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Square-mean almost automorphic mild solutions to some stochastic differential equations in a Hilbert space

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Abstract

This article deals primarily with the existence and uniqueness of square-mean almost automorphic mild solutions for a class of stochastic differential equations in a real separable Hilbert space. We study also some properties of square-mean almost automorphic functions including a compostion theorem. To establish our main results, we use the Banach contraction mapping principle and the techniques of fractional powers of an operator.

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1 Introduction

In this article, we investigate the existence and uniqueness of square-mean almost automorphic solutions to the class of stochastic differential equations in the abstract form:

$$d[x(t) - f(t, B_1x(t))] = [Ax(t) + g(t, B_2x(t))]dt + h(t, B_3x(t))dW(t), \quad t \in \mathbb{R}, \quad (1.1)$$

where $A : D(A) \subset L^2(\mathbb{P}, \mathbb{H}) \to L^2(\mathbb{P}, \mathbb{H})$ is the infinitesimal generator of an analytic semigroup of linear operators $\{T(t)\}_{t\geq 0}$ on $L^2(\mathbb{P}, \mathbb{H})$, B_i, $i = 1, 2, 3$, are bounded linear operators that can be viewed as control terms, and $W(t)$ is a two-sided standard one-dimensional Brownian motion defined on the filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \mathcal{F}_t)$, where $\mathcal{F}_t = \sigma\{W(u) - W(v); u, v \leq t\}$. Here, f, g, and h are appropriate functions to be specified later.

The concept of almost automorphy is an important generalization of the classical almost periodicity. They were introduced by Bochner [1,2]; for more details about this topic, we refer the reader to [3,4]. In recent years, the existence of almost periodic and almost automorphic solutions on different kinds of deterministic differential equations have been considerably investigated in lots of publications [5-15] because of its significance and applications in physics, mechanics, and mathematical biology.

Recently, the existence of almost periodic or pseudo almost periodic solutions to some stochastic differential equations have been considered in many publications, such

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as [16-22] and references therein. In a very recent article [23], the authors introduced a new concept of square-mean almost automorphic stochastic process. This paper generalizes the concept of quadratic mean almost periodic processes introduced by Bezandry and Diagana [18]. The authors established the existence and uniqueness of square-mean almost automorphic mild solutions to the following stochastic differential equations:

$$dx(t) = Ax(t)dt + f(t)dt + W(t)dW(t), \quad t \in \mathbb{R},$$
$$dx(t) = Ax(t)dt + f(t, x(t))dt + g(t, x(t))\, dW(t), \quad t \in \mathbb{R},$$

in a Hilbert space $L^2(\mathbb{P}, \mathbb{H})$, where A is an infinitesimal generator of a C_0-semigroup $\{T(t)\}_{t \geq 0}$, and $W(t)$ is a two-sided standard one-dimensional Brown motion defined on the filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \mathcal{F}_t)$, where $\mathcal{F}_t = \sigma\{W(u) - W(v); u, v \leq t\}$.

Motivated by the above mentioned studies [18,23], the main purpose of this article is to investigate the existence and uniqueness of square-mean almost automorphic solutions to the problem (1.1). Note that (1.1) is more general than the problem studied in [23]. We first use a sharper definition (Definition 2.1) of square-mean almost automorphic process than the Definition 2.5 in [23]. We then present some additional properties of square-mean almost automorphic processes (see Lemmas 2.4-2.5). Our main result is established by using fractional powers of linear operators and Banach contraction principle. The obtained result can be seen as a contribution to this emerging field since it improves and generalizes the results in [23].

The rest of this article is organized as follows. In section 2, we recall and prove some basic definitions, lemmas, and preliminary facts which will be used throughout this article. We also prove some additional properties of square-mean almost automorphic functions. In Section 3, we prove the existence and uniqueness of square-mean almost automorphic mild solutions to (1.1).

2 Preliminaries

In this section, we introduce some basic definitions, notations, lemmas, and technical results which are used in the sequel. For more details on this section, we refer the reader to [23,24].

Throughout the article, we assume that $(\mathbb{H}, ||\cdot||\langle\cdot,\cdot\rangle)$ and $(\mathbb{K}, ||\cdot||_{\mathbb{K}}, \langle\cdot,\cdot\rangle_{\mathbb{K}})$ are two REAL separable Hilbert spaces. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space. The notation $L^2(\mathbb{P}, \mathbb{H})$ stands for the space of all $\mathbb{H}$-valued random variables x such that

$$E\|x\|^2 = \int_\Omega \|x\|^2 d\mathbb{P} < \infty.$$

For $x \in L^2(\mathbb{P}, \mathbb{H})$, let

$$||x||_2 = \left(\int_\Omega ||x||^2 d\mathbb{P}\right)^{\frac{1}{2}}.$$

Then, it is routine to check that $L^2(\mathbb{P}, \mathbb{H})$ is a Hilbert space equipped with the norm $||\cdot||_2$. We let $L(\mathbb{K}, \mathbb{H})$ denote the space of all the linear-bounded operators from $\mathbb{K}$ into $\mathbb{H}$, equipped with the usual operator norm $||\cdot||_{L(\mathbb{K},\mathbb{H})}$. In addition, $W(t)$ is a two-sided standard one-dimensional Brownian motion defined on the filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \mathcal{F}_t)$, where $\mathcal{F}_t = \sigma\{W(u) - W(v); u, v \leq t\}$.

