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Partial stochastic asymptotic stability of neutral stochastic functional differential equations with Markovian switching by boundary condition

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Abstract

The partial stochastic asymptotic stability of neutral stochastic functional differential equations with Markovian switching (NSFDEwMSs) has been investigated, the sufficient conditions for partial stochastic asymptotic stability have been given. We have used the boundary condition and $LV \leq 0$ to replace the condition $LV < 0$ and have generalized some results of Sharov and Ignatyev to cover a class of much more general NSFDEwMSs.

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1 Introduction

In recent years, Lyapunov's second method has been a very important method to study the stability of a stochastic differential equation, which has attracted much attention since it is not only academically challenging, but also of practical importance, and has played an important role in many fields such as insurance, finance, population dynamic, control (see, *e.g.*, Øsendal [1]). Recently, much work has been done on stochastic differential equations with Markovian switching (see, *e.g.*, Mao and Yuan [2]). In particular, significant work has appeared on the stability of stochastic differential delay equations with Markovian switching (see, *e.g.*, Kolmanovskii *et al.* [3], Luo [4], Luo *et al.* [5], Mao *et al.* [6], Yuan and Mao [7], Yuan *et al.* [8], Mao *et al.* [9], Liu [10, 11]).

On the other hand, many dynamical systems depend not only on present and past states, but also involve derivatives with delays. Motivated by chemical engineering systems as well as the theory of aero elasticity, many researchers have been interested in the neutral stochastic functional differential equations, and the neutral part, which has been used in investigating the qualitative properties of derivatives of delays for dynamical systems, has been added to all kinds of equations. In particular, the significant work has been done (see, *e.g.*, Mao [12], Luo *et al.* [13]).

The global stability of SDEwMSs with respect to sample paths or moments, which is too strong for many cases, appeared in many more papers. So, the notion of the partial stability (see, *e.g.*, Peiffer and Rouche [14], Rouche *et al.* [15]) has been involved, and the second

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method of Lyapunov as an indispensable tool has been used to investigate the partial stability (see, *e.g.*, Sontag and Wang [16], Vorotnikov [17], Vorotnikov and Rumyantsev [18], Ignatyev [19, 20]).

However, in the process of investigating the qualitative properties of equilibria and boundedness properties of motions of dynamical systems, the partial stability as a new concept plays the key role, and the systems are often determined by all kinds of equations, including stochastic differential equations, of course. Motivated by the previously mentioned topic, and since the results of the partial stochastic asymptotic stability of neutral stochastic functional differential equations with Markovian switching are very few, we make the attempt to study this topic to fill the gap.

This paper is organized as follows. In Section 2, we present some basic preliminaries and the form of neutral stochastic functional differential equations with Markovian switching. In Section 3, the sufficient conditions for partial stochastic asymptotic stability have been obtained and proofs have been given. In Section 4, some well-known results are generalized in the remarks. In Section 5, there is the discussion about the main results.

2 Preliminaries and definitions

Let $\{\Omega, \mathcal{F}, \{\mathcal{F}\}_{t\geq 0}, \mathbf{P}\}$ be a complete probability space with a filtration satisfying the usual conditions, *i.e.*, the filtration is continuous on the right and $\mathcal{F}_0$ contains all P-zero sets. $B(t)$ is an m-dimensional Brownian motion defined on the probability space. Let $C([-\tau, 0]; R^n)$ denote the family of functions φ from $[-\tau, 0]$ to R^n that are right-continuous and have limits on the left. $C([-\tau, 0]; R^n)$ is equipped with the norm $\|\varphi\| = \sup_{-\tau \leq s \leq 0} |\varphi(s)|$ and $|x| = \sqrt{x^T x}$ for any $x \in R^n$. If A is a vector or matrix, its trace norm is denoted by $|A| = \sqrt{\text{trace}(A^T A)}$, while its operator norm is denoted by $\|A\| = \sup\{|Ax| : |x| = 1\}$. Denote $C_{\mathcal{F}_0}^b([-\tau, 0]; R^n)$ to be the family of all $\mathcal{F}_0$-measurable bounded $C([-\tau, 0]; R^n)$-valued random variables $\xi = \{\xi(\theta) : -\tau \leq \theta \leq 0\}$. Let $p > 0$, $t \geq 0$, $L_{\mathcal{F}_t}^p([-\tau, 0]; R^n)$ denote the family of all $\mathcal{F}_t$ measurable, $C([-\tau, 0]; R^n)$-valued random variables $\varphi = \{\varphi(\theta) : -\tau \leq \theta \leq 0\}$ such that $\sup_{-\tau \leq \theta \leq 0} E|\varphi(\theta)|^p < \infty$.

