Kenary et al. Advances in Difference Equations 2012, 2012:125
http://www.advancesindifferenceequations.com/content/2012/1/125

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

Approximate *m*-Lie homomorphisms and approximate Jordan *m*-Lie homomorphisms associated to a parametric additive functional equation

Hassan Azadi Kenary[1], Hamid Rezaei[2], Madjid Eshaghi Gordji[3], Choonkil Park[4*] and Sang Og Kim[5]

* Correspondence: baak@hanyang.ac.kr
[4]Research Institute for Natural Sciences, Hanyang University, Seoul 133-791, Korea
Full list of author information is available at the end of the article

Abstract

Using fixed point method, we establish the Hyers-Ulam stability of *m*-Lie homomorphisms and Jordan *m*-Lie homomorphisms in *m*-Lie algebras associated to the following generalized Jensen functional equation

$$\sum_{i=1}^{m} \mu f(x_i) = \frac{1}{2m}\left[\sum_{i=1}^{m} f\left(\mu m x_i + \sum_{j=1,\, i\neq j}^{m} x_j\right) + f\left(\sum_{i=1}^{m} \mu x_i\right)\right]$$

for a fixed positive integer m with $m \geq 2$.

Mathematics Subject Classification (2010): Primary 17A42, 39B82, 39B52.

Keywords: *m*-Lie algebra, homomorphism, Jordan homomorphism, Hyers-Ulam stability, fixed point approach, Jensen-type functional equation

1. Introduction

Let n be a natural number greater or equal to 3. The notion of an n-Lie algebra was introduced by V.T. Filippov in 1985 [1]. The Lie product is taken between n elements of the algebra instead of two. This new bracket is n-linear, anti-symmetric and satisfies a generalization of the Jacobi identity. For $n = 3$, this product is a special case of the Nambu bracket, well-known in physics, which was introduced by Nambu [2] in 1973, as a generalization of the Poisson bracket in Hamiltonian mechanics.

An n-Lie algebra is a natural generalization of a Lie algebra. Namely:

A vector space V together with a multi-linear, antisymmetric n-ary operation $[\ ]: \Lambda^n V \to V$ is called an n-Lie algebra, $n \geq 3$, if the n-ary bracket is a derivation with respect to itself, i.e.,

$$[[x_1, \cdots, x_n], x_{n+1}, \cdots, x_{2n-1}] = \sum_{i=1}^{n} [x_1, \cdots, x_{i-1}[x_i, x_{n+1}, \cdots, x_{2n-1}], \cdots, x_n] \tag{1.1}$$

where $x_1, x_2, \cdots, x_{2n-1} \in V$. The equation (1.1) is called the generalized Jacobi identity. The meaning of this identity is similar to that of the usual Jacobi identity for a Lie algebra (which is a 2-Lie algebra).

In [1] and several subsequent papers [3-5], a structure theory of finite-dimensional n-Lie algebras over a field $\mathbb{F}$ of characteristic 0 was developed.

n-ary algebras have been considered in physics in the context of Nambu mechanics [2,6] and, recently (for n = 3), in the search for the effective action of coincident $M2$-branes in M-theory initiated by the Bagger-Lambert-Gustavsson (BLG) model [7,8] (further references on the physical applications of n-ary algebras are given in [9]).

From now on, we only consider n-Lie algebras over the field of complex numbers. An n-Lie algebra A is a normed n-Lie algebra if there exists a norm $||\ ||$ on A such that $||[x_1, x_2, \cdots, x_n]|| \leq ||x_1|| ||x_2|| \cdots ||x_n||$ for all $x_1, x_2, \cdots, x_n \in A$. A normed n-Lie algebra A is called a Banach n-Lie algebra if $(A, ||\ ||)$ is a Banach space.

Let $(A, [\ ]_A)$ and $(B, [\ ]_B)$ be two Banach n-Lie algebras. A $\mathbb{C}$ -linear mapping H: $(A, [\ ]_A) \to (B, [\ ]_B)$ is called an n-Lie homomorphism if

$$H([x_1 x_2 \cdots x_n]_A) = [H(x_1)H(x_2)\cdots H(x_n)]_B$$

for all $x_1, x_2, \cdots, x_n \in A$. A $\mathbb{C}$ -linear mapping H: $(A, [\ ]_A) \to (B, [\ ]_B)$ is called a Jordan n-Lie homomorphism if

$$H([xx\cdots x]_A) = [H(x)H(x)\cdots H(x)]_B$$

for all $x \in A$.

