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Positive periodic solution of higher-order functional difference equation

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Abstract

Based on a fixed point theorem in a cone, a new sufficient condition for the existence of a positive periodic solution to a class of higher-order functional difference equations is established in this article. The result obtained in this article is different from the existing results in previous literature.
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1 Introduction

The existence of positive periodic solutions of discrete mathematical models such as the discrete model of blood cell production and the single-species discrete periodic population model has been studied extensively in recent years (see [1-8], for example). Most of these discrete mathematical models are first-order functional difference equations. Relatively, few articles focused on the existence of positive periodic solutions of higher-order functional difference equations. In 2010, Wang and Chen [9] have studied the existence of positive periodic solutions for the following general higher-order functional difference equation

$$x(n+m+k) - ax(n+m) - bx(n+k) + abx(n) = f(n, x(n-\tau(n))) \tag{1}$$

where $a \neq 1$, $b \neq 1$ are positive constants, τ: $Z \to Z$ and $\tau(n + \omega) = \tau(n)$, $f(n + \omega, u) = f(n, u)$ for any $u \in R$, ω, m, $k \in N$ where N denotes the set of positive integers. Based on fixed point theorem in a cone [10,11], some new sufficient conditions on the existence of positive periodic solutions to the higher-order functional difference equation (1) are obtained. However, the main results in [9] require that a should be positive constant, l should satisfy condition $l = \omega$ where $l = \dfrac{\omega}{(m,\omega)}$ and (m, ω) are the greatest common divisor of m and ω. In fact, in most cases, m and ω do not satisfy such severe constraint $l = \omega$. In general, $l \leq \omega$. In this article, we consider the following higher-order functional difference equation

$$x(n+m+k) - a(n+m)x(n+m) - bx(n+k) + a(n)bx(n) = f(n, x(n-\tau(n))) \tag{2}$$

where $b \neq 1$ is positive constant, a: $Z \to R_+$ with $a(n) \neq 1$ and $a(n + \omega) = a(n)$, τ: $Z \to Z$ and $\tau(n + \omega) = \tau(n)$, $f(n + \omega, u) = f(n, u)$ for any $u \in R$, $k, \omega, m \in N$ where N denotes the set of positive integers.

The purpose of this article is to consider the existence of positive periodic solution of higher-order functional difference equation (2), we will remove the constrains on a and l in [9]. We will replace constant a in [9] with function $a(n)$. At same time, we will remove the unreasonable assumption $l = w$. Based on a fixed point theorem in a cone, a new sufficient condition is established for the existence of positive periodic solutions for higher-order functional difference equation.

2 Some preparation

Let X be the set of all real ω periodic sequences, then X is a Banach space with the maximum norm $||x|| = \max_{n\in[0,\omega-1]} |x(n)|$.

Lemma 1 (Deimling [10]) *Let X be a Banach space and K be a cone in X. Suppose* Ω_1 *and* Ω_2 *are open subsets of X such that* $0 \in \Omega_1 \subset \bar{\Omega}_1 \subset \Omega_2$ *and suppose that*

$$\Phi : K \cap (\bar{\Omega}_2 \backslash \Omega_1) \to K$$

is a completely continuous operator such that

(i) $||\Phi u|| \le ||u||$ *for* $u \in K \cap \partial\Omega_1$ *and there exists* $\psi \in K\backslash\{0\}$ *such that* $x \neq \Phi x + \lambda\psi$ *for* $x \in K \cap \partial\Omega_2$ *and* $\lambda > 0$; *or*

(ii) $||\Phi u|| \le ||u||$ *for* $u \in K \cap \partial\Omega_2$ *and there exists* $\psi \in K\backslash\{0\}$ *such that* $x \neq \Phi x + \lambda\psi$ *for* $x \in K \cap \partial\Omega_1$ *and* $\lambda > 0$.

Then, Φ *has a fixed point in* $K \cap (\bar{\Omega}_2 \backslash \Omega_1)$.

