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Disturbance observer based H_∞ control for flexible spacecraft with time-varying input delay

Ran Zhang[1], Tao Li[2*] and Lei Guo[1*]

[*]Correspondence:
litaojia79@yahoo.com.cn;
lguo66@yahoo.com.cn
[1]School of Instrumentation Science and Opto-Electronics Engineering, Beihang University, Beijing, 100191, China
[2]School of Information and Control, Nanjing University of Information Science and Technology, Nanjing, 210044, China

Abstract

In this paper, a composite disturbance-observer-based control (DOBC) and H_∞ control scheme is applied to solve the spacecraft attitude control with time-varying input delay. Compared with some existing results, distinct features of the proposed method are that the delay-dependent disturbance observer and H_∞ controller are used to estimate and compensate the main disturbance caused by flexible appendages and to attenuate exogenous bounded disturbances with attenuation level, respectively. The proposed design is obtained by combining the augmented Lyapunov functional with linear matrix inequality technique. The effectiveness of the proposed design method is illustrated via a numerical example.

Keywords: flexible spacecraft; disturbance observer; time-varying delay; H_∞ control; LMI

Introduction

Flexible spacecraft plays an important role in communication, remote sensing and a variety of space related research works [1–4]. However, the design of these appendages often involves the need for large, complex and light-weight space structures to achieve increased functionality at a reduced launch cost [5]. During the control of the rigid body attitude, the unwanted excitation of flexible modes, together with other external disturbances, measurement and actuator error, may degrade the performance of attitude control systems [6]. On the other hand, spacecrafts usually operate in the presence of various disturbances, which include radiation torque, gravitational torque, aerodynamic torque and non-environmental torques, *etc.* [7]. The problem of disturbance rejection is particularly pronounced in the case of low-earth-orbiting satellites that operate in altitude ranges where their dynamics are substantially affected by most of the disturbances mentioned above [2, 8]. In modern control theory, anti-disturbance control methodologies can be divided into two main types. One is the disturbance attenuation method such as H_∞ control [9–11]. The other is the disturbance rejection method which may compensate the disturbance via the compensator [12–14]. However, these approaches only deal with one type of disturbance. In practice, together with the rapid development of sensor and data processing technologies, the disturbances or noise from different sources (*e.g.*, sensor and actuator noise, friction, vibration) can be characterized by different mathematical models. Also, disturbance can represent the unmodeled dynamics and system uncertainties.

For the case of multiple disturbances, a composite hierarchical anti-disturbance control was proposed [7, 15] to guarantee the simultaneous disturbance attenuation and rejection performance.

In practical application, input delay always exists in a flexible spacecraft due to the physical structure and energy consumption of the actuators. Although it is not the most important factor to affect the attitude control, it still leads to substantial performance deterioration and even to instability of the system [16, 17]. Hence, anti-disturbance control algorithms for such systems that explicitly take input time delay into account are of practical interest. Up to now, the issue of anti-disturbance control problems for flexible spacecraft subject to both disturbance and input time delay has not been fully investigated and remains to be open and challenging.

Motivated by the preceding discussion, in this paper, a composite attitude controller design approach is designed for a flexible spacecraft based on DOBC and H_∞ state-feedback control. By constructing an augmented Lyapunov functional with slack variables, new delay-dependent DOBC and H_∞ controller are obtained in terms of linear inequality matrices. The resultant DOBC can reject the effect of vibrations from flexible appendages, and H_∞ state-feedback control can attenuate the influence of the norm bounded disturbances. Moreover, compared with the existing results [6, 7, 16], (I) Input time delay is considered in the designed controller, which is more practical than the methods in [6, 7]; (II) An augmented Lyponov function is used to design H_∞ controller, which may lead to a more relaxed design than the methods in [16]. Finally, a numerical example is shown to demonstrate the good performance of our method.

Notation: Throughout this paper, R^n denotes the n-dimensional Euclidean space; the space of square-integrable vector functions over $[0,\infty)$ is denoted by $l_2[0,\infty)$; the superscripts '$\top$' and '-1' stand for matrix transposition and matrix inverse, respectively; $P > (\geq 0)$ means that P is real symmetric and positive definite (semidefinite). In symmetric block matrices or complex matrix expressions, $\mathrm{diag}\{\cdots\}$ stands for a block-diagonal matrix, and $*$ represents a term that is induced by symmetry. For a vector $\nu(t)$, its norm is given by $\|\nu(t)\|_2^2 = \int_0^\infty \nu^\top(t)\nu(t)\,dt$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for related algebraic operations.

