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Some identities on Bernoulli and Euler polynomials arising from the orthogonality of Laguerre polynomials

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Abstract
In this paper, we derive some interesting identities on Bernoulli and Euler polynomials by using the orthogonal property of Laguerre polynomials.

1 Introduction

The generalized Laguerre polynomials are defined by

$$\frac{\exp\left(-\frac{xt}{1-t}\right)}{(1-t)^{\alpha+1}} = \sum_{n=0}^{\infty} L_n^{\alpha}(x)t^n \quad (\alpha \in \mathbb{Q} \text{ with } \alpha > -1). \tag{1.1}$$

From (1.1), we note that

$$L_n^{\alpha}(x) = \sum_{r=0}^{n} \frac{(-1)^r \binom{n+\alpha}{n-r} x^r}{r!} \quad \left(\text{see } [1\text{–}3]\right). \tag{1.2}$$

By (1.2), we see that $L_n^{\alpha}(x)$ is a polynomial with degree n. It is well known that Rodrigues' formula for $L_n^{\alpha}(x)$ is given by

$$L_n^{\alpha}(x) = \frac{x^{-\alpha}e^x}{n!}\left(\frac{d^n}{dx^n}\left(e^{-x}x^{n+\alpha}\right)\right) \quad \left(\text{see } [1\text{–}3]\right). \tag{1.3}$$

From (1.3) and a part of integration, we note that

$$\int_0^{\infty} x^{\alpha} e^{-x} L_m^{\alpha}(x) L_n^{\alpha}(x)\, dx = \frac{\Gamma(\alpha+n+1)}{n!}\delta_{m,n}, \tag{1.4}$$

where $\delta_{m,n}$ is a Kronecker symbol. As is well known, Bernoulli polynomials are defined by the generating function to be

$$\frac{t}{e^t-1}e^{xt} = e^{B(x)t} = \sum_{n=0}^{\infty} B_n(x)\frac{t^n}{n!} \quad \left(\text{see } [1\text{–}29]\right), \tag{1.5}$$

with the usual convention about replacing $B^n(x)$ by $B_n(x)$.

In the special case, $x = 0$, $B_n(0) = B_n$ are called the nth Bernoulli numbers. By (1.5), we get

$$B_n(x) = \sum_{l=0}^{n} \binom{n}{l} B_{n-l} x^l \quad \bigl(\text{see [1–29]}\bigr). \tag{1.6}$$

The Euler numbers are defined by

$$E_0 = 1, \qquad (E+1)^n + E_n = 2\delta_{0,n} \quad \bigl(\text{see [27, 28]}\bigr), \tag{1.7}$$

with the usual convention about replacing E^n by E_n.

In the viewpoint of (1.6), the Euler polynomials are also defined by

$$E_n(x) = (E+x)^n = \sum_{l=0}^{n} \binom{n}{l} E_{n-l} x^l \quad \bigl(\text{see [11–24]}\bigr). \tag{1.8}$$

From (1.7) and (1.8), we note that the generating function of the Euler polynomial is given by

$$\frac{2}{e^t+1} e^{xt} = e^{E(x)t} = \sum_{n=0}^{\infty} E_n(x) \frac{t^n}{n!} \quad \bigl(\text{see [15–29]}\bigr). \tag{1.9}$$

By (1.5) and (1.9), we get

$$\frac{2}{e^t+1} e^{xt} = \frac{1}{t}\left(2 - 2\frac{2}{e^t+1}\right)\left(\frac{te^{xt}}{e^t-1}\right) = -2\sum_{n=0}^{\infty}\left(\sum_{l=0}^{n} \frac{E_{l+1}}{l+1}\binom{n}{l} B_{n-l}(x)\right)\frac{t^n}{n!}. \tag{1.10}$$

Thus, by (1.10), we see that

$$E_n(x) = -2\sum_{l=0}^{n} \binom{n}{l} \frac{E_{l+1}}{l+1} B_{n-l}(x). \tag{1.11}$$

By (1.7) and (1.8), we easily get

$$\frac{t}{e^t-1} e^{xt} = \frac{t}{2}\left(\frac{2e^{xt}}{e^t+1}\right) + \left(\frac{t}{e^t-1}\right)\left(\frac{2e^x t}{e^t+1}\right). \tag{1.12}$$