The study of stability problems had been formulated by Ulam [10] during a talk in 1940: Under what condition does there exist a homomorphism near an approximate homomorphism? In the following year, Hyers [11] was answered affirmatively the question of Ulam for Banach spaces, which states that if $\varepsilon > 0$ and f: $X \to Y$ is a mapping with X a normed space and Y a Banach spaces such that

$$\|f(x+y) - f(x) - f(y)\| \leq \varepsilon \tag{1.2}$$

for all $x, y \in X$, then there exists a unique additive map T: $X \to Y$ such that

$$\|f(x) - T(x)\| \leq \varepsilon$$

for all $x \in X$. A generalized version of the theorem of Hyers for approximately linear mappings was presented by Rassias [12] in 1978 by considering the case when inequality (1.2) is unbounded.

In 2003, Cădariu and Radu applied the fixed point method to the investigation of the Jensen functional equation [13] (see also [14-16]). They could present a short and a simple proof (different of the "*direct method* ", initiated by Hyers in 1941) for the Hyers-Ulam stability of Jensen functional equation [13] and for quadratic functional equation [14].

Park and Rassias [17] proved the stability of homomorphisms in C^*-algebras and Lie C^*-algebras and also of derivations on C^*-algebras and Lie C^*-algebras for the Jensen-type functional equation

$$\mu f\left(\frac{x+y}{2}\right) + \mu f\left(\frac{x-y}{2}\right) - f(\mu x) = 0$$

for all $\mu \in \mathbb{T}^1 := \{\lambda \in \mathbb{C} : |\lambda| = 1\}$.

In this paper, by using fixed point method, we establish the Hyers-Ulam stability of n-Lie homomorphisms and Jordan n-Lie homomorphisms in n-Lie Banach algebras associated to the following generalized Jensen-type functional equation

$$\sum_{i=1}^{m} \mu f(x_i) - \frac{1}{2m}\left[\sum_{i=1}^{m} f\left(\mu m x_i + \sum_{j=1,\ i\neq j}^{m} \mu x_j\right) + f\left(\sum_{i=1}^{m} \mu x_i\right)\right] = 0$$

for all

$$\mu \in \mathbb{T}^1_{\frac{1}{n_o}} := \left\{e^{i\theta} :\ 0 \le \theta \le \frac{2\pi}{n_o}\right\} \cup \{1\},$$

where $m \geq 2$.

Throughout this paper, assume that $(A, [\ ]_A)$ and $(B, [\ ]_B)$ are two m-Lie Banach algebras.

2. Main results

Before proceeding to the main results, we recall a fundamental result in fixed point theory.

Theorem 2.1. [18]*Let* (Ω, d) *be a complete generalized metric space and* $T: \Omega \to \Omega$ *be a strictly contractive function with Lipschitz constant L. Then for each given* $x \in \Omega$, *either*

$$d(T^m x,\ T^{m+1} x) = \infty \text{ for all } m \geq 0,$$

or other exists a natural number m_0 *such that*

- $d\ (T^{\ m}x,\ T^{\ m+1}x) < \infty$ *for all* $m \geq m_0$;
- *the sequence* $\{T^{\ m}x\}$ *is convergent to a fixed point* y^* *of T;*
- y^* *is the unique fixed point of T in* $\Lambda = \{y \in \Omega: d(T^{\ m0}\ x, y) < \infty\}$;
- $d(y,\ y^*) \leq \frac{1}{1-L} d(y,\ Ty)$ *for all* $y \in \Lambda$.

Theorem 2.2. *Let V and W be real vector spaces. A mapping f: V → W satisfies the following functional equation*

$$\sum_{i=1}^{m} f(x_i) = \frac{1}{2m}\left[\sum_{i=1}^{m} f\left(m x_i + \sum_{j=1,\ i\neq j}^{m} x_j\right) + f\left(\sum_{i=1}^{m} x_i\right)\right]$$

if and only f f is additive.