Let $\{r(t), t \in R_+ = [0, +\infty)\}$ be a right-continuous Markov chain on the probability space $\{\Omega, \mathcal{F}, \{\mathcal{F}\}_{t\geq 0}, \mathbf{P}\}$ taking values in a finite state space $S = \{1, 2, \dots, N\}$ with a generator $\Gamma = (\gamma_{ij})_{N \times N}$ given by

$$P\big(r(t + \Delta) = j | r(t) = i\big) = \begin{cases} \gamma_{ij}\Delta + o(\Delta), & \text{if } i \neq j, \\ 1 + \gamma_{ii}\Delta + o(\Delta), & \text{if } i = j, \end{cases}$$

where $\Delta > 0$. Here $\gamma_{ij} \geq 0$ is the transition rate from i to j, if $i \neq j$, while

$$\gamma_{ii} = -\sum_{j \neq i} \gamma_{ij}.$$

We assume that the Markov chain $r(\cdot)$ is independent of the Brownian motion $B(\cdot)$. It is known that almost every sample path of $r(t)$ is a right-continuous step function with a finite number of simple jumps in any finite sub-interval of R_+.

Consider the following neutral stochastic functional differential equation with Markovian switching:

$$d\big[x(t) - G(x_t)\big] = f\big(t, x_t, r(t)\big) dt + g\big(t, x_t, r(t)\big) dB(t), \quad t > 0, \tag{2.1}$$

with the initial condition $x_0 = \xi = (\xi_1, \xi_2)^T \in C_{\mathcal{F}_0}^b([-\tau, 0]; R^n)$, where $\xi_1 \in R^k$ and $\xi_2 \in R^p$, $k + p = n$, which is independent of $B(\cdot)$, and $x_t = \{x(t + \theta) : -\tau \le \theta \le 0\}$. Let $\widetilde{x(t)} = x(t) - G(x_t)$. Here, we furthermore assume that

$$f : R_+ \times C([-\tau, 0]; R^n) \times S \to R^n, \qquad g : R_+ \times C([-\tau, 0]; R^n) \times S \to R^{n \times m},$$

$$G : C([-\tau, 0]; R^n) \to R^n.$$

For our purpose, we impose the following assumptions which can ensure the existence and uniqueness of a solution, denoted by $x(t)$ on $t > 0$, for equation (2.1). Let f and g satisfy the local Lipschitz and linear growth condition. That is, for each $d > 0$, there exist constants $h > 0$, $C_d > 0$ and $\kappa \in (0, 1)$ such that

$$\left| f(t, \varphi_1, i) - f(t, \varphi_2, i) \right|^2 \vee \left| g(t, \varphi_1, i) - g(t, \varphi_2, i) \right|^2 \le C_d \|\varphi_1 - \varphi_2\|^2,$$

$$\left| f(t, \varphi, i) \right|^2 \vee \left| g(t, \varphi, i) \right|^2 \le h\left(1 + \|\varphi\|^2\right),$$

$$\left| G(\varphi) \right| \le \kappa \|\varphi\|.$$

Denote $x = (x_1, x_2)^T \in R^n$, where $x_1 \in R^k$ and $x_2 \in R^p$, $k + p = n$, and the definitions of $\widetilde{x} = (\widetilde{x_1}, \widetilde{x_2})^T \in R^n$ and $G(x_t) = (G_1(x_t), G_2(x_t))^T \in R^n$ are similar to $x = (x_1, x_2)^T$. The domain $B_K = \{x \in R^n : |x_1| < K, |x_2| < \infty\}$, and the stopping time σ_B is the first exit time from the B_K of the sample path of the process $x(t)$. Let $K \to \infty$, then $\sigma_B \to \infty$. Denote the set of functions $\mathcal{C} := \{\phi : R^+ \to R^+, \text{ continuous, monotonically increasing and } \phi(0) = 0\}$.

