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Permanence and existence of positive periodic solution for a multi-species cooperation system with continuous time delays, feedback control, and periodic external source

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Abstract

In this paper, sufficient conditions which guarantee the existence of positive periodic solutions for a multi-species cooperation system are obtained. The permanence of solutions is studied. The proof is based on Schauder's fixed-point theorem. Also, we give a illustrative example in order to indicate the validity of the assumptions.
MSC: 54H99

Keywords: Schauder's fixed-point theorem; periodic solution; multi-species system; feedback control

1 Introduction

May suggested the following system equations [1] as the mathematical modeling of the pair of mutualist:

$$\dot{x} = \rho_1 x \left[1 - \frac{x}{a_1 + b_1 y} - c_1 x \right],$$

$$\dot{y} = \rho_2 y \left[1 - \frac{y}{a_2 + b_2 x} - c_2 y \right],$$

where $x(t)$, $y(t)$ are densities of the species X, Y at time t, ρ_i, a_i, b_i, c_i, $i = 1, 2$ are positive constants. Subsequently, the nonautonomous version was argued by Cui and Chen [2]. Cui in [3] proposed the following generalization for the N-species cooperation system with continuous time delays:

$$\frac{dx_i}{dt} = \rho_i(t)x_i(t) \left[1 - \frac{x_i(t)}{a_i(t) + \sum_{j=1, j \neq i}^n b_j(t) \int_{-\tau}^0 V_j(\theta)x_j(t+\theta)\,d\theta} - c_i(t)x_i(t) \right],$$

$$x_i(t) = \varphi_i(t), \text{ for } \tau \leq t < 0, \tag{1.1}$$

where $\int_{-\tau}^0 V_j(\theta)\,d\theta = 1, j = 1, 2, \ldots, N$.

On the other hand, in the more realistic situation, the cooperation systems or ecosystems are continuously perturbed *via* unpredictable forces. These perturbations are gen-

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erally results of the change in the system's parameters. In the language of the control theory, these perturbation functions may be regarded as control variables, and consequently, one should ask the question that whether or not an ecosystem can withstand those unpredictable perturbations which persist for a finite periodic time. During the last decade, many scholars did works on the feedback control ecosystems. Some results can be found in [3–7] and the references therein. Chen, Lio, and Huang [6] studied the dynamical behavior of the following non-autonomous N-species cooperation system with continuous time delay and feedback control:

$$\frac{dx_i}{dt} = \rho_i(t)x_i(t)\left[1 - \frac{x_i(t)}{a_i(t) + \sum_{j=1,j\neq i}^{n} b_{ij}(t)\int_{-\tau_{ij}}^{0} V_{ij}(\theta)x_j(t+\theta)\,d\theta} - c_i(t)x_i(t)\right]$$

$$- d_i(t)u_i(t)x_i(t) - e_i(t)x_i(t)\int_{-\tau_i}^{0} W_i(\theta)u_i(t+\theta)\,d\theta, \tag{1.2}$$

$$\frac{du_i}{dt} = -\alpha_i(t)u_i(t) + \beta_i(t)x_i(t) + \gamma_i(t)\int_{-\eta_i}^{0} U_i(\theta)x_i(t+\theta)\,d\theta, \tag{1.3}$$

where x_i and u_i are the density of ith cooperation species and control variable, respectively. a_i, b_{ij}, c_i, d_i, ρ_i, α_i, β_i, and γ_i, are all continuous real-valued functions which are bounded above and below by positive constants. Also,

$$\int_{-\tau_{ij}}^{0} V_i(\theta)\,d\theta = \int_{-\tau_i}^{0} W_i(\theta)\,d\theta = \int_{-\eta_i}^{0} U_i(\theta)\,d\theta = 1, \quad i,j = 1,2,\ldots,N.$$

Very recently, Chen and Xie [7] obtained a set of sufficient conditions for permanence of the system above. The study was based on a new integral inequality and the results showed that the feedback control variables have no influence on the permanence of the system. In the present paper, we study the sufficient condition for existence and permanence of the positive periodic solutions of generalized version of the N-species cooperation system (1.2), (1.3) while for each i, time variation of the ith species is affected by external periodic source $S_i(t)$, *i.e.*,

$$\frac{dx_i}{dt} = \rho_i(t)x_i(t)\left[1 - \frac{x_i(t)}{a_i(t) + \sum_{j=1,j\neq i}^{n} b_{ij}(t)\int_{-\tau_{ij}}^{0} V_{ij}(\theta)x_j(t+\theta)\,d\theta} - c_i(t)x_i(t)\right]$$

$$- d_i(t)u_i(t)x_i(t) - e_i(t)x_i(t)\int_{-\tau_i}^{0} W_i(\theta)u_i(t+\theta)\,d\theta - S_i(t), \tag{1.4}$$

$$\frac{du_i}{dt} = -\alpha_i(t)u_i(t) + \beta_i(t)x_i(t) + \gamma_i(t)\int_{-\eta_i}^{0} U_i(\theta)x_i(t+\theta)\,d\theta. \tag{1.5}$$

Our key tool is the following fixed-point theorem for a proper compact integral operator on the convex subset of infinite dimension Banach space which is originally due to Schauder [8].

Theorem 1.1 (Schauder) *Let X be a Banach space and Λ be a closed, bounded, and convex subset of X. If $\Gamma : \Lambda \mapsto \Lambda$ is a compact operator, then Γ has at least one fixed point on Λ.*

Besides, we also invoke the following weak version of Arzela-Ascoli theorem [9].

Theorem 1.2 (Arzela-Ascoli) *Let $\{\xi_n(t)\}$ be a sequence of real functions on $[0, T]$ which is uniformly bounded and equicontinuous. Then $\{\xi_n(t)\}$ has a uniformly convergent subsequence.*

Also, we set,

$$\mathcal{C}_T = \{\xi \,|\, \xi \text{ is a continuous } T\text{-periodic function on } R\},$$

and for $\xi \in \mathcal{C}_T$ we define $\|\xi\| = \sup_{t \in [0,T]} |\xi(t)|$,

$$\mathcal{B} = \mathcal{C}_T^N = \left\{ \Psi = (\xi_1, \xi_2, \ldots, \xi_N) : \xi_i \in \mathcal{C}_T, i = 1, 2, \ldots, N \right\},$$

and for $\Psi \in \mathcal{B}$ we define $|\Psi(t)| = \sum_{i=1}^N |\xi_i(t)|$ and $\|\Psi\|_{\mathcal{B}} = \sum_{i=1}^N \|\xi_i\|$.

