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Further results of the estimate of growth of entire solutions of some classes of algebraic differential equations

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Abstract

In this article, by means of the normal family theory we estimate the growth order of entire solutions of some algebraic differential equations and improve the related results of Bergweiler, Barsegian, and others. We also estimate the growth order of entire solutions of a type system of a special algebraic differential equations. We give some examples to show that our results are sharp in special cases.

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1. Introduction and main results

Let $f(z)$ be a meromorphic function in the complex plane. We use the standard notation of the Nevanlinna theory of meromorphic functions and denotes the order of $f(z)$ by $\lambda(f)$ (see [1-3]).

Let $\mathbb{C}$ be the whole complex domain. Let D be a domain in $\mathbb{C}$ and $\mathcal{F}$ be a family of meromorphic functions defined in D. $\mathcal{F}$ is said to be normal in D, in the sense of Montel, if each sequence $\{f_n\} \subset \mathcal{F}$ has a subsequence $\{f_{n_j}\}$ which converse spherically locally uniformly in D, to a meromorphic function or ∞ (see [1]).

In general, it is not easy to have an estimate on the growth of an entire or meromorphic solution of a nonlinear algebraic differential equation of the form

$$P(z, w, w', \ldots, w^{(k)}) = 0, \tag{1.1}$$

where P is a polynomial in each of its variables.

A general result was obtained by Gol'dberg [4]. He obtained

Theorem 1.1. *All meromorphic solutions of algebraic differential equation (1.1) have finite order of growth, when* $k = 1$.

For a half century Bank and Kaufman [5] and Barsegian [6] gave some extensions or different proofs, but the results have not changed. Barsegian [7] and Bergweiler [8] have extended Gol'dberg's result to certain algebraic differential equations of higher order. In 2009, Yuan et al. [9], improved their results and gave a general estimate of order of $w(z)$, which depends on the degrees of coefficients of differential polynomial for $w(z)$. In order to state these results, we must introduce some notations: $m \in \mathbb{N} =$

{1, 2, 3,...}, $r_j \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$ for $j = 1, 2,..., m$, and put $r = (r_1, r_2,..., r_m)$. Define $M_r[w](z)$ by

$$M_r[w](z) := [w'(z)]^{r_1}[w''(z)]^{r_2} \cdots [w^{(m)}(z)]^{r_m},$$

with the convention that $M_{\{0\}}[w] = 1$. We call $p(r) := r_1 + 2r_2 + ... + mr_m$ the weight of $M_r[w]$. A differential polynomial $P[w]$ is an expression of the form

$$P[w](z) := \sum_{r \in I} a_r(z, w(z)) M_r[w] \tag{1.2}$$

where the a_r are rational in two variables and I is a finite index set. The weight $\deg P[w]$ of $P[w]$ is given by $\deg P[w] := \max_{r \in I} p(r)$. $\deg_{z,\infty} a_r$ denotes the degree at infinity in variable z concerning $a_r(z, w)$. $\deg_{z,\infty} a := \max_{r \in I} \max\{\deg_{z,\infty} a_r, 0\}$.

Theorem 1.2. [9]*Let $w(z)$ be a meromorphic function in the complex plane, $n \in \mathbb{N}$, $P[w]$ be a polynomial with the form (1.2) $n > \deg P[w]$. If $w(z)$ satisfies the differential equation $[w'(z)]^n = P[w]$, then the growth order $\lambda := \lambda(w)$ of $w(z)$ satisfies*

$$\lambda \le 2 + \frac{2\deg_{z,\infty} a}{n - \deg P[w]}.$$

Recently Qi et al. [10] further improved Theorem 1.2 as below.