Denote by $C^{1,2}(R_+ \times R^n \times S; R_+)$ the family of all non-negative functions $V(t, x, i)$ on $R_+ \times R^n \times S$, which are twice continuously differential with respect to x and once continuously differential with respect to t. For any $(t, x, i) \in R_+ \times R^n \times S$, define an operator LV by

$$\begin{aligned}
LV(t, x_t, i) = {}& V_t\left(t, \widetilde{x(t)}, i\right) + V_x\left(t, \widetilde{x(t)}, i\right) f(t, x_t, i) \\
& + \frac{1}{2} \operatorname{trace}\left[g^T(t, x_t, i) V_{xx}\left(t, \widetilde{x(t)}, i\right) g(t, x_t, i) \right] \\
& + \sum_{j=1}^{N} \gamma_{ij} V\left(t, \widetilde{x(t)}, j\right),
\end{aligned}$$

where

$$V_t = \frac{\partial V(t, x, i)}{\partial t}, \qquad V_x = \left(\frac{\partial V(t, x, i)}{\partial x_1}, \dots, \frac{\partial V(t, x, i)}{\partial x_n} \right), \qquad V_{xx} = \left(\frac{\partial^2 V(t, x, i)}{\partial x_i \partial x_j} \right)_{n \times n}.$$

Definition 2.1 The trivial solution $x(t)$ of equation (2.1) is said to be partial stochastic stability with respect to $x_1(t)$ if, for any $\epsilon > 0$ and $\epsilon_1 > 0$, there exists a $\delta = \delta(\epsilon, \epsilon_1) > 0$ such that

$$P\left\{ \sup_{t \ge 0} |x_1(t)| > \epsilon \right\} < \epsilon_1, \tag{2.2}$$

whenever $\|\xi\| < \delta$.

Definition 2.2 The trivial solution $x(t)$ of equation (2.1) is said to be partial stochastic asymptotic stability with respect to $x_1(t)$ if it is the $x_1(t)$-stochastic stability and for any $\epsilon \in (0, 1)$, there exists a $\delta_0 = \delta_0(\epsilon)$ such that

$$P\left\{\lim_{t \to \infty} |x_1(t)| = 0\right\} \geq 1 - \epsilon, \tag{2.3}$$

whenever $\|\xi\| < \delta_0$.

3 Main results

First of all, we should present Sharov's theorem [21].

Theorem 3.1 (Sharov [21]) *If in the domain $B_K \times R_+$ there exists a function $V(x, t)$ which satisfies the following conditions*:

(I) $\alpha(|x_1|) \leq V(t, x) \leq \beta(|x_1|)$, *where $\alpha, \beta \in \mathcal{C}$;*
(II) $LV \leq -\gamma(|x_1|)$, *where $\gamma \in \mathcal{C}$;*
then the trivial solution $x(t) \equiv 0$ of system (2.1) is $x_1(t)$-stochastic asymptotic stability.

The condition of Theorem 3.1 is more difficult to be satisfied, which is demonstrated by the following example.

Example 3.1 Consider the system of stochastic differential equations

$$\begin{aligned} dx_1 &= -ax_1^3 x_2^2 \, dt + \sqrt{a} x_1^2 x_2 \, dB_1(t), \\ dx_2 &= bx_1 \, dt + cx_2 \, dB_2(t), \end{aligned} \tag{3.1}$$

where a, b, c are constants and $abc \neq 0$. The system has the trivial solution $x_1 = 0$, $x_2 = 0$.
Denote $V := x_1^2$, we have $LV = -2ax_1^4 x_2^2 + 2ax_1^4 x_2^2 = 0$.

Now, by Theorem 3.1, we cannot give whether the solution of (3.1) is stochastic asymptotic stability or not. So, we present the main results of this paper as follows.

Theorem 3.2 *Let there exist a non-negative functional $V(t, x, i) \in C^{1,2}(R_+ \times B_K \times S; R_+)$ such that*

(I) $\mu_1(|x_1|) \leq V(t, x, i) \leq \mu_2(|x_1|)$, *where $\mu_1, \mu_2 \in \mathcal{C}$;*
(II) $LV \leq 0$;
(III) (*Boundary condition*) *For any sufficiently small $\eta > 0$, $\theta > 0$, any solution $x(t)$ of equation (2.1), beginning in the domain $\{\eta < |x_1(t)| < \theta\}$, is such that*

$$\rho = \inf\{t : x_1(t) = \eta\} < \infty, \quad a.s.$$

Then the trivial solution of equation (2.1) is said to be partial stochastic asymptotic stability with respect to $x_1(t)$.

