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Exponential synchronization of the coupling delayed switching complex dynamical networks via impulsive control

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Abstract

In this paper, we investigate the exponential synchronization issue of coupling delayed switching complex dynamical networks via impulsive control. Basing on the Lyapunov functional method and establishing a new impulsive delay differential inequality, we derive some sufficient conditions which depend on delay and impulses to guarantee the exponential synchronization of the coupling delay switching complex dynamical network. Finally, numerical simulations are given to illustrate the effectiveness of the obtained results.

Keywords: exponential synchronization; complex dynamical network; switching topology; delayed coupling

1 Introduction

During the last two decades, synchronization and control problems of complex dynamical networks have been focused on in many different fields such as mathematics, engineering, social and economic science, *etc.* [1–8]. Many effective methods, like feedback control, adaptive control, sampled-data control and impulsive control, are used to stabilize and synchronize a coupled complex dynamical network. At the same time, a wide variety of synchronization criteria have also been presented for different network coupling such as switch topology, time delays, impulsive characters, *etc.*

Up to now, plenty of researchers have devoted much effort to guarantee synchronization of complex dynamical networks with fixed topology [9–16]. However, in real situations, many complex systems may be subject to abrupt changes in their connection structure or network mode switching caused by some phenomena such as link failures, component failures or repairs, changing subsystem interconnections, and abrupt environmental disturbance, *etc.* Although some synchronization criteria of networks with uncertain topological structure and continuous time-varying topology have been studied, those methods may not work for the network topology when it becomes discontinued or changes very quickly [17, 18]. Hence, to study the synchronization of the switched networks is still very useful and meaningful. Because of this reason, the synchronization of a complex network with switching topology has attracted researchers' interest [17–21]. Wang *et al.* [17] provided several synchronization criteria for switched networks, in which synchronization could be evaluated by the time average of the second smallest eigenvalue that corresponded to the Laplacians matrix of switching topology. Authors in [18] studied the local and global exponential synchronization of switched networks with time-varying coupling delays, whose inner and outer coupling matrices take values in two finite sets of matrices via a switching signal. An adaptive controller was designed to synchronize a switched network under arbitrary switching in [19]. Yu *et al.* [20] explored the synchronization of switched neural networks, and some sufficient conditions were given to guarantee the global synchronization. Jia *et al.* [21] investigated the leader-following problem of network, in which the network topology is assumed to be arbitrarily switched among a finite set of topologies, and time-varying delay exists in the coupling of agents.

In many systems, the impulsive effects are common phenomena due to instantaneous perturbations at certain moments. In general, there are two kinds of impulse in terms of synchronization in complex dynamical networks: desynchronizing impulse and synchronizing impulse [22]. In previous literature, most of the results were devoted to investigating the desynchronizing impulse (the impulsive effect can suppress the synchronization of the complex dynamical networks) [23–26]. The global exponential synchronization was studied for linear coupled neural networks with impulsive disturbances in [23]. Zhu *et al.* gave some global impulsive exponential synchronization criteria for time-delayed coupled chaotic systems [24]. In [25], some impulsive control schemes were given to guarantee the consensus of nonlinear multi-agent systems with switching topology. Yang and Cao [26] studied the exponential synchronization of a coupling delay complex dynamical network with impulsive effects and proved that the network can achieve synchronization for a desynchronizing impulse. All of them have a common feature that the network must be synchronous. As we all know that the network is not always synchronous, there are some factors that will lead to an unstable network such as the change of topology structure, time delays and low strength of the coupling. Impulsive control (synchronizing impulse) may give an efficient method to deal with a dynamical system which is unstable. It is worth mentioning that synchronization and the control problems in complex networks with fix topology and synchronizing impulse have been widely studied [23, 27–29], but research into switched topology and synchronizing impulse is rare.

In this paper, we investigate the problem of exponential synchronization of a switching complex dynamical network via impulsive control. The contribution of this paper is to propose a new impulsive delay differential inequality. By utilizing the Lyapunov stability and impulsive control theory on delayed dynamical networks, some sufficient conditions of exponential synchronization for a switching complex dynamical network are presented. It shows that impulsive controller (synchronizing impulsive) can control the coupling delay switching complex dynamical network to a homogenous solution. Numerical simulations are given to show the validity of the developed results.