Let $0 \in \rho(A)$ where $\rho(A)$ is the resolvent set of A; then, it is possible to define the fractional power $(-A)^{\alpha}$, for $0 < \alpha \leq 1$, as a closed linear invertible operator on its domain $D((-A)^{\alpha})$. Furthermore, the subspace $D((-A)^{\alpha})$ is dense in $L^2(\mathbb{P}, \mathbb{H})$ and the expression

$$\|x\|_{\alpha} = \|(-A)^{\alpha}x\|_2, \quad x \in D((-A)^{\alpha}),$$

defines a norm on $D((-A)^{\alpha})$. Hereafter, we denote by $L^2(\mathbb{P}, \mathbb{H}_{\alpha})$ the Banach space $D((-A)^{\alpha})$ with norm $\|x\|_{\alpha}$.

The following properties hold by Pazy [25].

Lemma 2.1 *Let $0 < \gamma \leq \mu \leq 1$. Then, the following properties hold:*

(i) $L^2(\mathbb{P}, \mathbb{H}_{\mu})$ is a Banach space and $L^2(\mathbb{P}, \mathbb{H}_{\mu}) \hookrightarrow L^2(\mathbb{P}, \mathbb{H}_{\gamma})$ is continuous.
(ii) The function $s \to (-A)^{\mu}T(s)$ is continuous in the uniform operator topology on $(0, \infty)$, and there exists $M_{\mu} > 0$ such that $\|(-A)^{\mu}T(t)\| \leq M_{\mu}e^{-\delta t}t^{-\mu}$ for each $t > 0$.
(iii) For each $x \in D((-A)^{\mu})$ and $t \geq 0$, $(-A)^{\mu}T(t)x = T(t)(-A)^{\mu}x$.
(iv) $(-A)^{-\mu}$ is a bounded linear operator in $L^2(\mathbb{P}, \mathbb{H})$ with $D((-A)^{\mu}) = Im((-A)^{-\mu})$.

Definition 2.1 ([23]) *A stochastic process $x : \mathbb{R} \to L^2(\mathbb{P}, \mathbb{H})$ is said to be stochastically continuous if*

$$\lim_{t \to s} E\|x(t) - x(s)\|^2 = 0.$$

Definition 2.2 *(compare with* [23]) *A stochastically continuous stochastic process $x : \mathbb{R} \to L^2(\mathbb{P}, \mathbb{H})$ is said to be square-mean almost automorphic if for every sequence of real numbers $\{s'_n\}_{n \in \mathbb{N}}$, there exist a subsequence $\{s_n\}_{n \in \mathbb{N}}$ and a stochastic process*

$$\lim_{n \to \infty} E\|x(t + s_n) - y(t)\|^2 = 0 \quad \textit{and} \quad \lim_{n \to \infty} E\|y(t - s_n) - x(t)\|^2 = 0$$

hold for each $t \in \mathbb{R}$.

The collection of all square-mean almost automorphic stochastic processes $x : \mathbb{R} \to L^2(\mathbb{P}, \mathbb{H})$ is denoted by $AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}))$.

Lemma 2.2 ([23]) *If x, x_1 and x_2 are all square-mean almost automorphic stochastic processes, then the following hold true:*

(i) $x_1 + x_2$ is square-mean almost automorphic.
(ii) λx is square-mean almost automorphic for every scalar λ.
(iii) There exists a constant $M > 0$ such that $\sup_{t \in \mathbb{R}}\|x(t)\|_2 \leq M$. That is, x is bounded in $L^2(\mathbb{P}, \mathbb{H})$.

Lemma 2.3 ([23]) *$(AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H})), \|\cdot\|_{\infty})$ is a Banach space when it is equipped with the norm:*

$$\|x\|_{\infty} := \sup_{t \in \mathbb{R}} \|x(t)\|_2 = \sup_{t \in \mathbb{R}} (E\|x(t)\|^2)^{\frac{1}{2}},$$

for $x \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}))$.

Let $L^2(\mathbb{P}, \tilde{\mathbb{H}})$ be defined as $L^2(\mathbb{P}, \mathbb{H})$ and note that $L^2(\mathbb{P}, \mathbb{H})$, $L^2(\mathbb{P}, \tilde{\mathbb{H}})$ are Banach spaces; then, we state the following lemmas (cf. [3,13]):

Lemma 2.4 *Let* $f \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}))$. *Then, we have*

(I) $h(t) := f(-t) \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}))$.
(II) $f_a(t) := f(t + a) \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}))$.

Lemma 2.5 *Let* $\mathcal{L} \in L(L^2(\mathbb{P}, \tilde{\mathbb{H}}), L^2(\mathbb{P}, \mathbb{H}))$ *and assume that* $f \in AA(\mathbb{R}; L^2(\mathbb{P}, \tilde{\mathbb{H}}))$. *Then,* $\mathcal{L}f \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}))$.

Definition 2.3 ([23]) *A function* $f : \mathbb{R} \times L^2(\mathbb{P}, \mathbb{H}) \to L^2(\mathbb{P}, \mathbb{H})$, $(t,x) \to f(t,x)$, *which is jointly continuous, is said to be square-mean almost automorphic in* $t \in \mathbb{R}$ *for each* $x \in L^2(\mathbb{P}, \mathbb{H})$ *if for every sequence of real numbers* $\{s'_n\}_{n\in\mathbb{N}}$, *there exist a subsequence* $\{s_n\}_{n\in\mathbb{N}}$ *and a stochastic process* $\tilde{f} : \mathbb{R} \times L^2(\mathbb{P}, \mathbb{H}) \to L^2(\mathbb{P}, \mathbb{H})$ *such that*

$$\lim_{n\to\infty} E\,||f(t + s_n, x) - \tilde{f}(t, x)||^2 = 0 \quad \text{and} \quad \lim_{n\to\infty} E\,||\tilde{f}(t + s_n, x) - f(t, x)||^2 = 0$$

for each $t \in \mathbb{R}$ *and each* $x \in L^2(\mathbb{P}, \mathbb{H})$.