Problem formulation and preliminaries

Similarly to the references [7, 16], the single-axis model can be derived from the nonlinear attitude dynamics of the flexible spacecraft. In this paper, the problem is simplified and only considers the single-axis rotational manoeuvre. It is assumed that this model includes one rigid body and one flexible appendage, and the relative elastic spacecraft model is described as

$$\begin{cases} J\ddot{\theta}(t) + F\ddot{\eta}(t) = u(t) + w_1(t), \\ \ddot{\eta}(t) + C_m\dot{\eta}(t) + \Lambda\eta(t) + F^\top\ddot{\theta}(t) = 0, \end{cases} \tag{1}$$

where $\theta(t)$ is the attitude angle, J is the moment of inertia of the spacecraft, $\eta(t)$ is the flexible modal coordinate, F is the rigid-elastic coupling matrix, $u(t)$ is the control torque generated by the reaction wheels that are installed in the flexible spacecraft. $w_1(t)$ represents the merged disturbance torque including the space environmental torques, unmodeled uncertainties and noises from sensors and actuators and belongs to $l_2[0,1)$ and

$\|w_1(t)\| \leq \delta_1$. $C_m = \operatorname{diag}\{2\xi_1\omega_1, \dots, 2\xi_n\omega_n\}$ is a modal damping matrix, where ξ_i $(i = 1, \dots, n)$ is the damping ratio and ω_i $(i = 1, \dots, n)$ is the modal frequency. $\Lambda = \operatorname{diag}\{\omega_1^2, \dots, \omega_n^2\}$ is a stiffness matrix. Since vibration energy is concentrated in low frequency modes in a flexible structure, its reduced order model can be obtained by modal truncation. In this paper, only the first two bending modes are taken into account. Then we can get

$$\left(J - FF^\top\right)\ddot{\theta} = F\left(C_m\dot{\eta}(t) + \Lambda\eta(t)\right) + u(t) + w_1(t). \tag{2}$$

Denote $x(t) = [\theta(t)\ \dot{\theta}(t)]^\top$, then (2) can be transformed into the state-space form

$$\dot{x}(t) = Ax(t) + Bu(t) + Bw_1(t) + Bw_2(t), \tag{3}$$

where $w_2(t) = F(C_m\dot{\eta}(t) + \Lambda\eta(t))$ is as the disturbance due to elastic vibration of the flexible appendages, and where $\dot{w}_2(t)$ is supposed to belong to $l_2[0, 1)$ and $\|\dot{w}_2(t)\| \leq \delta_2$ and

$$A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ (J - FF^\top)^{-1} \end{bmatrix}.$$

According to system (3), we formulate the disturbance observer as

$$\begin{cases} \dot{p}(t) = -LB(p(t) + Lx(t)) - L(Ax(t) + Bu(t)), \\ \hat{w}_2(t) = p(t) + Lx(t), \end{cases} \tag{4}$$

where $p(t)$ is an auxiliary variable, L is the gain of the observer to be designed. The estimation error of the disturbance observer is defined as $e(t) = w_2(t) - \hat{w}_2(t)$. Then we have

$$\dot{e}(t) = \dot{w}_2(t) - LBe(t) - LBw_1(t). \tag{5}$$

The first step of DOBC framework is to estimate the disturbance via the disturbance observer. According to the practical situation of the flexible spacecraft, we should design an appropriate L such that $e(t) \to 0$. In the DOBC scheme, a general controller including time delay is constructed as

$$u\left(t - d(t)\right) = -\hat{w}_2(t) + Kx\left(t - d(t)\right), \tag{6}$$

where K is the gain of controller and needs to be designed. $d(t)$ is time-varying delay and satisfies $0 \leq d(t) \leq d_M$ and $\dot{d}(t) \leq \mu \leq 1$. It can be seen that the composite hierarchical controller consists of two parts: the inner loop is the disturbance observer and feedforward compensation, and the outside loop is the attitude controller. Thus, the composite controller can effectively control the spacecraft attitude and attenuate disturbances [7]. Then, with the control law (6), the augmented system can be expressed as follows:

$$\dot{z}(t) = \bar{A}z(t) + \bar{A}_d z\left(t - d(t)\right) + \bar{B}w(t) \tag{7}$$

and the reference output is chosen as

$$y(t) = Cz(t) + C_d w(t), \tag{8}$$

where

$$z(t)=\begin{bmatrix}x(t)\\ e(t)\end{bmatrix},\qquad \bar{A}=\begin{bmatrix}A & B\\ 0 & -LB\end{bmatrix},\qquad \bar{A}_d=\begin{bmatrix}BK & 0\\ 0 & 0\end{bmatrix},$$
$$\bar{B}=\begin{bmatrix}B & 0\\ -LB & I\end{bmatrix},\qquad w(t)=\begin{bmatrix}w_1(t)\\ \dot{w}_2(t)\end{bmatrix},$$
$$C=[C_1,C_2],\qquad C_d=[C_{d1},C_{d2}].$$

Meanwhile, it is noted that $w(t)$ belongs to $l_2[0,\infty)$ and $\|w(t)\|\leq\delta$, where $\delta=\max\{\delta_1,\delta_2\}$.
For the system described by (7)-(8), the objectives of this paper are as follows:

- Designing the observer gain L and controller gain K makes the closed-loop system (7)-(8) asymptotically stable;
- The performance of system (7)-(8) satisfies $\|y(t)\|_2<\gamma\|w(t)\|_2$ for any nonzero $w(t)\in l_2[0,\infty)$ under the zero initial condition.