2 Model and preliminaries

The switching complex dynamical networks investigated in this paper consist of N nodes, whose state is described as

$$\begin{cases} \dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} g_{ij}^{\sigma(t)}\Gamma x_j(t-\tau), & t \neq t_k, \\ \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-) = b_k x_i(t_k^-), & t = t_k, \\ \quad k \in Z^+, i = 1, 2, \ldots, N, \end{cases} \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n$ is the state vector of node i; $f : R^n \to R^n$ is a continuous vector value function, $c > 0$ is the coupling strength, τ is a coupling delay;

$\Gamma = \operatorname{diag}(\gamma_1, \gamma_2, \ldots, \gamma_n)$ is an inner coupling matrix between the two connected nodes; $\sigma(t) : [0,\infty) \to \aleph = \{1, 2, \ldots, m\}$ is a switching signal, which is a piecewise constant function; $G^{\sigma(t)} = (g_{ij}^{\sigma(t)}) \in R^{N\times N}$ is a Laplacian matrix associated with the switching function $\sigma(t)$, in which the entries of matrix $G^{\sigma(t)}$ are defined as follows: if nodes i and j ($i \neq j$) are connected, then $g_{ij}^{\sigma(t)} > 0$; otherwise, $g_{ij}^{\sigma(t)} = 0$, and the diagonal entries of matrix $G^{\sigma(t)}$ are defined by $g_{ii}^{\sigma(t)} = -\sum_{j=1,j\neq i}^{N} g_{ij}^{\sigma(t)}$. Note that the coupling matrix $G^{\sigma(t)}$ is not assumed to be irreducible; b_k is the ith node impulsive gain at $t = t_k$. The discrete set $\{t_k\}$ satisfies $0 \leq t_0 < t_1 < \cdots < t_k < \cdots$, $t_k \to +\infty$ as $k \to +\infty$, note $x(t_k^-) = \lim_{t\to t_k^-} x(t)$, and $x(t_k^+) = \lim_{t\to t_k^+} x(t) = x(t_k)$.

We assume that the network (1) satisfies the following initial conditions: $x_i^0(t) = (x_{i1}^0(t), x_{i2}^0(t), \ldots, x_{in}^0(t))^T \in C([t_0 - \tau, t_0], R^n)$.

To discuss exponential synchronization, we define the set

$$s(t) = \sum_{j=1}^{N} \xi_j x_j(t), \tag{2}$$

which is the synchronization state for the network (1), where $\xi_i > 0$ and $\sum_{j=1}^{N} \xi_j = 1$.

Remark 1 In general, the synchronization state $s(t)$ may be an equilibrium point, a periodic orbit, or a chaotic attractor. In this paper, we did not need $(\xi_1, \xi_2, \ldots, \xi_N)$ to be the left eigenvector of coupling matrix G corresponding to eigenvalue 0.

Definition 1 The network (1) is said to achieve exponentially synchronization if there exist some constants $\varepsilon > 0$ and $M > 0$ such that

$$\lim_{t\to\infty} \|x_i(t) - s(t)\| \leq M e^{-\varepsilon(t-t_0)}$$

for all initial conditions $x_i^0(t) \in C([t_0 - \tau, t_0], R^n)$ and $i = 1, 2, \ldots, N$.

Definition 2 [16, 26] Let $P = \operatorname{diag}(p_1, p_2, \ldots, p_n)$ be a positive definite diagonal matrix, and let $\Delta = \operatorname{diag}(\delta_1, \delta_2, \ldots, \delta_n)$ be a diagonal matrix. QUAD(Δ, P) denotes a class of continuous functions $f(x) : R^n \to R^n$ satisfying

$$(x-y)^T P\bigl(f(x) - f(y) - \Delta(x-y)\bigr) \leq -\alpha(x-y)^T P(x-y)$$

for some $\alpha > 0$, all $x, y \in R^n$ and $t > t_0$.

