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Existence of positive solutions for eigenvalue problem of nonlinear fractional differential equations

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Abstract

In this article, by using the fixed point theorem, existence of positive solutions for eigenvalue problem of nonlinear fractional differential equations

$$\begin{cases} D^{\alpha}_{0+}u(t) + \lambda a(t)f(t, u(t)) = 0, \ 0 < t < 1, \\ u(0) = u(1) = 0 \end{cases}$$

is considered, where $1 < \alpha < 2$ is a real number, D^{α}_{0+} is the standard Riemann-Liouville derivate, λ is a positive parameter and $a(t) \in C([0, 1], [0, \infty))$, $f(t, u) \in C([0, 1] \times [0, \infty), [0, \infty))$.

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1 Introduction

Fractional differential equations have been of great interest recently. It is caused both by the intensive development of the theory of fractional calculus itself and by the applications of such constructions in various sciences such as physics, mechanics, chemistry, engineering, etc. For details, see [1-6] and references therein.

Recently, many results were obtained dealing with the existence and multiplicity of solutions of nonlinear fractional differential equations by the use of techniques of nonlinear analysis, see [7-22] and the reference therein. Bai and Lu [7] studied the existence of positive solutions of nonlinear fractional differential equation

$$\begin{cases} D^{\alpha}_{0+}u(t) + f(t, u(t)) = 0, \ 0 < t < 1, \\ u(0) = u(1) = 0, \end{cases} \tag{1.1}$$

where $1 < \alpha \leq 2$ is a real number, D^{α}_{0+} is the standard Riemann-Liouville differentiation, and $f : [0, 1] \times [0, \infty) \to [0, \infty)$ is continuous. They derived the corresponding Green function and obtained some properties as follows.

Proposition 1 The Green function $G(t, s)$ satisfies the following conditions:

(R1) $G(t, s) \in C([0,1] \times [0,1])$, and $G(t, s) > 0$ for $t, s \in (0, 1)$;

(R2) There exists a positive function $\gamma \in C(0, 1)$ such that

$$\min_{\frac{1}{4} \le t \le \frac{3}{4}} G(t,s) \ge \gamma(s) \max_{0 \le t \le 1} G(t,s) \ge \gamma(s) G(s,s), s \in (0,1), \tag{1.2}$$

where

$$G(t,s) = \begin{cases} \frac{[t(1-s)]^{\alpha-1} - (t-s)^{\alpha-1}}{\Gamma(\alpha)}, & 0 \le s \le t \le 1, \\ \frac{[t(1-s)]^{\alpha-1}}{\Gamma(\alpha)}, & 0 \le t \le s \le 1. \end{cases} \tag{1.3}$$

It is well known that the cone plays a very important role in applying the Green function in research area. In [7], the authors cannot acquire a positive constant taken instead of the role of positive function $\gamma(s)$ with $1 < \alpha < 2$ in (1.2). In [9], Jiang and Yuan obtained some new properties of the Green function and established a new cone. The results can be stated as follows.

Proposition 2 The Green function $G(t, s)$ defined by (1.3) has the following properties: $G(t, s) = G(1 - s, 1 - t)$ and

$$\frac{\alpha-1}{\Gamma(\alpha)} t^{\alpha-1}(1-t)(1-s)^{\alpha-1} s \le G(t,s) \le \frac{1}{\Gamma(\alpha)} t^{\alpha-1}(1-t)(1-s)^{\alpha-2} \quad \forall t,s \in (0,1). \tag{1.4}$$

Proposition 3 The function $G^*(t, s) := t^{2-\alpha} G(t, s)$ has the following properties:

$$q(t)\Phi(s) \le G^*(t,s) \le \Phi(s) \quad \forall t,s \in [0,1]. \tag{1.5}$$

where $q(t) = (\alpha - 1)t(1 - t), \Phi(s) = \frac{1}{\Gamma(\alpha)} s(1-s)^{\alpha-1}$.