Proof. It is easy to prove the theorem. □

We start our work with the main theorem of the our paper.

Theorem 2.3. *Let* $n_0 \in \mathbb{N}$ *be a fixed positive integer. Let f: A → B be a mapping for which there exists a function* $\varphi: A^m \to [0, \infty)$ *such that*

$$\left\|\mu \sum_{i=1}^{m} \mu f(x_i) - \frac{1}{2m}\left[\sum_{i=1}^{m} f\left(\mu m x_i + \sum_{j=1,\ i\neq j}^{m} \mu x_j\right) + f\left(\sum_{i=1}^{m} \mu x_i\right)\right]\right\| \leq \varphi(x_1,\ x_2,\ \cdots,\ x_m), \tag{2.1}$$

$$\| f([x_1 x_2 \cdots x_n]_A) - [f(x_1) f(x_2) \cdots f(x_m)]_B \| \leq \varphi(x_1,\ x_2,\ \cdots,\ x_m) \tag{2.2}$$

for all $\mu \in \mathbb{T}^1_{\frac{1}{n_0}}$ *and all* $x_1, \cdots, x_m \in A$. *If there exists an* $L < 1$ *such that*

$$\varphi(x_1, x_2, \cdots, x_m) \leq mL\varphi\left(\frac{x_1}{m}, \frac{x_2}{m}, \cdots, \frac{x_m}{m}\right) \tag{2.3}$$

for all $x_1, \cdots, x_m \in A$, *then there exists a unique m-Lie homomorphism* $H: A \to B$ *such that*

$$||f(x) - H(x)|| \leq \frac{\varphi(x, 0, 0, \cdots, 0)}{m - mL} \tag{2.4}$$

for all $x \in A$.

Proof. Let Ω be the set of all functions from A into B and let

$$d(g, h) := \inf\{C \in \mathbb{R}^+ : ||g(x) - h(x)||_B \leq C\phi(x, 0, \cdots, 0), \forall x \in A\}.$$

It is easy to show that (Ω, d) is a generalized complete metric space [19].

Now we define the mapping $J: \Omega \to \Omega$ by

$$J(h)(x) = \frac{1}{m}h(mx)$$

for all $x \in A$.

Note that for all $g, h \in \Omega$,

$$\begin{aligned} d(g, h) < C &\Rightarrow \| g(x) - h(x) \| \leq C\phi(x, 0, \cdots, 0) \\ &\Rightarrow \left\| \frac{1}{m}g(mx) - \frac{1}{m}h(mx) \right\| \leq \frac{C\varphi(mx, 0, \cdots, 0)}{|m|^{\ell}} \\ &\Rightarrow \left\| \frac{1}{m}g(mx) - \frac{1}{m}h(mx) \right\| \leq LC\varphi(x, 0, \cdots, 0) \\ &\Rightarrow d(J(g), J(h)) \leq LC \end{aligned}$$

for all $x \in A$. Hence we see that

$$d(J(g), J(h)) \leq Ld(g, h)$$

for all $g, h \in \Omega$. It follows from (2.3) that

$$\lim_{k\to\infty} \frac{\varphi(m^k x_1, m^k x_2, \cdots, m^k x_m)}{m^k} \leq \lim_{k\to\infty} L^k \varphi(x_1, \cdots, x_m) = 0 \tag{2.5}$$

for all $x_1, \cdots, x_m \in A$. Putting $\mu = 1$, $x_1 = x$ and $x_j = 0$ $(j = 2, \cdots, n)$ in (2.1), we obtain

$$\left\| \frac{f(mx)}{m} - f(x) \right\| \leq \frac{\varphi(x, 0, \cdots, 0)}{m}$$

for all $x \in A$. Therefore,

$$d(f, J(f)) \leq \frac{1}{m} < \infty. \tag{2.6}$$

By Theorem 2.1, J has a unique fixed point in the set $X_1 := \{h \in \Omega : d(f, h) < \infty\}$. Let H be the fixed point of J. H is the unique mapping with