Theorem 2.1 ([23]) *Let* $f : \mathbb{R} \times L^2(\mathbb{P}, \mathbb{H}) \to L^2(\mathbb{P}, \mathbb{H})$, $(t, x) \to f(t, x)$ *be square-mean almost automorphic in* $t \in \mathbb{R}$ *for each* $x \in L^2(\mathbb{P}, \mathbb{H})$, *and assume that f satisfies Lipschitz condition in the following sense:*

$$E\,||f(t, x) - f(t, y)||^2 \le \tilde{M} E\,||x - y||^2$$

for all $x, y \in L^2(\mathbb{P}, \mathbb{H})$ *and for each* $t \in \mathbb{R}$, *where* $\tilde{M} > 0$ *is independent of t. Then, for any square-mean almost automorphic process* $x : \mathbb{R} \to L^2(\mathbb{P}, \mathbb{H})$, *the stochastic process* $F : \mathbb{R} \to L^2(\mathbb{P}, \mathbb{H})$ *given by* $F(t) = f(t, x(t))$ *is square-mean almost automorphic.*

Definition 2.4 *An* $\mathcal{F}_t$*-progressively measurable stochastic process* $\{x(t)\}_{t\in\mathbb{R}}$ *is called a mild solution of problem (1.1) on R if the function* $s \to AT(t - s)f(s, B_1x(s))$ *is integrable on* $(-\infty, t)$ *for each* $t \in \mathbb{R}$, *and x(t) satisfies the corresponding stochastic integral equation*

$$x(t) = T(t-a)[x(a) - f(a, B_1x(a))] + f(t, B_1x(t)) + \int_a^t AT(t-s)f(s, B_1x(s))\,ds \\ + \int_a^t T(t-s)g(s, B_2x(s))\,ds + \int_a^t T(t-s)h(s, B_3x(s))\,dW(s)$$

for all $t \ge a$ *and for each* $a \in \mathbb{R}$.

3 Main results

In this section, we investigate the existence of a square-mean almost automorphic solution for the problem (1.1). We first list the following basic assumptions:

(H1) The operator $A : D(A) \subset L^2(\mathbb{P}, \mathbb{H}) \to L^2(\mathbb{P}, \mathbb{H})$ is the infinitesimal generator of an analytic semigroup of linear operators $\{T(t)\}_{t\ge0}$ on $L^2(\mathbb{P}, \mathbb{H})$ and M, δ are positive numbers such that $||T(t)|| \le Me^{-\delta t}$ for $t \ge 0$.

(H2) The operators $B_i : L^2(\mathbb{P}, \mathbb{H}_\alpha) \to L^2(\mathbb{P}, \mathbb{H})$ for $i = 1, 2, 3$, are bounded linear operators and $\varpi := \max_{i=1,2,3}\{||B_i||_{L(L^2(\mathbb{P},\mathbb{H}_\alpha), L^2(\mathbb{P},\mathbb{H}))}\}$.

(H3) There exists a positive number $\beta \in (0, 1)$ such that $f : \mathbb{R} \times L^2(\mathbb{P}, \mathbb{H}) \to L^2(\mathbb{P}, \mathbb{H}_\beta)$ is square-mean almost automorphic in $t \in \mathbb{R}$ for each $\varphi \in L^2(\mathbb{P}, \mathbb{H})$. Let $L_f > 0$ be such that for each $(t, \varphi), (t, \psi) \in \mathbb{R} \times L^2(\mathbb{P}, \mathbb{H})$

$$E\,||\,(-A)^{\beta}f(t,\ \varphi)-(-A)^{\beta}f(t,\ \psi)\,||^2\ \le L_f E\,||\,\varphi-\psi\,||^2.$$

(H4) The functions $g:\mathbb{R}\times L^2(\mathbb{P},\mathbb{H})\to L^2(\mathbb{P},\mathbb{H})$ and $h:\mathbb{R}\times L^2(\mathbb{P},\mathbb{H})\to L^2(\mathbb{P},\mathbb{H})$ are square-mean almost automorphic in $t\in\mathbb{R}$ for each $\varphi\in L^2(\mathbb{P},\mathbb{H})$. Moreover, g and h satisfy Lipschitz conditions in ϕ uniformly for t, that is, there exist positive numbers L_g, and L_h such that

$$E\,||\,g(t,\ \varphi)-g(t,\ \psi)\,||^2\ \le L_g E\,||\,\varphi-\psi\,||^2$$

and

$$E\,||\,h(t,\ \varphi)-h(t,\ \psi)\,||^2\ \le L_g E\,||\,\varphi-\psi\,||^2$$

for all $t\in\mathbb{R}$ and each $\phi,\psi\in L^2(\mathbb{P},\mathbb{H})$.

Theorem 3.1 *Let* $\alpha\in(0,\frac{1}{2})$*and* $\alpha<\beta<1$. If *the conditions (H1)-(H4) are satisfied, then the problem (1.1) has a unique square-mean almost automorphic mild solution* $x(\cdot)\in AA(\mathbb{R};L^2(\mathbb{P},\mathbb{H}_\alpha))$*provided that*

$$L_0=4\varpi^2\Big\{||\,(-A)^{\alpha-\beta}\,||^2L_f+M_{1-\beta+\alpha}^2\delta^{2(\alpha-\beta)}[\Gamma(\beta-\alpha)]^2L_f+M_\alpha^2\delta^{2(\alpha-1)}[\Gamma(1-\alpha)]^2L_g+M_\alpha^2L_h(2\delta)^{2\alpha-1}\Gamma(1-2\alpha)\Big\}<1, \tag{3.1}$$

where $\Gamma(\cdot)$ *is the gamma function.*

Proof: Let $\Lambda:AA(\mathbb{R};L^2(\mathbb{P},\mathbb{H}_\alpha))\to AA(\mathbb{R};L^2(\mathbb{P},\mathbb{H}_\alpha))$ be the operator defined by

$$\begin{aligned}\Lambda x(t)=&f(t,B_1x(t))+\int_{-\infty}^t AT(t-s)f(s,B_1x(s))\,ds\\ &+\int_{-\infty}^t T(t-s)g(s,B_2x(s))ds+\int_{-\infty}^t T(t-s)h\,(s,B_3x(s))\,dW(s),\quad t\in\mathbb{R}.\end{aligned}$$