Thus, by (1.12), we see that

$$B_n(x) = \sum_{k=0, k\neq 1}^{n} \binom{n}{k} B_k E_{n-k}(x). \tag{1.13}$$

Throughout this paper, we assume that $\alpha \in \mathbb{Q}$ with $\alpha > -1$. Let $\mathbb{P}_n = \{p(x) \in \mathbb{Q}[x] \mid \deg p(x) \leq n\}$ be the inner product space with the inner product

$$\langle p(x), q(x) \rangle = \int_0^{\infty} x^{\alpha} e^{-x} p(x) q(x)\, dx,$$

where $p(x), q(x) \in \mathbb{P}_n$. From (1.4), we note that $\{L_0^\alpha(x), L_1^\alpha(x), \ldots, L_n^\alpha(x)\}$ is an orthogonal basis for $\mathbb{P}_n$.

In this paper, we give some interesting identities on Bernoulli and Euler polynomials which can be derived by an orthogonal basis $\{L_0^\alpha(x), L_1^\alpha(x), \ldots, L_n^\alpha(x)\}$ for $\mathbb{P}_n$.

2 Some identities on Bernoulli and Euler polynomials

Let $p(x) \in \mathbb{P}_n$. Then $p(x)$ can be generated by $\{L_0^\alpha(x), L_1^\alpha(x), \ldots, L_n^\alpha(x)\}$ in $\mathbb{P}_n$ to be

$$p(x) = \sum_{k=0}^{n} C_k L_k^\alpha(x), \tag{2.1}$$

where

$$\begin{aligned}\langle p(x), L_k^\alpha(x)\rangle &= C_k \langle L_k^\alpha(x), L_k^\alpha(x)\rangle \\ &= C_k \int_0^\infty x^\alpha e^{-x} L_k^\alpha(x) L_k^\alpha(x)\, dx \\ &= C_k \frac{\Gamma(\alpha+k+1)}{k!}.\end{aligned} \tag{2.2}$$

From (2.2), we note that

$$\begin{aligned}C_k &= \frac{k!}{\Gamma(\alpha+k+1)} \langle p(x), L_k^\alpha(x)\rangle \\ &= \frac{k!}{\Gamma(\alpha+k+1)} \frac{1}{k!} \int_0^\infty \left(\frac{d^k}{dx^k} x^{k+\alpha} e^{-x}\right) p(x)\, dx \\ &= \frac{1}{\Gamma(\alpha+k+1)} \int_0^\infty \left(\frac{d^k}{dx^k} x^{k+\alpha} e^{-x}\right) p(x)\, dx.\end{aligned} \tag{2.3}$$

Let us take $p(x) = \sum_{m=0,m\neq 1}^{n} \binom{n}{m} B_m E_n - m(x) \in \mathbb{P}_n$. Then, from (2.3), we have

$$\begin{aligned}C_k &= \frac{1}{\Gamma(\alpha+k+1)} \int_0^\infty \left(\frac{d^k}{dx^k} x^{k+\alpha} e^{-x}\right) \sum_{m=0,m\neq 1}^{n} \binom{n}{m} B_n E_{n-m}(x)\, dx \\ &= \frac{(-1)^k}{\Gamma(\alpha+k+1)} \sum_{m=0,m\neq 1}^{n-k} \sum_{l=k}^{n-m} \binom{n}{m}\binom{n-m}{l} B_m E_{n-m-l} \frac{l!}{(l-k)!} \int_0^\infty x^{l+\alpha} e^{-x}\, dx \\ &= \frac{(-1)^k}{\Gamma(\alpha+k+1)} \sum_{m=0,m\neq 1}^{n-k} \sum_{l=k}^{n-m} \binom{n}{m}\binom{n-m}{l} B_m E_{n-m-l} \frac{l!}{(l-k)!} \Gamma(l+\alpha+1) \\ &= (-1)^k \sum_{m=0,m\neq 1}^{n-k} \sum_{l=k}^{n-m} \binom{n}{m}\binom{n-m}{l} B_m E_{n-m-l} \frac{l!}{(l-k)!} \frac{(l+\alpha)(l+\alpha-1)\cdots\alpha}{(\alpha+k)(\alpha+k-1)\cdots\alpha} \\ &= (-1)^k n! \sum_{m=0,m\neq 1}^{n-k} \sum_{l=k}^{n-m} \frac{B_m}{m!} \frac{E_{n-m-l}}{(n-m-l)!} \binom{l+\alpha}{l-k}.\end{aligned} \tag{2.4}$$

Therefore, by (2.1) and (2.4), we obtain the following theorem.