Theorem 1.3. *Let $w(z)$ be a meromorphic function in the complex plane and all zeros of $w(z)$ have multiplicity at least k ($k \in \mathbb{N}$), $P[w]$ be a polynomial with the form (1.2) and $nkq > \deg P[w]$ ($n \in \mathbb{N}$). If $w(z)$ satisfies the differential equation $[Q(w^{(k)}(z))]^n = P[w]$, then the growth order $\lambda := \lambda(w)$ of $w(z)$ satisfies*

$$\lambda \le 2 + \frac{2\deg_{z,\infty} a}{nqk - \deg P[w]},$$

where $Q(z)$ is a polynomial with degree q.

In this article, we first give a small upper bound for entire solutions.

Theorem 1.4. *Let $w(z)$ be an entire function in the complex plane and all zeros of $w(z)$ have multiplicity at least k ($k \in \mathbb{N}$), $P[w]$ be a polynomial with the form (1.2) and $nkq > \deg P[w]$ ($n \in \mathbb{N}$). If $w(z)$ satisfies the differential equation $[Q(w^{(k)}(z))]^n = P[w]$, then the growth order $\lambda := \lambda(w)$ of $w(z)$ satisfies*

$$\lambda \le 1 + \frac{\deg_{z,\infty} a}{nqk - \deg P[w]},$$

where $Q(z)$ is a polynomial with degree q.

Example 1 For $n = 2$, entire function $w(z) = e^{z^2}$ satisfies the following algebraic differential equation

$$(w'')^2 = 4w^2 + 16z^2w^2 + 8z^3w'w,$$

we know $\deg_{z,\infty} a = 3$, $\deg P[w] = 2$, So $\lambda = 2 \le 1 + \frac{3}{2 \times 2 - 1} = 2$. This example illustrates that Theorem 1.4 is an extending result of Theorem 1.3 and our result is sharp in the special cases.

By Theorem 1.4, we immediately have the following corollaries.

Corollary 1.5. *Let $w(z)$ be an entire function in the complex plane and all zeros of $w(z)$ have multiplicity at least k ($k \in \mathbb{N}$), $P[w]$ be a differential polynomial with constant coefficients in variable w or $\deg_{z,\infty} a_t \leq 0 (t \in I)$ in the (1.2) and $nkq > \deg P[w]$ ($n \in \mathbb{N}$). If $w(z)$ satisfies the differential equation $[Q(w^{(k)}(z))]^n = P[w]$, then the growth order $\lambda := \lambda(w)$ of $w(z)$ satisfies $\lambda \leq 1$, where $Q(z)$ is a polynomial with degree q.*

Corollary 1.6. *Let $w(z)$ be an entire function in the complex plane and all zeros of $w(z)$ have multiplicity at least k ($k \in \mathbb{N}$), $P[w]$ be a polynomial with the form (1.2) and $nk > \deg P[w]$ ($n \in \mathbb{N}$). If $w(z)$ satisfies the differential equation $[H(w(z))]^n = P[w]$, then the growth order $\lambda := \lambda(w)$ of $w(z)$ satisfies*

$$\lambda \leq 1 + \frac{\deg_{z,\infty} a}{nk - \deg P[w]},$$

where $H(w(z)) = w^{(k)}(z) + b_{k-1}w^{(k-1)}(z) + b_{k-2}w^{(k-2)}(z) + \ldots + b_1 w(z) + b_0$ and $b_{k-1},\ldots, b_0$ are constants.

In 2009, Gu et al. [11] investigated the growth order of solutions of a type systems of algebraic differential equations of the form

$$\begin{cases} (w'_2)^{m_1} = a(z)w_1^{(n)}, \\ \left(w_1^{(n)}\right)^{m_2} = P[w_2] \end{cases} \tag{1.3}$$

where m_1, m_2 are the non-negative integer, $a(z)$ is a polynomial, $P[w_2]$ is defined by (1.2).

They obtained the following result.