To obtain our main results, we need the following lemma.

Lemma 1 [18] *For any functions* $W_1(t),W_2(t)\in R$ *satisfying* $W_1(t)\geq 0$, $W_2(t)\geq 0$, *if* $0\leq d_m\leq d(t)\leq d_M$, *then the following inequality is true*:

$$\frac{W_1(t)}{d(t)-d_m}+\frac{W_2(t)}{d_M-d(t)}\geq\min\left\{\frac{3W_1(t)+W_2(t)}{d_M-d_m},\frac{W_1(t)+3W_2(t)}{d_M-d_m}\right\}.$$

Main results

In this section, we consider the augmented system (7)-(8). We give the design method based on LMI to compute the controller gain and the observer gain simultaneously.

Theorem 1 *Given scalars* $\gamma>0$, $\mu\leq 1$. *For any delay* $d(t)$ *satisfying* $0<d(t)\leq d_M$ *and* $\dot{d}(t)\leq\mu$, *system* (7)-(8) *is asymptotically stable and satisfies* $\|y(t)\|_2<\gamma\|w(t)\|_2$ *for any nonzero* $w(t)\in l_2[0,\infty)$ *under the zero initial condition if there exist matrices* $P_1>0$, $Q_1>0$, $Q_2>0$, $R>0$, P_2 *and* P_3 *such that the following inequalities hold*:

$$\Omega=\begin{bmatrix}\Omega_{11}-\frac{3}{d_M}R & \Omega_{12} & \Omega_{13}+\frac{3}{d_M}R & 0 & P_2^\top\bar{B} & C^\top\\ * & \Omega_{22} & P_3^\top\bar{A}_d & 0 & P_3^\top\bar{B} & 0\\ * & * & \Omega_{33}-\frac{4}{d_M}R & \frac{1}{d_M}R & 0 & 0\\ * & * & * & -Q_1-\frac{1}{d_M}R & 0 & 0\\ * & * & * & * & -\gamma^2 I & C_d^\top\\ * & * & * & * & * & -I\end{bmatrix}<0, \tag{9a}$$

$$\bar{\Omega}=\begin{bmatrix}\Omega_{11}-\frac{1}{d_M}R & \Omega_{12} & \Omega_{13}+\frac{1}{d_M}R & 0 & P_2^\top\bar{B} & C^\top\\ * & \Omega_{22} & P_3^\top\bar{A}_d & 0 & P_3^\top\bar{B} & 0\\ * & * & \Omega_{33}-\frac{4}{d_M}R & \frac{3}{d_M}R & 0 & 0\\ * & * & * & -Q_1-\frac{3}{d_M}R & 0 & 0\\ * & * & * & * & -\gamma^2 I & C_d^\top\\ * & * & * & * & * & -I\end{bmatrix}<0, \tag{9b}$$

where

$$\Omega_{11} = P_2^\top \bar{A} + \bar{A}^\top P_2 + Q_1 + Q_2,$$

$$\Omega_{12} = P_1 - P_2^\top + \bar{A}^\top P_3,$$

$$\Omega_{13} = P_2^\top \bar{A}_d,$$

$$\Omega_{22} = -P_3 - P_3^\top + d_M R,$$

$$\Omega_{33} = -(1-\mu)Q_2.$$

Proof The first step is to analyze the asymptotic stability of system (7). Consider system (7) in the absence of $w(t)$, that is,

$$\dot{z}(t) = \bar{A}z(t) + \bar{A}_d z\bigl(t - d(t)\bigr). \tag{10}$$

Choose the following Lyapunov-Krasovskii functional:

$$\begin{aligned} V(t) &= \xi^\top(t)EP\xi(t) + \int_{t-d_M}^{t} z^\top(s)Q_1 z(s)\,ds + \int_{t-d(t)}^{t} z^\top(s)Q_2 z(s)\,ds \\ &\quad + \int_{-d_M}^{0}\int_{t+\vartheta}^{t} \dot{z}^\top(s)R\dot{z}(s)\,ds\,d\vartheta, \end{aligned} \tag{11}$$

where

$$E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \qquad P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \qquad \xi(t) = \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix},$$

$$P_1 > 0, \qquad Q_1 > 0, \qquad Q_2 > 0, \qquad R > 0.$$

Then along the solution of the system in (7), the time derivative of $V(t)$ is given by

$$\begin{aligned} \dot{V}(t) &= 2\xi^\top(t)P^\top \begin{bmatrix} \dot{z}(t) \\ -\dot{z}(t) + \bar{A}z(t) + \bar{A}_d z(t-d(t)) \end{bmatrix} \\ &\quad + z^\top(t)(Q_1+Q_2)z(t) - z^\top(t-d_M)Q_1 z(t-d_M) \\ &\quad - (1-\mu)z^\top\bigl(t-d(t)\bigr)Q_2 z\bigl(t-d(t)\bigr) + d_M\dot{x}^\top(t)R\dot{x}(t) - \int_{t-d_M}^{t} \dot{z}^\top(s)R\dot{z}(s)\,ds. \end{aligned} \tag{12}$$