The purpose of this article is to establish the existence of positive solutions for eigenvalue problem of nonlinear fractional differential equations

$$\begin{cases} D_{0+}^{\alpha} u(t) + \lambda a(t) f(t, u(t)) = 0, & 0 < t < 1, \\ u(0) = u(1) = 0 \end{cases} \tag{1.6}$$

where $1 < \alpha < 2$ is a real number, D_{0+}^{α} is the standard Riemann-Liouville derivate, λ is a positive parameter and $a(t) \in C([0,1], [0, \infty))$, $f(t, u) \in C([0,1] \times [0, \infty), [0, \infty))$.

2 The preliminary lemmas

For the convenience of the reader, we present here the necessary definitions from fractional calculus theory. These definitions can be found in the recent literature.

Definition 2.1 The fractional integral of order $\alpha > 0$ of a function $y : (0, \infty) \to R$ is given by

$$I_{0+}^{\alpha} y(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} y(s) ds$$

provided the right side is pointwise defined on $(0, \infty)$.

Definition 2.2 The fractional derivative of order $\alpha > 0$ of a function $y : (0, \infty) \to R$ is given by

$$D_{0+}^{\alpha} y(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_0^t \frac{y(s)}{(t-s)^{\alpha-n+1}} ds$$

where $n = [\alpha] + 1$, provided the right side is pointwise defined on $(0, \infty)$.

Lemma 2.1 Let α >0. If we assume $u \in C(0, 1) \cap L(0, 1)$, then the fractional differential equation

$$D_{0+}^{\alpha} u(t) = 0$$

has $u(t) = C_1 t^{\alpha-1} + C_2 t^{\alpha-2} + \ldots + C_N t^{\alpha-N}$, $C_i \in R$, $i = 1, 2, \ldots, N$, where N is the smallest integer greater than or equal to α, as unique solutions.

Lemma 2.2 [7] Assume that $u \llcorner C(0, 1) \cap L(0, 1)$ with a fractional derivative of order α >0 that belongs to $u \in C(0, 1) \cap L(0, 1)$. Then

$$I_{0+}^{\alpha} D_{0+}^{\alpha} u(t) = u(t) + C_1 t^{\alpha-1} + C_2 t^{\alpha-2} + \cdots + C_N t^{\alpha-N}$$

for some $C_i \in R$, $i = 1, 2, \ldots, N$.

Lemma 2.3 [7] Given $y \in C[0,1]$ and $1 < \alpha \leq 2$, the unique solution of

$$\begin{cases} D_{0+}^{\alpha} u(t) + y(t) = 0, & 0 < t < 1, \\ u(0) = u(1) = 0, \end{cases} \tag{2.1}$$

is

$$u(t) = \int_0^1 G(t, s) y(s) ds,$$

where

$$G(t, s) = \begin{cases} \frac{[t(1-s)]^{\alpha-1} - (t-s)^{\alpha-1}}{\Gamma(\alpha)}, & 0 \leq s \leq t \leq 1, \\ \frac{[t(1-s)]^{\alpha-1}}{\Gamma(\alpha)}, & 0 \leq t \leq s \leq 1. \end{cases} \tag{2.2}$$

Lemma 2.4 [10] Let K be a cone in Banach space E. Suppose that $T : \overline{K}_r \to K$ is a completely continuous operator.

(i) If there exists $u_0 \in K \setminus \{\theta\}$ such that $u - Tu \neq \mu u_0$ for any $u \in \partial K_r$ and $\mu \geq 0$, then $i(T, K_r, K) = 0$.

(ii) If $Tu \neq \mu u$ for any $u \in \partial K_r$ and $\mu \geq 1$, then $i(T, K_r, K) = 1$.

Lemma 2.5 [8] Let P be a cone in Banach space X. Suppose that $T : P \to P$ is a completely continuous operator. If there exists a bounded open set $\Omega(P)$ such that each solution of

$$u = \sigma T u, \quad u \in P, \sigma \in [0, 1]$$

satisfies $u \in \Omega(P)$, then the fixed point index $i(T, \Omega(P), P) = 1$.