Theorem 1.7. *Let $w = (w_1, w_2)$ be the meromorphic solution vector of a type systems of algebraic differential equations of the form (1.3), if $m_1 m_2 > \deg P(w_2)$, then the growth orders $\lambda(w_i)$ of $w_i(z)$ for $i = 1,2$ satisfy*

$$\lambda(w_1) = \lambda(w_2) \leq 2 + \frac{2(\nu + \deg_{z,\infty} a)}{m_1 m_2 - \deg P(w_2)}$$

where $\nu = \deg (a(z))^{m_2}$.

Qi et al. [10] also consider the similar result to Theorem 1.7 for the systems of the algebraic differential equations

$$\begin{cases} \left(Q(w_2^{(k)}(z))\right)^{m_1} = a(z)w_1^{(n)} \\ \left(w_1^{(n)}\right)^{m_2} = P(w_2), \end{cases} \tag{1.4}$$

where $Q(z)$ is a polynomial with degree q.

They obtained the following result.

Theorem 1.8. *Let $w = (w_1, w_2)$ be a meromorphic solution of a type systems of algebraic differential equations of the form (1.4), if $m_1 m_2 qk > \deg P(w_2)$, and all zeros of $w_2(z)$ have multiplicity at least k ($k \in \mathbb{N}$), then the growth orders $\lambda(w_i)$ of $w_i(z)$ for $i = 1,2$ satisfy*

$$\lambda(w_1) = \lambda(w_2) \leq 2 + \frac{2(\nu + \deg_{z,\infty} a)}{m_1 m_2 qk - \deg P(w_2)},$$

where $\nu = \deg (a(z))^{m_2}$.

Similarly we have a small upper bounded estimate for entire solutions below.

Theorem 1.9. *Let* $w = (w_1, w_2)$ *be an entire solution of a type systems of algebraic differential equations of the form (1.4), if* $m_1m_2qk > \deg P(w_2)$*, and all zeros of* $w_2(z)$ *have multiplicity at least* k ($k \in \mathbb{N}$)*, then the growth orders* $\lambda(w_i)$ *of* $w_i(z)$ *for* $i = 1, 2$ *satisfy*

$$\lambda(w_1) = \lambda(w_2) \leq 1 + \frac{\nu + \deg_{z,\infty} a}{m_1 m_2 qk - \deg P(w_2)},$$

where $\nu = \deg\left(a(z)\right)^{m_2}$.

By Theorem 1.9, we immediately obtain a corollary below.

Corollary 1.10. *Let* $w = (w_1, w_2)$ *be an entire solution of a type systems of algebraic differential equations of the form*

$$\begin{cases} (H(w_2))^{m_1} = a(z)w_1^{(n)} \\ \left(w_1^{(n)}\right)^{m_2} = p(w_2), \end{cases} \tag{1.5}$$

where $H(w(z)) = w^{(k)}(z)+b_{k-1}w^{(k-1)}(z)+b_{k-2}w^{(k-2)}(z)+...+b_0$ *and* $b_{k-1}, ..., b_0$ *are constants. If* $m_1m_2qk > \deg P(w_2)$*, and all zeros of* $w_2(z)$ *have multiplicity at least* k ($k \in \mathbb{N}$)*, then the growth orders* $\lambda(w_i)$ *of* $w_i(z)$ *for* $i = 1, 2$ *satisfy*

$$\lambda(w_1) = \lambda(w_2) \leq 1 + \frac{\nu + \deg_{z,\infty} a}{m_1 m_2 qk - \deg P(W_2)},$$

where $\nu = \deg\left(a(z)\right)^{m_2}$.

Example 2 Set $w_1(z) = e^z + c$, $w_2(z) = e^z$ satisfy a type systems of algebraic differential equations of the form

$$\begin{cases} (w_2^{(k)}) = w_1^{(n)} \\ (w_1^{(n)})^5 = (w_2)^3(w'_2)^2 \end{cases}, \tag{1.6}$$

where c is a constant, $m_1 = 1$, $m_2 = 5$, $\nu = 0$, $\deg_{z,\infty} a = 0$, and $\deg P(w_2) = 2$. The (1.6) satisfies the $m_1m_2 = 5 > 2 = \deg P(w_2)$. So $\lambda(w_1) = \lambda(w_2) = 1 \leq 1$. So the conclusion of Theorem 1.9, Corollary 1.10 may occur and our results are sharp in the special cases.