It is noted that

$$\begin{aligned} -\int_{t-d_M}^{t} \dot{z}^\top(s)R\dot{z}(s)\,ds &= -\int_{t-d_M}^{t-d(t)} \dot{z}^\top(s)R\dot{z}(s)\,ds - \int_{t-d(t)}^{t} \dot{z}^\top(s)R\dot{z}(s)\,ds \\ &\le -\frac{1}{d_M - d(t)}\left(\int_{t-d_M}^{t-d(t)} \dot{z}(s)\,ds\right)^\top R\left(\int_{t-d_M}^{t-d(t)} \dot{z}(s)\,ds\right) \\ &\quad - \frac{1}{d(t)}\left(\int_{t-d(t)}^{t} \dot{z}(s)\,ds\right)^\top R\left(\int_{t-d(t)}^{t} \dot{z}(s)\,ds\right). \end{aligned} \tag{13}$$

Letting $W_1(t) = (\int_{t-d(t)}^{t} \dot{z}(s)\,ds)^\top R(\int_{t-d(t)}^{t} \dot{z}(s)\,ds)$ and $W_2(t) = (\int_{t-d_M}^{t-d(t)} \dot{z}(s)\,ds)^\top R(\int_{t-d_M}^{t-d(t)} \dot{z}(s)\,ds)$, from Lemma 1, one obtains

$$-\frac{W_1(t)}{d(t)} - \frac{W_2(t)}{d_M - d(t)} \le \max\left\{-\frac{3W_1(t) + W_2(t)}{d_M}, -\frac{W_1(t) + 3W_2(t)}{d_M}\right\}, \tag{14}$$

where

$$\begin{aligned} &-\frac{3W_1(t) + W_2(t)}{d_M} \\ &\quad= \begin{bmatrix} z(t) \\ z(t-d(t)) \\ z(t-d_M) \end{bmatrix}^\top \begin{bmatrix} -\frac{3}{d_M}R & \frac{3}{d_M}R & 0 \\ \frac{3}{d_M}R & -\frac{4}{d_M}R & \frac{1}{d_M}R \\ 0 & \frac{1}{d_M}R & -\frac{1}{d_M}R \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-d(t)) \\ z(t-d_M) \end{bmatrix}, \end{aligned} \tag{15}$$

$$\begin{aligned} &-\frac{W_1(t) + 3W_2(t)}{d_M} \\ &\quad= \begin{bmatrix} z(t) \\ z(t-d(t)) \\ z(t-d_M) \end{bmatrix}^\top \begin{bmatrix} -\frac{1}{d_M}R & \frac{1}{d_M}R & 0 \\ \frac{1}{d_M}R & -\frac{4}{d_M}R & \frac{3}{d_M}R \\ 0 & \frac{3}{d_M}R & -\frac{3}{d_M}R \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-d(t)) \\ z(t-d_M) \end{bmatrix}. \end{aligned} \tag{16}$$

According to (12), (13), (14), it is clear that

$$\begin{aligned} \dot{V}(t) &\le \eta^\top(t) \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & 0 \\ * & \Omega_{22} & \Omega_{23} & 0 \\ * & * & \Omega_{33} & 0 \\ * & * & * & -Q_1 \end{bmatrix} \eta(t) \\ &\quad + \max\left\{-\frac{3W_1(t) + W_2(t)}{d_M}, -\frac{W_1(t) + 3W_2(t)}{d_M}\right\}, \end{aligned} \tag{17}$$

where $\eta(t) = [z^\top(t)\ \dot{z}^\top(t)\ z^\top(t-d(t))\ z^\top(t-d_M)]^\top$ and Ω_{11}, Ω_{12}, Ω_{13}, Ω_{22}, Ω_{33} are defined in (9a)-(9b). Applying the Schur complement to (9a)-(9b) gives

$$\eta^\top(t) \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & 0 \\ * & \Omega_{22} & \Omega_{23} & 0 \\ * & * & \Omega_{33} & 0 \\ * & * & * & -Q_1 \end{bmatrix} \eta(t) - \frac{3W_1(t) + W_2(t)}{d_M} < 0 \tag{18}$$

and

$$\eta^\top(t) \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & 0 \\ * & \Omega_{22} & \Omega_{23} & 0 \\ * & * & \Omega_{33} & 0 \\ * & * & * & -Q_1 \end{bmatrix} \eta(t) - \frac{W_1(t) + 3W_2(t)}{d_M} < 0, \tag{19}$$

which implies $\dot{V}(t) < 0$. Hence, system (7) is asymptotically stable. Next, we shall establish the H_∞ performance of the time-delay system (7)-(8) under the zero initial condition. Let

$$J(t) = \int_0^t \left[y^\top(s)y(s) - \gamma^2 w^\top(s)w(s)\right] ds. \tag{20}$$

