{aligned}$$

which is a contradiction. Similarly when $p \neq g(p)$ and $p = h(p)$ or when $p \neq h(p)$ and $p = g(p)$, we arrive at a contradiction following the similar arguments to those given above. Hence in all cases, we conclude that, $f(p) = g(p) = h(p) = p$. It is obvious that every fixed point of f is a fixed point of g and h and conversely. □

Theorem 2.4. *Let f, g, and h be self maps on a complete G-metric space X satisfying*

$$G(fx, gy, hz) \leq kU(x, y, z), \tag{2.3}$$

where $k \in [0, \frac{1}{3})$*and*

$$\begin{aligned} U(x, y, z) \quad = \quad & \max\{G(y, fx, fx) + G(x, gy, gy), G(z, gy, gy) \\ & + G(y, hz, hz), G(z, fx, fx) + G(x, hz, hz)\} \end{aligned}$$

for all $x, y, z \in X$. *Then f, g, and h have a unique common fixed point in X. Moreover, any fixed point of f is a fixed point g and h and conversely.*

Proof. Suppose x_0 is an arbitrary point in X. Define $\{x_n\}$ by $x_{3n+1} = fx_{3n}$, $x_{3n+2} = gx_{3n+1}$, $x_{3n+3} = hx_{3n+2}$. We have

$$\begin{aligned} G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \quad & = \quad G(fx_{3n}, gx_{3n+1}, hx_{3n+2}) \\ & \leq \quad kU(x_{3n}, x_{3n+1}, x_{3n+2}) \end{aligned}$$

for $n = 0, 1, 2, \ldots$, where

$$\begin{aligned} & U(x_{3n}, x_{3n+1}, x_{3n+2}) \\ = \quad & \max\{G(x_{3n+1}, fx_{3n}, fx_{3n}) + G(x_{3n}, gx_{3n+1}, gx_{3n+1}), \\ & G(x_{3n+2}, gx_{3n+1}, gx_{3n+1}) + G(x_{3n+1}, hx_{3n+2}, hx_{3n+2}), \\ & G(x_{3n+2}, fx_{3n}, fx_{3n}) + G(x_{3n}, hx_{3n+2}, hx_{3n+2})\} \\ = \quad & \max\{G(x_{3n+1}, x_{3n+1}, x_{3n+1}) + G(x_{3n}, x_{3n+2}, x_{3n+2}), \\ & G(x_{3n+2}, x_{3n+2}, x_{3n+2}) + G(x_{3n+1}, x_{3n+3}, x_{3n+3}), \\ & G(x_{3n+2}, x_{3n+1}, x_{3n+1}) + G(x_{3n}, x_{3n+3}, x_{3n+3})\} \\ \leq & \max\{G(x_{3n}, x_{3n+1}, x_{3n+2}), G(x_{3n+1}, x_{3n+2}, x_{3n+3}), \\ & G(x_{3n+2}, x_{3n+1}, x_{3n+1}) + G(x_{3n}, x_{3n+3}, x_{3n+3})\}. \end{aligned}$$

Now if $U(x_{3n}, x_{3n+1}, x_{3n+2}) = G(x_{3n}, x_{3n+1}, x_{3n+2})$, then

$$G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \leq kG(x_{3n}, x_{3n+1}, x_{3n+2}).$$

Also if $U(x_{3n}, x_{3n+1}, x_{3n+2}) = G(x_{3n+1}, x_{3n+2}, x_{3n+3})$, then

$$G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \leq kG(x_{3n+1}, x_{3n+2}, x_{3n+3}),$$

which implies that $G(x_{3n+1}, x_{3n+2}, x_{3n+3}) = 0$, and $x_{3n+1} = x_{3n+2} = x_{3n+3}$ and the result follows immediately.

Finally $U(x_{3n}, x_{3n+1}, x_{3n+2}) = G(x_{3n+2}, x_{3n+1}, x_{3n+1}) + G(x_{3n}, x_{3n+3}, x_{3n+3})$, implies

$$\begin{aligned} & G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \\ \leq \quad & k[G(x_{3n+2}, x_{3n+1}, x_{3n+1}) + G(x_{3n}, x_{3n+3}, x_{3n+3})] \\ \leq \quad & k[G(x_{3n}, x_{3n+1}, x_{3n+2}) + G(x_{3n}, x_{3n+1}, x_{3n+1}) + G(x_{3n+1}, x_{3n+3}, x_{3n+3})] \\ \leq \quad & k[G(x_{3n}, x_{3n+1}, x_{3n+2}) + G(x_{3n}, x_{3n+1}, x_{3n+2}) + G(x_{3n+1}, x_{3n+2}, x_{3n+3})] \\ = \quad & 2kG(x_{3n}, x_{3n+1}, x_{3n+2}) + kG(x_{3n+1}, x_{3n+2}, x_{3n+3}) \end{aligned}$$

which further implies that

$$(1-k)G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \leq 2kG(x_{3n}, x_{3n+1}, x_{3n+2}).$$

Thus,

$$G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \leq \lambda G(x_{3n}, x_{3n+1}, x_{3n+2}),$$

where $\lambda = \dfrac{2k}{1-k}$. Obviously $0 < \lambda < 1$.

Hence,

$$G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \leq kG(x_{3n}, x_{3n+1}, x_{3n+2}).$$

Similarly it can be shown that

$$G(x_{3n+2}, x_{3n+3}, x_{3n+4}) \leq kG(x_{3n+1}, x_{3n+2}, x_{3n+3})$$

and

$$G(x_{3n+3}, x_{3n+4}, x_{3n+5}) \leq kG(x_{3n+2}, x_{3n+3}, x_{3n+4}).$$

Therefore, for all n,

$$\begin{aligned} G(x_{n+1}, x_{n+2}, x_{n+3}) &\leq kG(x_n, x_{n+1}, x_{n+2}) \\ &\leq \cdots \leq k^{n+1}G(x_0, x_1, x_2). \end{aligned}$$

Following similar arguments to those given in Theorem 2.1, $G(x_n, x_m, x_l) \to 0$ as n, m, $l \to \infty$. Hence, $\{x_n\}$ is a G-Cauchy sequence. By G-completeness of X, there exists $u \in X$ such that $\{x_n\}$ converges to u as $n \to \infty$. We claim that $fu = u$. If not, then consider

$$G(fu, x_{3n+2}, x_{3n+3}) = G(fu, gx_{3n+1}, hx_{3n+2}) \leq kU(u, x_{3n+1}, x_{3n+2}),$$

where

$$\begin{aligned} &U(u, x_{3n+1}, x_{3n+2}) \\ = \; &\max\{G(x_{3n+1}, fu, fu) + G(u, gx_{n+1}, gx_{n+1}), G(x_{3n+2}, gx_{3n+1}, gx_{3n+1}) \\ &+ G(x_{3n+1}, hx_{3n+2}, hx_{3n+2}), G(x_{3n+2}, fu, fu) + G(u, hx_{3n+2}, hx_{3n+2})\} \\ = \; &\max\{G(x_{3n+1}, fu, fu) + G(u, x_{n+2}, x_{n+2}), G(x_{3n+2}, x_{3n+2}, x_{3n+2}) \\ &+ G(x_{3n+1}, x_{3n+3}, x_{3n+3}), G(x_{3n+2}, fu, fu) + G(u, x_{3n+3}, x_{3n+3})\} \end{aligned}$$