Theorem 2.1 *For $n \in \mathbb{Z}_+$, we have*

$$\begin{aligned}
&\sum_{m=0,m\neq 1}^{n} \binom{n}{m} B_m E_{n-m}(x) \\
&\quad = n! \sum_{k=0}^{n-k} (-1)^k \left(\sum_{m=0,m\neq 1}^{n} \sum_{l=k}^{n-m} \frac{B_m}{m!} \frac{E_n - m - l}{(n-m-l)!} \binom{l+\alpha}{l-k} \right) L_k^{\alpha}(x).
\end{aligned}$$

From (1.13), we can derive the following corollary.

Corollary 2.2 *For $n \in \mathbb{Z}_+$, we have*

$$B_n(x) = n! \sum_{k=0}^{n} (-1)^k \left(\sum_{m=0,m\neq 1}^{n-k} \sum_{l=k}^{n-m} \frac{B_m}{m!} \frac{E_n - m - l}{(n-m-l)!} \binom{l+\alpha}{l-k} \right) L_k^{\alpha}(x).$$

Let us take $p(x) = \sum_{l=0}^{n} \binom{n}{l} \frac{E_{l+1}}{l+1} B_{n-l}(x)$. By the same method, we get

$$\begin{aligned}
C_k &= \frac{1}{\Gamma(\alpha+k+1)} \int_0^\infty \left(\frac{d^k}{dx^k} x^{k+\alpha} e^{-x} \right) \sum_{l=0}^{n} \binom{n}{l} \frac{E_{l+1}}{l+1} B_{n-l}(x)\, dx \\
&= \frac{1}{\Gamma(\alpha+k+1)} \sum_{l=0}^{n-k} \sum_{m=0}^{n-l} \binom{n}{l} \binom{n-l}{m} \frac{E_{l+1}}{l+1} B_{n-l-m} \int_0^\infty \left(\frac{d^k}{dx^k} x^{k+\alpha} e^{-x} \right) x^m\, dx \\
&= \frac{(-1)^k}{\Gamma(\alpha+k+1)} \sum_{l=0}^{n-k} \sum_{m=k}^{n-l} \binom{n}{l} \binom{n-l}{m} \frac{E_{l+1}}{l+1} B_{n-l-m} \frac{m!}{(m-k)!} \Gamma(m+\alpha+1) \\
&= (-1)^k \sum_{l=0}^{n-k} \sum_{m=k}^{n-l} \binom{n}{l} \binom{n-l}{m} \frac{m!}{(m-k)!} \frac{E_{l+1}}{(l+1)} B_{n-l-m} \frac{(\alpha+m)(\alpha+m-1)\cdots\alpha}{(\alpha+k)(\alpha+k-1)\cdots\alpha} \\
&= (-1)^k n! \sum_{l=0}^{n-k} \sum_{m=k}^{n-l} \binom{\alpha+m}{m-k} \frac{E_{l+1}}{(l+1)!} \frac{B_{n-l-m}}{(n-l-m)!}.
\end{aligned} \tag{2.5}$$

Therefore, by (1.11), (2.1), and (2.5), we obtain the following theorem.

Theorem 2.3 *For $n \in \mathbb{Z}_+$, we have*

$$-\frac{E_n(x)}{2} = n! \sum_{k=0}^{n} (-1)^k \left(\sum_{l=0}^{n-k} \sum_{m=k}^{n-l} \binom{\alpha+m}{m-k} \frac{E_{m+1}}{(m+1)!} \frac{B_{n-m-l}}{(n-m-l)!} \right) L_k^{\alpha}(x).$$

For $n \in \mathbb{N}$ with $n \geq 2$ and $m \in \mathbb{Z}_+$ with $n - m \geq 0$, we have

$$\begin{aligned}
B_{n-m}(x) B_m(x) &= \sum_r \left\{ \binom{n-m}{2r} m + \binom{m}{2r} (n-m) \right\} \frac{B_{2r} B_{n-2r}(x)}{n-2r} \\
&\quad + (-1)^{m+1} \frac{(n-m)!m!}{n!} B_n \in \mathbb{P}_n \quad \bigl(\text{see [8]}\bigr).
\end{aligned} \tag{2.6}$$