It can be shown that for any nonzero $w(t) \in l_2[0,\infty)$ and $t > 0$,

$$J(t) \le \int_0^t \big[y^\top(s)y(s) - \gamma^2 w^\top(s)w(s) + \dot{V}(s)\big]\,ds.$$

It is noted that

$$y^\top(s)y(s) - \gamma^2 w^\top(s)w(s) = \phi^\top(t)\big[\operatorname{diag}\{0,0,0,0,-\gamma^2 I\} + Y^\top Y\big]\phi(t) \tag{21}$$

and

$$\begin{aligned}\dot{V}(s) \le \phi^\top(s)&\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & 0 & P_2^\top \bar{B} \\ * & \Omega_{22} & \Omega_{23} & 0 & P_3^\top \bar{B} \\ * & * & \Omega_{33} & 0 & 0 \\ * & * & * & -Q_1 & 0 \\ * & * & * & * & 0 \end{bmatrix}\phi(s) \\ &+ \max\left\{-\frac{3W_1(s) + W_2(s)}{d_M}, -\frac{W_1(s) + 3W_2(s)}{d_M}\right\},\end{aligned} \tag{22}$$

where $\phi(t) = [\eta(t), w(t)]$ and $Y = [C\ 0\ 0\ 0\ C_d]$. According to (21) and (22), $J(t) < 0$ follows from (9a)-(9b), which implies that $\|y(t)\|_2 < \gamma\|w(t)\|_2$ holds for any nonzero $w(t) \in l_2[0,\infty)$. □

In Theorem 1, a new Lyapunov-Krasovskii functional is constructed by employing slack variables P_2 and P_3. It is noted that P_2 and P_3 are useless for reducing the conservatism of stability conditions in [19]. However, they can provide a more relaxed design of the H_∞ controller later on since P_2 need only be an invertible matrix rather than a positive definite matrix.

On the basis of Theorem 1, we will present a design method of the disturbance observer based H_∞ controller in the following.

Theorem 2 *Given scalars $\gamma > 0$, $\mu \le 1$, λ_1 and λ_2. For any delay $d(t)$ satisfying $0 < d(t) \le d_M$, system (7) is asymptotically stable and satisfies $\|y(t)\|_2 < \gamma\|w(t)\|_2$ for any nonzero $w(t) \in l_2[0,\infty)$ under the zero initial condition if there exist matrices $\hat{P}_{11} > 0$, $P_{12} > 0$, $\hat{Q}_{11} > 0$, $\hat{Q}_{21} > 0$, $Q_{12} > 0$, $Q_{22} > 0$, $\hat{R}_1 > 0$, $R_2 > 0$ and invertible matrix $\bar{P}_{21}$ and any matrices M, N, P_{22} such that the following inequalities hold*:

$$\begin{bmatrix} \hat{\Pi}_{111} & B & \hat{\Pi}_{13} & 0 & BM + \frac{3}{d_M}\hat{R}_1 & 0 & 0 & 0 & B & 0 & \hat{P}_{21}^\top C_1^\top \\ * & \Pi_{221} & \lambda_1 B^\top & \Pi_{24} & 0 & 0 & 0 & 0 & NB & P_{22}^\top & C_2^\top \\ * & * & \hat{\Pi}_{33} & 0 & \lambda_1 BM + \frac{3}{d_M}\hat{R}_1 & 0 & 0 & 0 & \lambda_1 B & 0 & 0 \\ * & * & * & \hat{\Pi}_{44} & 0 & 0 & 0 & 0 & -\lambda_2 NB & \lambda_2 P_{22}^\top & 0 \\ * & * & * & * & \hat{\Pi}_{55} & 0 & \frac{1}{d_M}\hat{R}_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \hat{\Pi}_{66} & 0 & \frac{1}{d_M}R_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & \hat{\Pi}_{771} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \hat{\Pi}_{881} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\gamma^2 I & 0 & C_{d1}^\top \\ * & * & * & * & * & * & * & * & * & -\gamma^2 I & C_{d2}^\top \\ * & * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{23a}$$