First, we prove that Λx is well defined. Indeed, let $x\in AA(\mathbb{R};L^2(\mathbb{P},\mathbb{H}_\alpha))$, then $s\to B_ix(s)$ is in $AA(\mathbb{R};L^2(\mathbb{P},\mathbb{H}))$ as $B_i\in L(L^2(\mathbb{P},\mathbb{H}_\alpha),\ L^2(\mathbb{P},\ \mathbb{H}),\ i=1,2,3$ in virtue of Lemma 2.5, and hence, by Theorem 2.1, the function $s\to(-A)^\beta f(s,B_1x(s))$ belongs to $AA(\mathbb{R};L^2(\mathbb{P},\mathbb{H}))$ whenever $B_1x\in AA(\mathbb{R};L^2(\mathbb{P},\mathbb{H}))$. Thus, using Lemma 2.2 (iii), it follows that there exists a constant $N_f>0$ such that $\sup_{t\in\mathbb{R}}E||(-A)^\beta f(t,B_1x(t))||^2\le N_f$. Moreover, from the continuity of $s\to AT(t-s)$ and $s\to T(t-s)$ in the uniform operator topology on $(-\infty,t)$ for each $t\in\mathbb{R}$ and the estimate

$$\begin{aligned}&E\left\|\int_{-\infty}^t AT(t-s)f(s,B_1x(s))ds\right\|_\alpha^2\\
\le\ &E\left(\int_{-\infty}^t ||(-A)^{1-\beta+\alpha}T(t-s)(-A)^\beta f(s,B_1x(s))||\,ds\right)^2\\
\le\ &M_{1-\beta+\alpha}^2E\left(\int_{-\infty}^t e^{-\delta(t-s)}(t-s)^{\beta-\alpha-1}||(-A)^\beta f(s,B_1x(s))||\,ds\right)^2\\
\le\ &M_{1-\beta+\alpha}^2\left(\int_{-\infty}^t e^{-\delta(t-s)}(t-s)^{\beta-\alpha-1}ds\right)\\
&\times\left(\int_{-\infty}^t e^{-\delta(t-s)}(t-s)^{\beta-\alpha-1}E||(-A)^\beta f(s,B_1x(s))||^2ds\right)\\
\le\ &M_{1-\beta+\alpha}^2\left(\int_{-\infty}^t e^{-\delta(t-s)}(t-s)^{\beta-\alpha-1}ds\right)^2\sup_{t\in\mathbb{R}}E||(-A)^\beta f(t,B_1x(t))||^2\\
\le\ &M_{1-\beta+\alpha}^2N_f\delta^{2(\alpha-\beta)}[\Gamma(\beta-\alpha)]^2,\end{aligned}$$

it follows that $s \to AT(t - s)f(s, B_1x(s))$, $s \to T(t - s)g(s, B_2x(s))$ and $s \to T(t - s)h(s, B_3x(s))$ are integrable on $(-\infty, t)$ for every $t \in \mathbb{R}$, therefore, Λx is well defined.

Next, we show that $\Lambda x(t) \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}_\alpha))$. Let us consider the nonlinear operator $\Lambda_1 x$, $\Lambda_2 x$, and $\Lambda_3 x$ acting on the Banach space $AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}_\alpha))$ defined by

$$\Lambda_1 x(t) = \int_{-\infty}^{t} AT(t-s)f(s, B_1x(s))\, ds,$$
$$\Lambda_2 x(t) = \int_{-\infty}^{t} T(t-s)g(s, B_2x(s))\, ds$$

and

$$\Lambda_3 x(t) = \int_{-\infty}^{t} T(t-s)h(s, B_2x(s))\, dW(s),$$

respectively. Now, let us prove that $\Lambda_1 x(t) \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}_\alpha))$. Let $\{s'_n\}_{n\in\mathbb{N}}$ be an arbitrary sequence of real numbers. Since $F(\cdot) = (-A)^\beta f(\cdot, B_1x(\cdot)) \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}))$, there exists a subsequence $\{s_n\}_{n\in\mathbb{N}}$ of $\{s'_n\}_{n\in\mathbb{N}}$ such that for certain stochastic process $\tilde{F}$

$$\lim_{n\to\infty} E\,||\, F(t+s_n) - \tilde{F}(t)\,||^2 = 0 \quad \textit{and} \quad \lim_{n\to\infty} E\,||\,\tilde{F}(t+s_n) - F(t)\,||^2 = 0 \tag{3.2}$$

hold for each $t \in \mathbb{R}$. Moreover, if we let $\widetilde{\Lambda_1}x(t) = \int_{-\infty}^{t} (-A)^{1-\beta}T(t-s)\tilde{F}(s)ds$, then by using Cauchy-Schwarz inequality, we have