Let $d \in N$. Consider the equation

$$x(n + d) = cx(n) + \gamma(n) \tag{3}$$

where $\gamma \in X$. Set (d, ω) as the greatest common divisor of d and ω, $p = \omega/(d, \omega)$.

Lemma 2 [9]*Assume that* $0 < c \neq 1$, *then (3) has a unique periodic solution*

$$x(n) = [c^{-p} - 1]^{-1} \sum_{i=1}^{p} c^{-i} \gamma(n + (i - 1)d).$$

Let $y(n) = x(n + k) - a(n)x(n)$, $\bar{a} = \max_{1\le n\le\omega} a(n)$, $\underline{a} = \min_{1\le n\le\omega} a(n)$, then (2) can be rewritten as

$$\begin{cases} x(n + k) = \underline{a}x(n) + y(n) + [a(n) - \underline{a}]x(n), \\ y(n + m) = by(n) + f(n, x(n - \tau(n))). \end{cases} \tag{4}$$

Let $h = \dfrac{\omega}{(k,\omega)}, l = \dfrac{\omega}{(m,\omega)}$. Assume that $x \in X$ solution of (2), then $y \in X$. From Lemma 2, we have

$$x(n) = [\underline{a}^{-h} - 1]^{-1} \sum_{i=1}^{h} \underline{a}^{-1} \{y(n + (i-1)k) + [a(n + (i-1)k) - \underline{a}]x(n + (i-1)k)\},$$

$$y(n) = [b^{-l} - 1]^{-1} \sum_{i=1}^{l} b^{-i} f(n + (i-1)m, x(n + (i-1)m - \tau(n + (i-1)m))).$$

If $f(n, x(n - \tau(n))) \geq 0$ and $0 < b < 1$, then $y(n) \geq 0$.

We introduce the following conditions:

(H) $0 < a(n) < 1$, $0 < b < 1$, $h = \omega$ and f: $R \times (0, +\infty) \to [0, +\infty)$ is continuous.

Define the operator T by

$$\begin{aligned}(Tx)(n) = {} & \frac{\underline{a}^h b^l}{(1-\underline{a}^h)(1-b^l)} \sum_{i=1}^{h} \underline{a}^{-i} \sum_{j=1}^{l} b^{-j} f(n + (i-1)k + (j-1)m, \\ & x(n + (i-1)k + (j-1)m - \tau(n + (i-1)k + (j-1)m))) \\ & + \frac{\underline{a}^h}{(1-\underline{a}^h)} \sum_{i=1}^{h} \underline{a}^{-i} [a(n + (i-1)k) - \underline{a}]x(n + (i-1)k).\end{aligned}$$

Define the cone by

$$K = \{x \in X, x(n) \geq \delta ||x||\}$$

where $\delta = \underline{a}^h b^l (1 - \underline{a}^h)(1 - b^l)/\omega$.

Lemma 3 *Assume that (H) holds and* $0 < r_1 < r_2$, *then* $T : \bar{K}_{r_2} \backslash K_{r_1} \to K$ *is completely continuous, where* $K_r = \{x \in K: ||x|| < r\}$ *and* $\bar{K}_r = \{x \in K : ||x|| \leq r\}$.

Proof Since $0 < a(n) < 1$, then $0 < \underline{a} < 1$. Noting that $0 < b < 1$ and $f(n, x(n - \tau(n))) \geq 0$, we have $y(n) \geq 0$. So $(Tx)(n) \geq 0$ on $[0, \omega - 1]$. Since $\tau(n + \omega) = \tau(n)$ and $f(n + \omega, u) = f(n, u)$ for any $u > 0$, $(Tx)(n + \omega) = (Tx)(n)$ for $x \in X$. Since $l = \dfrac{\omega}{(m,\omega)} \leq \omega$ we have

$$\sum_{j=1}^{l} f(n + (j-1)m, x(n + (j-1)m - \tau(n + (j-1)m))) \leq \sum_{j=1}^{\omega} f(j, x(j - \tau(j))).$$