Let us take $p(x) = B_{n-m}(x)B_m(x) \in \mathbb{P}_n$. Then $p(x)$ can be generated by an orthogonal basis $\{L_0^\alpha(x), L_1^\alpha(x), \ldots, L_n^\alpha(x)\}$ in $\mathbb{P}_n$ to be

$$p(x) = \sum_{k=0}^{n} C_k L_k^\alpha(x). \tag{2.7}$$

From (2.3), (2.6), and (2.7), we note that

$$\begin{aligned}
C_k &= \frac{1}{\Gamma(\alpha+k+1)} \int_0^\infty \left(\frac{d^k}{dx^k} x^{k+\alpha} e^{-x} \right) p(x)\,dx \\
&= \frac{1}{\Gamma(\alpha+k+1)} \sum_{r=0}^{[\frac{n}{2}]} \left\{ \binom{n-m}{2r} m + \binom{m}{2r}(n-m) \right\} \\
&\quad \times \frac{B_{2r}}{n-2r} \int_0^\infty \left(\frac{d^k}{dx^k} x^{k+\alpha} e^{-x} \right) B_{n-2r}(x)\,dx \\
&= \frac{1}{\Gamma(\alpha+k+1)} \sum_{r=0}^{[\frac{n}{2}]} \left\{ \binom{n-m}{2r} m + \binom{m}{2r}(n-m) \right\} \frac{B_{2r}}{n-2r} \\
&\quad \times \sum_{l=0}^{n-2r} \binom{n-2r}{l} B_{n-2r-l} \int_0^\infty \left(\frac{d^k}{dx^k} x^{k+\alpha} e^{-x} \right) x^l\,dx \\
&= \frac{1}{\Gamma(\alpha+k+1)} \sum_{r=0}^{[\frac{n}{2}]} \sum_{l=0}^{n-2r} \left\{ \binom{n-m}{2r} m + \binom{m}{2r}(n-m) \right\} \binom{n-2r}{l} \\
&\quad \times \frac{B_{2r}B_{n-2r-l}}{n-2r} \int_0^\infty \left(\frac{d^k}{dx^k} x^{k+\alpha} e^{-x} \right) x^l\,dx \\
&= \frac{(-1)^k}{\Gamma(\alpha+k+1)} \sum_{r=0}^{[\frac{n-k}{2}]} \sum_{l=k}^{n-2r} \left\{ \binom{n-m}{2r} m + \binom{m}{2r}(n-m) \right\} \binom{n-2r}{l} \\
&\quad \times \frac{B_{2r}B_{n-2r-l}\,l!}{(n-2r)(l-k)!} \Gamma(\alpha+l+1).
\end{aligned} \tag{2.8}$$

It is easy to show that

$$\begin{aligned}
\frac{\Gamma(\alpha+l+1)}{\Gamma(\alpha+k+1)(l-k)!} &= \frac{(\alpha+l)(\alpha+l-1)\cdots\alpha\Gamma(\alpha)}{(\alpha+k)(\alpha+k-1)\cdots\alpha\Gamma(\alpha)(l-k)!} \\
&= \frac{(\alpha+l)(\alpha+l-1)\cdots(\alpha+k+1)}{(\alpha-k)!} = \binom{\alpha+l}{l-k}.
\end{aligned} \tag{2.9}$$

By (2.8) and (2.9), we get

$$\begin{aligned}
C_k &= (-1)^k \sum_{r=0}^{[\frac{n-k}{2}]} \sum_{l=k}^{n-2r} \left\{ \binom{n-m}{2r} m + \binom{m}{2r}(n-m) \right\} \\
&\quad \times \binom{n-2r}{l} \binom{\alpha+l}{l-k} \frac{l!B_{2r}B_{n-2r-l}}{(n-2r)}.
\end{aligned} \tag{2.10}$$

Therefore, by (2.7) and (2.10), we obtain the following theorem.

Theorem 2.4 *For $n \in \mathbb{N}$ with $n \geq 2$ and $m \in \mathbb{Z}_+$ with $n - m \geq 0$, we have*

$$B_{n-m}(x)B_m(x) = \sum_{k=0}^{n}(-1)^k\left\{\sum_{r=0}^{[\frac{n-k}{2}]}\sum_{l=k}^{n-2r}\left(\binom{n-m}{2r}m + \binom{m}{2r}(n-m)\right) \times \binom{n-2r}{l}\binom{\alpha+l}{l-k}\frac{l!B_{2r}B_{n-2r-l}}{(n-2r)}\right\}L_k^{\alpha}(x).$$