$$\begin{aligned}
& E\,||\,\Lambda_1 x(t+s_n) - \widetilde{\Lambda_1}x(t)\,||_\alpha^2 \\
= \;& E\left\| \int_{-\infty}^{t+s_n} AT(t+s_n-s)f(s, B_1x(s))ds - \int_{-\infty}^{t} (-A)^{1-\beta}T(t-s)\tilde{F}(s)ds \right\|_\alpha^2 \\
= \;& E\left\| \int_{-\infty}^{t} (-A)^{1-\beta}T(t-s)F(s+s_n)ds - \int_{-\infty}^{t} (-A)^{1-\beta}T(t-s)\tilde{F}(s)ds \right\|_\alpha^2 \\
\le \;& E\left(\int_{-\infty}^{t} ||(-A)^{1-\beta+\alpha}T(t-s)\,||\;||\,F(s+s_n) - \tilde{F}(s)\,||\, ds \right)^2 \\
\le \;& M_{1-\beta+\alpha}^2 E\left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{\beta-\alpha-1}\,||\,F(s+s_n) - \tilde{F}(s)\,||\, ds \right)^2 \\
\le \;& M_{1-\beta+\alpha}^2 \left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{\beta-\alpha-1} ds \right) \\
& \times \left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{\beta-\alpha-1} E\,||\,F(s+s_n) - \tilde{F}(s)\,||^2\, ds \right) \\
\le \;& M_{1-\beta+\alpha}^2 \left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{\beta-\alpha-1} ds \right)^2 \sup_{t\in\mathbb{R}} E\,||\,F(t+s_n) - \tilde{F}(t)\,||^2 \\
\le \;& M_{1-\beta+\alpha}^2 \delta^{2(\alpha-\beta)}[\Gamma(\beta-\alpha)]^2 \sup_{t\in\mathbb{R}} E\,||\,F(t+s_n) - \tilde{F}(t)\,||^2.
\end{aligned}$$

Thus, by (3.2), we immediately obtain that

$$\lim_{n\to\infty} E\,||\,\Lambda_1 x(t+s_n) - \widetilde{\Lambda_1}x(t)\,||_\alpha^2 = 0,$$

for each $t \in \mathbb{R}$, and we can show in a similar way that

$$\lim_{n\to\infty} E\,||\,\widetilde{\Lambda_1}x(t-s_n) - \Lambda_1 x(t)\,||_\alpha^2 = 0,$$

for each $t \in \mathbb{R}$. Thus, we conclude that $\Lambda_1 x(t) \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}_\alpha))$.

Similarly, by using Theorem 2.1, one easily sees that $s \to g\ (s, B_2 x(s))$ belongs to $AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}))$ whenever $B_2 x \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}))$. Since $G(\cdot) = g(\cdot, B_2 x(\cdot)) \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}))$ for every sequence of real numbers $\{s'_n\}_{n\in\mathbb{N}}$, there exists a subsequence $\{s_n\}_{n\in\mathbb{N}} \subset \{s'_n\}_{n\in\mathbb{N}}$ such that for certain stochastic process $\tilde{G}$

$$\lim_{n\to\infty} E\,||\,G(t+s_n) - \tilde{G}(t)\,||^2 = 0 \quad and \quad \lim_{n\to\infty} E\,||\,\tilde{G}(t+s_n) - G(t)\,||^2 = 0 \tag{3.3}$$

hold for each $t \in \mathbb{R}$. Moreover, if we let $\widetilde{\Lambda_2} x(t) = \int_{-\infty}^{t} T(t-s)\tilde{G}(s)ds$, then by using Cauchy-Schwarz inequality, we get

$$\begin{aligned}
& E\,||\,\Lambda_2 x(t+s_n) - \widetilde{\Lambda_2} x(t)\,||_\alpha^2 \\
= \;& E\left\| \int_{-\infty}^{t+s_n} T(t+s_n-s) g(s, B_2 x(s)) ds - \int_{-\infty}^{t} T(t-s)\tilde{G}(s)ds \right\|_\alpha^2 \\
\le \;& E\left(\int_{-\infty}^{t} ||(-A)^\alpha T(t-s)[G(s+s_n) - \tilde{G}(s)]\,||\, ds \right)^2 \\
\le \;& M_\alpha^2 E\left(\int_{-\infty}^{t} e^{-\delta(t-s)} (t-s)^{-\alpha} ||\,G(s+s_n) - \tilde{G}(s)\,||\, ds \right)^2 \\
\le \;& M_\alpha^2 \left(\int_{-\infty}^{t} e^{-\delta(t-s)} (t-s)^{-\alpha} ds \right) \left(\int_{-\infty}^{t} e^{-\delta(t-s)} (t-s)^{-\alpha} E\,||\,G(s+s_n) - \tilde{G}(s)\,||^2 ds \right) \\
\le \;& M_\alpha^2 \left(\int_{-\infty}^{t} e^{-\delta(t-s)} (t-s)^{-\alpha} ds \right)^2 \sup_{t\in\mathbb{R}} E\,||\,G(t+s_n) - \tilde{G}(t)\,||^2 \\
\le \;& M_\alpha^2 \delta^{2(\alpha-1)} [\Gamma(1-\alpha)]^2 \sup_{t\in\mathbb{R}} E\,||\,G(t+s_n) - \tilde{G}(t)\,||^2.
\end{aligned}$$

Thus, by (3.3), we immediately obtain that

$$\lim_{n\to\infty} E\,||\,\Lambda_2 x(t+s_n) - \widetilde{\Lambda_2} x(t)\,||_\alpha^2 = 0,$$

for each $t \in \mathbb{R}$, and we can show in a similar way that

$$\lim_{n\to\infty} E\,||\,\widetilde{\Lambda_2} x(t+s_n) - \Lambda_2 x(t)\,||_\alpha^2 = 0,$$

for each $t \in \mathbb{R}$. Thus, we conclude that $\Lambda_2 x(t) \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}_\alpha))$.