Before giving the proof of Theorem 3.1, we should present the following lemma.

Lemma 3.1 *Let $V(t, x, i)$ be a function in the class $C^{1,2}(R_+ \times B_K \times S; R_+)$, bounded in the domain $(R_+ \times B_K \times S)$, and suppose that $LV \leq 0$ in this domain. Then the process*

$V(\sigma_B \wedge t, x\widetilde{(\sigma_B \wedge t)}, r(\sigma_B \wedge t))$ *is a supermartingale so that* $EV(\sigma_B \wedge t, x\widetilde{(\sigma_B \wedge t)}, r(\sigma_B \wedge t)) \le V(0, x_0, i_0)$, *for* $x(t) \in B_K$.

Proof As the generalized Itô formula, we have

$$V\big(\sigma_B \wedge t, x\widetilde{(\sigma_B \wedge t)}, r(\sigma_B \wedge t)\big) = V(0, x_0, i_0) + \int_0^{\sigma_B \wedge t} LV\big(s, x_s, r(s)\big)\, ds + M(\sigma_B \wedge t),$$

where $M(\sigma_B \wedge t)$ is the martingale.

For any s_1, $0 \le s_1 \le \sigma_B \wedge t$ and $LV \le 0$, we derive that

$$
\begin{aligned}
EV&\big(\sigma_B \wedge t, x\widetilde{(\sigma_B \wedge t)}, r(\sigma_B \wedge t)|\mathcal{F}_{s_1}\big) \\
&= V(0, x_0, i_0) + E\bigg(\int_0^{\sigma_B \wedge t} LV\big(s, x_s, r(s)\big)\, ds \Big| \mathcal{F}_{s_1}\bigg) + E\big(M(\sigma_B \wedge t)|\mathcal{F}_{s_1}\big) \\
&\le V(0, x_0, i_0) + E\bigg(\int_0^{s_1} LV\big(s, x_s, r(s)\big)\, ds \Big| \mathcal{F}_{s_1}\bigg) + M(s_1) \\
&= V(0, x_0, i_0) + \int_0^{s_1} LV\big(s, x_s, r(s)\big)\, ds + M(s_1) \\
&= V\big(s_1, x\widetilde{(s_1)}, r(s_1)\big).
\end{aligned}
$$

So, $V(\sigma_B \wedge t, x\widetilde{(\sigma_B \wedge t)}, r(\sigma_B \wedge t))$ is a supermartingale. Moreover, due to $LV \le 0$ in the domain,

$$
\begin{aligned}
EV&\big(\sigma_B \wedge t, x\widetilde{(\sigma_B \wedge t)}, r(\sigma_B \wedge t)\big) \\
&= V(0, x_0, i_0) + E\bigg(\int_0^{\sigma_B \wedge t} LV\big(s, x_s, r(s)\big)\, ds\bigg) + E\big(M(\sigma_B \wedge t)\big) \\
&\le V(0, x_0, i_0),
\end{aligned}
$$

so the proof is completed. $\qquad\square$

Proof of Theorem 3.2 By Lemma 3.1 and Doob's supermartingale convergence theorem, the process $V(\sigma_B \wedge t, x\widetilde{(\sigma_B \wedge t)}, r(t))$ is a supermartingale, and we can get

$$\lim_{t \to \infty} V\big(\sigma_B \wedge t, x\widetilde{(\sigma_B \wedge t)}, r(t)\big) = c, \quad \text{a.s.} \tag{3.2}$$

Denote $\Omega_1 = \{\omega : \sigma_B = \infty\}$. Owing to $LV \le 0$ and the partial stochastic stability theorem (Sharov [21]), the solution of equation (2.1) is the partial stochastic stability with respect to $x_1(t)$, so we have

$$\lim_{\xi \to 0} P(\Omega_1) = 1. \tag{3.3}$$

Combining this with condition (III) and for all the paths of Ω_1, then

$$\inf_{t>0}\big|x_1(t)\big| = 0. \tag{3.4}$$

By the lemma of Has'minskii [22], that is, the coefficients of equation (2.1) satisfy the local Lipschitz and linear growth condition in every domain bounded, and the process $x(t)$ is regular, then the set $\{\omega : x_1(t, \omega) = 0\}$ is inaccessible to any sample path of the process if $\xi_1 \neq 0$, where $\xi = (\xi_1, \xi_2)^T$, so we get

$$\liminf_{t \to \infty} |x_1(t)| = 0. \tag{3.5}$$

Noting the similar definition of $\widetilde{x(t)} = x(t) - G(x_t)$, obviously,

$$-\left|\widetilde{x_1(t)}\right| \geq -\left(|x_1(t)| + |G_1(x_t)|\right) \geq -\left(|x_1(t)| + \kappa \|x_{t1}\|\right),$$

and we get that

$$\sup_{-\tau \leq t < \infty} \left(-\left|\widetilde{x_1(t)}\right|\right) \geq \sup_{-\tau \leq t < \infty} \left[-(1 + \kappa)|x_1(t)|\right].$$