$$\begin{bmatrix} \hat{\Pi}_{112} & B & \hat{\Pi}_{13} & 0 & BM + \frac{1}{d_M}\hat{R}_1 & 0 & 0 & 0 & B & 0 & \hat{P}_{21}^\top C_1^\top \\ * & \hat{\Pi}_{222} & \lambda_1 B^\top & \Pi_{24} & 0 & 0 & 0 & 0 & NB & P_{22}^\top & C_2^\top \\ * & * & \hat{\Pi}_{33} & 0 & \lambda_1 BM + \frac{1}{d_M}\hat{R}_1 & 0 & 0 & 0 & \lambda_1 B & 0 & 0 \\ * & * & * & \hat{\Pi}_{44} & 0 & 0 & 0 & 0 & -\lambda_2 NB & \lambda_2 P_{22}^\top & 0 \\ * & * & * & * & \hat{\Pi}_{55} & 0 & \frac{3}{d_M}\hat{R}_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \hat{\Pi}_{66} & 0 & \frac{3}{d_M}R_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & \hat{\Pi}_{772} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \hat{\Pi}_{882} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\gamma^2 I & 0 & C_{d1}^\top \\ * & * & * & * & * & * & * & * & * & -\gamma^2 I & C_{d2}^\top \\ * & * & * & * & * & * & * & * & * & * & -I \end{bmatrix}$$

$$< 0, \tag{23b}$$

where

$$\hat{\Pi}_{111} = A\hat{P}_{21} + \hat{P}_{21}^\top A^\top + \hat{Q}_{11} + \hat{Q}_{21} - \frac{3}{d_M}\hat{R}_1,$$

$$\hat{\Pi}_{112} = A\hat{P}_{21} + \hat{P}_{21}^\top A^\top + \hat{Q}_{11} + \hat{Q}_{21} - \frac{1}{d_M}\hat{R}_1,$$

$$\hat{\Pi}_{13} = \hat{P}_{11} - \hat{P}_{21} + \lambda_1 \hat{P}_{21}^\top A^\top,$$

$$\hat{\Pi}_{221} = -NB - B^\top N^\top + Q_{12} + Q_{22} - \frac{3}{d_M}R_2,$$

$$\hat{\Pi}_{222} = -NB - B^\top N^\top + Q_{12} + Q_{22} - \frac{1}{d_M}R_2,$$

$$\hat{\Pi}_{24} = P_{12} - P_{22}^\top - \lambda_2 B^\top N^\top,$$

$$\hat{\Pi}_{33} = -\lambda_1 \hat{P}_{21} - \lambda_1 \hat{P}_{21}^\top + d_M \hat{R}_1,$$

$$\hat{\Pi}_{44} = -\lambda_2 P_{22} - \lambda_2 P_{22}^\top + d_M R_2,$$

$$\hat{\Pi}_{55} = -(1-\mu)\hat{Q}_{21} - \frac{4}{d_M}\hat{R}_1,$$

$$\hat{\Pi}_{66} = -(1-\mu)Q_{22} - \frac{4}{d_M}R_2,$$

$$\hat{\Pi}_{771} = \hat{Q}_{11} - \frac{1}{d_M}\hat{R}_1, \qquad \hat{\Pi}_{881} = Q_{12} - \frac{1}{d_M}R_2,$$

$$\hat{\Pi}_{772} = \hat{Q}_{11} - \frac{3}{d_M}\hat{R}_1, \qquad \hat{\Pi}_{882} = Q_{12} - \frac{3}{d_M}R_2.$$

Moreover, the controller gain matrix K and observer gain matrix L are given by

$$K = M\bar{P}_{21}^{-1}, \qquad L = P_{22}^{-\top}N. \tag{24}$$

Proof Suppose the inequality (9a)-(9b) holds and let

$$\begin{aligned} P_1 &= \begin{bmatrix} P_{11} & 0 \\ 0 & P_{12} \end{bmatrix}, \qquad P_2 = \begin{bmatrix} P_{21} & 0 \\ 0 & P_{22} \end{bmatrix}, \qquad P_3 = \begin{bmatrix} \lambda_1 P_{21} & 0 \\ 0 & \lambda_2 P_{22} \end{bmatrix}, \\ Q_1 &= \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{12} \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} Q_{21} & 0 \\ 0 & Q_{22} \end{bmatrix}, \qquad R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}. \end{aligned} \tag{25}$$

Substitute (25) into (9a)-(9b) and let

$$\Pi_{111} = P_{21}^{\top}A + A^{\top}P_{21} + Q_{11} + Q_{21} - \frac{3}{d_M}R_1,$$

$$\Pi_{112} = P_{21}^{\top}A + A^{\top}P_{21} + Q_{11} + Q_{21} - \frac{1}{d_M}R_1,$$

$$\Pi_{13} = P_{11} - P_{21}^{\top} + \lambda_1 A^{\top}P_{21},$$

$$\Pi_{14} = P_{21}^{\top}BK,$$

$$\Pi_{221} = -P_{22}^{\top}LB - B^{\top}L^{\top}P_{22} + Q_{12} + Q_{22} - \frac{3}{d_M}R_1,$$

$$\Pi_{222} = -P_{22}^{\top}LB - B^{\top}L^{\top}P_{22} + Q_{12} + Q_{22} - \frac{1}{d_M}R_1,$$