2. Preliminary lemmas

In order to prove our result, we need the following lemmas. The first one extends a famous result by Zalcman [12] concerning normal families. Zalcman's lemma is a very important tool in the study of normal families. It has also undergone various extensions and improvements. The following is one up-to-date local version, which is due to Pang and Zaclman [13].

Lemma 2.1 [13,14] Let $\mathcal{F}$ be a family of meromorphic (analytic) functions in the unit disc Δ with the property that for each $f \in \mathcal{F}$, all zeros of multiplicity at least k. Suppose that there exists a number $A \geq 1$ such that $|f^{(k)}(z)| \leq A$ whenever $f \in \mathcal{F}$ and $f = 0$. If $\mathcal{F}$ is not normal in Δ, then for $0 \leq \alpha \leq k$, there exist

1. a number $r \in (0,1)$;
2. a sequence of complex numbers z_n, $|z_n| < r$;

3. a sequence of functions $f_n \in \mathcal{F}$;

4. a sequence of positive numbers $\rho_n \to 0^+$;

such that $g_n(\xi) = \rho_n^{-\alpha} f_n(z_n + \rho_n \xi)$ converges locally uniformly (with respect to the spherical metric) to a non-constant meromorphic (entire) function $g(\zeta)$ on $\mathbb{C}$, and moreover, the zeros of $g(\zeta)$ are of multiplicity at least k, $g^{\#}(\zeta) \le g^{\#}(0) = kA + 1$. In particular, g has order at most 2. In particular, we may choose w_n and ρ_n, such that

$$\rho_n \le \frac{2}{\left[f_n^{\#}(w_n)\right]^{\frac{1}{1+|\alpha|}}}, \quad f_n^{\#}(w_n) \ge f_n^{\#}(0).$$

Here, as usual, $g^{\#}(\xi) = \frac{|g'(\xi)|}{1+|g(\xi)|^2}$ is the spherical derivative. For $0 \le \alpha < k$, the hypothesis on $f^{(k)}(z)$ can be dropped, and $kA + 1$ can be replaced by an arbitrary positive constant.

Lemma 2.2 [15] Let $f(z)$ be holomorphic in whole complex plane with growth order $\lambda := \lambda(f) > 1$, then for each $0 < \mu < \lambda - 1$, there exists a sequence $a_n \to \infty$, such that

$$\lim_{n\to\infty} \frac{f^{\#}(a_n)}{|a_n|^{\mu}} = +\infty. \tag{2.1}$$

3. Proof of the results

Proof of Theorem 1.4 Suppose that the conclusion of theorem is not true, then there exists an entire solution $w(z)$ satisfies the equation $[Q(w(z))]^n = P[w]$. such that

$$\lambda > 1 + \frac{\deg_{z,\infty} a}{nqk - \deg P[w]}. \tag{3.1}$$

By Lemma 2.2 we know that for each $0 < \rho < \lambda - 1$, there exists a sequence of points $a_m \to \infty (m \to \infty)$, such that (2.1) is right. This implies that the family $\{w_m(z) := w(a_m + z)\}_{m\in\mathbb{N}}$ is not normal at $z = 0$. By Lemma 2.1, there exist sequences $\{b_m\}$ and $\{\rho_m\}$ such that

$$|a_m - b_m| < 1, \quad \rho_m \to 0, \tag{3.2}$$

and $g_m(\zeta) := w_m(b_m - a_m + \rho_m\zeta) = w(b_m + \rho_m\zeta)$ converges locally uniformly to a non-constant entire function $g(\zeta)$, which order is at most 2, all zeros of $g(\zeta)$ have multiplicity at least k. In particular, we may choose b_m and ρ_m, such that

$$\rho_m \le \frac{2}{w^{\#}(b_m)}, \quad w^{\#}(b_m) \ge w^{\#}(a_m). \tag{3.3}$$