Therefore,

$$\inf_{-\tau \leq t < \infty} \left|\widetilde{x_1(t)}\right| \leq \inf_{-\tau \leq t < \infty} (1 + \kappa)|x_1(t)|,$$

and (3.4), we have

$$\inf_{t > 0} \left|\widetilde{x_1(t)}\right| = 0. \tag{3.6}$$

Similar to the proof of (3.5), we get

$$\liminf_{t \to \infty} \left|\widetilde{x_1(t)}\right| = 0. \tag{3.7}$$

Combining this with condition (I) ($V(t, x, i) \leq \mu_2(|x_1|)$) and (3.7), we obtain

$$\liminf_{t \to \infty} V\left(t, \widetilde{x(t)}, r(t)\right) = 0. \tag{3.8}$$

Owing to (3.2), the limit

$$\lim_{t \to \infty} V\left(\sigma_B \wedge t, \widetilde{x(\sigma_B \wedge t)}, r(t)\right) = \lim_{t \to \infty} V\left(t, \widetilde{x(t)}, r(t)\right) = c \tag{3.9}$$

exists for almost all paths in Ω_1. By (3.8) and (3.9), we have the limit of the above equation $c = 0$, a.s., since the condition (I) ($\mu_1(|x_1|) \leq V(t, x, i)$). For all the paths of Ω_1, we can imply that

$$\lim_{t \to \infty} \left|\widetilde{x_1(t)}\right| = 0. \tag{3.10}$$

Moreover,

$$\limsup_{t \to \infty} \left|\widetilde{x_1(t)}\right| = 0. \tag{3.11}$$

Noting the similar definition of $\widetilde{x(t)} = x(t) - G(x_t)$, obviously,

$$\left|x_1(t)\right| \leq \left|\widetilde{x_1(t)}\right| + \left|G_1(x_t)\right| \leq \left|\widetilde{x_1(t)}\right| + \kappa \|x_{t1}\|,$$

we get that

$$\sup_{-\tau \leq t < \infty} (1 - \kappa)\left|x_1(t)\right| \leq \sup_{-\tau \leq t < \infty} \left|\widetilde{x_1(t)}\right|.$$

By (3.11), then

$$\limsup_{t \to \infty} \left|x_1(t)\right| = 0. \tag{3.12}$$

Combining this with conditions (3.5) and (3.12) yields

$$\lim_{t \to \infty} \left|x_1(t)\right| = 0. \tag{3.13}$$

So, we have obtained that the trivial solution of equation (2.1) is partial stochastic asymptotic stability with respect to x_1 with (3.3) and (3.13). □

4 Remarks

Remark 4.1 When $G(x_t) \equiv 0$, $\tau \equiv 0$ and $r(t) \equiv 0$, equation (2.1) reduces to

$$dx(t) = f\big(t, x(t)\big)\, ds + g\big(t, x(t)\big)\, dB(t), \quad t > 0, \tag{4.1}$$

which has recently been studied in Ignatyev [23], that is to say, Theorem 3.1 of Ignatyev [23] has been generalized.

Remark 4.2 The operator LV of equation (4.1) of Theorem 3.1 (Sharov [21]) must be negative, but we can obtain the partial stochastic asymptotic stability of equation (2.1) under LV being non-positive ($LV \leq 0$), which makes the conditions of the solution more feasible to be satisfied.

5 Discussion of main results

Lyapunov methods have been developed to research the conditions of the partial asymptotic stochastic stability of neutral stochastic functional differential equations with Markovian switching. We have the non-positive Lyapunov operator and boundary condition to weaken the conditions of the previous theorems, but there is a small problem that the boundary condition of Theorem 3.2 is a stochastic domain. We will obtain the equivalent and practical results for the boundary condition in the future investigation.

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