It is easy to show that

$$\frac{t^2e^{t(x+y)}}{(e^t-1)^2} = (x+y-1)\frac{t^2e^{t(x+y-1)}}{e^t-1} - \frac{t^2 d}{dt}\left(\frac{e^{t(x+y-1)}}{e^t-1}\right). \tag{2.11}$$

From (2.11), we have

$$\sum_{k=0}^{n}\binom{n}{k}B_k(x)B_{n-k}(y) = (1-n)B_n(x+y) + (x+y-1)nB_{n-1}(x+y) \quad \bigl(\text{see [11]}\bigr). \tag{2.12}$$

Let $x = y$. Then by (2.12), we get

$$\sum_{k=0}^{n}\binom{n}{k}B_k(x)B_{n-k}(x) = (1-n)B_n(2x) + (2x-1)B_{n-1}(2x). \tag{2.13}$$

Let us take $p(x) = \sum_{k=0}^{n}\binom{n}{k}B_k(x)B_{n-k}(x) \in \mathbb{P}_n$. Then $p(x)$ can be generated by an orthogonal basis $\{L_0^{\alpha}(x), L_1^{\alpha}(x), \dots, L_n^{\alpha}(x)\}$ in $\mathbb{P}_n$ to be

$$p(x) = \sum_{k=0}^{n}\binom{n}{k}B_k(x)B_{n-k}(x) = \sum_{k=0}^{n}C_kL_k^{\alpha}(x). \tag{2.14}$$

From (2.3), (2.13), and (2.14), we can determine the coefficients C_k's to be

$$\begin{aligned}C_k &= \frac{1}{\Gamma(\alpha+k+1)}\int_0^{\infty}\left(\frac{d^k}{dx^k}x^{k+\alpha}e^{-x}\right)p(x)\,dx \\ &= \frac{1}{\Gamma(\alpha+k+1)}\left\{(1-n)\int_0^{\infty}\left(\frac{d^k}{dx^k}x^{k+\alpha}e^{-x}\right)B_n(2x)\,dx \right. \\ &\quad \left. + n\int_0^{\infty}\left(\frac{d^k}{dx^k}x^{k+\alpha}e^{-x}\right)(2x-1)B_{n-1}(2x)\,dx\right\}.\end{aligned} \tag{2.15}$$

By simple calculation, we get

$$\begin{aligned}&\frac{1}{\Gamma(\alpha+k+1)}\int_0^{\infty}\left(\frac{d^k}{dx^k}x^{k+\alpha}e^{-x}\right)(2x-1)B_{n-1}(2x)\,dx \\ &= 2(-1)^k\sum_{l=k-1}^{n-1}\binom{n-1}{l}2^lB_{n-1-l}\binom{\alpha+l+1}{l-k+1}(l+1)! \\ &\quad + (-1)^{k+1}\sum_{l=k}^{n-1}\binom{n-1}{l}2^lB_{n-1-l}\binom{\alpha+l}{l-k}l!\end{aligned} \tag{2.16}$$

and

$$\begin{aligned}&\frac{1}{\Gamma(\alpha+k+1)}\int_0^\infty\left(\frac{d^k}{dx^k}x^{k+\alpha}e^{-x}\right)B_n(2x)\,dx\\&\quad=(-1)^k\sum_{l=k}^{n}\binom{n}{l}2^lB_{n-l}l!\binom{l+\alpha}{l-k}.\end{aligned}\tag{2.17}$$

Therefore, by (2.13), (2.14), (2.15), (2.16), and (2.17), we obtain the following theorem.

Theorem 2.5 *For $n \in \mathbb{Z}_+$, we get*

$$\begin{aligned}&\sum_{k=0}^{n}\binom{n}{k}B_k(x)B_{n-k}(x)\\&\quad=(1-n)\sum_{k=0}^{n}\left\{(-1)^k\sum_{l=k}^{n}\binom{n}{l}2^lB_{n-l}l!\binom{l+\alpha}{l-k}\right\}L_k^\alpha(x)\\&\qquad+n\sum_{k=0}^{n}(-1)^k\left\{\sum_{l=k-1}^{n-1}\binom{n-1}{l}2^{l+1}B_{n-1-l}(l+1)!\binom{\alpha+l+1}{l-k+1}\right.\\&\qquad\left.-\sum_{l=k}^{n-1}\binom{n-1}{l}2^lB_{n-l-1}l!\binom{\alpha+l}{l-k}\right\}L_k^\alpha(x).\end{aligned}$$

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the manuscript and typed, read, and approved the final manuscript.

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