On the other hand, from (H), $h = \dfrac{\omega}{(k,\omega)} = \omega$, we have

$$\sum_{i=1}^{h} f(n + (i-1)k, x(n + (i-1)k - \tau(n + (i-1)k))) = \sum_{i=1}^{\omega} f(i, x(i - \tau(i)))$$

and

$$\sum_{i=1}^{h}[a(n+(i-1)k)-\underline{a}]x(n+(i-1)k)=\sum_{i=1}^{\omega}[a(i)-\underline{a}]x(i).$$

For any $x \in \bar{K}_{r_2} \backslash K_{r_1}$,

$$\begin{aligned}
(Tx)(n) &= \frac{\underline{a}^h b^l}{(1-\underline{a}^h)(1-b^l)}\sum_{i=1}^{h}\underline{a}^{-i}\sum_{j=1}^{l}b^{-j}f(n+(i-1)k+(j-1)m,\\
&\quad x(n+(i-1)k+(j-1)m-\tau(n+(i-1)k+(j-1)m)))\\
&\quad + \frac{\underline{a}^h}{(1-\underline{a}^h)}\sum_{i=1}^{h}\underline{a}^{-i}[a(n+(i-1)k)-\underline{a}]x(n+(i-1)k)\\
&\le \frac{\underline{a}^h b^l}{(1-\underline{a}^h)(1-b^l)}\underline{a}^{-h}b^{-l}\sum_{i=1}^{h}\sum_{j=1}^{l}\{f(n+(i-1)k+(j-1)m,\\
&\quad x(n+(i-1)k+(j-1)m-\tau(n+(i-1)k+(j-1)m)))\}\\
&\quad + \frac{\underline{a}^h}{(1-\underline{a}^h)}\underline{a}^{-h}\sum_{i=1}^{h}[a(n+(i-1)k)-\underline{a}]x(n+(i-1)k)\\
&= \frac{\underline{a}^h b^l}{(1-\underline{a}^h)(1-b^l)}\underline{a}^{-h}b^{-l}\sum_{j=1}^{l}\sum_{i=1}^{h}\{f(n+(i-1)k+(j-1)m,\\
&\quad x(n+(i-1)k+(j-1)m-\tau(n+(i-1)k+(j-1)m)))\}\\
&\quad + \frac{\underline{a}^h}{(1-\underline{a}^h)}\underline{a}^{-h}\sum_{i=1}^{\omega}(a(i)-\underline{a})x(i)\\
&\le \frac{1}{(1-\underline{a}^h)(1-b^l)}\sum_{j=1}^{\omega}\sum_{i=1}^{\omega}f(i,x(i-\tau(i)))\\
&\quad + \frac{1}{(1-\underline{a}^h)}\sum_{i=1}^{\omega}(a(i)-\underline{a})x(i)\\
&\le \frac{\omega}{(1-\underline{a}^h)(1-b^l)}\sum_{i=1}^{\omega}f(i,x(i,\tau(i)))\\
&\quad + \frac{1}{(1-\underline{a}h)}\sum_{i=1}^{\omega}(a(i)-\underline{a})x(i)\\
&\le \frac{\omega}{(1-\underline{a}^h)(1-b^l)}\sum_{i=1}^{\omega}\{f(i,x(i,\tau(i)))+(a(i)-\underline{a})x(i)\}.
\end{aligned}$$

So

$$||Tx|| \le \frac{\omega}{(1-\underline{a}^h)(1-b^l)}\sum_{i=1}^{\omega}\{f(i,x(i-\tau(i)))+(a(i)-\underline{a})x(i)\}. \tag{5}$$