Clearly, $(\mathcal{C}_T, \|\cdot\|)$ and $(\mathcal{B}, \|\cdot\|_{\mathcal{B}})$ are Banach spaces.

$$\lambda_i = \exp\left(\int_0^T \rho_i(t)\, dt\right), \qquad \kappa_i = \exp\left(\int_0^T \alpha_i(t)\, dt\right),$$

$$A_i = \int_0^T \left\{ \beta_i(t) + \gamma_i(t) \right\} dt, \qquad B_i = \int_0^T \left\{ d_i(t) + e_i(t) \right\} dt,$$

$$J_i(Q) = \int_0^T \rho_i(s) \left[\frac{1}{a_i(s) + Q \sum_{j=1, j \neq i}^n b_{ij}(s)} + c_i(s) \right] ds, \quad Q \in \mathbb{R},$$

$$I_i(Q) = \frac{1}{\inf_{t \in [0,T]} [a_i(t) + Q \sum_{j=1, j \neq i}^n b_{ij}(t)]} + \|c_i\|.$$

Throughout this paper, we assume that

(H_1) $a_i, b_{ij}, c_i, d_i, e_i, \rho_i, \alpha_i, \beta_i,$ and $\gamma_i, i, j = 1, 2, \ldots, N$ are all positive continuous real-valued functions.

(H_2) $\int_{-\tau_{ij}}^0 V_i(\theta)\, d\theta = \int_{-\tau_i}^0 W_i(\theta)\, d\theta = \int_{-\eta_i}^0 U_i(\theta)\, d\theta = 1, i, j = 1, 2, \ldots, N.$

The following section is arranged based on two main steps: In step 1, we obtain sufficient conditions with guarantee the existence of periodic solutions of each equation of system (1.4)-(1.5) for each $1 \leq i \leq N$. In the proof of existence, we will use the method of Green's functions according to Mokhtarzadeh *et al.* [10]. In step 2, we follow Chen [11] to construct a bounded, closed, and convex set in a product space and apply Schauder's fixed-point theorem.

2 Main results

In this section, we shall study the existence of periodic solutions of the multi-species system (1.4)-(1.5). To do this, we transform this system of couple equations into one integral equation. For each $1 \leq i \leq N$, we introduce the following integral operator Γ_i on the Banach space $(\mathcal{C}_T, \|\cdot\|)$,

$$\Gamma_i : \mathcal{C}_T \to \mathcal{C}_T,$$

$$(\Gamma_i \xi)(t) = \int_0^T G_i(t, s) \left\{ \beta_i(s)\xi(s) + \gamma_i(s) \int_{-\tau_i}^0 U_i(\theta)\xi(s + \theta)\, d\theta \right\} ds, \tag{2.1}$$

The kernel of the integral operator (2.1) is in fact the Green's function of Eq. (1.2) and is given by

$$
G_i(t,s) = \begin{cases}
\dfrac{\exp(\int_0^T \alpha_i(\theta)\,d\theta)}{\exp(\int_0^T \alpha_i(\theta)\,d\theta)-1} \exp(\int_t^s \alpha_i(\theta)\,d\theta), & 0 \leq s \leq t \leq T, \\[3mm]
\dfrac{1}{\exp(\int_0^T \alpha_i(\theta)\,d\theta)-1} \exp(\int_t^s \alpha_i(\theta)\,d\theta), & 0 \leq t \leq s \leq T,
\end{cases}
$$

where $\int_0^T \alpha_i \neq 0$; see [12].

Lemma 2.1 *Let $1 \leq i \leq N$ and α_i, β_i, γ_i, and U_i are belong to $\mathcal{C}_T$ as well as $\int_0^T \alpha_i \neq 0$. Suppose that u_i is a continuous real function such that for some $x_i \in \mathcal{C}_T$, $\Gamma_i x_i = u_i$. Then u_i is a T-periodic solution of Eq. (1.5).*

The proof of Lemma 2.1 is similar to the proof of Lemma 3.1 of [10].

According to Lemma 2.1, it may be deduced that the existence problem of T-periodic solution of system (1.4), (1.5) is equivalent to that of the T-periodic solution of the following equation:

$$
\frac{dx_i}{dt} = \rho_i(t)x_i(t)\left[1 - \frac{x_i(t)}{a_i(t) + \sum_{j=1,j\neq i}^n b_{ij}(t)\int_{-\tau_{ij}}^0 V_{ij}(\theta)x_j(t+\theta)\,d\theta} - c_i(t)x_i(t)\right]
$$

$$
- d_i(t)(\Gamma_i x_i)(t)x_i(t) - e_i(t)x_i(t)\int_{-\tau_i}^0 W_i(\theta)(\Gamma_i x_i)(t+\theta)\,d\theta - S_i(t). \tag{2.2}
$$

For any $\Psi = (\xi_1, \xi_2, \ldots, \xi_N)$ belong to $\mathcal{B}$, and $1 \leq i \leq N$, we define the following integral operator:

$$
\mathcal{F}_i^\Psi : \mathcal{C}_T \to \mathcal{C}_T,
$$

$$
\left(\mathcal{F}_i^\Psi x_i\right)(t) = \left(\Xi_i^\Psi x_i\right)(t) + \left(\Xi_i^\Gamma x_i\right)(t) + \Xi_i^S(t). \tag{2.3}
$$

Wherein,

$$
\left(\Xi_i^\Psi x_i\right)(t) = \int_0^T H_i(t,s)\rho_i(s)x_i^2(s)\left[\frac{1}{a_i(s) + \sum_{j=1,j\neq i}^n b_{ij}(s)\int_{-\tau_{ij}}^0 V_{ij}(\theta)\xi_j(s+\theta)\,d\theta} + c_i(s)\right]ds,
$$

$$
\left(\Xi_i^\Gamma x_i\right)(t) = \int_0^T H_i(t,s)x_i(s)\left\{d_i(s)(\Gamma_i x_i)(s) + e_i(s)\int_{-\tau_i}^0 W_i(\theta)(\Gamma_i x_i)(s+\theta)\,d\theta\right\}ds,
$$

and,

$$
\Xi_i^S(t) = \int_0^T H_i(t,s)S_i(s)\,ds.
$$

Kernel is given by

$$
H_i(t,s) = \begin{cases}
\dfrac{1}{\exp(\int_0^T \rho_i(\theta)\,d\theta)-1} \exp(-\int_t^s \rho_i(\theta)\,d\theta), & 0 \leq s \leq t \leq T, \\[3mm]
\dfrac{\int_0^T \rho_i(\theta)\,d\theta}{\exp(\int_0^T \rho_i(\theta)\,d\theta)-1} \exp(-\int_t^s \rho_i(\theta)\,d\theta), & 0 \leq t \leq s \leq T.
\end{cases}
$$