3 The main results

Let $E = C[0,1]$ be endowed with the ordering $u \leq v$ if $u(t) \leq v(t)$ for all $t \in [0,1]$, and the maximum norm, $\|u\| = \max_{0 \leq t \leq 1} |u(t)|$. Define the cone $P \subset E$ by $P = \{u \in E | u(t) \geq 0\}$, and

$$K = \{u \in P | u(t) \geq q(t) ||u||\}.$$

where $q(t)$ is defined by (1.5).

It is easy to see that P and K are cones in E. For any $0 < r < R < +\infty$, let $K_r = \{u \in K | \, ||u|| < r\}$, $\partial K_r = \{u \in K | \, ||u|| = r\}$, $\overline{K}_r = \{u \in K | \, ||u|| \leq r\}$ *and* $\overline{K}_R \backslash K_r = \{u \in K \, | \, r \leq ||u|| \leq R\}$.

For convenience, we introduce the following notations

$$g_0 = \lim_{u\to 0^+}\frac{g(u)}{u},\quad g_\infty = \lim_{u\to +\infty}\frac{g(u)}{u},$$

$$\int_{\frac{1}{4}}^{\frac{3}{4}} G^*(\tau,s)a(s)q_1(s)ds = \max_{\frac{1}{4}\le t\le\frac{3}{4}}\int_{\frac{1}{4}}^{\frac{3}{4}} G^*(t,s)a(s)q_1(s)ds.$$

We assume the following conditions hold throughout the article:

(H1) $a(t) \in C([0, 1], [0, \infty))$, $a(t) \not\equiv 0$;

(H2) $f(t, u) \in C([0, 1] \times [0, \infty), [0, \infty))$, and there exist $g \in C([0, +\infty), [0, +\infty))$, $q_1, q_2 \in C((0, 1), (0, +\infty))$ such that

$$q_1(t)g(u) \le f(t, t^{\alpha-2}u) \le q_2(t)g(u),\quad t \in (0,1),\ u \in [0,+\infty),$$

where $\int_0^1 q_i(s)ds < +\infty, i = 1, 2$.

By similar arguments to Lemma 4.1 and Theorem 1.3 of [9], we obtain the following result.

Lemma 3.1 Assume that (H1)(H2) hold. Let

$$Tu(t) := \lambda \int_0^1 G^*(t,s)a(s)f(s, s^{\alpha-2}u(s))ds, \tag{3.1}$$

then $T : K \to K$ is completely continuous. Moreover, if u is a fixed point of T in $\overline{K}_R\backslash K_r$, then $y = t^{\alpha-2}u$ is a positive solution of BVP (1.6).

By similar arguments to Theorems 2 and 3 of [18], we obtain the following result.

Theorem 3.1 Assume that (H1)(H2) hold. Then, for each λ satisfying

$$\frac{16}{[(\alpha-1)\int_{\frac{1}{4}}^{\frac{3}{4}} G^*(\tau,s)a(s)q_1(s)ds]g_\infty} < \lambda < \frac{1}{\left(\int_0^1 \Phi(s)a(s)q_2(s)ds\right) g_0}, \tag{3.2}$$

there exists at least one positive solution of BVP (1.6) in K.

Theorem 3.2 Assume that (H1)(H2) hold. Then, for each λ satisfying

$$\frac{16}{[(\alpha-1)\int_{\frac{1}{4}}^{\frac{3}{4}} G^*(\tau,s)a(s)q_1(s)ds]g_0} < \lambda < \frac{1}{\left(\int_0^1 \Phi(s)a(s)q_2(s)ds\right) g_\infty}, \tag{3.3}$$

there exists at least one positive solution of BVP (1.6) in K.