On taking limit $n \to \infty$, we obtain that

$$G(fu, u, u) \leq kU(u, u, u),$$

where

$$\begin{aligned} U(u, u, u) &= \max\{G(u, fu, fu) + G(u, u, u), G(u, u, u) + G(u, u, u) \\ &\quad G(u, fu, fu) + G(u, u, u)\} = G(fu, fu, u). \end{aligned}$$

Thus

$$G(fu, u, u) \leq kG(fu, fu, u) \leq 2kG(fu, u, u),$$

gives a contradiction. Hence, $fu = u$. Similarly it can be shown that $gu = u$ and $hu = u$. To prove the uniqueness, suppose that if v is another common fixed point of f, g, and h, then

$$G(u, v, v) = G(fu, gv, hv) \leq kU(u, v, v),$$

where

$$\begin{aligned} U(u, v, v) &= \max\{G(v, fu, fu) + G(u, gv, gv), G(v, gv, gv) + G(v, hv, hv), \\ &\quad G(v, fu, fu) + G(u, hv, hv)\} \\ &= \max\{G(v, u, u) + G(u, v, v), G(v, v, v) + G(v, v, v), \\ &\quad G(v, u, u) + G(u, v, v)\} \\ &= G(v, u, u) + G(u, v, v). \end{aligned}$$

Hence,

$$G(u, v, v) \leq k[G(v, u, u) + G(u, v, v)] \leq 3kG(u, v, v),$$

which gives that $G(u, v, v) = 0$, and $u = v$. Therefore, u is a unique common fixed point of f, g, and h.

Now suppose that for some p in X, we have $f(p) = p$. We claim that $p = g(p) = h(p)$, if not then in case when $p \neq g(p)$ and $p \neq h(p)$, we obtain

$$G(p, gp, hp) = G(fp, gp, hp) \leq kU(p, p, p),$$

where

$$\begin{aligned} U(p, p, p) &= \max\{G(p, fp, fp) + G(p, gp, gp), G(p, gp, gp) \\ &\quad + G(p, hp, hp), G(p, fp, fp) + G(p, hp, hp)\} \\ &= \max\{G(p, p, p) + G(p, gp, gp), G(p, gp, gp) \\ &\quad + G(p, hp, hp), G(p, p, p) + G(p, hp, hp)\} \\ &= \max\{G(p, gp, gp), G(p, gp, gp) + G(p, hp, hp), G(p, hp, hp)\}. \end{aligned}$$

If $U(p, p, p) = G(p, gp, gp)$, then

$$G(p, gp, hp) \leq kG(p, gp, gp) \leq kG(p, gp, hp),$$

a contradiction.

Also for $U(p, p, p) = G(p, gp, gp) + G(p, hp, hp)$, we obtain

$$\begin{aligned} G(p, gp, hp) &\leq k[G(p, gp, gp) + G(p, hp, hp)] \\ &\leq 2kG(p, gp, hp), \end{aligned}$$

a contradiction. If $U(p, p, p) = G(p, hp, hp)$, then

$$G(p, gp, hp) \leq kG(p, hp, hp) \leq kG(p, gp, hp),$$

a contradiction. Similarly when $p \neq g(p)$ and $p = h(p)$ or when $p \neq h(p)$ and $p = g(p)$, we arrive at a contradiction following the similar arguments to those given above. Hence, in all cases, we conclude that $p = gp = hp$. □

Corollary 2.5. *Let f, g, and h be self maps on a complete G-metric space X satisfying*

$$G(f^m x, g^m y, h^m z) \leq kU(x, y, z), \tag{2.4}$$

where $k \in [0, \frac{1}{3})$*and*

$$\begin{aligned} U(x, y, z) &= \max\{G(y, f^m x, f^m x) + G(x, g^m y, g^m y), G(z, g^m y, g^m y) \\ &\quad + G(y, h^m z, h^m z), G(z, f^m x, f^m x) + G(x, h^m z, h^m z)\} \end{aligned}$$

for all x, y, z ∈ X. Then f, g, and h have a unique common fixed point in X. Moreover, any fixed point of f is a fixed point g and h and conversely.

Proof. It follows from Theorem 2.4 that f^m, g^m, and h^m have a unique common fixed point p. Now $f(p) = f(f^m(p)) = f^{m+1}(p) = f^m(f(p))$, $g(p) = g(g^m(p)) = g^{m+1}(p) = g^m(g(p))$ and $h(p) = h(h^m(p)) = h^{m+1}(p) = h^m(h(p))$ implies that $f(p)$, $g(p)$ and $h(p)$ are also fixed points for f^m, g^m and h^m.

We claim that $p = g(p) = h(p)$, if not then in case when $p \neq g(p)$ and $p \neq h(p)$, we obtain

$$\begin{aligned} G(p, gp, hp) &= G(f^m p, g^m(gp), h^m(hp)) \\ &\leq kU(p, gp, hp) \\ &= k\max\{G(gp, f^m p, f^m p) + G(p, g^m(gp), g^m(gp)), \\ &\quad G(hp, g^m(gp), g^m(gp)) + G(gp, h^m(hp), h^m(hp)), \\ &\quad G(hp, f^m p, f^m p) + G(p, h^m(hp), h^m(hp)\} \\ &= k\max\{G(gp, p, p) + G(p, gp, gp), G(hp, gp, gp) + G(gp, hp, hp), \\ &\quad G(hp, p, p) + G(p, hp, hp)\} \\ &\leq 2kG(p, gp, hp). \end{aligned}$$

a contradiction. Similarly when $p \neq g(p)$ and $p = h(p)$ or when $p \neq h(p)$ and $p = g(p)$, we arrive at a contradiction following the similar arguments to those given above. Hence, in all cases, we conclude that, $f(p) = g(p) = h(p) = p$. □

Theorem 2.6. *Let f, g, and h be self maps on a complete G-metric space X satisfying*

$$G(fx, gy, hz) \leq kU(x, y, z), \tag{2.5}$$

where $k \in [0, \frac{1}{3})$*and*

$$\begin{aligned} U(x, y, z) &= \max\{G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx), \\ &\quad G(x, gy, gy) + G(y, gy, gy) + G(z, gy, gy), \\ &\quad G(x, hz, hz) + G(y, hz, hz) + G(z, hz, hz)\} \end{aligned}$$

for all x, y, z ∈ X. Then f, g, and h have a common fixed point in X. Moreover, any fixed point of f is a fixed point g and h and conversely.