Lemma 2.2 *Let (H_1) and (H_2) hold and a_i, b_{ij}, c_i, d_i, ρ_i, α_i, β_i, γ_i, $i, j = 1, 2, \ldots, N$ as well as x are all belong to $\mathcal{C}_T$ as well as $\int_0^T \rho \neq 0$. Then $\mathcal{F}_i^\Psi x$ is T-periodic function and satisfies the following differential equation:*

$$
\left(\mathcal{F}_i^\Psi x \right)'(t) = \rho_i(t) \left[\left(\mathcal{F}_i^\Psi x \right)(t) - \frac{x^2(t)}{a_i(t) + \sum_{j=1, j\neq i}^n b_{ij}(t) \int_{-\tau_{ij}}^0 V_{ij}(\theta) \xi_j(t + \theta)\, d\theta} - x^2(t) c_i(t) \right]
$$
$$
- x(t) \left\{ d_i(t)(\Gamma_i x)(s) + e_i(t) \int_{-\tau_i}^0 W_i(\theta)(\Gamma_i x)(t + \theta)\, d\theta \right\}
$$
$$
- S_i(t). \tag{2.4}
$$

Proof Appealing to presses of the proof of Lemma 2.1 for operators $(\Xi_i^\Psi x_i)(t)$, $(\Xi_i^\Gamma x_i)(t)$ and $\Xi_i^S(t)$ one obtains

$$
\left(\Xi_i^\Psi x \right)'(t) = \rho_i(t) \left(\Xi_i^\Psi x \right)(t)
$$
$$
- \rho_i(t) x^2(t) \left[\frac{1}{a_i(t) + \sum_{j=1, j\neq i}^n b_{ij}(t) \int_{-\tau_{ij}}^0 V_{ij}(\theta) \xi_j(t + \theta)\, d\theta} + c_i(t) \right],
$$
$$
\left(\Xi_i^\Gamma x_i \right)'(t) = \rho_i(t)(\Xi_i x)(t) - x(t) \left\{ d_i(t)(\Gamma_i x)(t) + e_i(t) \int_{-\tau_i}^0 W_i(\theta)(\Gamma_i x)(t + \theta)\, d\theta \right\},
$$

and,

$$
\left(\Xi_i^S \right)'(t) = \rho_i(t) \left(\Xi_i^S \right)(t) - S_i(s).
$$

The sum of terms above and taking the equality (2.3) into account leads to (2.4). □

Corollary 2.3 *Let a_i, b_{ij}, c_i, d_i, ρ_i, α_i, β_i, γ_i, $i, j = 1, 2, \ldots, n$ and x all belong to $\mathcal{C}_T$ as well as $\int_0^T \rho \neq 0$. Let that $x_i \in \mathcal{C}_T$ be a fixed point of the operator $\mathcal{F}_i^\Psi$, i.e., $\mathcal{F}_i^\Psi x_i = x_i$, then x_i is a solution of Eq. (2.2).*

Theorem 2.4 *Let*

$$
\Lambda_i = \left\{ \xi \in \mathcal{C}_T : \| \xi - \Xi_i^S \| \leq Q \| \Xi_i^S \| \right\},
$$
$$
Q_i = \frac{\inf_{t \in [0, T]} | \Xi_i^S(t) |}{1 + \sup_{t \in [0, T]} | \Xi_i^S(t) |}, \qquad Q = \min\{ Q_1, Q_2, \ldots, Q_N \}.
$$

Assume (H_1) and (H_2) hold, assume further that (H_3) for $i = 1, 2, \ldots, N$

$$
J_i(Q) + A_i B_i \leq \left(1 - \frac{1}{\lambda_i} \right) \left(1 - \frac{1}{\kappa_i} \right) \frac{Q}{(Q + 1)^2 \| \Xi_i^S \|}.
$$

Then the integral operator $\mathcal{F}_i^\Psi$ maps Λ_i into Λ_i and has at least one fixed point.

Proof Let $\xi_i \in \Lambda_i$, then

$$
\left| \xi_i(t) - \Xi_i^S(t) \right| \leq Q \| \Xi_i^S \|,
$$

thus

$$\left| \xi_i(t) \right| \leq (Q+1) \left\| \Xi_i^S \right\|.$$

On the other hand,

$$
\begin{aligned}
\left| \xi_i(t) \right| &\geq \left| \Xi_i^S(t) \right| - Q \left\| \Xi_i^S \right\| \geq \inf_{t \in [0,T]} \left| \Xi_i^S(t) \right| - Q \sup_{t \in [0,T]} \left| \Xi_i^S(t) \right| \\
&\geq \inf_{t \in [0,T]} \left| \Xi_i^S(t) \right| - Q_i \sup_{t \in [0,T]} \left| \Xi_i^S(t) \right| \\
&= \inf_{t \in [0,T]} \left| \Xi_i^S(t) \right| - \frac{\inf_{t \in [0,T]} |\Xi_i^S(t)| \sup_{t \in [0,T]} |\Xi_i^S(t)|}{1 + \sup_{t \in [0,T]} |\Xi_i^S(t)|} \\
&= Q_i \geq Q.
\end{aligned}
$$

Consequently,

$$Q \leq \xi_i(t) \leq (Q+1) \left\| \Xi_i^S \right\|, \quad \text{for any } t \in [0,T]. \tag{2.5}$$

In addition, since ρ_i and α_i $(i = 1, 2, \ldots, N)$ are positive functions, we have

$$\frac{1}{\lambda_i - 1} \leq H_i(t,s) \leq \frac{\lambda_i}{\lambda_i - 1}, \quad s, t \in [0,T],$$

and

$$\frac{1}{\kappa_i - 1} \leq G_i(t,s) \leq \frac{\kappa_i}{\kappa_i - 1}, \quad s, t \in [0,T].$$

Therefore, for $x_i \in \Lambda_i$, $\Psi = (\xi_1, \xi_2, \ldots, \xi_N)$ belong to $\bigotimes_{i=1}^{N} \Lambda_i$ and any $t \in [0, T]$, we obtain

$$
\begin{aligned}
\left| (\Xi_i^\Psi x_i)(t) \right| &\leq \frac{\lambda_i}{\lambda_i - 1} \|x_i\|^2 \\
&\quad \times \int_0^T \rho_i(s) \left[\frac{1}{a_i(s) + \sum_{j=1, j \neq i}^n b_{ij}(s) \int_{-\tau_{ij}}^0 V_{ij}(\theta) \xi_j(s+\theta) \, d\theta} + c_i(s) \right] ds \\
&\leq \frac{\lambda_i}{\lambda_i - 1} (Q+1)^2 \left\| \Xi_i^S \right\|^2 \int_0^T \rho_i(s) \left[\frac{1}{a_i(s) + Q \sum_{j=1, j \neq i}^n b_{ij}(s)} + c_i(s) \right] ds \\
&\leq \frac{\lambda_i}{\lambda_i - 1} (Q+1)^2 \left\| \Xi_i^S \right\|^2 J_i(Q). \tag{2.6}
\end{aligned}
$$