Now, by using Theorem 2.1, one easily sees that $s \to h\ (s,\ B_3 x(s))$ is in $AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}))$ whenever $B_3 x(t) \in AA(\mathbb{R}\mathbb{P}; L^2(\mathbb{P}, \mathbb{H}))$. Since $H(\cdot) = h(\cdot, B_2 x(\cdot)) \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}))$, for every sequence of real numbers $\{s'_n\}_{n\in\mathbb{N}}$, there exists a subsequence $\{s_n\}_{n\in\mathbb{N}} \subset \{s'_n\}_{n\in\mathbb{N}}$ such that for certain stochastic process $\tilde{H}$

$$\lim_{n\to\infty} E\,||\,H(t+s_n) - \tilde{H}(t)\,||^2 = 0 \quad and \quad \lim_{n\to\infty} E\,||\,\tilde{H}(t+s_n) - H(t)\,||^2 = 0 \tag{3.4}$$

hold for each $t \in \mathbb{R}$. The next step consists of showing that $\Lambda_3 x(t) \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}_\alpha))$. Let $\widetilde{W}(\sigma) := W(\sigma + s_n) - W(s_n)$ for each $\sigma \in \mathbb{R}$. Note that $\widetilde{W}$ is also a Brownian motion and has the same distribution as W. Moreover, if we let $\widetilde{\Lambda_3} x(t) = \int_{-\infty}^{t} T(t-s)\tilde{H}(s)dW(s)$, then by making a change of variables $\sigma = s - s_n$ we get

$$\begin{aligned}
& E||\Lambda_3 x(t+s_n) - \widetilde{\Lambda_3} x(t)\,||_\alpha^2 \\
= \;& E\left\| \int_{-\infty}^{t+s_n} T(t+s_n-s) H(s) dW(s) - \int_{-\infty}^{t} T(t-s)\tilde{H}(s) dW(s) \right\|_\alpha^2 \\
= \;& E\left\| \int_{-\infty}^{t} T(t-\sigma)[H(\sigma+s_n) - \tilde{H}(\sigma)] d\widetilde{W}(\sigma) \right\|_\alpha^2
\end{aligned}$$

Thus, using an estimate on Ito integral established in Ichikawa [26], we obtain that

$$\begin{aligned}
& E\|\Lambda_3 x(t+s_n) - \widetilde{\Lambda_3}x(t)\|_\alpha^2 \\
\leq\ & E\left(\int_{-\infty}^{t} \|(-A)^\alpha T(t-\sigma)[H(\sigma+s_n) - \tilde{H}(\sigma)]\|^2 ds\right) \\
\leq\ & M_\alpha^2 \int_{-\infty}^{t} e^{-2\delta(t-s)}(t-s)^{-2\alpha} E\|H(\sigma+s_n) - \tilde{H}(\sigma)\|^2 ds \\
\leq\ & M_\alpha^2 (2\delta)^{2\alpha-1}\Gamma(1-2\alpha)\sup_{t\in\mathbb{R}} E\|H(t+s_n) - \tilde{H}(t)\|^2.
\end{aligned}$$

Thus, by (3.4), we immediately obtain that

$$\lim_{n\to\infty} E\|\Lambda_3 x(t+s_n) - \widetilde{\Lambda_3}x(t)\|_\alpha^2 = 0,$$

for each $t \in \mathbb{R}$. Arguing in a similar way, we infer that

$$\lim_{n\to\infty} E\|\widetilde{\Lambda_3}x(t+s_n) - \Lambda_3 x(t)\|_\alpha^2 = 0,$$

for each $t \in \mathbb{R}$. Thus, we conclude that $\Lambda_3 x(t) \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}_\alpha))$. Since $f(\cdot, B_1x(\cdot)) \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}_\beta)) \subset AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}_\alpha))$, and in view of the above, it is clear that Λ maps $AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}_\alpha))$ into itself.

Now the remaining task is to prove that is a contraction mapping on $AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}_\alpha))$. Indeed, for each $t \in \mathbb{R}$, $x, y \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}_\alpha))$, we see that

$$\begin{aligned}
& E\|(\Lambda x)(t) - (\Lambda y)(t)\|_\alpha^2 \\
=\ & E\Big\| f(t, B_1x(t)) - f(t, B_1y(t)) + \int_{-\infty}^{t} AT(t-s)[f(s, B_1x(s)) - f(s, B_1y(s))]ds \\
& + \int_{-\infty}^{t} T(t-s)[g(s, B_2x(s)) - g(s, B_2y(s))]ds \\
& + \int_{-\infty}^{t} T(t-s)[h(s, B_3x(s)) - h(s, B_3y(s))]dW(s)\Big\|_\alpha^2 \\
\leq\ & 4E\|f(t,\ B_1x(t)) - f(t, B_1y(t))\|_\alpha^2 \\
& + 4E\left\|\int_{-\infty}^{t} AT(t-s)[f(s,\ B_1x(s)) - f(s,\ B_1y(s))]ds\right\|_\alpha^2 \\
& + 4E\left\|\int_{-\infty}^{t} T(t-s)[g(s,\ B_2x(s)) - g(s,\ B_2y(s))]ds\right\|_\alpha^2 \\
& + 4E\left\|\int_{-\infty}^{t} T(t-s)[h(s,\ B_3x(s)) - h(s,\ B_3y(s))]dW(s)\right\|_\alpha^2 \\
\leq\ & 4\|(-A)^{\alpha-\beta}\|^2 E\|(-A)^\beta f(t,\ B_1x(t)) - (-A)^\beta f(t,\ B_1y(t))\|^2 \\
& + 4E\left(\int_{-\infty}^{t} \|(-A)^{1-\beta+\alpha}T(t-s)[(-A)^\beta f(s,\ B_1x(s)) - (-A)^\beta f(s,\ B_1y(s))]\|\, ds\right)^2 \\
& + 4E\left(\int_{-\infty}^{t} \|(-A)^\alpha T(t-s)[g(s,\ B_2x(s)) - g(s,\ B_2y(s))]\|\, ds\right)^2 \\
& + 4E\left\|\int_{-\infty}^{t} T(t-s)[h(s,\ B_3x(s)) - h(s,\ B_3y(s))]dW(s)\right\|_\alpha^2.
\end{aligned}$$