Remark 2 It is easy to verify that the function class QUAD exists in almost all the well-known chaotic systems with or without time delays such as Lorenz systems, Rössler system, Chen system, Chua's circuit, delayed Hopfield neural networks and delayed cellular neural networks (cNN), *etc.*

Define error state $e_i(t) = x_i(t) - s(t)$ $(1 \leq i \leq N)$. It is easy to verify that $\sum_{i=1}^{N} \xi_i e_i(t) = 0$ and the dynamical equation of $s(t)$ and $e_i(t)$ satisfies

$$\dot{s}(t) = \sum_{i=1}^{N} \xi_i \left[f\bigl(x_i(t)\bigr) + c \sum_{j=1}^{N} g_{ij}^{\sigma(t)} \Gamma x_j(t-\tau) \right], \tag{3}$$

$$\begin{cases} \dot{e}_i(t) = \tilde{f}(e_i(t)) + c\sum_{j=1}^{N} g_{ij}^{\sigma(t)}\Gamma e_j(t-\tau) + J, & t \neq t_k, \\ \Delta e_i(t_k) = e_i(t_k^+) - e_i(t_k^-) = b_k e_i(t_k^-), & t = t_k, \\ \quad k \in Z^+, i = 1,2,\ldots,N, \end{cases} \tag{4}$$

where $\tilde{f}(e_i(t)) = f(x_i(t)) - f(s(t))$ and $J = \sum_{j=1}^{N} \xi_j \cdot \{[f(s(t)) - f(x_j(t))] - \sum_{k=1}^{N} g_{jk}^{\sigma(t)}\Gamma x_k(t-\tau)\}$.

In order to derive the main results, it is necessary to propose the following lemmas.

Lemma 1 *Let $u(t) : [t_0 - \tau, \infty) \to [0, \infty)$ satisfy the scalar impulsive differential inequality*

$$\begin{cases} \dot{u}(t) \leq pu(t) + qu(t-\tau), & t \neq t_k, t \geq t_0, \\ u(t_k) \leq \alpha_k u(t_k^-), \qquad u(t) = \phi(t), & t \in [t_0 - \tau, t_0], \end{cases} \tag{5}$$

where $p, q > 0$, $\alpha_k > 0$, $u(t)$ is continuous at $t \neq t_k$, $t \geq t_0$, $u(t_k) = u(t_k^+) = \lim_{t\to t_k^+} u(t)$ and $u(t_k^-) = \lim_{t\to t_k^-} u(t)$ exists, $\phi \in C([t_0 - \tau, t_0], R^+)$. Then

$$u(t) \leq \left(\prod_{i=1}^{k} \alpha_i\right) e^{(p+q)(t-t_0+k\tau)} \Big[\sup_{t_0-\tau\leq s\leq t_0} \phi(s)\Big]$$

for $t \in [t_k, t_{k+1})$.

The proof is given in the Appendix.

Lemma 2 [21] *For real constant matrices $Z_1, Z_2, Z_3 \in R^{N\times N}$ with $Z_1 = Z_1^T$, $Z_3 = Z_3^T$, and diagonal matrix $\Lambda = \operatorname{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$, where $\lambda_i \in R$, the matrices $K = \operatorname{diag}(K_1, K_2, \ldots, K_N)$ and*

$$\Omega = \begin{bmatrix} I_N \otimes Z_1 & \Lambda \otimes Z_2 \\ \Lambda \otimes Z_2^T & I_N \otimes Z_3 \end{bmatrix}$$

share the same eigenvalues, where I_N indicates the N dimensional identity matrix and $K_i = \left[\begin{smallmatrix} Z_1 & \lambda_i Z_2 \\ \lambda_i Z_2^T & Z_3 \end{smallmatrix}\right]$.

3 Main result

In this section, we investigate the exponential synchronization of error system (4), in which coupling matrix G^σ is divided into two cases: symmetric or asymmetric. Some new criteria are presented for the exponential synchronization of the network (1) based on the Lyapunov functional method, linear matrix inequality approach and establishing an impulsive delay differential inequality.