Also, we have

$$
\begin{aligned}
\left| (\Gamma_i \xi)(t) \right| &\leq \frac{\kappa_i}{\kappa_i - 1} \|x_i\| \int_0^T \left\{ \beta_i(s) + \gamma_i(s) \int_{-\tau_i}^0 U_i(\theta) \, d\theta \right\} ds \\
&\leq \frac{\kappa_i}{\kappa_i - 1} \|x_i\| A_i. \tag{2.7}
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\left|\left(\Xi_i^\Gamma x_i\right)(t)\right| &\le \frac{\lambda_i}{\lambda_i - 1}\|x_i\| \int_0^T \left\{ d_i(s)(\Gamma_i x_i)(s) + e_i(s)\int_{-\tau_i}^0 W_i(\theta)(\Gamma_i x_i)(s+\theta)\,d\theta \right\} ds \\
&\le \frac{\lambda_i}{\lambda_i-1}\frac{\kappa_i}{\kappa_i-1}\|x_i\|^2 A_i \int_0^T \left\{ d_i(s) + e_i(s)\int_{-\tau_i}^0 W_i(\theta)\,d\theta \right\} ds \\
&\le \frac{\lambda_i \kappa_i}{(\lambda_i-1)(\kappa_i-1)}(Q+1)^2 \left\|\Xi_i^S\right\|^2 A_i B_i.
\end{aligned}
\tag{2.8}
$$

In these regards, based on (2.6) and (2.8) one obtains

$$
\begin{aligned}
\left|\left(\mathcal{F}_i^\Psi x_i\right)(t) - \Xi_i^S(t)\right| &= \left|\left(\Xi_i^\Psi x_i\right)(t) - \left(\Xi_i^\Gamma x_i\right)(t)\right| \\
&\le \left|\left(\Xi_i^\Psi x_i\right)(t)\right| + \left|\left(\Xi_i^\Gamma x_i\right)(t)\right| \\
&\le \frac{\lambda_i}{\lambda_i-1}(Q+1)^2\left\|\Xi_i^S\right\|^2\left(J_i(Q) + \frac{\kappa_i}{\kappa_i-1}A_i B_i \right) \\
&\le \frac{\lambda_i \kappa_i}{(\lambda_i-1)(\kappa_i-1)}(Q+1)^2\left\|\Xi_i^S\right\|^2\left(J_i(Q) + A_i B_i \right) \\
&\le Q\left\|\Xi_i^S\right\|.
\end{aligned}
\tag{2.9}
$$

This shows that $\mathcal{F}_i^\Psi x_i$ is belong to Λ_i ($i = 1, 2, \dots, N$) and, therefore, the integral operator $\mathcal{F}_i^\Psi$ maps Λ_i into Λ_i.

In addition, based on inequality (2.9), we have

$$
\left|\left(\mathcal{F}_i^\Psi x_i\right)(t)\right| \le (Q+1)\left\|\Xi_i^S\right\|.
$$

Thus,

$$
\begin{aligned}
\left|\left(\mathcal{F}_i^\Psi x\right)'(t)\right| &\le \|\rho_i\|(Q+1)\left\|\Xi_i^S\right\| + (Q+1)^2\left\|\Xi_i^S\right\|^2 \\
&\quad \times \left\{ I_i(Q) + \frac{\kappa_i}{\kappa_i-1}A_i\big\{\|d_i\| + \|e_i\|\big\} \right\} + \|S_i\|,
\end{aligned}
\tag{2.10}
$$

which shows that $|(\mathcal{F}_i^\Psi x)'(t)|$ is bounded by

$$
\begin{aligned}
M_i &= \|\rho_i\|(Q+1)\left\|\Xi_i^S\right\| \\
&\quad + (Q+1)^2\left\|\Xi_i^S\right\|^2\left\{ I_i(Q) + \frac{\kappa_i}{\kappa_i-1}A_i\big\{\|d_i\| + \|e_i\|\big\} \right\} + \|S_i\|.
\end{aligned}
$$

Let $M = \max\{M_1, M_2, \dots, M_N\}$. For any $t, s \in [0, T]$, one obtains

$$
\left|\left(\mathcal{F}_i^\Psi x\right)(t) - \left(\mathcal{F}_i^\Psi x\right)(s)\right| \le M|t - s|.
$$

This implies that $(\mathcal{F}_i^\Psi x)(t)$ is Lipschitzan with Lipschit constance M. In this way, for given $\varepsilon > 0$, if we consider $\delta(\varepsilon) = \frac{\varepsilon}{M}$, then

$$
\sup_{x \in \Lambda_i}\left|\left(\mathcal{F}_i^\Psi x\right)(t) - \left(\mathcal{F}_i^\Psi x\right)(s)\right| \le \varepsilon, \quad \text{whenever } t, s \in [0, T] \text{ and } |t - s| \le \delta(\varepsilon).
$$

Consequently, for any $1 \leq i \leq N$ the family $\{\mathcal{F}_i^\Psi x\}_{x \in \Lambda_i}$ is equicontinuous on $[0, T]$.

Suppose $\{x_{i,m}\}$ is a sequence on Λ_i, $1 \leq i \leq N$. Thus, $\{(\mathcal{F}_i^\Psi x_{i,m})(t)\}$ as a sequence of functions on $[0, T]$ is equicontinuous. Appealing to the Arzela-Ascoli theorem, there exist a subsequent denoted by $\{(\mathcal{F}_i^\Psi x_{i,m_k})(t)\}$, which is uniformly convergence on $[0, T]$. This means that $\{\mathcal{F}_i^\Psi x_{i,m_k}\}$ is convergent on Λ_i and consequently, $\mathcal{F}_i^\Psi : \Lambda_i \to \Lambda_i$ is compact. Appealing to Schauder's fixed-point theorem, $\mathcal{F}_i^\Psi$ has at least a fixed point on Λ_i.
$\qquad \square$

We emphasize that according to Corollary 2.3, for any $1 \leq i \leq N$, and $\Psi = (\xi_1, \xi_2, \ldots, x_i, \ldots, \xi_N)$ belong to $\bigotimes_{i=1}^N \Lambda_i$ fixed point of $\mathcal{F}_i^\Psi$ is the positive T-periodic solutions of Eq. (2.2) or equivalently, T-periodic solution of the nonlinear population system (1.3) and (1.4).