At the same time

$$(Tx)(n) \geq \frac{\underline{a}^h b^l}{(1-\underline{a}^h)(1-b^l)}\underline{a}^{-1}b^{-1}\sum_{i=1}^{h}\sum_{j=1}^{l}\{f(n+(i-1)k+(j-1)m,$$
$$x(n+(i-1)k+(j-1)m-\tau(n+(i-1)k+(j-1)m)))\}$$
$$+\frac{\underline{a}^h}{(1-\underline{a}^h)}\underline{a}^{-1}\sum_{i=1}^{h}[a(n+(i-1)k)-\underline{a}]x(n+(i-1)k)$$
$$=\frac{\underline{a}^h b^l}{(1-\underline{a}^h)(1-b^l)}\underline{a}^{-1}b^{-1}\sum_{j=1}^{l}\sum_{i=1}^{h}\{f(n+(i-1)k+(j-1)m,$$
$$x(n+(i-1)k+(j-1)m-\tau(n+(i-1)k+(j-1)m)))\}$$
$$+\frac{\underline{a}^h}{(1-\underline{a}^h)}\underline{a}^{-1}\sum_{i=1}^{\omega}(a(i)-\underline{a})x(i)$$
$$\geq \frac{\underline{a}^h}{(1-\underline{a}^h)}\frac{lb^l}{(1-b^l)}\sum_{i=1}^{\omega}f(i,x(i-\tau(i)))$$
$$+\frac{\underline{a}^h}{(1-\underline{a}^h)}\sum_{i=1}^{\omega}(a(i)-\underline{a})x(i)$$
$$\geq \frac{\underline{a}^h}{(1-\underline{a}^h)}\sum_{i=1}^{\omega}[\frac{b^l}{(1-b^l)}f(i,x(i,\tau(i)))+(a(i)-\underline{a})x(i)]$$
$$\geq \underline{a}^h\sum_{i=1}^{\omega}[b^l f(i,x(i-\tau(i)))+(a(i)-\underline{a})x(i)]$$
$$\geq \underline{a}^h b^l\sum_{i=1}^{\omega}[f(i,x(i-\tau(i)))+(a(i)-\underline{a})x(i)].$$

We have

$$(Tx)(n) \geq \delta||Tx||. \tag{6}$$

Thus $T:\bar{K}_{r_2}\backslash K_{r_1}\to K$ is well defined. Since X is finite-dimensional Banach space, one can easily show that T is completely continuous. This completes the proof.

We can easily obtain the following result.

Lemma 4 *The fixed point of T in K is a positive periodic solution of (2).*

3 Main result

Let

$$\varphi(s)=\max\{f(n,u),n\in[0,\omega-1],u\in[\delta s,s]\}$$
$$\psi(s)=\min\left\{\frac{f(n,u)}{u},n\in[0,\omega-1],u\in[\delta s,s]\right\}$$

Let $\bar{a}=\max_{1\leq n\leq\omega}a(n),\underline{a}=\min_{1\leq n\leq\omega}a(n)$.

Theorem 1 *Assume that (H) holds and there exist two positive constants α, β with $\alpha \neq \beta$ such that*

$$\varphi(\alpha) \le (\bar{a}-1)(b-1)\alpha,\ \psi(\beta) \ge (\underline{a}-1)(b-1) \tag{7}$$

Then (2) has at least one positive ω-periodic solution x with $\min\{\alpha, \beta\} \le ||x|| \le \max\{\alpha, \beta\}$.

Proof Without loss of generality, we assume that (H) holds, $\alpha < \beta$. Obviously, $0 < \bar{a} < 1, 0 < \underline{a} < 1$. We claim that:

(i) $||Tx|| \le ||x||, x \in \partial K_\alpha$,

(ii) $x \ne Tx + \lambda \cdot 1, \forall x \in \partial K_\beta, 1 \in K$ and $\lambda > 0$.