Set

$$L_1u(t) := \int_0^1 G^*(t,s)a(s)q_1(s)u(s)ds, \tag{3.4}$$

$$L_2u(t) := \int_0^1 G^*(t,s)a(s)q_2(s)u(s)ds. \tag{3.5}$$

It is clear that L_1, L_2 is a completely continuous linear operator and $L_1(P) \subset K$, $L_2(P) \subset K$. By virtue of the Krein-Rutman theorem and Proposition 3, we have the following lemma.

Lemma 3.2 Assume that (H1)(H2) hold. Then the spectral radius $r(L_1) >0$, $r(L_2) >0$ and L_1, L_2, respectively, has a positive eigenfunction ϕ_1, ϕ_2 corresponding to its first eigenvalue $\lambda_1 = (r(L_1))^{-1}$, $\lambda_2 = (r(L_1))^{-1}$, that is $\phi_1 = \lambda_1 L_1\phi_1$, $\phi_2 = \lambda_2 L_2\phi_2$.

Obviously, $L_1\phi_1 \in K$, $L_2\phi_2 \in K$, so $\phi_1 \in K$, $\phi_2 \in K$.

In the following, we will obtain some existence results under some conditions concerning the first eigenvalue with respect to linear operator L_1, L_2.

Assume that

(H3)$\frac{\lambda_1}{g_0} < \lambda < \frac{\lambda_2}{g_\infty}$;

(H3)$\frac{\lambda_1}{g_\infty} < \lambda < \frac{\lambda_2}{g_0}$.

Theorem 3.3 Assume (H1)(H2)(H3) hold. Then the BVP (1.6) has at least one positive solution.

Proof. First, by Lemma 3.1, we know that $T : K \to K$ is completely continuous.

By (H3), we have $g_0 > \frac{\lambda_1}{\lambda}$, there exists $r_1 > 0$ such that

$$g(u) \geq \frac{\lambda_1}{\lambda} u, \quad 0 < u \leq r_1.$$

Then for $u \in \partial K_{r_1}$, we have

$$\begin{aligned}(Tu)(t) &= \lambda \int_0^1 G^*(t,s)a(s)f(s, s^{\alpha-2}u(s))ds \\ &\geq \lambda \int_0^1 G^*(t,s)a(s)q_1(s)g(u(s))ds \\ &\geq \lambda_1 \int_0^1 G^*(t,s)a(s)q_1(s)u(s)ds = \lambda_1(L_1u)(t).\end{aligned}$$

By Lemma 3.2, $\phi_1 = \lambda_1 L_1 \phi_1$. We may suppose that T has no fixed points on ∂K_{r_1} (otherwise, the proof is finished). Now we show that

$$u - Tu \neq \mu\varphi_1, \quad u \in \partial K_{r1}, \quad \mu \geq 0. \tag{3.6}$$

Assume by contradicts that there exist $u \in \partial K_{r_1}$ and $\mu_1 \geq 0$ such that $u_1 - Tu_1 = \mu_1\phi_1$, then $\mu_1 > 0$ and $u_1 = Tu_1 + \mu_1\phi_1 \geq \mu_1\phi_1$. Let $\bar{\mu} = \sup\{\mu \mid u_1 \geq \mu\varphi_1\}$, then $\bar{\mu} \geq \mu_1, u_1 \geq \bar{\mu}\varphi_1$ and $Tu_1 \geq \lambda_1\bar{\mu}L_1\varphi_1 = \bar{\mu}\varphi_1$. Thus,

$$u_1 = Tu_1 + \mu_1\varphi_1 \geq \bar{\mu}\varphi_1 + \mu_1\varphi_1 = (\bar{\mu} + \mu_1)\varphi_1,$$

which contradicts the definition of $\bar{\mu}$ So (3.6) is true and by lemma 2.4 we have

$$i(T, K_{r_1}, K) = 0. \tag{3.7}$$

On the other hand, by (H3), we have $g_\infty < \frac{\lambda 2}{\lambda}$, there exists $0 < \sigma < 1$ and $r_2 > r_1 > 0$ such that