Proof. Suppose x_0 is an arbitrary point in X. Define $\{x_n\}$ by $x_{3n+1} = fx_{3n}$, $x_{3n+2} = gx_{3n+1}$, $x_{3n+3} = hx_{3n+2}$. We have

$$\begin{aligned} G(x_{3n+1}, x_{3n+2}, x_{3n+3}) &= G(fx_{3n}, gx_{3n+1}, hx_{3n+2}) \\ &\leq kU(x_{3n}, x_{3n+1}, x_{3n+2}) \end{aligned}$$

for $n = 0, 1, 2, \ldots$, where

$$\begin{aligned} &U(x_{3n}, x_{3n+1}, x_{3n+2}) \\ = &\max\{G(x_{3n}, fx_{3n}, fx_{3n}) + G(x_{3n+1}, fx_{3n}, fx_{3n}) + G(x_{3n+2}, fx_{3n}, fx_{3n}), \\ &G(x_{3n}, gx_{3n+1}, gx_{3n+1}) + G(x_{3n+1}, gx_{3n+1}, gx_{3n+1}) + G(x_{3n+2}, gx_{3n+1}, gx_{3n+1}), \\ &G(x_{3n}, hx_{3n+2}, hx_{3n+2}) + G(x_{3n+1}, hx_{3n+2}, hx_{3n+2}) + G(x_{3n+2}, hx_{3n+2}, hx_{3n+2})\} \\ = &\max\{G(x_{3n}, x_{3n+1}, x_{3n+1}) + G(x_{3n+1}, x_{3n+1}, x_{3n+1}) + G(x_{3n+2}, x_{3n+1}, x_{3n+1}), \\ &G(x_{3n}, x_{3n+2}, x_{3n+2}) + G(x_{3n+1}, x_{3n+2}, x_{3n+2}) + G(x_{3n+2}, x_{3n+2}, x_{3n+2}), \\ &G(x_{3n}, x_{3n+3}, x_{3n+3}) + G(x_{3n+1}, x_{3n+3}, x_{3n+3}) + G(x_{3n+2}, x_{3n+3}, x_{3n+3})\} \\ = &\max\{G(x_{3n}, x_{3n+1}, x_{3n+1}) + G(x_{3n+2}, x_{3n+1}, x_{3n+1}), \\ &G(x_{3n}, x_{3n+2}, x_{3n+2}) + G(x_{3n+1}, x_{3n+2}, x_{3n+2}), \\ &G(x_{3n}, x_{3n+3}, x_{3n+3}) + G(x_{3n+1}, x_{3n+3}, x_{3n+3}) + G(x_{3n+2}, x_{3n+3}, x_{3n+3})\} \end{aligned}$$

Now if $U(x_{3n}, x_{3n+1}, x_{3n+2}) = G(x_{3n}, x_{3n+1}, x_{3n+1}) + G(x_{3n+2}, x_{3n+1}, x_{3n+1})$, then

$$\begin{aligned} G(x_{3n+1}, x_{3n+2}, x_{3n+3}) &\leq k[G(x_{3n}, x_{3n+1}, x_{3n+1}) + G(x_{3n+2}, x_{3n+1}, x_{3n+1})] \\ &\leq k[G(x_{3n}, x_{3n+1}, x_{3n+2}) + G(x_{3n}, x_{3n+1}, x_{3n+2})] \\ &\leq 2kG(x_{3n}, x_{3n+1}, x_{3n+2}). \end{aligned}$$

Also if $U(x_{3n}, x_{3n+1}, x_{3n+2}) = G(x_{3n}, x_{3n+2}, x_{3n+2}) + G(x_{3n+1}, x_{3n+2}, x_{3n+2})$, then

$$\begin{aligned} G(x_{3n+1}, x_{3n+2}, x_{3n+3}) &\leq k[G(x_{3n}, x_{3n+2}, x_{3n+2}) + G(x_{3n+1}, x_{3n+2}, x_{3n+2})] \\ &\leq k[G(x_{3n}, x_{3n+1}, x_{3n+2}) + G(x_{3n}, x_{3n+1}, x_{3n+2})] \\ &\leq 2kG(x_{3n}, x_{3n+1}, x_{3n+2}). \end{aligned}$$

Finally for $U(x_{3n}, x_{3n+1}, x_{3n+2}) = G(x_{3n}, x_{3n+3}, x_{3n+3}) + G(x_{3n+1}, x_{3n+3}, x_{3n+3}) + G(x_{3n+2}, x_{3n+3}, x_{3n+3})$, implies

$$\begin{aligned} &G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \\ \leq\ & k[G(x_{3n}, x_{3n+3}, x_{3n+3}) + G(x_{3n+1}, x_{3n+3}, x_{3n+3}) + G(x_{3n+2}, x_{3n+3}, x_{3n+3})] \\ \leq\ & k[2G(x_{3n}, x_{3n+1}, x_{3n+2}) + G(x_{3n}, x_{3n+1}, x_{3n+1}) + G(x_{3n+1}, x_{3n+3}, x_{3n+3})] \\ \leq\ & k[G(x_{3n}, x_{3n+1}, x_{3n+2}) + G(x_{3n}, x_{3n+1}, x_{3n+2}) + G(x_{3n+1}, x_{3n+2}, x_{3n+3})] \\ \leq\ & 2kG(x_{3n}, x_{3n+1}, x_{3n+2}) + kG(x_{3n+1}, x_{3n+2}, x_{3n+3})] \end{aligned}$$

implies that

$$(1-k)G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \leq 2kG(x_{3n}, x_{3n+1}, x_{3n+2}).$$

Thus,

$$G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \leq \lambda G(x_{3n}, x_{3n+1}, x_{3n+2}),$$

where $\lambda = \dfrac{2k}{1-k}$. Obviously $0 < \lambda < 1$.