Let $\Psi = (\xi_1, \xi_2, \ldots, \xi_N) \in \bigotimes_{i=1}^N \Lambda_i$ and (H_1), (H_2), and (H_3) hold. Based on Theorem 2.4 for $i = 1, 2, \ldots, N$, operator $\mathcal{F}_i^\Psi$ has at least a fixed point in Λ_i. Let $\mathrm{Fix}(\mathcal{F}_i^\Psi) \subseteq \Lambda_i$ denote the set of fixed points of $\mathcal{F}_i^\Psi$. Applying the Axiom of Choice, we chose a representative point, say $x_{i\Psi}$, in $\mathrm{Fix}(\mathcal{F}_i^\Psi)$ *i.e.*,

$$\left(\mathcal{F}_i^\Psi x_{i\Psi}\right)(t) = x_{i\Psi}(t), \quad t \in [0, T], i = 1, 2, \ldots, N.$$

We introduce the following operator on $\bigotimes_{i=1}^N \Lambda_i$:

$$\mathfrak{T} : \bigotimes_{i=1}^N \Lambda_i \to \bigotimes_{i=1}^N \Lambda_i,$$

$$\Psi(t) = \left(\xi_1(t), \xi_2(t), \ldots, \xi_N(t)\right) \to \left(x_{1\Psi}(t), x_{2\Psi}(t), \ldots, x_{N\Psi}(t)\right). \tag{2.11}$$

Theorem 2.5 *Let (H_1), (H_2), and (H_3) hold. Then operator $\mathfrak{T} : \bigotimes_{i=1}^N \Lambda_i \to \bigotimes_{i=1}^N \Lambda_i$ defined by (2.11) has at least a fixed point on $\bigotimes_{i=1}^N \Lambda_i$.*

Proof It is obvious that $\mathfrak{T}$ is bounded on B. Also, according to inequality (2.10), we have

$$\left|x'_{i\Psi}(t)\right| \leq M, \quad t \in [0, T], i = 1, 2, \ldots, N.$$

Thus,

$$\left|x_{i\Psi}(t) - x_{i\Psi}(s)\right| \leq M|t - s|, \quad t, s \in [0, T], i = 1, 2, \ldots, N.$$

For given $\varepsilon > 0$, if we consider $\delta(\varepsilon) = \frac{\varepsilon}{MN}$, then

$$\sup_{\Psi \in \bigotimes_{i=1}^N \Lambda_i} \left|(\mathfrak{T}\Psi)(t) - (\mathfrak{T}\Psi)(s)\right| \leq \varepsilon, \quad \text{whenever } t, s \in [0, T] \text{ and } |t - s| \leq \delta(\varepsilon),$$

which shows that $\mathfrak{T}(\bigotimes_{i=1}^N \Lambda_i)$ is equicontinuous on [0,T]. In this regard, any sequence, say $\{X_k\}$ in $\bigotimes_{i=1}^N \Lambda_i$ satisfies all the conditions of the Arzela-Ascoli theorem on [0,T]. Hence, $\{X_k\}$ has a subsequence $\{X_{k_m}\}$ such that is uniformly convergent on [0,T]. This shows that $\mathfrak{T}(\bigotimes_{i=1}^N \Lambda_i)$ is relatively compact in B.

In the sequel, we show that $\mathfrak{T}$ is continuous. We define the following map on $\bigotimes_{i=1}^{N} \Lambda_i \times [0, T]$,

$$Z_i(\Psi, t) = \rho_i(t) \left[1 - \frac{\xi_i(t)}{a_i(t) + \sum_{j=1, j \neq i}^{n} b_{ij}(t) \int_{-\tau_{ij}}^{0} V_{ij}(\theta) \xi_j(t + \theta) \, d\theta} - c_i(t) \xi_i(t) \right]$$

$$- d_i(t)(\Gamma_i \xi_i)(t) - e_i(t) \int_{-\tau_i}^{0} W_i(\theta)(\Gamma_i \xi_i)(t + \theta) \, d\theta. \qquad (2.12)$$

Wherein, $\Psi = (\xi_1, \xi_2, \ldots, \xi_N)$. For each $1 \leq i, j \leq N$, the partial derivative $\partial Z_i(\Psi, t)/\partial \xi_j$ exist and straightforward calculation shows

$$\left| \frac{\partial Z_i(\Psi, t)}{\partial \xi_i} \right| \leq |\rho_i(t)| \left\{ \left| \frac{1}{a_i(t) + \sum_{j=1, j \neq i}^{n} b_{ij}(t) \int_{-\tau_{ij}}^{0} V_{ij}(\theta) \xi_j(t + \theta) \, d\theta} \right| + |c_i(t)| \right\}$$

$$+ \left| d_i(t) \int_{0}^{T} G_i(t, s) \{ \beta_i(s) + \gamma_i(s) \} \, ds \right|$$

$$+ \left| e_i(t) \int_{-\tau_i}^{0} W_i(\theta) \left\{ \int_{0}^{T} G_i(t, s) \{ \beta_i(s) + \gamma_i(s) \} \, ds \right\} d\theta \right|$$

$$\leq \|\rho_i\| I_i(Q) + \frac{\kappa_i}{\kappa_i - 1} A_i \{ \|d_i\| + \|e_i\| \}. \qquad (2.13)$$

Also, in the case $k \neq i$,

$$\left| \frac{\partial Z_i(\Psi, t)}{\partial \xi_k} \right| \leq \frac{\rho_i(t) \xi_k(t) b_{ik}(t)}{(a_i(t) + \sum_{j=1, j \neq i}^{n} b_{ij}(t) \int_{-\tau_{ij}}^{0} V_{ij}(\theta) \xi_j(t + \theta) \, d\theta)^2}$$

$$\leq (Q + 1) \| \rho_i b_{ik} \Xi_i^S \| (I_i(Q) - \|c_i\|)^2. \qquad (2.14)$$

We set $\sigma = \max\{g_i, h_{ij} : i, j = 1, 2, \ldots, N\}$, wherein,

$$g_i = \|\rho_i\| I_i(Q) + \frac{\kappa_i}{\kappa_i - 1} A_i \{ \|d_i\| + \|e_i\| \},$$

and,

$$h_{ij} = (Q + 1) \| \rho_i b_{ik} \Xi_i^S \| (I_i(Q) - \|c_i\|)^2.$$

Therefore,

$$\left| \frac{\partial Z_i(\Psi, t)}{\partial \xi_j} \right| \leq \sigma, \quad \text{whenever } t \in [0, T] \text{ and } i, j = 1, 2, \ldots, N. \qquad (2.15)$$

Based on property (2.15) and by induction, one can obtain

$$\left| Z_i(\xi_1, \xi_2, \ldots, \xi_N, t) - Z_i(\phi_1, \phi_2, \ldots, \phi_N, t) \right| \leq P(\sigma) \sum_{i=1}^{N} |\xi_i(t) - \phi_i(t)|, \qquad (2.16)$$

where $P(\sigma)$ is a specific polynomial in terms of σ.