From (7), we have that

$$f(n,x) \le (\bar{a}-1)(b-1)\alpha, \quad \forall 0 \le n \le \omega-1, \quad \forall \delta\alpha \le x \le \alpha, \tag{8}$$

$$f(n,x) \ge (\underline{a}-1)(b-1)x, \quad \forall 0 \le n \le \omega-1, \quad \forall \delta\beta \le x \le \beta. \tag{9}$$

In order to prove (i), let $x \in \partial K_\alpha$, then $||x|| = \alpha$ and $\delta\alpha \le x(n) \le \alpha$ for $0 \le n \le \omega - 1$. So

$$\begin{aligned}
(Tx)(n) = {} & \frac{\underline{a}^h b^l}{(1-\underline{a}^h)(1-b^l)} \sum_{i=1}^{h} \underline{a}^{-i} \sum_{j=1}^{l} b^{-j} f(n+(i-1)k+(j-1)m, \\
& x(n+(i-1)k+(j-1)m-\tau(n+(i-1)k+(j-1)m))) \\
& + \frac{\underline{a}^h}{(1-\underline{a}^h)} \sum_{i=1}^{h} \underline{a}^{-i}[a(n+(i-1)k)-\underline{a}]x(n+(i-1)k) \\
\le {} & \frac{\underline{a}^h b^l}{(1-\underline{a}^h)(1-b^l)} \sum_{i=1}^{h} \underline{a}^{-i} \sum_{j=1}^{l} b^{-j}\{(\bar{a}-1)(b-1)\alpha\} \\
& + \frac{\underline{a}^h}{(1-\underline{a}^h} \sum_{i=1}^{h} \underline{a}^{-i}[\bar{a}-\underline{a}]||x|| \\
\le {} & \left\{ \frac{b^l}{(1-b^l)}(1-b) \sum_{j=1}^{l} b^{-j} \right\} \frac{\underline{a}^h}{(1-\underline{a}^h)} \sum_{i=1}^{h} \underline{a}^{-i}\{(1-\bar{a})\alpha\} \\
& + \frac{\underline{a}^h}{(1-\underline{a}^h)} \sum_{i=1}^{h} \underline{a}^{-i}[\bar{a}-\underline{a}]\alpha \\
= {} & \frac{\underline{a}^h}{(1-\underline{a}^h)} \sum_{i=1}^{h} \underline{a}^{-i}[1-\underline{a}]\alpha \\
= {} & \alpha.
\end{aligned}$$

It follows that

$$||Tx|| \le ||x||, x \in \partial K_\alpha. \tag{10}$$

Next, let $\psi = 1 \in K$ in Lemma 1, we prove (ii). If not, there exists $u_o \in \partial K_\beta$ and $\lambda_o > 0$ such that

$$u_0 = (Tu_0)(n) + \lambda_0. \tag{11}$$

Since $u_o \in \partial K_\beta$, then $||u_o|| = \beta$ and $\delta\beta \le u_o(n) \le \beta$. Put $u_o(n) = \min\{u_o(i)|0 \le i \le \omega - 1\}$ for some $n \in [0, \omega - 1]$. Noting that $u_o(n) > 0$ and $0 < \underline{a} < 1$, we have

$$\begin{aligned}
u_0(n) &= (Tu_0)(n) + \lambda_0 \\
&= \frac{\underline{a}^h b^l}{(1-\underline{a}^h)(1-b^l)} \sum_{i=1}^{h} \underline{a}^{-i} \sum_{j=1}^{l} b^{-j} f(n+(i-1)k+(j-1)m, \\
&\quad u_0(n+(i-1)k+(j-1)m-\tau(n+(i-1)k+(j-1)m))) \\
&\quad + \frac{\underline{a}^h}{(1-\underline{a}^h)} \sum_{i=1}^{h} \underline{a}^{-i} [a(n+(i-1)k)-\underline{a}] u_0(n+(i-1)k) + \lambda_0 \\
&\geq \frac{\underline{a}^h b^l}{(1-\underline{a}^h)(1-b^l)} \sum_{i=1}^{h} \underline{a}^{-i} \sum_{j=1}^{l} b^{-j} \{ f(n+(i-1)k+(j-1)m, \\
&\quad u_0(n+(i-1)k+(j-1)m-\tau(n+(i-1)k+(j-1)m)))\} + \lambda_0 \\
&\geq \frac{\underline{a}^h b^l}{(1-\underline{a}^h)(1-b^l)} \sum_{i=1}^{h} \underline{a}^{-i} \sum_{j=1}^{l} b^{-j} (\underline{a}-1)(b-1) u_0(n+(i-1)k \\
&\quad + (j-1)m-\tau(n+(i-1)k+(j-1)m)) + \lambda_0 \\
&\geq u_0(n) + \lambda_0
\end{aligned}$$

which implies that $u_o(n) > u_o(n)$, a contradiction.