Hence,

$$G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \leq kG(x_{3n}, x_{3n+1}, x_{3n+2}).$$

Similarly it can be shown that

$$G(x_{3n+2}, x_{3n+3}, x_{3n+4}) \leq kG(x_{3n+1}, x_{3n+2}, x_{3n+3})$$

and

$$G(x_{3n+3}, x_{3n+4}, x_{3n+5}) \leq kG(x_{3n+2}, x_{3n+3}, x_{3n+4}).$$

Therefore, for all n,

$$\begin{aligned} G(x_{n+1}, x_{n+2}, x_{n+3}) &\leq kG(x_n, x_{n+1}, x_{n+2}) \\ &\leq \cdots \leq k^{n+1}G(x_0, x_1, x_2). \end{aligned}$$

Following similar arguments to those given in Theorem 2.1, $G(x_n, x_m, x_l) \to 0$ as n, m, $l \to \infty$. Hence, $\{x_n\}$ is a G-Cauchy sequence. By G-completeness of X, there exists $u \in X$ such that $\{x_n\}$ converges to u as $n \to \infty$. We claim that $fu = gu = u$. If not, then consider

$$G(fu, gu, x_{3n+3}) = G(fu, gu, hx_{3n+2}) \leq kU(u, u, x_{3n+2}),$$

where

$$\begin{aligned} & U(u, x_{3n+1}, x_{3n+2}) \\ = \ & \max\{G(u, fu, fu) + G(u, fu, fu) + G(x_{3n+2}, fu, fu), \\ & G(u, gu, gu) + G(u, gu, gu) + G(x_{3n+2,} gu, gu), \\ & G(u, hx_{3n+2}, hx_{3n+2}) + G(u, hx_{3n+2}, hx_{3n+2}) + G(x_{3n+2}, hx_{3n+2}, hx_{3n+2})\} \\ = \ & \max\{2G(u, fu, fu) + G(x_{3n+2}, fu, fu), 2G(u, gu, gu) + G(x_{3n+2}, gu, gu), \\ & 2G(u, x_{3n+3}, x_{3n+3}) + G(x_{3n+2}, x_{3n+3}, x_{3n+3})\}. \end{aligned}$$

On taking limit as $n \to \infty$, we obtain that

$$G(fu, gu, u) \leq kU(u, u, u),$$

where

$$\begin{aligned} U(u, u, u) &= \max\{2G(u, fu, fu) + G(u, fu, fu), \\ & \quad 2G(u, gu, gu) + G(u, gu, gu), 2G(u, u, u) + G(u, u, u)\} \\ &= \max\{3G(u, fu, fu), 3G(u, gu, gu)\}. \end{aligned}$$

Now for $U(u, u, u) = 3G(fu, fu, fu)$, then

$$G(fu, gu, u) \leq 3kG(fu, fu, u) \leq 3kG(fu, gu, u),$$

a contradiction. Hence, $fu = gu = u$. Also for $U(u, u, u) = 3G(u, gu, gu)$,

$$G(fu, gu, u) \leq 3kG(u, gu, gu) \leq 3kG(fu, gu, u),$$

a contradiction. Hence, $fu = gu = u$. Similarly it can be shown that $gu = u$ and $hu = u$.

Now suppose that for some p in X, we have $f(p) = p$. We claim that $p = g(p) = h(p)$, if not then in case when $p \neq g(p)$ and $p \neq h(p)$, we obtain

$$G(p, gp, hp) = G(fp, gp, hp) \leq kU(p, p, p),$$

where

$$\begin{aligned} U(p, p, p) &= \max\{G(p, fp, fp) + G(p, fp, fp), G(p, fp, fp), \\ & \quad G(p, gp, gp) + G(p, gp, gp) + G(p, gp, gp), \\ & \quad G(p, hp, hp) + G(p, hp, hp) + G(p, hp, hp)\} \\ &= \max\{3G(p, p, p), 3G(p, gp, gp), 3G(p, hp, hp)\} \\ &= \max\{3G(p, gp, gp), 3G(p, hp, hp)\}. \end{aligned}$$

If $U(p, p, p) = 3G(p, gp, gp)$, then

$$G(p, gp, hp) \leq 3kG(p, gp, gp) \leq 3kG(p, gp, hp),$$

a contradiction. Also, $U(p, p, p) = 3G(p, hp, hp)$ gives

$$G(p, gp, hp) \leq 3kG(p, hp, hp) \leq 3kG(p, gp, hp),$$

a contradiction. Similarly when $p \neq g(p)$ and $p = h(p)$ or when $p \neq h(p)$ and $p = g(p)$, we arrive at a contradiction following the similar arguments to those given above. Hence in all cases, we conclude that $p = gp = hp$. □

Remark 2.7. Let f, g, and h be self maps on a complete G-metric space X satisfying (2.5). Then f, g and h have a unique common fixed point in X provided that $0 \leq k < \frac{1}{4}$.

Proof. Existence of common fixed points of f, g, and h follows from Theorem 2.6. To prove the uniqueness, suppose that if v is another common fixed point of f, g, and h, then

$$G(u, v, v) = G(fu, gv, hv) \leq kU(u, v, v),$$

where

$$\begin{aligned} U(u, v, v) &= \max\{G(u, fu, fu) + G(v, fu, fu) + G(v, fu, fu), \\ &\quad G(u, gv, gv), G(v, gv, gv) + G(v, gv, gv), \\ &\quad G(u, hv, hv) + G(v, hv, hv) + G(v, hv, hv)\} \\ &= \max\{G(u, u, u) + G(v, u, u) + G(v, u, u), \\ &\quad G(u, v, v) + G(v, v, v) + G(v, v, v), \\ &\quad G(u, v, v) + G(v, v, v) + G(v, v, v)\} \\ &= \max\{2G(v, u, u), G(u, v, v)\}. \end{aligned}$$

$U(u, v, v) = 2G(v, u, u)$, implies that

$$G(u, v, v) \leq 2kG(v, u, u) \leq 4kG(u, v, v),$$

which gives $u = v$. And $U(u, v, v) = G(u, v, v)$, gives

$$G(u, v, v) \leq kG(u, v, v),$$

$U = v$. Hence, u is a unique common fixed point of f, g, and h. □

Corollary 2.8. *Let f, g, and h be self maps on a complete G-metric space X satisfying*

$$G(f^m x, g^m y, h^m z) \leq kU(x, y, z), \tag{2.6}$$

where $k \in [0, \frac{1}{4})$and

$$\begin{aligned} U(x, y, z) &= \max\{G(x, f^m x, f^m x) + G(y, f^m x, f^m x) + G(z, f^m x, f^m x), \\ &\quad G(x, g^m y, g^m y) + G(y, g^m y, g^m y) + G(z, g^m y, g^m y), \\ &\quad G(x, h^m z, h^m z) + G(y, h^m z, h^m z) + G(z, h^m z, h^m z)\} \end{aligned}$$

for all $x, y, z \in X$. Then f, g and h have a unique common fixed point in X. Moreover, any fixed point of f is a fixed point g and h and conversely.