Let $\{\Psi_k\}$ be a sequence belong to $\bigotimes_{i=1}^{N} \Lambda_i$ and $\Psi_k = (\xi_{1k}, \xi_{2k}, \ldots, \xi_{Nk}) \rightarrow \Phi = (\phi_1, \phi_2, \ldots, \phi_N)$ as $k \mapsto \infty$. Let $\mathfrak{T} \Psi_k = X_{\Psi_k} = (x_{1\Psi_k}, x_{2\Psi_k}, \ldots, x_{N\Psi_k}) \in \mathfrak{T}(\bigotimes_{i=1}^{N} \Lambda_i)$. According to rel-

ative compactness of $\bigotimes_{i=1}^{N} \Lambda_i$ in B, there exist a subsequence $\{X_{\Psi_q}\}$ of $\{X_{\Psi_k}\}$ and $Y = (y_1, y_2, \ldots, y_N) \in B$ such that $X_{\Psi_q} \mapsto Y$, uniformly, in B as $q \mapsto \infty$. It is obvious that for any q, we have

$$\frac{dx_{i\Psi_q}}{dt} = x_{i\Psi_q}(t)\rho_i(t)\left[1 - \frac{x_{i\Psi_q}(t)}{a_i(t) + \sum_{j=1,j\neq i}^{n} b_{ij}(t) \int_{-\tau_{ij}}^{0} V_{ij}(\theta)\xi_{jq}(t+\theta)\,d\theta} - c_i(t)x_{i\Psi_q}(t)\right]$$

$$- d_i(t)(\Gamma_i x_{i\Psi_q})(t)x_{i\Psi_q}(t) - e_i(t)x_{i\Psi_q}(t)\int_{-\tau_i}^{0} W_i(\theta)(\Gamma_i x_{i\Psi_q})(t+\theta)\,d\theta - S_i(t)$$

$$= x_{i\Psi_q}(t)Z_i\big(\xi_{1q}(t), \ldots, \xi_{i-1q}(t), x_{i\Psi_q}(t), \xi_{i+1q}(t), \ldots, \xi_{Nq}(t), t\big). \tag{2.17}$$

Or equivalently,

$$\big(\mathcal{F}_i^{\Psi_q} x_{i\Psi_q}\big)(t) = x_{i\Psi_q}(t), \quad t \in [0, T], i = 1, 2, \ldots, N.$$

Based on property (2.16), we have

$$\big|Z_i\big(\xi_{1q}(t), \xi_{2q}(t), \ldots, \xi_{i-1q}(t), x_{i\Psi_q}(t), \xi_{i+1q}(t), \ldots, \xi_{Nq}(t), t\big)$$

$$- Z_i\big(\phi_1(t), \phi_2(t), \ldots, \phi_{i-1}(t), y_i(t), \phi_{i+1}(t), \ldots, \phi_N(t), t\big)\big|$$

$$\leq P(\sigma)\left\{\big|x_{i\Psi_q}(t) - y_i(t)\big| + \sum_{j=1,j\neq i}^{N}\big|\xi_{jq}(t) - \phi_j(t)\big|\right\}. \tag{2.18}$$

Therefore,

$$x_{i\Psi_q}(t)Z_i\big(\xi_{1q}(t), \xi_{2q}(t), \ldots, \xi_{i-1q}(t), x_{i\Psi_q}(t), \xi_{i+1q}(t), \ldots, \xi_{Nq}(t), t\big)$$

converges to

$$y_i(t)Z_i\big(\phi_1(t), \phi_2(t), \ldots, \phi_{i-1}(t), y_i(t), \phi_{i+1}(t), \ldots, \phi_N(t), t\big)$$

uniformly, as $q \mapsto \infty$ for all $t \in [0, T]$ and $i = 1, 2, \ldots, N$.

Thus, derivative dy_i/dt exist on $[0, T]$ and

$$\frac{dx_{i\Psi_q}}{dt} \mapsto \frac{dy_i}{dt}, \quad \text{as } q \mapsto \infty.$$

Or equivalently,

$$dy_i/dt = y_i(t)Z_i\big(\phi_1(t), \phi_2(t), \ldots, \phi_{i-1}(t), y_i(t), \phi_{i+1}(t), \ldots, \phi_N(t), t\big)$$

for all $t \in [0, T]$ and $i = 1, 2, \ldots, N$.

Consequently, based on definition of map $\mathfrak{T}$ we have $\mathfrak{T}\Phi = Y$. Also, $\Psi_q \mapsto \Phi$. Thus, we obtain

$$\lim_{q \mapsto \infty} \mathfrak{T}\Psi_q = Y = \mathfrak{T}\Phi = \mathfrak{T}\lim_{q \mapsto \infty}\Phi_q.$$

Hence, $\lim_{k \mapsto \infty} \mathfrak{T}\Psi_k = Y$, which it shows that $\mathfrak{T}$ is continuous on $\bigotimes_{i=1}^{N} \Lambda_i$.

Therefore, applying Schauder's fixed-point theorem, map $\mathfrak{T}$ has a fixed point $\Phi = (\phi_1(t), \ldots, \phi_N(t)) \in \bigotimes_{i=1}^{N} \Lambda_i$, i.e.,

$$\mathfrak{T}\Phi = \Phi.$$

Or equivalently,

$$\left(\mathcal{F}_i^\Phi \phi_i\right)(t) = \phi_i(t), \quad t \in [0, T], i = 1, 2, \ldots, N.$$

Finally, this indicate that $(\phi_1(t), \ldots, \phi_N(t))$ is a positive periodic solution of system of Eq. (2.2) or equivalently, T-periodic solution of the multi-species cooperation system (1.3) and (1.4). This completes the proof of the theorem. $\qquad\square$

2.1 Permanence of system

Let $\tau = \{\tau_{ij}, \tau_i, \eta_i, i, j = 1, 2, \ldots, N\}$. We consider the system (1.4), (1.5) together with the following initial conditions:

$$x_i(\theta) = \varphi_i(\theta) \geq 0, \quad \theta \in [-\tau, 0], \varphi(0) > 0,$$

$$u_i(\theta) = \psi_i(\theta) \geq 0, \quad \theta \in [-\tau, 0], \psi(0) > 0. \tag{2.19}$$

Lemma 2.6 (see [6]) *If $a > 0$, $b > 0$ and $\dot{u} \geq b - au$, when $t \geq t_0$ and $u(t_0) > 0$, we have*

$$u(t) \geq \frac{b}{a} \left[1 + \left(\frac{au(t_0)}{b} - 1 \right) \exp\left(-a(t - t_0)\right) \right]^{-1}.$$