$$\Pi_{24} = P_{12} - P_{22}^{\top} - \lambda_2 B^{\top}L^{\top}P_{22},$$

$$\Pi_{33} = -\lambda_1 P_{21} - \lambda_1 P_{21}^{\top} + d_M R_1,$$

$$\Pi_{55} = -(1-\mu)Q_{21} - \frac{4}{d_M}R_1,$$

$$\Pi_{771} = Q_{11} - \frac{1}{d_M}R_1,$$

$$\Pi_{772} = Q_{11} - \frac{3}{d_M}R_1.$$

Thus, we have the following inequalities hold:

$$\begin{bmatrix}
\Pi_{111} & P_{21}^{\top}B & \Pi_{13} & 0 & \Pi_{14} + \frac{3}{d_M}R_1 & 0 & 0 & 0 & P_{21}^{\top}B & 0 & C_1^{\top} \\
* & \Pi_{221} & \lambda_1 B^{\top}P_{21} & \Pi_{24} & 0 & 0 & 0 & 0 & P_{22}^{\top}LB & P_{22}^{\top} & C_2^{\top} \\
* & * & \Pi_{33} & 0 & \lambda_1\Pi_{14} + \frac{3}{d_M}R_1 & 0 & 0 & 0 & \lambda_1 P_{21}^{\top}B & 0 & 0 \\
* & * & * & \hat{\Pi}_{44} & 0 & 0 & 0 & 0 & -\lambda_2 P_{22}^{\top}LB & \lambda_2 P_{22}^{\top} & 0 \\
* & * & * & * & \Pi_{55} & 0 & \frac{1}{d_M}R_1 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \hat{\Pi}_{66} & 0 & \frac{1}{d_M}R_2 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Pi_{771} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \hat{\Pi}_{881} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\gamma^2 I & 0 & C_{d1}^{\top} \\
* & * & * & * & * & * & * & * & * & -\gamma^2 I & C_{d2}^{\top} \\
* & * & * & * & * & * & * & * & * & * & -I
\end{bmatrix}
< 0, \tag{26a}$$

$$\begin{bmatrix}
\Pi_{112} & P_{21}^{\top}B & \Pi_{13} & 0 & \Pi_{14} + \frac{1}{d_M}R_1 & 0 & 0 & 0 & P_{21}^{\top}B & 0 & C_1^{\top} \\
* & \Pi_{222} & \lambda_1 B^{\top}P_{21} & \Pi_{24} & 0 & 0 & 0 & 0 & P_{22}^{\top}LB & P_{22}^{\top} & C_2^{\top} \\
* & * & \Pi_{33} & 0 & \lambda_1\Pi_{14} + \frac{1}{d_M}R_1 & 0 & 0 & 0 & \lambda_1 P_{21}^{\top}B & 0 & 0 \\
* & * & * & \hat{\Pi}_{44} & 0 & 0 & 0 & 0 & -\lambda_2 P_{22}^{\top}LB & \lambda_2 P_{22}^{\top} & 0 \\
* & * & * & * & \Pi_{55} & 0 & \frac{3}{d_M}R_1 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \hat{\Pi}_{66} & 0 & \frac{3}{d_M}R_2 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Pi_{772} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \hat{\Pi}_{882} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\gamma^2 I & 0 & C_{d1}^{\top} \\
* & * & * & * & * & * & * & * & * & -\gamma^2 I & C_{d2}^{\top} \\
* & * & * & * & * & * & * & * & * & * & -I
\end{bmatrix}
< 0, \tag{26b}$$

where $\hat{\Pi}_{44}$, $\hat{\Pi}_{66}$, $\hat{\Pi}_{881}$ and $\hat{\Pi}_{882}$ are defined in (23a)-(23b). Then pre-multiply (26a)-(26b) by $\mathrm{diag}\{P_{21}^{-\top}, I, P_{21}^{-\top}, I, P_{21}^{-\top}, I, P_{21}^{-\top}, I, I, I, I\}$ and post-multiply by $\mathrm{diag}\{P_{21}^{-1}, I, P_{21}^{-1}, I, P_{21}^{-1}, I, P_{21}^{-1},$

$I, I, I, I\}$ and define some matrices as follows:

$$\hat{P}_{21} = \bar{P}_{21}^{-1}, \qquad \hat{P}_{11} = P_{21}^{-\top}\bar{P}_{11}P_{21}^{-1}, \qquad \hat{Q}_{11} = P_{21}^{-\top}\bar{Q}_{11}P_{21}^{-1},$$
$$\hat{Q}_{12} = P_{21}^{-\top}\bar{Q}_{12}P_{21}^{-1}, \qquad \hat{R}_1 = P_{21}^{-\top}\bar{R}_1P_{21}^{-1}.$$

From (9a)-(9b), it is clear that (23a)-(23b) holds. As a result, the closed-loop system (7)-(8) is asymptotically stable and satisfies $\|y(t)\|_2 < \gamma\|w(t)\|_2$. The proof is thus completed. □

Compared with the design method in [16], the matrices $\bar{P}_{21}$ and P_{22} are invertible matrices instead of positive definite matrices, which makes the design more flexible. Moreover, the augmented Lyapunov functional method also can be extended to the systems without time delay.