Proof. It follows from Theorem 2.6, that f^m, g^m, and h^m have a unique common fixed point p. Now $f(p) = f(f^m(p)) = f^{m+1}(p) = f^m(f(p))$, $g(p) = g(g^m(p)) = g^{m+1}(p) = g^m(g(p))$ and $h(p) = h(h^m(p)) = h^{m+1}(p) = h^m(h(p))$ implies that $f(p)$, $g(p)$ and $h(p)$ are also fixed points for f^m, g^m, and h^m. Now we claim that $p = g(p) = h(p)$, if not then in case when $p \neq g(p)$ and $p \neq h(p)$, we obtain

$$\begin{aligned} G(p, gp, hp) &= G(f^m p, g^m(gp), h^m(hp)) \\ &\leq kU(p, gp, hp) \\ &= k\max\{G(p, f^m p, f^m p) + G(gp, f^m p, f^m p) + G(hp, f^m p, f^m p), \\ &\quad G(p, g^m(gp), g^m(gp)) + G(gp, g^m(gp), g^m(gp)) + G(hp, g^m(gp), g^m(gp)), \\ &\quad G(p, h^m(hp), h^m(hp)), G(gp, h^m(hp), h^m(hp)) + G(hp, h^m(hp), h^m(hp))\} \\ &= k\max\{G(p, p, p) + G(gp, p, p) + G(hp, p, p), \\ &\quad G(p, gp, gp) + G(gp, gp, gp) + G(hp, gp, gp), \\ &\quad G(p, hp, hp)), G(gp, hp, hp) + G(hp, hp, hp)\} \\ &= k\max\{G(gp, p, p) + G(hp, p, p), G(p, gp, gp) + G(hp, gp, gp), \\ &\quad G(p, hp, hp)) + G(gp, hp, hp)\}. \end{aligned}$$

Now if $U(p, gp, hp) = G(gp, p, p) + G(hp, p, p)$, then

$$\begin{aligned} G(p, gp, hp) &\leq k[G(gp, p, p) + G(hp, p, p)] \\ &\leq 2kG(p, gp, hp), \end{aligned}$$

a contradiction. Also if $U(p, gp, hp) = G(p, gp, gp) + G(hp, gp, gp)$, then

$$\begin{aligned} G(p, gp, hp) &\leq k[G(p, gp, gp) + G(hp, gp, gp)] \\ &\leq 2kG(p, gp, hp), \end{aligned}$$

a contradiction. Finally, if $U(p, gp, hp) = G(p, hp, hp) + G(gp, hp, hp)$, then

$$\begin{aligned} G(p, gp, hp) &\leq k[G(p, hp, hp) + G(gp, hp, hp)] \\ &\leq 2kG(p, gp, hp), \end{aligned}$$

a contradiction.

Also similarly when $p \neq g(p)$ and $p = h(p)$ or when $p \neq h(p)$ and $p = g(p)$, we arrive at a contradiction following the similar arguments to those given above. Hence, in all cases, we conclude that $f(p) = g(p) = h(p) = p$ □

Example 2.9. Let $X = [0, 1]$ and $G(x, y, z) = \max\{|x - y|, |y - z|, |z - x|\}$ be a G-metric on X. Define $f, g, h : X \to X$ by

$$f(x) = \begin{cases} \frac{x}{12} & \text{for } x \in [0, \frac{1}{2}) \\ \frac{x}{10} & \text{for } x \in [\frac{1}{2}, 1], \end{cases}$$

$$g(x) = \begin{cases} \frac{x}{8} & \text{for } x \in [0, \frac{1}{2}) \\ \frac{x}{6} & \text{for } x \in [\frac{1}{2}, 1], \end{cases}$$

and

$$h(x) = \begin{cases} \frac{x}{5} & \text{for } x \in [0, \frac{1}{2}) \\ \frac{x}{3} & \text{for } x \in [\frac{1}{2}, 1]. \end{cases}$$

Note that f, g and h are discontinuous maps. Also $fg(\frac{1}{2}) = f(\frac{1}{12}) = \frac{1}{144}$, $gh(\frac{1}{2}) = g(\frac{1}{6}) = \frac{1}{48}$, $gh(\frac{1}{2}) = g(\frac{1}{6}) = \frac{1}{48}$, $hg(\frac{1}{2}) = h(\frac{1}{12}) = \frac{1}{60}$, and $fh(\frac{1}{2}) = f(\frac{1}{6}) = \frac{1}{72}$, $hf(\frac{1}{2}) = h(\frac{1}{20}) = \frac{1}{100}$, which shows that f, g and h does not commute with each other.

Note that for $x, y, z \in [0, \frac{1}{2})$,

$$[G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx)] = \frac{11x}{12} + \left|y - \frac{x}{12}\right| + \left|z - \frac{x}{12}\right|,$$

$$[G(x, gy, gy) + G(y, gy, gy) + G(z, gy, gy)] = \left|x - \frac{y}{8}\right| + \frac{7y}{8} + \left|z - \frac{y}{8}\right|,$$

and

$$[G(x, hz, hz) + G(y, hz, hz) + G(z, hz, hz)] = \left|x - \frac{z}{5}\right| + \left|y - \frac{z}{5}\right| + \frac{4z}{5}.$$

Now

$$\begin{aligned} G(fx, gy, hz) &= \max\left\{\left|\frac{x}{12} - \frac{y}{8}\right|, \left|\frac{y}{8} - \frac{z}{5}\right|, \left|\frac{z}{5} - \frac{x}{12}\right|\right\} \\ &= \frac{1}{8}\max\left\{\left|\frac{2x}{3} - y\right|, \left|y - \frac{8z}{5}\right|, \left|\frac{8z}{5} - \frac{2x}{3}\right|\right\}. \end{aligned}$$

For $U(x, y, z) = G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx)$, we obtain

$$\begin{aligned} G(fx, gy, hz) &= \frac{1}{8}\max\left\{\left|\frac{2x}{3} - y\right|, \left|y - \frac{8z}{5}\right|, \left|\frac{8z}{5} - \frac{2x}{3}\right|\right\} \\ &\le \frac{1}{8}\left[\frac{11x}{12} + \left|y - \frac{x}{12}\right| + \left|z - \frac{x}{12}\right|\right] \\ &= \frac{1}{8}[G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx)]. \end{aligned}$$

In case $U(x, y, z) = G(x, gy, gy) + G(y, gy, gy) + G(z, gy, gy)$, then

$$\begin{aligned} G(fx, gy, hz) &= \frac{1}{8}\max\left\{\left|\frac{2x}{3} - y\right|, \left|y - \frac{8z}{5}\right|, \left|\frac{8z}{5} - \frac{2x}{3}\right|\right\} \\ &\le \frac{1}{4}\left[\left|x - \frac{y}{8}\right| + \frac{7y}{8} + \left|z - \frac{y}{8}\right|\right] \\ &= \frac{1}{4}[G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx)]. \end{aligned}$$

And for $U(x, y, z) = G(x, hz, hz) + G(y, hz, hz) + G(z, hz, hz)$, we have

$$\begin{aligned} G(fx, gy, hz) &= \frac{1}{8}\max\left\{\left|\frac{2x}{3} - y\right|, \left|y - \frac{8z}{5}\right|, \left|\frac{8z}{5} - \frac{2x}{3}\right|\right\} \\ &\le \frac{1}{4}\left[\left|x - \frac{z}{5}\right| + \left|y - \frac{z}{5}\right| + \frac{4z}{5}\right] \\ &= \frac{1}{4}[G(x, hz, hz) + G(y, hz, hz) + G(z, hz, hz)]. \end{aligned}$$