$$H(mx) = mH(x)$$

such that there exists $C \in (0, \infty)$ satisfying

$$\| f(x) - H(x) \| \le C\varphi(x, 0, \cdots, 0)$$

for all $x \in A$. On the other hand, we have $\lim_{k\to\infty} d(J^k(f), H) = 0$ and so

$$\lim_{k\to\infty} \frac{1}{m^k} f(m^k x) = H(x) \tag{2.7}$$

for all $x \in A$. By Theorem 2.1, we have

$$d(f, H) \le \frac{1}{1-L} d(f, J(f)). \tag{2.8}$$

It follows from (2.6) and (2.8) that

$$d(f, H) \le \frac{1}{m - mL}.$$

This implies the inequality (2.4). By (2.2), we have

$$\begin{aligned}
&\| H([x_1 x_2 \cdots x_m]_A) - [H(x_1)H(x_2)H(x_3)\cdots H(x_m)]_B \| \\
&= \lim_{k\to\infty} \left\| \frac{H([m^k x_1 m^k x_2 \cdots m^k x_m]_A)}{m^{mk}} - \frac{([H(m^k x_1)H(m^k x_2)H(m^k x_3)\cdots H(m^k x_m)]_B)}{m^{mk}} \right\| \\
&\le \lim_{m\to\infty} \frac{\varphi(m^k x_1, m^k x_2, \cdots, m^k x_m)}{m^{mk}} = 0
\end{aligned}$$

for all $x_1, \cdots, x_m \in A$. Hence

$$H([x_1 x_2 \cdots x_m]_A) = [H(x_1)H(x_2)H(x_3)\cdots H(x_m)]_B$$

for all $x_1, \cdots, x_m \in A$.

On the other hand, it follows from (2.1), (2.5) and (2.7) that

$$\begin{aligned}
&\left\| \sum_{i=1}^{m} H(x_i) - \frac{1}{2m}\left[\sum_{i=1}^{m} H\left(mx_i + \sum_{j=1,\, i\ne j}^{m} x_j \right) + H\left(\sum_{i=1}^{m} x_i \right) \right] \right\|_B \\
&= \lim_{k\to\infty} \frac{1}{m^k} \left\| \sum_{i=1}^{m} f(m^k x_i) - \frac{1}{2m}\left[\sum_{i=1}^{m} f\left(m^{k+1} x_i + \sum_{j=1,\, i\ne j}^{m} m^k x_j \right) + f\left(\sum_{i=1}^{m} m^k x_i \right) \right] \right\| \\
&\le \lim_{m\to\infty} \frac{\varphi(m^k x_1, m^k x_2, \cdots, m^k x_m)}{m^k} = 0
\end{aligned}$$

for all $x_1, \cdots, x_m \in A$. Then

$$\sum_{i=1}^{m} H(x_i) = \frac{1}{2m}\left[\sum_{i=1}^{m} H\left(mx_i + \sum_{j=1,\, i\ne j}^{m} x_j \right) + H\left(\sum_{i=1}^{m} x_i \right) \right]$$

for all $x_1, \cdots, x_m \in A$. So by Theorem 2.1, H is additive. Letting $x_i = x$ for all $i = 1, 2, \cdots, n$ in (2.1), we obtain

$$\| \mu f(x) - f(\mu x) \| \le \varphi(x, x, \cdots, x)$$

for all $x \in A$. It follows that

$$\begin{aligned} \| H(\mu x) - \mu H(x) \| &= \lim_{k\to\infty} \frac{\| f(\mu m^k x) - \mu f(m^k x) \|}{m^k} \\ &\le \lim_{k\to\infty} \frac{\varphi(m^k x, m^k x, \cdots, m^k x)}{m^k} = 0 \end{aligned}$$

for all $\mu \in \mathbb{T}^1_{\frac{1}{n_0}}$ and all $x \in A$. One can show that the mapping H: $A \to B$ is $\mathbb{C}$-linear.