Theorem 2.7 *Let (H_1), (H_2), (H_3) hold and $(x_1, x_2, \ldots, x_N) \in \bigotimes_{i=1}^{N} \Lambda_i$ be any solution differential equation (2.2) and $(u_1, u_2, \ldots, u_N)$ with*

$$u_i(t) = \int_0^T G_i(t, s) \left\{ \beta_i(s)\xi(s) + \gamma_i(s) \int_{-\tau_i}^{0} U_i(\theta)\xi(s + \theta) \, d\theta \right\} ds, \tag{2.20}$$

be the solution of system (1.5) with initial condition (2.19). Then solution $(x_1, x_2, \ldots, x_N, u_1, u_2, \ldots, u_N)$ of system (1.4), (1.5) is permanent, i.e., there exist $\widetilde{\tau} > 0$ and $m_i > 0$, M_i, $\widetilde{m}_i > 0$, and $\widetilde{M}_i$ ($i = 1, 2, \ldots, N$) such that

$$m_i \leq x_i(t) \leq M_i, \qquad \widetilde{m}_i \leq u_i(t) \leq \widetilde{M}_i.$$

Proof Since $S_i \geq 0$ for any $0 \leq i \leq N$, appealing to the proof of the Lemma 2.3 given in [6] we can immediately demonstrate that there exist a $\widetilde{\tau} > 0$ such that

$$x_i \leq M_i^\varepsilon, \qquad u_i \leq N_i^\varepsilon, \quad \text{for } t \geq \widetilde{\tau}, \tag{2.21}$$

where

$$M_i^\varepsilon > M_i^*, \qquad N_i^\varepsilon > N_i^*,$$

$$M_i^* = \frac{\rho_i^u}{\rho_i^l c_i^l}, \qquad N_i^* = \frac{(\beta_i^u + \gamma_i^u) M_i^*}{\alpha_i^l} \tag{2.22}$$

while $f^u = \sup_t f(t)$ and $f^l = \inf_t f(t)$.

On the other hand, from Lemma 2.1 and inequality (2.5), we have

$$
\begin{aligned}
\frac{du_i}{dt} &= -\alpha_i(t)u_i(t) + \beta_i(t)x_i(t) + \gamma_i(t)\int_{-\eta_i}^{0} U_i(\theta)x_i(t+\theta)\,d\theta \\
&\leq -\alpha_i^u + \left(\beta_i^l + \gamma_i^l\right)Q,
\end{aligned}
$$

then by Lemma 2.6, for arbitrary $\epsilon > 0$, there exist $\tau' > \widetilde{\tau}$ such that

$$
u_i(t) \geq \frac{(\beta_i^l + \gamma_i^l)Q}{\alpha_i^u} - \epsilon \quad \text{for } \tau' > \widetilde{\tau}.
$$

Since ϵ is arbitrary small, one may assume that

$$
\epsilon \leq \frac{(\beta_i^l + \gamma_i^l)Q}{\alpha_i^u}.
$$

Thus,

$$
u_i(t) \geq \frac{1}{2}\frac{(\beta_i^l + \gamma_i^l)Q}{\alpha_i^u} \equiv \widetilde{m}_i \quad \text{for } \tau' > \widetilde{\tau}. \qquad \square
$$

Remark 2.8

For the system without the external source, the sets Λ_i must be replaced by

$$
B_R = \left\{\xi \in \mathcal{C}_T : \|\xi\| \leq R\right\}.
$$

Similar calculation shows that condition (H_3) is reduced to the following one:

$$
\left(\frac{\lambda_i}{\lambda_i - 1}\right)\left[J_i(0) + \left(\frac{\kappa_i}{\kappa_i - 1}\right)A_iB_i\right] \leq \frac{1}{R}.
$$

3 Illustrative example

Consider the following 2-species cooperation system with delay, feedback control, and external source

$$
\begin{aligned}
\frac{dN_1}{dt} &= \rho_1(t)N_1(t)\left[1 - \frac{N_1(t)}{a_1(t) + b_{12}(t)\int_{-\tau_{12}}^{0} V_{12}(\theta)N_2(t+\theta)\,d\theta} - c_1(t)N_1(t)\right] \\
&\quad - d_1(t)u_1(t)N_1(t) - e_1(t)N_1(t)\int_{-\tau_1}^{0} W_1(\theta)u_1(t+\theta)\,d\theta - S_1(t),
\end{aligned}
\tag{3.1}
$$

$$
\frac{du_1}{dt} = -\alpha_1(t)u_1(t) + \beta_1(t)N_1(t) + \gamma_1(t)\int_{-\tau_1}^{0} U_1(\theta)N_1(t+\theta)\,d\theta,
\tag{3.2}
$$

$$
\begin{aligned}
\frac{dN_2}{dt} &= \rho_2(t)N_2(t)\left[1 - \frac{N_2(t)}{a_2(t) + b_{21}(t)\int_{-\tau_{21}}^{0} V_{21}(\theta)N_1(t+\theta)\,d\theta} - c_2(t)N_2(t)\right] \\
&\quad - d_2(t)u_2(t)N_2(t) - e_2(t)N_2(t)\int_{-\tau_2}^{0} W_2(\theta)u_2(t+\theta)\,d\theta - S_2(t),
\end{aligned}
\tag{3.3}
$$

$$
\frac{du_2}{dt} = -\alpha_2(t)u_2(t) + \beta_2(t)N_2(t) + \gamma_2(t)\int_{-\tau_2}^{0} U_2(\theta)N_2(t+\theta)\,d\theta.
\tag{3.4}
$$

Wherein,

$$\rho_1(t) = \frac{5 - \sin(2\pi t) - \cos(4\pi t)}{5 - \sin(2\pi t) + \cos(2\pi t)}, \qquad \rho_2(t) = \frac{7 - \sin(2\pi t) - \cos(4\pi t)}{6 - 3\sin(2\pi t) + \cos(2\pi t)},$$

$$S_1(t) = \frac{2 - \sin(2\pi t)}{2 + \sin(2\pi t)}, \qquad S_2(t) = \frac{2 - \sin(2\pi t)}{2 + \cos(2\pi t)},$$

$$H_1(t,s) = \begin{cases} 0.5270 \exp(\int_s^t \rho_1(\theta)\,d\theta), & 0 \le s \le t \le T, \\ 1.5270 \exp(\int_s^t \rho_1(\theta)\,d\theta), & 0 \le t \le s \le T, \end{cases}$$

$$H_2(t,s) = \begin{cases} 0.3110 \exp(\int_s^t \rho_2(\theta)\,d\theta), & 0 \le s \le t \le T, \\ 1.3110 \exp(\int_s^t \rho_2(\theta)\,d\theta), & 0 \le t \le s \le T, \end{cases}$$

$$\alpha_1(t) = \frac{15 + \sin(2\pi t) + \cos(2\pi t)}{10 - \sin(2\pi t) - \cos(2\pi t)}, \qquad \alpha_2(t) = \frac{12 + \sin(2\pi t) - \cos(2\pi t)}{5 - \sin(2\pi t) - \cos(2\pi t)},$$

$$G_1(t,s) = \begin{cases} 1.2813 \exp(\int_t^s \alpha_1(\theta)\,d\theta), & 0 \le s \le t \le T, \\ 0.2813 \exp(\int_t^s \alpha_1(\theta)\,d\theta), & 0 \le t \le s \le T, \end{cases}$$