Case 1. Asymmetric connected of switching topology

Theorem 1 *The network* (1) *is exponential synchronization if there exist positive definite diagonal matrices $P = \operatorname{diag}(p_1, p_2, \ldots, p_n)$ and $Q = \operatorname{diag}(q_1, q_2, \ldots, q_n)$, and positive constants β and η such that*

(1) $f(x) \in \mathrm{QUAD}(\Delta, P)$,

(2) $2(\delta_j - \alpha)p_j \Xi + \dfrac{(cp_j\gamma_j)^2}{q_j}\big(G^\sigma\big)^T \Xi G^\sigma - \beta p_j \Xi \leq 0, \quad j = 1,2,\ldots,n, \forall\sigma \in \aleph,$

(3) $\quad (\beta + a)\left(1 + \frac{\tau}{T_{\min}}\right) + \frac{2\ln|1+b|}{T_{\max}} \leq -\eta,$

where $a = \{\frac{\max q_i}{\min p_j} | i,j = 1,2,\ldots,N\}$, $\Xi = \operatorname{diag}(\xi_1, \xi_2, \ldots, \xi_N)$, $|1+b| = \max\{|1+b_k| | k \in Z^+\}$, $T_{\min} = \min\{t_k - t_{k-1} | k \in Z^+\}$, *and* $T_{\max} = \max\{t_k - t_{k-1} | k \in Z^+\}$.

Proof Condition (3) of Theorem 1 implies that the impulsive gains $b_k \in (-2, 0)$.

Choose the Lyapunov function as follows:

$$V(t) = \sum_{i=1}^{N} \xi_i e_i^T(t) P e_i(t).$$

Then the derivative of $V(t)$ with respect to time t along the solution of Eq. (4) can be calculated as follows:

$$\begin{aligned}
\dot{V}(t) &= 2\sum_{i=1}^{N} \xi_i e_i^T(t) P \dot{e}_i(t) \\
&= 2\sum_{i=1}^{N} \xi_i e_i^T(t) P \Bigg[f\big(x_i(t)\big) - f\big(s(t)\big) - \Delta\big(x_i(t) - s(t)\big) \\
&\quad + \Delta\big(x_i(t) - s(t)\big) + c\sum_{j=1}^{N} g_{ij}^{\sigma(t)} \Gamma e_j(t-\tau) + J \Bigg].
\end{aligned}$$

Since $\sum_{i=1}^{N} \xi_i e_i(t) = 0$, we have

$$\sum_{i=1}^{N} \xi_i e_i^T(t) P J = \sum_{i=1}^{N} \xi_i e_i^T(t) P \left[\sum_{j=1}^{N} \xi_j \big(f\big(s(t)\big) - f\big(x_j(t)\big)\big) - c\sum_{j=1}^{N}\sum_{k=1}^{N} \xi_j g_{jk}^{\sigma(t)} \Gamma e_k(t) \right] = 0.$$

Considering the time intervals in which the σth topology is being activated and using the QUAD condition, we have

$$\begin{aligned}
\dot{V}(t) &\leq \sum_{i=1}^{N} 2\xi_i e_i^T(t) P(-\alpha I_n + \Delta) e_i(t) + \sum_{i=1}^{N}\sum_{j=1}^{N} 2c\xi_i g_{ij}^{\sigma} e_i^T(t) P\Gamma e_j(t-\tau) \\
&\quad - \sum_{i=1}^{N} \xi_i e_i^T(t-\tau) Q e_i(t-\tau) + \sum_{i=1}^{N} \xi_i e_i^T(t-\tau) Q e_i(t-\tau) \\
&= \sum_{j=1}^{n} 2(\delta_j - \alpha) p_j \tilde{e}_j^T(t) \Xi \tilde{e}_j(t) + 2c\sum_{j=1}^{n} p_j \gamma_j \tilde{e}_j^T(t) \Xi G^{\sigma} \tilde{e}_j(t-\tau) \\
&\quad - \sum_{j=1}^{n} q_j \tilde{e}_j^T(t-\tau) \Xi \tilde{e}_j(t-\tau) + \sum_{i=1}^{N} \xi_i e_i^T(t-\tau) Q e_i(t-\tau) \\
&= \sum_{j=1}^{n} \begin{bmatrix} \tilde{e}_j^T(t) & \tilde{e}_j^T(t-\tau) \end{bmatrix} \begin{bmatrix} 2(\delta_j - \alpha) p_j \Xi & c p_j \gamma_j \Xi G^{\sigma} \\ c p_j \gamma_j (G^{\sigma})^T \Xi & -q_j \Xi \end{bmatrix} \begin{bmatrix} \tilde{e}_j(t) \\ \tilde{e}_j(t-\tau) \end{bmatrix} \\
&\quad + \sum_{i=1}^{N} \xi_i e_i^T(t-\tau) Q e_i(t-\tau),
\end{aligned} \tag{6}$$