According to (2.1) and (3.1)-(3.3), we can get the following conclusion:

For any fixed constant $0 \le \rho < \lambda - 1$, we have

$$\lim_{m\to\infty} b_m^{\rho} \rho_m = 0. \tag{3.4}$$

In the differential equation $[Q(w^{(k)}(z))]^n = P[w(z)]$, we now replace z by $b_m + \rho_m\zeta$. Assuming that $P[w]$ has the form (1.2). Then we obtain

$$(Q(w^{(k)}(b_m + \rho_m\zeta)))^n = \sum_{r\in I} a_r(b_m + \rho_m\zeta, g_m(\zeta))\rho_m^{-p(r)} M_r[g_m](\zeta),$$

where

$$Q(w^{(k)}(b_m+\rho_m\zeta)) = \rho_m^{-qk}\left[(g_m^{(k)})^q(\zeta)+\rho_m^k a_{q-1}(g_m^{(k)})^{q-1}(\zeta)+ + \cdots + \rho_m^{(q-1)k}a_1 g_m^{(k)}(\zeta)+\rho_m^{qk}a_0\right].$$

Hence we deduce that

$$\rho_m^{-nqk}\left[(g_m^{(k)})^q(\zeta)+\rho_m^k a_{q-1}(g_m^{(k)})^{q-1}(\zeta)+\cdots+\rho_m^{qk}a_0\right]^n = \sum_{r\in I} a_r(b_m+\rho_m\zeta, g_m(\zeta))\rho_m^{-p(r)}M_r[g_m](\zeta).$$

Therefore

$$\left[(g_m^{(k)})^q(\zeta)+\rho_m^k a_{q-1}(g_m^{(k)})^{q-1}(\zeta)+\cdots+\rho_m^{qk}a_0\right]^n = \sum\nolimits_{r\in I}\frac{a_r(b_m+p_m\zeta, g_m(\zeta))}{b_m^{\deg_{z,\infty}a_r}}\left[b_m^{\frac{\deg_{z,\infty}a_r}{nqk-p(r)}}\rho_m\right]^{nqk-p(r)}M_r\left[g_m\right](\zeta). \tag{3.5}$$

Because $0\le\rho=\frac{\deg_{z,\infty}a_r}{nqk-p(r)}\le\frac{\deg_{z,\infty}a}{nqk-\deg P[w]}<\lambda-1, p(r)<nqk$, for every fixed $\zeta\in\mathbb{C}$, if ζ is not the zero of $g(\zeta)$, by (3.4) then we can get $g^{(k)}(\zeta)=0$ from (3.5). By the all zeros of $g(\zeta)$ have multiplicity at least k, this is a contradiction.

The proof of Theorem 1.4 is complete.

Proof of Theorem 1.9 By the first equation of the systems of algebraic differential equations (1.4), we know

$$w_1^{(n)}=\frac{(Q(w_2^{(k)}(z)))^{m_1}}{a(z)}.$$

Therefore we have

$$\lambda(w_1)=\lambda(w_2).$$

If w_2 is a rational function, then w_1 must be a rational function, so that the conclusion of Theorem 2 is right. If w_2 is a transcendental meromorphic function, by the systems of algebraic differential equations (1.3), then we have

$$(Q(w_2^{(k)}))^{m_1m_2}=(a(z))^{m_2}P(w_2). \tag{3.6}$$

Suppose that the conclusion of Theorem 2 is not true, then there exists an entire vector $w(z) = (w_1(z),w_2(z))$ which satisfies the system of equations (1.4) such that

$$\lambda:=\lambda(w_2)>1+\frac{\nu+\deg_{z,\infty}a}{m_1m_2qk-\deg P(w_2)}, \tag{3.7}$$