We first evaluate the first term of the right-hand side as follows:

$$\begin{aligned}
& 4\|(-A)^{\alpha-\beta}\|^2 E\|(-A)^\beta f(t,\ B_1x(t)) - (-A)^\beta f(t,\ B_1y(t))\|^2 \\
\leq\ & 4\|(-A)^{\alpha-\beta}\|^2 L_f E\|B_1x(t) - B_1y(t)\|^2 \\
\leq\ & 4\|(-A)^{\alpha-\beta}\|^2 L_f \varpi^2 \sup_{t\in\mathbb{R}} E\|x(t) - y(t)\|_\alpha^2.
\end{aligned}$$

As regards the second term, by Cauchy-Schwarz inequality, we have

$$\begin{aligned}
& 4E\left(\int_{-\infty}^{t} ||(-A)^{1-\beta+\alpha}T(t-s)[(-A)^{\beta}f(s,\ B_1x(s))-(-A)^{\beta}f(s,\ B_1y(s))]\,||\,ds\right)^2 \\
\le\ & 4M_{1-\beta+\alpha}^2E\left(e^{-\delta(t-s)}(t-s)^{\beta-\alpha-1}||\,(-A)^{\beta}f(s,\ B_1x(s))-(-A)^{\beta}f(s,\ B_1y(s))\,||\,ds\right)^2 \\
\le\ & 4M_{1-\beta+\alpha}^2E\left[\left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{\beta-\alpha-1}ds\right)\right. \\
& \left.\times\left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{\beta-\alpha-1}||\,(-A)^{\beta}f(s,\ B_1x(s))-(-A)^{\beta}f(s,\ B_1y(s))\,||^2ds\right)\right] \\
\le\ & 4M_{1-\beta+\alpha}^2\left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{\beta-\alpha-1}ds\right) \\
& \times\left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{\beta-\alpha-1}E\,||\,(-A)^{\beta}f(s,\ B_1x(s))-(-A)^{\beta}f(s,\ B_1y(s))\,||^2ds\right) \\
\le\ & 4M_{1-\beta+\alpha}^2L_f\left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{\beta-\alpha-1}ds\right) \\
& \times\left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{\beta-\alpha-1}E\,||\,B_1x(s)-B_1y(s)\,||^2ds\right) \\
\le\ & 4M_{1-\beta+\alpha}^2L_f\varpi^2\left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{\beta-\alpha-1}ds\right)^2\sup_{t\in\mathbb{R}}E\,||\,x(t)-y(t)\,||_{\alpha}^2 \\
\le\ & 4M_{1-\beta+\alpha}^2L_f\varpi^2\delta^{2(\alpha-\beta)}[\Gamma(\beta-\alpha)]^2\sup_{t\in\mathbb{R}}E\,||\,x(t)-y(t)\,||_{\alpha}^2.
\end{aligned}$$

As regards the third term, we use again Cauchy-Schwarz inequality and obtain

$$\begin{aligned}
& 4E\left(\int_{-\infty}^{t} ||(-A)^{\alpha}T(t-s)[g(s,\ B_2x(s))-g(s,\ B_2y(s))]\,||\,ds\right)^2 \\
\le\ & 4M_{\alpha}^2E\left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{-\alpha}||\,g(s,\ B_2x(s))-g(s,\ B_2y(s))\,||\,ds\right)^2 \\
\le\ & 4M_{\alpha}^2E\left[\left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{-\alpha}ds\right)\right. \\
& \left.\times\left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{-\alpha}||\,g(s,\ B_2x(s))-g(s,\ B_2y(s))\,||^2ds\right)\right] \\
\le\ & 4M_{\alpha}^2\left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{-\alpha}ds\right) \\
& \times\left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{-\alpha}E\,||\,g(s,\ B_2x(s))-g(s,\ B_2y(s))\,||^2ds\right) \\
\le\ & 4M_{\alpha}^2L_g\left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{-\alpha}ds\right) \\
& \times\left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{-\alpha}E\,||\,B_2x(s)-B_2y(s)\,||^2ds\right) \\
\le\ & 4M_{\alpha}^2L_g\varpi^2\left(\int_{-\infty}^{t} e^{-\delta(t-s)}(t-s)^{-\alpha}ds\right)^2\sup_{t\in\mathbb{R}}E\,||\,x(t)-y(t)\,||_{\alpha}^2 \\
\le\ & 4M_{\alpha}^2L_g\varpi^2\delta^{2(\alpha-1)}[\Gamma(1-\alpha)]^2\sup_{t\in\mathbb{R}}E\,||\,x(t)-y(t)\,||_{\alpha}^2.
\end{aligned}$$

As far as the last term is concerned, by the Ito integral, we get

$$\begin{aligned}
& 4E\left\|\int_{-\infty}^{t} T(t-s)[h(s,\ B_3x(s))-h(s,\ B_3y(s))]dW(s)\right\|_{\alpha}^2 \\
\le\ & 4E\left(\int_{-\infty}^{t} ||(-A)^{\alpha}T(t-s)[h(s,\ B_3x(s))-h(s,\ B_3y(s))]\,||^2ds\right) \\
\le\ & 4M_{\alpha}^2\int_{-\infty}^{t} e^{-2\delta(t-s)}(t-s)^{-2\alpha}E\,||\,h(s,\ B_3x(s))-h(s,\ B_3y(s))\,||^2ds \\
\le\ & 4M_{\alpha}^2L_h\int_{-\infty}^{t} e^{-2\delta(t-s)}(t-s)^{-2\alpha}E\,||\,B_3x(s)-B_3y(s)\,||^2ds \\
\le\ & 4M_{\alpha}^2L_h\varpi^2\left(\int_{-\infty}^{t} e^{-2\delta(t-s)}(t-s)^{-2\alpha}ds\right)\sup_{t\in\mathbb{R}}E\,||\,x(t)-y(t)\,||_{\alpha}^2 \\
\le\ & 4M_{\alpha}^2L_h\varpi^2(2\delta)^{2\alpha-1}\Gamma(1-2\alpha)\sup_{t\in\mathbb{R}}E\,||\,x(t)-y(t)\,||_{\alpha}^2.
\end{aligned}$$