Hence H: $A \to B$ is an m-Lie homomorphism satisfying (2.4), as desired. □

Corollary 2.4. *Let θ and p be nonnegative real numbers such that $p < 1$. Suppose that a mapping f: $A \to B$ satisfies*

$$\left\| \mu \sum_{i=1}^{m} \mu f(x_i) - \frac{1}{2m} \left[\sum_{i=1}^{m} f\left(\mu m x_i + \sum_{j=1, i \neq j}^{m} \mu x_j \right) + f\left(\sum_{i=1}^{m} \mu x_i \right) \right] \right\| \le \theta \sum_{i=1}^{m} (\| x_i \|^p), \tag{2.9}$$

$$\| f([x_1 x_2 \cdots x_n]_A) - [f(x_1) f(x_2) \cdots f(x_m)]_B \| \le \theta \sum_{i=1}^{m} (\| x_i \|^p) \tag{2.10}$$

for all $\mu \in \mathbb{T}^1_{\frac{1}{n_0}}$ and all $x_1, \cdots, x_m \in A$. Then there exists a unique m-Lie homomorphism H: $A \to B$ such that

$$\| f(x) - H(x) \| \le \frac{\theta \| x \|^p}{(m - m^p)} \tag{2.11}$$

for all $x \in A$.

Proof. Putting $\varphi(x_1, x_2, \cdots, x_m) := \theta \sum_{i=1}^{m} (||x_i||^p)$ for all $x_1, \cdots, x_n \in A$ and letting $L = m^{p-1}$ in Theorem 2.3, we obtain (2.11). □

Similarly, we have the following and we will omit the proof.

Theorem 2.5. *Let f: $A \to B$ be a mapping for which there exists a function ϕ: $A^m \to [0, \infty)$ satisfying (2.1) and (2.2). If there exists an $L < 1$ such that*

$$\varphi\left(\frac{x_1}{m}, \frac{x_2}{m}, \cdots, \frac{x_m}{m}\right) \le \frac{L}{m} \varphi(x_1, x_2, \cdots, x_m)$$

for all $x_1, \cdots, x_m \in A$, then there exists a unique m-Lie homomorphism H: $A \to B$ such that

$$\| f(x) - H(x) \| \le \frac{L\varphi(x, 0, 0, \cdots, 0)}{m - mL}$$

for all $x \in A$.

Corollary 2.6. *Let θ and p be nonnegative real numbers such that $p > 1$. Suppose that a mapping f: $A \to B$ satisfies (2.9) and (2.10). Then there exists a unique m-Lie homomorphism H: $A \to B$ such that*

$$\| f(x) - H(x) \| \le \frac{m\theta \| x \|^p}{m^{p+1} - m^2} \tag{2.12}$$

for all $x \in A$.

Proof. Putting $\varphi(x_1, x_2, \cdots, x_m) := \theta \sum_{i=1}^{m} (||x_i||^p)$ for all $x_1, \cdots, x_n \in A$ and letting $L = m^{1-p}$ in Theorem 2.5, we obtain (2.12). □

Theorem 2.7. *Let* $n_0 \in \mathbb{N}$ *be a fixed positive integer. Let* $f: A \to B$ *be a mapping for which there exists a function* $\phi: A^n \to [0, \infty)$ *such that*

$$\left\| \mu \sum_{i=1}^{m} \mu f(x_i) - \frac{1}{2m} \left[\sum_{i=1}^{m} f\left(\mu m x_i + \sum_{j=1, i \neq j}^{m} \mu x_j \right) + f\left(\sum_{i=1}^{m} \mu x_i \right) \right] \right\| \le \varphi(x_1, x_2, \cdots, x_m), \quad (2.13)$$

$$\| f([xx\cdots x]_A) - [f(x)f(x)\cdots f(x)]_B \| \le \varphi(x, x, \cdots, x) \quad (2.14)$$

for all $\mu \in \mathbb{T}^1_{\frac{1}{n_0}}$ *and all* $x_1, \cdots, x_m \in A$. *If there exists an* $L < 1$ *such that*

$$\varphi(x_1, x_2, \cdots, x_m) \le mL\varphi\left(\frac{x_1}{m}, \frac{x_2}{m}, \cdots, \frac{x_m}{m}\right)$$

for all $x_1, \cdots, x_m \in A$, *then there exists a unique Jordan m-Lie homomorphism* $H: A \to B$ *such that*

$$\| f(x) - H(x) \| \le \frac{\varphi(x, 0, \cdots, 0)}{m - mL} \quad (2.15)$$

for all $x \in A$.