By Lemma 2.2 we know that for each $0<\rho<\lambda-1$, there exists a sequence of points $a_m\to\infty$ $(m\to\infty)$, such that (2.1) is right. This implies that the family $\{w_m(z):=w(a_m+z)\}_{m\in\mathbb{N}}$ is not normal at $z = 0$. By Lemma 2.1, there exist sequences $\{b_m\}$ and $\{\rho_m\}$ such that

$$|a_m-b_m|<1,\quad \rho_m\to 0, \tag{3.8}$$

and $g_m(\zeta) := w_{2,m}(b_m - a_m + \rho_m\zeta) = w_2(b_m + \rho_m\zeta)$ converges locally uniformly to a nonconstant entire function $g(\zeta)$, which order is at most 2, all zeros of $g(\zeta)$ have multiplicity at least k. In particular, we may choose b_m and ρ_m, such that

$$\rho_m \leq \frac{2}{w_2^{\#}(b_m)}, \quad w_2^{\#}(b_m) \geq w_2^{\#}(a_m). \tag{3.9}$$

According to (3.6) and (3.7)-(3.9), we can get the following conclusion:
For any fixed constant $0 \leq \rho < \lambda - 1$, we have

$$\lim_{m\to\infty} b_m^{\rho}\rho_m = 0. \tag{3.10}$$

In the differential equation (3.6) we now replace z by $b_m + \rho_m\zeta$, then we obtain

$$(Q(w_2^{(k)}(b_m + \rho_m\zeta)))^{m_1 m_2} = \sum_{r\in I} a(b_m + \rho_m\zeta)^{m_2} a_r(b_m + \rho_m\zeta, g_m(\zeta))\rho_m^{-p(r)} M_r[g_m](\zeta).$$

where

$$Q(w_2^{(k)}(b_m + \rho_m\zeta)) = \rho_m^{-qk}\left[(g_m^{(k)})^q(\zeta) + \rho_m^k a_{q-1}(g_m^{(k)})^{q-1}(\zeta) + \cdots + \rho_m^{qk} a_1 g_m^{(k)}(\zeta)\right].$$

Namely

$$\left[(g_m^{(k)})^q(\zeta) + \rho_m^k a_{q-1}(g_m^{(k)})^{q-1}(\zeta) + \cdots + \rho_m^{qk} a_1 g_m^{(k)}(\zeta)\right]^{m_1 m_2} = \sum_{r\in I} \frac{a(b_m + \rho_m\zeta)^{m_2} a_r(b_m + \rho_m\zeta, g_m(\zeta))}{b_m^{a+\deg_{z,\infty} a_r}}\{b_m^{\frac{a+\deg_{z,\infty} a_r}{m_1 m_2 qk - p(r)}}\rho_m\}^{m_1 m_2 qk - p(r)} M_r[g_m](\zeta). \tag{3.11}$$

For every fixed $\zeta \in \mathbb{C}$, if ζ is not zero of $g(\zeta)$, for $m \to \infty$ and $0 \leq \rho = \frac{a+\deg_{z,\infty} a_r}{m_1 m_2 qk - p(r)} \leq \frac{a+\deg_{z,\infty} a}{m_1 m_2 qk - \deg P(w_2)} < \lambda - 1$ then we have $(g^{(k)})^{m_1 m_2} = 0$, which contradicts with all zeros of $g(\zeta)$ have multiplicity at least k. So $\lambda(w_2) \leq 1 + \frac{a+\deg_{z,\infty} a}{m_1 m_2 qk - \deg P(w_2)}$.

The proof of Theorem 1.9 is complete.

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Authors' contributions
JQ carried out the main part of this manuscript. YL and WY participated discussion and corrected the main theorem. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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