Thus, by combining, it follows that, for each $t \in \mathbb{R}$,

$$\begin{aligned} & E\|(\Lambda x)(t) - (\Lambda y)(t)\|_\alpha^2 \\ \leq\ & 4\varpi^2 \Big\{ \|(-A)^{\alpha-\beta}\|^2 L_f + M_{1-\beta+\alpha}^2 \delta^{2(\alpha-\beta)} [\Gamma(\beta-\alpha)]^2 L_f + M_\alpha^2 \delta^{2(\alpha-1)} [\Gamma(1-\alpha)]^2 L_g \\ & + M_\alpha^2 L_h (2\delta)^{2\alpha-1} \Gamma(1-2\alpha) \Big\} \sup_{t\in\mathbb{R}} E\|x(t) - y(t)\|_\alpha^2, \end{aligned}$$

that is,

$$\|(\Lambda x)(t) - (\Lambda y)(t)\|_{2,\alpha}^2 \leq L_0 \sup_{t\in\mathbb{R}} \|x(t) - y(t)\|_{2,\alpha}^2. \tag{3.5}$$

Note that

$$\sup_{t\in\mathbb{R}} \|x(t) - y(t)\|_{2,\alpha}^2 \leq \left(\sup_{t\in\mathbb{R}} \|x(t) - y(t)\|_{2,\alpha} \right)^2, \tag{3.6}$$

and (3.5) together with (3.6) gives, for each $t \in \mathbb{R}$,

$$\|(\Lambda x)(t) - (\Lambda y)(t)\|_{2,\alpha} \leq \sqrt{L_0} \|x - y\|_{\infty,\alpha}.$$

Hence, we obtain

$$\|\Lambda x - \Lambda y\|_{\infty,\alpha} = \sup_{t\in\mathbb{R}} \|(\Lambda x)(t) - (\Lambda y)(t)\|_{2,\alpha} \leq \sqrt{L_0} \|x - y\|_{\infty,\alpha}.$$

which implies that Λ is a contraction by (3.1). Therefore, by the Banach contraction principle, we conclude that there exists a unique fixed point $x(\cdot)$ for Λ in $AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}_\alpha))$, such that $\Lambda x = x$, that is

$$\begin{aligned} x(t) = & f(t, B_1 x(t)) + \int_{-\infty}^t AT(t-s) f(s, B_1 x(s))\, ds \\ & + \int_{-\infty}^t T(t-s) g(s, B_2 x(s)) ds + \int_{-\infty}^t T(t-s) h(s, B_3 x(s))\, dW(s) \end{aligned}$$

for all $t \in \mathbb{R}$. If we let $x(a) = f(a, B_1 x(a)) + \int_{-\infty}^a AT(a-s) f(s, B_1 x(s)) ds + \int_{-\infty}^a T(a-s) g(s, B_2 x(s)) ds + \int_{-\infty}^a T(a-s) h(s, B_3 x(s))\, dW(s)$, then

$$\begin{aligned} T(t-a)x(a) = & T(t-a) f(a, B_1 x(a)) + \int_{-\infty}^a AT(t-s) f(s, B_1 x(s))\, ds \\ & + \int_{-\infty}^a T(t-s) g(s, B_2 x(s)) ds + \int_{-\infty}^a T(t-s) h(s, B_3 x(s))\, dW(s). \end{aligned}$$

However, for $t \geq a$,

$$\begin{aligned} & \int_a^t T(t-s) h(s, B_3 x(s))\, dW(s) \\ = & \int_{-\infty}^t T(t-s) h(s, B_3 x(s))\, dW(s) - \int_{-\infty}^t T(t-s) h(s, B_3 x(s))\, dW(s) \\ = & x(t) - f(t, B_1 x(t)) - \int_{-\infty}^t AT(t-s) f(s, B_1 x(s))\, ds - \int_{-\infty}^t T(t-s) g\,(s, B_2 x(s))\, ds \\ & - T(t-a)[x(a) - f(a, B_1 x(a))] \\ & + \int_{-\infty}^a AT(t-s) f(s, B_1 x(s))\, ds + \int_{-\infty}^a T(t-s) g(s, B_2 x(s))\, ds \\ = & x(t) - T(t-a)[x(a) - f(a, B_1 x(a))] - f(t, B_1 x(t)) \\ & - \int_{-\infty}^t AT(t-s) f(s, B_1 x(s))\, ds - \int_{-\infty}^t T(t-s) g(s, B_2 x(s))\, ds. \end{aligned}$$

In conclusion, $x(t) = T(t-a)[x(a)-f(a, B_1x(a))]+f(t, B_1x(t))+\int_a^t AT(t-s)f(s, B_1x(s))\,ds+\int_a^t T(t-s)g(s, B_2x(s))\,ds+\int_a^t T(t-s)h(s, B_3x(s))\,dW(s)$ is a mild solution of equation (1.1) and $x(\cdot) \in AA(\mathbb{R}; L^2(\mathbb{P}, \mathbb{H}_\alpha))$. The proof is completed.

Remark 3.1 *The results of Theorem 3.1 can be used to study the existence and uniqueness of square-mean almost automorphic mild solutions to the example in* [18].

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Authors' contributions
YKC carried out the main proof of this manuscript, ZHZ drafted the manuscript and corrected the main theorems, GMN gave two lemmas, corrected the main theorems and improved this manuscript.

Competing interests
The authors declare that they have no competing interests.

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