Using Theorem 2, a feasible design algorithm can be summarized as follows:

(1) For given $\gamma > 0$, $\mu \leq 1$, λ_1, λ_2 and d_M, solve LMI (23a)-(23b) with $\hat{P}_{11} > 0$, $P_{12} > 0$, $\hat{Q}_{11} > 0$, $\hat{Q}_{21} > 0$, $Q_{12} > 0$, $Q_{22} > 0$, $\hat{R}_1 > 0$, $R_2 > 0$;
(2) Compute K and L through $K = M\bar{P}_{21}^{-1}$, $L = P_{22}^{-\top}N$;
(3) Construct the controller and observer as (4) and (6).

Numerical example

In this section, the composite control scheme will be applied to a spacecraft with one flexible appendage. Since low-frequency modes are generally dominant in a flexible system, only the lowest two bending modes have been considered for the implemented spacecraft model. Thus, we suppose that $\omega_1 = 3.17$ rad/s, $\omega_2 = 7.38$ rad/s with damping $\xi_1 = 0.0001$, $\xi_2 = 0.0015$. We suppose that $F = [F_1\ F_2]$, where the coupling coefficients of the first two bending modes are $F_1 = 1.27814$, $F_2 = 0.91756$, $J = 35.72$ kg/m^2 is the nominal principal moment of inertia of pitch axis. The flexible spacecraft is supposed to move in a circular orbit with the altitude of 500 km, then the orbit rate $n = 0.0011$ rad/s, the disturbance torques acting on the satellite are assumed to be $w_1(t) = 4.5 \times 10^{-5}(3\cos t + 1.5\sin t)$. The initial pitch attitude of the spacecraft is $\theta(0) = 0.08$ rad/s, $\dot{\theta}(0) = 0.001$ rad/s. And H_∞ performance index is supposed to be $\gamma = 1.5$ and time delay satisfies $d_M = 0.03$, $u = 0.1$. The tuning parameter is chosen as $\lambda_1 = -0.12$, $\lambda_2 = 0.1$, $C_1 = [0.1\ 0]$, $C_2 = -0.1$, $C_{d1} = 0$ and $C_{d2} = 0$. By using Theorem 2, the controller gain and observer gain are obtained as

$$K = \begin{bmatrix} -5.7323 & -5.4831 \end{bmatrix}, \qquad L = \begin{bmatrix} 0 & 105.4207 \end{bmatrix}.$$

Figures 1 and 2 show the attitude angle and attitude angle rate. From these, it is clear that the response performance can be guaranteed under the composite controller. Figure 3 shows the elastic vibration estimation error via disturbance observer, the effect of the elastic vibration can be rejected by feed-forward compensation.

Conclusion

In this paper, composite disturbance-observer-based control (DOBC) and H_∞ control scheme has been investigated. The LMI-based conditions are formulated for the existence of the admissible disturbance observer and controller, which ensures that the closed-loop system is asymptotically stable with a H_∞ disturbance attenuation level. A numerical simulation shows the performance of the attitude control system. Further improvement in a

Figure 1 The responses of attitude angle.

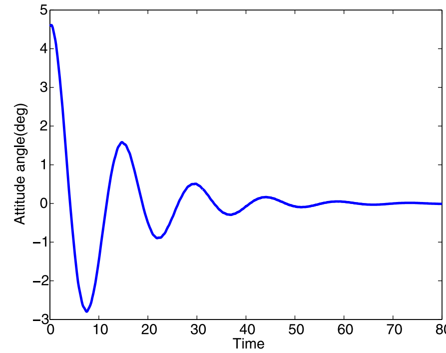


Figure 2 The responses of attitude angle rate.

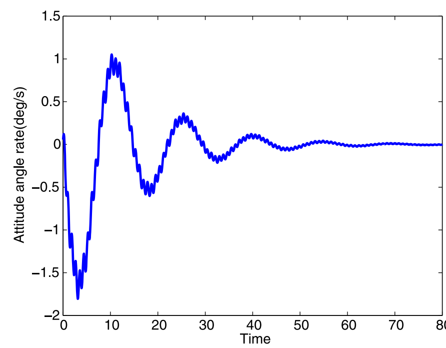
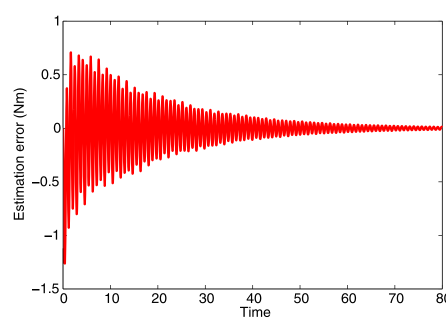


Figure 3 The responses of estimation error.



composite disturbance observer with output feedback control for flexible spacecrafts will be considered in our future work.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
RZ carried out the main part of this manuscript. The others participated in the discussion and gave the examples. All authors read and approved the final manuscript.

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