Proof. By the same reasoning as in the proof of Theorem 2.3, we can define the mapping

$$H(x) = \lim_{k\to\infty} \frac{1}{m^k} f(m^k x)$$

for all $x \in A$. Moreover, we can show that H is $\mathbb{C}$ -linear. By (2.14), we get that

$$\begin{aligned} &\| H([xx\cdots x]_A) - [H(x)H(x)\cdots H(x)]_B \| \\ &= \lim_{k\to\infty} \left\| \frac{1}{m^{mk}} H([m^k x \cdots m^k x]_A) - \frac{1}{m^{mk}} ([H(m^k x)H(m^k x)\cdots H(m^k x)]_B \right\| \\ &\le \lim_{k\to\infty} \frac{1}{m^{mk}} \varphi(m^k x, m^k x, \ldots, m^k x) = 0 \end{aligned}$$

for all $x \in A$. So

$$H([xx\cdots x]_A) = [H(x)H(x)\cdots H(x)]_B$$

for all $x \in A$. Hence H: $A \to B$ is a Jordan m-Lie homomorphism satisfying (2.15). □

Corollary 2.8. *Let* θ *and* p *be nonnegative real numbers such that* $p < 1$. *Suppose that a mapping* $f: A \to B$ *satisfies*

$$\left\| \mu \sum_{i=1}^{m} \mu f(x_i) - \frac{1}{2m} \left[\sum_{i=1}^{m} f\left(\mu m x_i + \sum_{j=1, i \neq j}^{m} \mu x_j \right) + f\left(\sum_{i=1}^{m} \mu x_i \right) \right] \right\| \le \theta \sum_{i=1}^{n} (\| x_i \|^p), \quad (2.16)$$

$$\| f([xx\cdots x]_A) - [f(x)f(x)\cdots f(x)]_B \| \le n\theta(\| x \|^p) \quad (2.17)$$

for all $\mu \in \mathbb{T}^1_{\frac{1}{n_0}}$ *and all* $x_1, \cdots, x_m \in A$. *Then there exists a unique Jordan m-Lie homomorphism H: A* $\to$ *B such that*

$$\| f(x) - H(x) \| \leq \frac{\theta \| x\|^p}{m - m^p}$$

for all $x \in A$.

Proof. The proof follows from Theorem 2.7 by putting $\varphi(x_1, x_2, \cdots, x_m) := \theta \sum_{i=1}^{m} (||x_i||^p)$ for all $x_1, \cdots, x_m \in A$ and letting $L = m^{p-1}$. □

Similarly, we have the following and we will omit the proof.

Theorem 2.9. *Let f: A* $\to$ *B be a mapping for which there exists a function* $\phi: A^m \to [0, \infty)$ *satisfying (2.13) and (2.14). If there exists an* $L < 1$ *such that*

$$\varphi\left(\frac{x_1}{m}, \frac{x_2}{m}, \cdots, \frac{x_m}{m}\right) \leq \frac{L}{m}\varphi(x_1, x_2, \cdots, x_m)$$

for all $x_1, \cdots, x_m \in A$, *then there exists a unique Jordan m-Lie homomorphism H: A* $\to$ *B such that*

$$\| f(x) - H(x) \| \leq \frac{L\varphi(x, 0, 0, \cdots, 0)}{m - mL}$$

for all $x \in A$.

Corollary 2.10. *Let* θ *and p be nonnegative real numbers such that* $p > 1$. *Suppose that a mapping f: A* $\to$ *B satisfies (2.16) and (2.17). Then there exists a unique Jordan m-Lie homomorphism H: A* $\to$ *B such that*

$$\| f(x) - H(x)\|_B \leq \frac{\theta \| x\|^p}{m^p - m} \tag{2.18}$$

for all $x \in A$.

Proof. Putting $\varphi(x_1, x_2, \cdots, x_m) := \theta \sum_{i=1}^{m} (||x_i||^p)$ for all $x_1, \cdots, x_n \in A$ and letting $L = m^{1-p}$ in Theorem 2.9, we obtain (2.18).