Thus, (2.5) is satisfied for $k = \frac{1}{4} < \frac{1}{3}$.
For $x, y, z \in [\frac{1}{2}, 1]$

$$G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx) = \frac{9x}{10} + \left|y - \frac{x}{10}\right| + \left|z - \frac{x}{10}\right|,$$
$$G(x, gy, gy) + G(y, gy, gy) + G(z, gy, gy) = \left|x - \frac{y}{6}\right| + \frac{5y}{6} + \left|z - \frac{y}{6}\right|,$$

and

$$G(x, hz, hz) + G(y, hz, hz) + G(z, hz, hz) = \left|x - \frac{z}{3}\right| + \left|y - \frac{z}{3}\right| + \frac{2z}{3}.$$

Now,

$$\begin{aligned} G(fx, gy, hz) &= \max\left\{\left|\frac{x}{10} - \frac{y}{6}\right|, \left|\frac{y}{6} - \frac{z}{3}\right|, \left|\frac{z}{3} - \frac{x}{10}\right|\right\} \\ &= \frac{1}{6}\max\left\{\left|\frac{3x}{5} - y\right|, |2z - y|, \left|2z - \frac{3x}{5}\right|\right\}, \end{aligned}$$

For $U(x, y, z) = G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx)$, we obtain

$$\begin{aligned} G(fx, gy, hz) &= \frac{1}{6}\max\left\{\left|\frac{3x}{5} - y\right|, |2z - y|, \left|2z - \frac{3x}{5}\right|\right\} \\ &\le \frac{1}{4}\left[\frac{9x}{10} + \left|y - \frac{x}{10}\right| + \left|z - \frac{x}{10}\right|\right] \\ &= \frac{1}{4}[G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx)]. \end{aligned}$$

In case, $U(x, y, z) = G(x, gy, gy) + G(y, gy, gy) + G(z, gy, gy)$, then

$$\begin{aligned} G(fx, gy, hz) &= \frac{1}{6}\max\left\{\left|\frac{3x}{5} - y\right|, |2z - y|, \left|2z - \frac{3x}{5}\right|\right\} \\ &\leq \frac{1}{4}\left[\left|x - \frac{y}{6}\right| + \frac{5y}{6} + \left|z - \frac{y}{6}\right|\right] \\ &= \frac{1}{4}[G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx)]. \end{aligned}$$

And $U(x, y, z) = G(x, hz, hz) + G(y, hz, hz) + G(z, hz, hz)$ gives that

$$\begin{aligned} G(fx, gy, hz) &= \frac{1}{6}\max\left\{\left|\frac{3x}{5} - y\right|, |2z - y|, \left|2z - \frac{3x}{5}\right|\right\} \\ &\leq \frac{1}{4}\left[\left|x - \frac{z}{3}\right| + \left|y - \frac{z}{3}\right| + \frac{2z}{3}\right] \\ &= \frac{1}{4}[G(x, hz, hz) + G(y, hz, hz) + G(z, hz, hz)]. \end{aligned}$$

Hence (2.5) is satisfied for $k = \frac{1}{4} < \frac{1}{3}$.
Now for $x \in [0, \frac{1}{2})$, $y, z \in [\frac{1}{2}, 1]$,

$$[G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx)] = \frac{11x}{12} + \left|y - \frac{x}{12}\right| + \left|z - \frac{x}{12}\right|,$$
$$[G(x, gy, gy) + G(y, gy, gy) + G(z, gy, gy)] = \left|x - \frac{y}{6}\right| + \frac{5y}{6} + \left|z - \frac{y}{6}\right|,$$

and

$$[G(x, hz, hz) + G(y, hz, hz) + G(z, hz, hz)] = \left|x - \frac{z}{3}\right| + \left|y - \frac{z}{3}\right| + \frac{2z}{3}.$$

Also

$$\begin{aligned} G(fx, gy, hz) &= \max\left\{\left|\frac{x}{12} - \frac{y}{6}\right|, \left|\frac{y}{6} - \frac{z}{3}\right|, \left|\frac{z}{3} - \frac{x}{12}\right|\right\} \\ &= \frac{1}{6}\max\left\{\left|y - \frac{x}{2}\right|, |2z - y|, \left|2z - \frac{x}{2}\right|\right\}. \end{aligned}$$

Now for $U(x, y, z) = G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx)$, then

$$\begin{aligned} G(fx, gy, hz) &= \frac{1}{6}\max\left\{\left|y - \frac{x}{2}\right|, |2z - y|, \left|2z - \frac{x}{2}\right|\right\} \\ &\leq \frac{1}{4}\left[\frac{11x}{12} + \left|y - \frac{x}{12}\right| + \left|z - \frac{x}{12}\right|\right] \\ &= \frac{1}{4}[G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx)]. \end{aligned}$$

In case $U(x, y, z) = G(x, gy, gy) + G(y, gy, gy) + G(z, gy, gy)$, then

$$\begin{aligned} G(fx, gy, hz) &= \frac{1}{6}\max\left\{\left|y - \frac{x}{2}\right|, |2z - y|, \left|2z - \frac{x}{2}\right|\right\} \\ &\leq \frac{1}{4}\left[\left|x - \frac{y}{6}\right| + \frac{5y}{6} + \left|z - \frac{y}{6}\right|\right] \\ &= \frac{1}{4}[G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx)]. \end{aligned}$$

And for $U(x, y, z) = G(x, hz, hz) + G(y, hz, hz) + G(z, hz, hz)$, we have

$$\begin{aligned} G(fx, gy, hz) &= \frac{1}{4}\max\left\{\left|y-\frac{x}{2}\right|, \left|2z-y\right|, \left|2z-\frac{x}{2}\right|\right\} \\ &\leq \frac{1}{4}\left[\left|x-\frac{z}{3}\right| + \left|y-\frac{z}{3}\right| + \frac{2z}{3}\right] \\ &= \frac{1}{4}[G(x, hz, hz) + G(y, hz, hz) + G(z, hz, hz)]. \end{aligned}$$

Thus, (2.5) is satisfied for $k = \frac{1}{4} < \frac{1}{3}$.