$$g(u) \leq \sigma \frac{\lambda_2}{\lambda} u, \quad u \geq r_2.$$

Let $L\phi = \sigma\lambda_2 L_2\phi$, $\phi \in C[0,1]$. Then $L : C[0,1] \to C[0,1]$ is a bounded linear operator and $L(K) \subset K$. Denote

$$M = (\max_{0 \leq t,s \leq 1} G^*(t,s)) \sup_{\varphi \in \partial K_{r_2}} \int_0^1 \lambda a(s)q_2(s)g(\varphi(s))ds.$$

It is clear that $M <+\infty$ Let

$$W = \{\varphi \in K | \varphi = \mu T\varphi,\ 0 \le \mu \le 1\}.$$

In the following, we prove that W is bounded.

For any $\phi \in W$, set $\bar{\varphi}(t) = \min\{\varphi(t), r_2\}$ and denote $E(\phi) = \{t \in [0,1] | \phi(t) > r_2\}$, then

$$\begin{aligned}
\varphi(t) = \mu(T\varphi)(t) \le (T\varphi)(t) &= \lambda \int_0^1 G^*(t,s)a(s)f(s, s^{\alpha-2}\varphi(s))ds \\
&\le \lambda \int_0^1 G^*(t,s)a(s)q_2(s)g(\varphi(s))ds \\
&= \lambda \int_{E(\varphi)} G^*(t,s)a(s)q_2(s)g(\varphi(s))ds + \lambda \int_{[0,1]\setminus E(\varphi)} G^*(t,s)a(s)q_2(s)g(\bar{\varphi}(s))ds \\
&\le \sigma\lambda_2 \int_0^1 G^*(t,s)a(s)q_2(s)\varphi(s)ds + \lambda \int_0^1 G^*(t,s)a(s)q_2(s)g(\bar{\varphi}(s))ds \\
&\le (L\varphi)(t) + M,\ t \in [0,1].
\end{aligned}$$

Thus $((I - L)\phi)(t) \le M$, $t \in [0,1]$: Since λ_2 is the first eigenvalue of L_2 and $0 < \sigma < 1$, the first eigenvalue of L, $(r(L))^{-1} > 1$. Therefore, the inverse operator $(I - L)^{-1}$ exists and

$$(I - L)^{-1} = I + L + L^2 + \cdots + L^n \cdots.$$

It follows from $L(K) \subset K$ that $(I - L)^{-1}(K) \subset K$. So we have $\phi(t) \le (I - L)^{-1}M$, $t \in [0,1]$ and W is bounded.

Choose $r_3 > \max\{r_2, \|(I - L)^{-1}M\|\}$. Then by lemma 2.5, we have

$$i(T, K_{r_3}, K) = 1. \tag{3.8}$$

By (3.7) and (3.8), one has

$$i(T, K_{r_3}\setminus\bar{K}_{r_1}, K) = i(T, K_{r_3}, K) - i(T, K_{r_1}, K) = 1.$$

Then T has at least one fixed point on $K_{r_3}\setminus\bar{K}_{r_1}$ By Lemma 3.1, this means that problem (1.6) has at least one positive solution. The proof is complete.

Theorem 3.4 Assume (H1)(H2)(H4) hold. Then the BVP (1.6) has at least one positive solution.

Theorem 3.5 Assume (H1)(H2) hold and $\frac{\lambda_1}{\lambda} < g_0 \le \infty, \frac{\lambda_1}{\lambda} < g_\infty \le \infty$. Moreover, the following condition holds:

(H5) there exists a $p > 0$ such that $0 \le u \le p$ implies $g(u) < M_1 p$, where $M_1 = [\lambda \int_0^1 \Phi(s)a(s)q_2(s)ds]^{-1}$.

Then the BVP (1.6) has at least two positive solution.