Author details
[1]Department of Mathematics, College of Sciences, Yasouj University, Yasouj 75914-353, Iran [2]Department of Mathematics, College of Sciences, Yasouj University, Yasouj 75914-353, Iran [3]Department of Mathematics, Semnan University, P. O. Box 35195-363, Semnan, Iran [4]Research Institute for Natural Sciences, Hanyang University, Seoul 133-791, Korea [5]Department of Mathematics, Hallym University, Chunchen 200-702, Korea

Authors' contributions
All authors conceived of the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

Received: 2 March 2012 Accepted: 24 July 2012 Published: 24 July 2012

References

1. Filippov, VT: *n*-Lie algebras. Sibirsk Mat Zh. **26**, 126–140 (1985)
2. Nambu, Y: Generalized Hamiltonian dynamics. Phys Rev D. **7**, 2405–2412 (1973). doi:10.1103/PhysRevD.7.2405
3. Filippov, VT: On *n*-Lie algebras of Jacobians. Sibirsk Mat Zh. **39**, 660–669 (1998)
4. Kasymov, SM: On the theory of *n*-Lie algebras. Algebra i Logika. **26**, 277–297 (1987)
5. Kasymov, SM: On nil-elements and nil-subsets of *n*-Lie algebras. Sibirsk Mat Zh. **32**, 77–80 (1991)

6. Takhtajan, L: On foundation of the generalized Nambu mechanics. Comm Math Phys. **160**, 295–316 (1994). doi:10.1007/BF02103278
7. Bagger, J, Lambert, N: Comments nn multiple M2-branes. J High Energy Phys **2008**, 15 (2008). Art. ID 105
8. Gustavsson, A: One-loop corrections to Bagger-Lambert theory. Nuclear Phys B. **807**, 315–333 (2009). doi:10.1016/j.nuclphysb.2008.09.003
9. Azcarraga, JA, Izquierdo, JM: *n*-ary algebras: a review with applications. J Phys A **43**, 1–117 (2010). Art. ID 293001
10. Ulam, SM: A Collection of Mathematical Problems. Interscience Tracts in Pure and Applied Mathematics. Interscience Publ. New York-London (1940)
11. Hyers, DH: On the stability of the linear functional equation. Proc Natl Acad Sci. **27**, 222–224 (1941). doi:10.1073/pnas.27.4.222
12. Rassias, TM: On the stability of the linear mapping in Banach spaces. Proc Amer Math Soc. **72**, 297–300 (1978). doi:10.1090/S0002-9939-1978-0507327-1
13. Cădariu, L, Radu, V: Fixed points and the stability of Jensen's functional equation. J Inequal Pure Appl Math **4** (2003). No. 1, Art. 4
14. Cădariu, L, Radu, V: Fixed points and the stability of quadratic functional equations. An Univ Timişoara Ser Mat Inform. **41**, 25–48 (2003)
15. Eshaghi Gordji, M, Ghaemi, MB, Kaboli Gharetapeh, S, Shams, S, Ebadian, A: On the stability of J^*-derivations. J Geom Phys. **60**, 454–459 (2010). doi:10.1016/j.geomphys.2009.11.004
16. Eshaghi Gordji, M, Najati, A: Approximately J^*-homomorphisms: a fixed point approach. J Geom Phys. **60**, 809–814 (2010). doi:10.1016/j.geomphys.2010.01.012
17. Park, C, Rassias, JM: Stability of the Jensen-type functional equation in C^*-algebras: a fixed point approach. Abstr Appl Anal **2009**, 17 (2009). Art. ID 360432
18. Diaz, JB, Margolis, B: A fixed point theorem of the alternative for contractions on a generalized complete metric space. Bull Amer Math Soc. **74**, 305–309 (1968). doi:10.1090/S0002-9904-1968-11933-0
19. Cădariu, L, Radu, V: On the stability of the Cauchy functional equation: a fixed point approach. Grazer Mathematische Berichte. **346**, 43–52 (2004)

doi:10.1186/1687-1847-2012-125
Cite this article as: Kenary *et al.*: **Approximate *m*-Lie homomorphisms and approximate Jordan *m*-Lie homomorphisms associated to a parametric additive functional equation.** *Advances in Difference Equations* 2012 **2012**:125.