Therefore, by Lemma 1, T has a fixed point $x \in K_\beta \backslash K_\alpha$. Furthermore, $\alpha \leq ||x|| \leq \beta$ and $x(n) \geq \delta\alpha$, which means that x is one positive periodic solution of (2). The proof is completed.

4 Example

Now, an example is given to demonstrate our result.

Example 1 Consider the difference equation

$$x(n+m+k) - a(n+m)x(n+m) - bx(n+k) + a(n)bx(n) = f(n, x(n-\tau(n))) \quad (12)$$

where $b = 1/2$, $m = 3$, $k = 5$, $\omega = 6$, τ: $Z \to Z$ and $\tau(n + \omega) = \tau(n)$, a: $Z \to R_+$ with $a(n) = \frac{1}{2} + \frac{1}{16} cos\frac{n\pi}{3}, f(n, u) = (1 - \frac{7}{16})(1 - \frac{1}{2})u^3[1 + \frac{1}{2}(-1)^n cos\frac{\pi u}{3}]$.

Obviously, $a(n + \omega) = a(n + 6) = a(n)$, $f(n + \omega, u) = f(n + 6, u) = f(n, u)$ for any $u \in R$. $h = \frac{\omega}{(k,\omega)} = \frac{6}{(5,6)} = 6, l = \frac{\omega}{m,\omega} = \frac{6}{(3,6)} = 2$. $\bar{a} = \max_{1\leq n\leq\omega} a(n) = \frac{9}{16}, \underline{a} = \min_{1\leq n\leq\omega} a(n) = \frac{7}{16}, \delta = \left(\frac{7}{16}\right)^6 \left(\frac{1}{2}\right)^2 \left[1 - \left(\frac{7}{16}\right)^6\right]\left[1 - \left(\frac{1}{2}\right)^2\right]/6$.

Let $\alpha = \frac{1}{2}$, then

$$\begin{aligned}
\varphi(\alpha) &= \varphi\left(\frac{1}{2}\right) \\
&\leq \left(1 - \frac{7}{16}\right)\left(1 - \frac{1}{2}\right)\left(\frac{1}{2}\right)^3 \left[1 + \frac{1}{2}\right] \\
&= \left(1 - \frac{7}{16}\right)\left(1 - \frac{1}{2}\right)\left(\frac{1}{2}\right)^2 \frac{3}{4} \\
&< \left(\frac{9}{16}\frac{1}{2}\right)\left(1 - \frac{1}{2}\right)\frac{1}{2} \\
&< \left(1 - \frac{9}{16}\right)\left(1 - \frac{1}{2}\right)\frac{1}{2}.
\end{aligned} \quad (13)$$

So $\varphi(\alpha) \leq (\bar{a} - 1)(b - 1)\alpha$.

Let $\beta = \frac{2}{\delta}$. If $u \in [\delta\beta, \beta]$, then $u \geq 2$. Furthermore,

$$\begin{aligned} \psi(\beta) &\geq \left(1 - \frac{7}{16}\right)\left(1 - \frac{1}{2}\right)\left(\frac{2^3}{2}\right)\left[1 - \frac{1}{2}\right] \\ &= 2\left(1 - \frac{7}{16}\right)\left(1 - \frac{1}{2}\right) \\ &> \left(1 - \frac{7}{16}\right)\left(1 - \frac{1}{2}\right). \end{aligned} \tag{14}$$

So $\psi(\beta) \geq (\underline{a} - 1)(b - 1)$.

By Theorem 1 in this article, (12) has at least one positive 6-periodic solution.

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Authors' contributions
All authors contributed equally to the manuscript and read and approved the final draft.

Competing interests
The authors declare that they have no competing interests.

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