For $x, y \in [0, \frac{1}{2})$ and $z \in [\frac{1}{2}, 1]$

$$\begin{aligned} G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx) &= \frac{11x}{12} + \left|y-\frac{x}{12}\right| + \left|z-\frac{x}{12}\right|, \\ G(x, gy, gy) + G(y, gy, gy) + G(z, gy, gy) &= \left|x-\frac{y}{8}\right| + \frac{7y}{8} + \left|z-\frac{y}{8}\right|, \\ G(x, hz, hz) + G(y, hz, hz) + G(z, hz, hz) &= \left|x-\frac{z}{3}\right| + \left|y-\frac{z}{3}\right| + \frac{2z}{3}, \end{aligned}$$

and

$$\begin{aligned} G(fx, gy, hz) &= \max\left\{\left|\frac{x}{12}-\frac{y}{8}\right|, \left|\frac{y}{8}-\frac{z}{3}\right|, \left|\frac{z}{3}-\frac{x}{12}\right|\right\} \\ &= \frac{1}{4}\max\left\{\left|\frac{y}{2}-\frac{x}{3}\right|, \left|\frac{4z}{3}-\frac{y}{2}\right|, \left|\frac{4z}{3}-\frac{x}{3}\right|\right\}. \end{aligned}$$

Now for $U(x, y, z) = G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx)$, we obtain

$$\begin{aligned} G(fx, gy, hz) &= \frac{1}{4}\max\left\{\left|\frac{y}{2}-\frac{x}{3}\right|, \left|\frac{4z}{3}-\frac{y}{2}\right|, \left|\frac{4z}{3}-\frac{x}{3}\right|\right\} \\ &\leq \frac{1}{4}\left[\frac{11x}{12} + \left|y-\frac{x}{12}\right| + \left|z-\frac{x}{12}\right|\right] \\ &= \frac{1}{4}[G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx)]. \end{aligned}$$

If $U(x, y, z) = G(x, gy, gy) + G(y, gy, gy) + G(z, gy, gy)$, then

$$\begin{aligned} G(fx, gy, hz) &= \frac{1}{4}\max\left\{\left|\frac{y}{2}-\frac{x}{3}\right|, \left|\frac{4z}{3}-\frac{y}{2}\right|, \left|\frac{4z}{3}-\frac{x}{3}\right|\right\} \\ &\leq \frac{1}{4}\left[\left|x-\frac{y}{8}\right| + \frac{7y}{8} + \left|z-\frac{y}{8}\right|\right] \\ &= \frac{1}{4}[G(x, fx, fx) + G(y, fx, fx) + G(z, fx, fx)]. \end{aligned}$$

For $U(x, y, z) = G(x, hz, hz) + G(y, hz, hz) + G(z, hz, hz)$, we have

$$\begin{aligned} G(fx, gy, hz) &= \frac{1}{4}\max\left\{\left|\frac{y}{2}-\frac{x}{3}\right|, \left|\frac{4z}{3}-\frac{y}{2}\right|, \left|\frac{4z}{3}-\frac{x}{3}\right|\right\} \\ &\leq \frac{1}{4}\left[\left|x-\frac{z}{3}\right| + \left|y-\frac{z}{3}\right| + \frac{2z}{3}\right] \\ &= \frac{1}{4}[G(x, hz, hz) + G(y, hz, hz) + G(z, hz, hz)]. \end{aligned}$$

Thus, (2.5) is satisfied for $k = \frac{1}{4} < \frac{1}{3}$. So all the conditions of Theorem 2.6 are satisfied for all $x, y, z \in X$. Moreover, 0 is the unique common fixed point of f, g, and h.

3. Probabilistic *G*-Metric Spaces

K. Menger introduced the notion of a probabilistic metric space in 1942 and since then the theory of probabilistic metric spaces has developed in many directions [8]. The idea of Menger was to use distribution functions instead of nonnegative real numbers as values of the metric. The notion of a probabilistic metric space corresponds to situations when we do not know exactly the distance between two points, but we know probabilities of possible values of this distance. A probabilistic generalization of metric spaces appears to be interest in the investigation of physical quantities and physiological thresholds. It is also of fundamental importance in probabilistic functional analysis.

Throughout this article, the space of all probability distribution functions (d.f.'s) is denoted by $\Delta^+ = \{F : \mathbb{R} \cup \{-\infty, +\infty\} \to [0, 1]$: F is left-continuous and nondecreasing on $\mathbb{R}$, $F(0) = 0$ and $F(+\infty) = 1\}$ and the subset $D^+ \subseteq \Delta^+$ is the set $D^+ = \{F \in \Delta^+ : l^- F(+\infty) = 1\}$. Here, $l^- f(x)$ denotes the left limit of the function f at the point x, $l^- f(x) = \lim_{t \to x^-} f(t)$. The space Δ^+ is partially ordered by the usual pointwise ordering of functions, i.e., $F \leq G$ if and only if $F(t) \leq G(t)$ for all t in $\mathbb{R}$. The maximal element for Δ^+ in this order is the d.f. given by

$$\varepsilon_0(t) = \begin{pmatrix} 0, \text{ if } t \leq 0, \\ 1, \text{ if } t > 0. \end{pmatrix}$$

Definition 3.1. [8] A mapping $T : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if T satisfies the following conditions

(a) T is commutative and associative;
(b) T is continuous;
(c) $T(a, 1) = a$ for all $a \in [0, 1]$;
(d) $T(a, b) \leq T(c, d)$ whenever $a \leq c$ and $c \leq d$, and $a, b, c, d \in [0, 1]$.

Two typical examples of continuous t-norm are $T_P(a, b) = ab$ and $T_M(a, b) = Min(a, b)$.

Now t-norms are recursively defined by $T^1 = T$ and

$$T^n(x_1, \ldots, x_{n+1}) = T(T^{n-1}(x_1, \ldots, x_n), x_{n+1})$$

for $n \geq 2$ and $x_i \in [0, 1]$, for all $i \in \{1, 2, ..., n + 1\}$.

We say that a t-norm T is of Hadžić type if the family $\{T^n\}_{n \in \mathbb{N}}$ is equicontinuous at $x = 1$, that is,

$$\forall \varepsilon \in (0, 1) \exists \delta \in (0, 1); a > 1 - \delta \Rightarrow T^n(a) > 1 - \varepsilon \quad (n \geq 1).$$

T_M is a trivial example of a t-norm of Hadžić type, but T_P is not of Hadžić type (see [9-11]).

Definition 3.2. A *Menger Probabilistic Metric space* (briefly, Menger PM-space) is a triple $(X, \mathcal{F}, T)$, where X is a nonempty set, T is a continuous t-norm, and $\mathcal{F}$ is a mapping from $X \times X$ into D^+ such that, if $F_{x, y}$ denotes the value of $\mathcal{F}$ at the pair (x, y), the following conditions hold: for all x, y, z in X,

(PM1) $F_{x, y}(t) = 1$ for all $t > 0$ if and only if $x = y$;

(PM2) $F_{x,y}(t) = F_{y,x}(t)$;
(PM3) $F_{x,z}(t+s) \geq T(F_{x,y}(t), F_{y,z}(s))$ for all $x, y, z \in X$ and $t, s \geq 0$.

Using PM-space we define probabilistic G-metric spaces.