$$G_2(t,s) = \begin{cases} 1.0893 \exp(\int_t^s \alpha_2(\theta)\,d\theta), & 0 \le s \le t \le T, \\ 0.0893 \exp(\int_t^s \alpha_2(\theta)\,d\theta), & 0 \le t \le s \le T, \end{cases}$$

$$\sup_{t \in [0,T]} \left| \Xi_1^S(t) \right| = 1.9994, \qquad \sup_{t \in [0,T]} \left| \Xi_2^S(t) \right| = 1.5138,$$

$$\inf_{t \in [0,T]} \left| \Xi_1^S(t) \right| = 0.69, \qquad \inf_{t \in [0,T]} \left| \Xi_2^S(t) \right| = 0.3591,$$

$$\lambda_1 = 2.8975, \quad \lambda_2 = 4.2146, \quad \kappa_1 = 4.5540, \quad \kappa_2 = 12.2089,$$

$$Q_1 = 0.23, \qquad Q_2 = 0.1428, \qquad Q = 0.23,$$

$$a_1(t) = 100 + \sin(2\pi t), \qquad a_2(t) = 220 - \cos(2\pi t),$$

$$c_1(t) = \frac{2 + \sin(2\pi t)}{200 + \cos(2\pi t)}, \qquad c_2(t) = \frac{2 - \cos(2\pi t)}{200 - \sin(2\pi t)},$$

$$b_{12}(t) = 30 + \sin(2\pi t), \qquad b_{21}(t) = 100 - \cos(2\pi t),$$

$$J_1(Q) = 0.02167, \qquad J_2(Q) = 0.02171,$$

$$\beta_1(t) = \frac{3 - \sin(2\pi t) - \cos(4\pi t)}{10 + \sin(4\pi t) + \cos(4\pi t)}, \qquad \beta_2(t) = \frac{3 - \cos(4\pi t)}{32 + 2\sin(2\pi t) + \cos(2\pi t)},$$

$$\gamma_1(t) = \frac{7 - \sin(2\pi t) + \cos(4\pi t)}{30 + 2\sin(4\pi t) + \cos(4\pi t)}, \qquad \gamma_2(t) = \frac{2 + \sin(4\pi t)}{29 - 2\sin(2\pi t) + 3\cos(2\pi t)},$$

$$d_1(t) = \frac{2 - \sin(4\pi t)}{21 + \sin(4\pi t) + \cos(8\pi t)}, \qquad d_2(t) = \frac{3 + \sin(4\pi t)}{40 + \sin(2\pi t) - 2\cos(2\pi t)},$$

$$e_1(t) = \frac{4 + \sin(2\pi t)}{20 - \sin(2\pi t) + \cos(4\pi t)}, \qquad e_2(t) = \frac{2 - \cos(4\pi t)}{72 + \sin(2\pi t) + 4\cos(2\pi t)},$$

$$A_1 = 0.5411, \qquad B_1 = 0.2970, \qquad A_2 = 0.1633, \qquad B_1 = 0.1029.$$

With due attention to the data above, we have

$$J_1(Q) + A_1 B_1 = 0.1823 \le \left(1 - \frac{1}{\lambda_1}\right)\left(1 - \frac{1}{\kappa_1}\right) \frac{Q}{(Q+1)^2 \|\Xi_1^S\|} = 0.5123,$$

and

$$J_2(Q) + A_2B_2 = 0.0385 \leq \left(1 - \frac{1}{\lambda_2}\right)\left(1 - \frac{1}{\kappa_2}\right)\frac{Q}{(Q+1)^2 \| \Xi_2^S \|} = 0.0531.$$

Therefore, the condition (H_3) in Theorem 2.4 is valid for our examples.

4 Conclusion

In this paper, we investigate the existence and permanence of periodic solutions for a multi-species cooperation system with continuous time delays, feedback control and periodic external source. Suppose that

(H_1) a_i, b_{ij}, c_i, d_i, e_i, ρ_i, α_i, β_i, and γ_i, are all positive continuous real-valued functions,

(H_2) $\int_{-\tau_{ij}}^0 V_i(\theta)\,d\theta = \int_{-\tau_i}^0 W_i(\theta)\,d\theta = \int_{-\eta_i}^0 U_i(\theta)\,d\theta = 1$,

(H_3) $J_i(Q) + A_iB_i \leq (1 - \frac{1}{\lambda_i})(1 - \frac{1}{\kappa_i})\frac{Q}{(Q+1)^2 \| \Xi_i^S \|}$

for $i = 1, 2, \ldots, N$, where

$$\lambda_i = \exp\left(\int_0^T \rho_i(t)\,dt\right), \qquad \kappa_i = \exp\left(\int_0^T \alpha_i(t)\,dt\right),$$

$$A_i = \int_0^T \left\{\beta_i(t) + \gamma_i(t)\right\}dt, \qquad B_i = \int_0^T \left\{d_i(t) + e_i(t)\right\}dt,$$

$$J_i(Q) = \int_0^T \rho_i(s)\left[\frac{1}{a_i(s) + Q\sum_{j=1,j\neq i}^n b_{ij}(s)} + c_i(s)\right]ds, \quad Q \in \mathbb{R},$$

$$I_i(Q) = \frac{1}{\inf_{t\in[0,T]}[a_i(t) + Q\sum_{j=1,j\neq i}^n b_{ij}(t)]} + \|c_i\|,$$

$$Q_i = \frac{\inf_{t\in[0,T]} |\Xi_i^S(t)|}{1 + \sup_{t\in[0,T]} |\Xi_i^S(t)|}, \qquad Q = \min\{Q_1, Q_2, \ldots, Q_N\},$$

$$\Xi_i^S(t) = \int_0^T H_i(t,s)S_i(s)\,ds.$$

Then the following multi-species cooperation system:

$$\frac{dx_i}{dt} = \rho_i(t)x_i(t)\left[1 - \frac{x_i(t)}{a_i(t) + \sum_{j=1,j\neq i}^n b_{ij}(t)\int_{-\tau_{ij}}^0 V_{ij}(\theta)x_j(t+\theta)\,d\theta} - c_i(t)x_i(t)\right]$$

$$- d_i(t)u_i(t)x_i(t) - e_i(t)x_i(t)\int_{-\tau_i}^0 W_i(\theta)u_i(t+\theta)\,d\theta - S_i(t),$$

$$\frac{du_i}{dt} = -\alpha_i(t)u_i(t) + \beta_i(t)x_i(t) + \gamma_i(t)\int_{-\eta_i}^0 U_i(\theta)x_i(t+\theta)\,d\theta$$

admits at least one T-periodic solution. In order to indicate the validity of the assumptions made in our results, we also treat a illustrative example.

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