where $\tilde{e}_j(t) = (e_{1j}(t), e_{2j}(t), \ldots, e_{Nj}(t))^T$ and $\Xi = \operatorname{diag}(\xi_1, \xi_2, \ldots, \xi_N)$.

According to Condition (2) of Theorem 1 and linear matrix inequality, it is not difficult to verify that

$$\begin{bmatrix} (2\delta_j - 2\alpha - \beta)p_j \Xi & cp_j\gamma_j \Xi G^\sigma \\ cp_j\gamma_j (G^\sigma)^T \Xi & -q_j \Xi \end{bmatrix} \le 0. \tag{7}$$

Substituting (7) into (6) yields

$$\begin{aligned} \dot{V}(t) &\le \sum_{i=1}^{N} \beta p_j \tilde{e}_j^T(t)\Xi \tilde{e}_j(t) + \sum_{i=1}^{N} \xi_i e_i^T(t-\tau)Qe_i(t-\tau) \\ &\le \beta \sum_{i=1}^{N} \xi_i e_i^T(t)Pe_i(t) + \frac{\max q_i}{\min p_j} \sum_{i=1}^{N} \xi_i e_i^T(t-\tau)Pe_i(t-\tau) \\ &= \beta V(t) + \frac{\max q_i}{\min p_j} V(t-\tau) \\ &= \beta V(t) + aV(t-\tau), \end{aligned} \tag{8}$$

where $a = \{\frac{\max q_i}{\min p_j} | i,j = 1,2,\ldots,N\}$.

On the other hand, from the construction of $V(t)$, we have

$$\begin{aligned} V(t_k) &= \sum_{i=1}^{N} \xi_i e_i^T(t_k)Pe_i(t_k) = \sum_{i=1}^{N} (1+b_k)^2 \xi_i e_i^T\bigl(t_k^-\bigr)Pe_i\bigl(t_k^-\bigr) \\ &= (1+b_k)^2 V\bigl(t_k^-\bigr). \end{aligned} \tag{9}$$

Hence, for $t \in [t_k, t_{k+1})$, by Lemma 1 and Eqs. (8)-(9), one can show that

$$V(t) \le \left(\prod_{i=1}^{k}(1+b_i)^2\right) e^{(\beta+a)(t-t_0+k\tau)} \Bigl(\sup_{t_0-\tau\le s\le t_0} V_s\Bigr). \tag{10}$$

Let $|1+b| = \max\{|1+b_k| | k \in Z^+\}$, $T_{\min} = \min\{t_k - t_{k-1} | k \in Z^+\}$, and $T_{\max} = \max\{t_k - t_{k-1} | k \in Z^+\}$, then

$$\begin{aligned} V(t) &\le \Bigl(\sup_{t_0-\tau\le s\le t_0} V(s)\Bigr) e^{(\beta+a)(t-t_0+k\tau)+2k\ln|1+b|} \\ &\le \Bigl(\sup_{t_0-\tau\le s\le t_0} V(s)\Bigr) e^{[(\beta+a)(1+\frac{\tau}{T_{\min}})+2\frac{\ln|1+b|}{T_{\max}}](t-t_0)}. \end{aligned}$$

Using Condition (3) of Theorem 1, we get

$$V(t) \le \Bigl(\sup_{t_0-\tau\le s\le t_0} V(s)\Bigr) e^{-\eta(t-t_0)}.$$

From the construction of $V(t)$, we have

$$V(t) \ge \xi_i e_i^T(t)Pe_i(t).$$

Hence, $\|e_i(t)\| \le (\frac{\sup_{t_0-\tau\le s\le t_0} V(s)}{p\xi_i})^{\frac{1}{2}} e^{-\frac{\eta}{2}(t-t_0)}$, where $p = \min\{p_j | j = 1,2,\ldots,n\}$.
The proof of Theorem 1 is completed. □

If the switching signal $\sigma(t) \equiv 1$, then the network (1) has only one coupling matrix G. Suppose G is irreducible and $\xi^T = (\xi_1, \xi_2, \ldots, \xi_N)$ is the left eigenvector of coupling matrix G corresponding to eigenvalue 0. By the proof of Theorem 1, we can derive the exponential synchronization criteria of the network (1) with only one topology, which is given as follows.