Definition 3.3. A *Menger Probabilistic G-Metric space* (briefly, Menger PGM-space) is a triple $(X, \mathcal{G}, T)$, where X is a nonempty set, T is a continuous t-norm, and $\mathcal{F}$ is a mapping from $X \times X \times X$ into D^+ such that, if $G_{x,y,z}$ denotes the value of $\mathcal{G}$ at the triple (x, y, z), the following conditions hold: for all x, y, z in X,

(PGM1) $G_{x,y,z}(t) = 1$ for all $t > 0$ if and only if $x = y = z$;
(PGM2) $G_{x,y,z}(t) < 1$ for all $t > 0$ if and only if $x \neq y$;
(PGM3) $G_{x,y,z}(t) = G_{y,x,z}(t) = G_{y,z,x}(t) = \ldots$;
(PGM4) $G_{x,y,z}(t+s) \geq T(G_{x,a,a}(t), G_{a,y,z}(s))$ for all $x, y, z, a \in X$ and $t, s \geq 0$.

Definition 3.4. A probabilistic G-metric is said to be symmetric if $G_{x,y,y}(t) = G_{y,x,x}(t)$ for all $x, y \in X$.

Example 3.5. Let $(X, \mathcal{F}, T)$ be a PM-space. Define

$$G_{x,y,z}(t) = T_M^2(F_{x,y}(t), F_{y,z}(t), F_{x,z}(t)).$$

Then, $(X, \mathcal{G}, T)$ is a PGM-space.

Now, we generalize the definition of G- Cauchy and G- convergent (see Definition 1.3) to Menger PGM-spaces.

Definition 3.6. Let $(X, \mathcal{G}, T)$ be a Menger PGM-space.

(1) A sequence $\{x_n\}_n$ in X is said to be *PG-convergent* to x in X if, for every $\varepsilon > 0$ and $\lambda > 0$, there exists positive integer N such that $G_{x,x_n,x_m}(\varepsilon) > 1 - \lambda$ whenever $m, n \geq N$.
(2) A sequence $\{x_n\}_n$ in X is called *PG-Cauchy sequence* if, for every $\varepsilon > 0$ and $\lambda > 0$, there exists positive integer N such that $G_{xn,x_m,x_l}(\varepsilon) > 1 - \lambda$ whenever $n, m, l \geq N$.
(3) A Menger PM-space $(X, \mathcal{G}, T)$ is said to be *complete* if and only if every PG-Cauchy sequence in X is PG-convergent to a point in X.

Definition 3.7. Let $(X, \mathcal{G}, T)$ be a Menger PGM space. For each p in X and $\lambda > 0$, the strong λ-neighborhood of p is the set

$$N_p(\lambda) = \{q \in X : G_{p,q,q}(\lambda) > 1 - \lambda\},$$

and the strong neighborhood system for X is the union $\bigcup_{p \in V} \mathcal{N}_p$ where $\mathcal{N}_p = \{N_p(\lambda) : \lambda > 0\}$.

4. Fixed Point Theorems in PGM-Spaces

Lemma 4.1. *Let $(X, \mathcal{G}, T)$ be a Menger PGM-space with T of Hadžić-type and $\{x_n\}$ be a sequence in X such that, for some $k \in (0, 1)$,*

$$G_{x_n,x_{n+1},x_{n+1}}(kt) \geq G_{x_{n-1},x_{n-1},x_n}(t) \quad (n \geq 1, t > 0).$$

Then, $\{x_n\}$ is a PG-Cauchy sequence.

Proof. Let T be Hadžić-type, then

$$\forall \varepsilon \in (0,1) \exists \delta \in (0,1); a > 1-\delta \Rightarrow T^N(a) > 1-\varepsilon, \quad (N \geq 1).$$

Since $(X, \mathcal{G}, T)$ is a Menger PGM-space, we have $\lim_{t\to\infty} G_{x_0,x_1,x_1}(t) = 1$ then there exists a $t_0 > 0$ such that $G_{x0,x_1,x_1}(t_0) > 1-\delta$, then

$$T^N(G_{x_0,x_1,x_1}(t_0)) > 1-\varepsilon, \quad \forall N \geq 1$$

Let $t > 0$. Since the series $\sum_{i=0}^{\infty} k^i t_0$ is convergent, there exists $n_1 \in \mathbb{N}$ such that for $n \geq n_0$ we have $\sum_{i=n}^{\infty} k^i t_0 < t$. Then, for all $n \geq n_1$ and m, $l \in \mathbb{N}$ (put $m + l - 1 = N$), we have

$$\begin{aligned}
G_{x_n,x_{n+m},x_{n+m+l}}(t) &\geq G_{x_n,x_{n+m},x_{n+m+l-1}}\left(\sum_{i=n}^{\infty} k^i t_0\right) \\
&\geq G_{x_n,x_{n+m},x_{n+m+l}}\left(\sum_{i=n}^{n+m+l-1} k^i t_0\right) \\
&\geq T_{i=n}^{n+m+l-1}(G_{x_n,x_{n+m},x_{n+m+l}}(k^i t_0)) \\
&\geq T_{i=n}^{n+m+l-1}(G_{x_i,x_{i+1},x_{i+l}}(k^i t_0)) \\
&= T_{i=0}^{m+l-1}(G_{x_{i+n},x_{i+n+1},x_{i+n+l}}(k^{i+n} t_0)) \\
&\geq T_{i=0}^{m+l-1}(G_{x_0,x_1,x_1}(t_0)) \\
&= T^N(G_{x_0,x_1,x_1}(t_0)) \\
&> 1-\varepsilon.
\end{aligned}$$

Hence, the sequence $\{x_n\}$ is *PG*-Cauchy. □

It is not difficult to see that more general fixed point results in probabilistic *G*-metric spaces can be proved in this manner. For example, we also have the following generalization of Theorem 2.1.

Theorem 4.2. *Let f, g, and h be self maps on a complete PGM-space* $(X, \mathcal{G}, T_M)$ *satisfying*

$$G_{fx,gy,hz}(t) \geq U_{x,y,z}\left(\frac{t}{k}\right) \tag{4.1}$$

where $k \in [0, \frac{1}{2})$ *and*

$$\begin{aligned}
U_{x,y,z}(t) = \; & T_M\{G_{x,y,z}(t), G_{fx,fx,x}(t), G_{y,gy,gy}(t), G_{z,hz,hz}(t), \\
& G_{x,gy,gy}(t), G_{y,hz,hz}(t), G_{z,fx,fx}(t)\}
\end{aligned}$$

for all x, y, z $\in$ *X. Then f, g, and h have a unique common fixed point in X. Moreover, any fixed point of f is a fixed point g and h and conversely.*

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Authors' contributions
All authors carried out the proof. All authors conceived of the study, and participated in its design and coordination. All authors read and approved the final manuscript.

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