Theorem 3.6 Assume (H1)(H2) hold and $0 \le g_0 < \frac{\lambda_2}{\lambda}$, $0 \le g_\infty < \frac{\lambda_2}{\lambda}$. Moreover, the following condition holds:

(H6) there exists a $p > 0$ such that $\frac{p(\alpha-1)}{16} \le u \le p$ implies $g(u) > M_2 p$, where $M_2 = [\lambda \int_{\frac{1}{4}}^{\frac{3}{4}} G^*(\frac{1}{2}, s)a(s)q_1(s)ds]^{-1}$. Then the BVP (1.6) has at least two positive solution.

Remark 3.1 Theorems 3.3, 3.4 extend and improve Theorem 1.5 in [9], Theorems 3.5 and 3.6 extend and improve Theorems 1.3 and 1.4 in [9], respectively.

4 Example

Consider the boundary value problem

$$\begin{cases} D_{0+}^{\alpha}u(t) + \lambda\mu(u^a(t) + u^b(t)) = 0, \quad 0 < t < 1,\ 0 < a < 1 < b < \frac{1}{2-\alpha}, 1 < \alpha < 2, \\ u(0) = u(1) = 0. \end{cases} \tag{3.9}$$

Then (3.9) have at least two positive solutions u_1 and u_2, for each $0 < \mu < \mu^*$, where μ^* is some positive constant.

Proof. We will apply Theorem 3.5. To this end we take $a(t) \equiv 1$, $f(t, u) = \mu(u^a(t) + u^b(t))$, then $f(t, t^{\alpha-2}y) = \mu(t^{a(\alpha-2)}y^a(t) + t^{b(\alpha-2)yb}(t))$.

Let $q_1(t) = t^{a(\alpha-2)}$, $q_2(t) = t^{b(\alpha-2)}$ and $g(y) = \mu(y^a + y^b)$, then $q_1(t)g(y) \le f(t, t^{\alpha-2}y) \le q_2(t)g(y)$ and $q_1, q_2 \in L^1(0, 1)$, $g \in C([0, +\infty), [0, +\infty))$. Thus (H1)(H2)is satisfied, and it is to see that $\frac{\lambda_1}{\lambda} < g_0 \le \infty, \frac{\lambda_1}{\lambda} < g_\infty \le \infty$.

Since

$$\begin{aligned} \frac{1}{\Gamma(\alpha)}\int_0^1 s(1-s)^{\alpha-1}q_2(s)ds &= \frac{1}{\Gamma(\alpha)}\int_0^1 s(1-s)^{\alpha-1}s^{b(\alpha-2)}ds \\ &= \frac{1}{\Gamma(\alpha)}\int_0^1 s^{(2+b(\alpha-2))-1}(1-s)^{\alpha-1}ds \\ &= \frac{1}{\Gamma(\alpha)}\frac{\Gamma(2+b(\alpha-2))\Gamma(\alpha)}{\Gamma(2+b(\alpha-2)+\alpha)} \\ &= \frac{\Gamma(2+b(\alpha-2))}{\Gamma(2+b(\alpha-2)+\alpha)}, \end{aligned}$$

then $M_1 = (\lambda\frac{\Gamma(2+b(\alpha-2))}{\Gamma(2+b(\alpha-2)+\alpha)})^{-1}$.

Let

$$\mu^* = F(p) = \sup_{y>0}\{F(y) : F(y) = M_1\frac{y}{y^a + y^b}\} = F((\frac{1-a}{b-1})^{\frac{1}{b-a}}),$$

where $p = (\frac{1-a}{b-1})^{\frac{1}{b-a}}$.

Then, for $\mu < \mu^*$, we have

$$g(y) = \mu(y^a + y^b) \le \mu(p^a + p^b) < M_1 p,\ 0 \le y \le p,$$

thus (H5) holds. Therefore, (3.9) have at least two positive solutions u_1 and u_2, for each $0 < \mu < \mu^*$.

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Authors' contributions

The authors declare that the study was realized in collaboration with the same responsibility. All authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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