Corollary 1 *The network* (1) *with only one topology is exponential synchronization if there exist positive definite diagonal matrices* $P = \operatorname{diag}(p_1, p_2, \ldots, p_n)$ *and* $Q = \operatorname{diag}(q_1, q_2, \ldots, q_n)$ *and positive constants* β *and* η *such that*

(1) $f(x) \in \mathrm{QUAD}(\Delta, P)$,

(2) $\begin{bmatrix} (2\delta_j - 2\alpha - \beta)p_j\Xi & cp_j\gamma_j\Xi G \\ cp_j\gamma_j G^T\Xi & -q_j\Xi \end{bmatrix} \le 0, \quad j = 1, 2, \ldots, n,$

(3) $(\beta + a)\left(1 + \frac{\tau}{T_{\min}}\right) + \frac{2\ln|1+b|}{T_{\max}} \le -\eta,$

where $a = \{\frac{\max q_i}{\min p_j} | i, j = 1, 2, \ldots, N\}$, $\Xi = \operatorname{diag}(\xi_1, \xi_2, \ldots, \xi_N)$, $|1+b| = \max\{|1+b_k| | k \in Z^+\}$, $T_{\min} = \min\{t_k - t_{k-1} | k \in Z^+\}$, *and* $T_{\max} = \max\{t_k - t_{k-1} | k \in Z^+\}$.

Remark 3 The result of Theorem 2 in [26] must satisfy $p > 0$, where $p = \min_{1 \le j \le n}\{(2\sigma - 2\delta_i - \gamma_i)p_j\}$. However, for almost chaotic systems there exists j such that $\delta_j - \sigma > 0$. It means that the condition of Theorem 2 (in [26]) is not true. In Corollary 1 of this paper, there exists $\beta > 0$ such that $2\delta_j - 2\alpha - \beta < 0$. So, Corollary 1 is more common than Theorem 2 in [26].

Case 2. Symmetric connected of switching topology

Theorem 2 *Suppose that* G^σ *is a symmetric matrix. If there exist positive constants* β *and* η *and positive definite diagonal matrices* $P = \operatorname{diag}(p_1, p_2, \ldots, p_n)$ *and* $Q = \operatorname{diag}(q_1, q_2, \ldots, q_n)$ *such that*

(1) $f(x) \in \mathrm{QUAD}(\Delta, P)$,

(2) $\xi\left[2P\Delta - (2\alpha + \beta)P\right] + \frac{(c\lambda)^2}{\xi} P\Gamma Q^{-1}\Gamma P \le 0,$

(3) $(\beta + a)\left(1 + \frac{\tau}{T_{\min}}\right) + 2\frac{\ln|1+b|}{T_{\max}} < -\eta,$

then the network (1) *is exponential synchronization, where* $a = \{\frac{\max q_i}{\min p_j} | i, j = 1, 2, \ldots, N\}$, $|1+b| = \max\{|1+b_k| | k \in Z^+\}$, $\xi = \min\{\xi_i | i = 1, 2, \ldots, N\}$, $T_{\min} = \min\{t_k - t_{k-1} | k \in Z^+\}$, $T = \max\{t_k - t_{k-1} | k \in Z^+\}$, $\lambda = \max\{|\lambda_i(\Xi G^\sigma)| | i = 1, 2, \ldots, n; \sigma \in \aleph\}$ *and* $\lambda_i(\Xi G^\sigma)$ *are the eigenvalues of matrices* ΞG^σ.

Proof Construct the following Lyapunov function:

$$V(t) = \sum_{i=1}^{N} \xi_i e_i^T(t) P e_i(t) = e^T(t)(\Xi \otimes P)e(t),$$

where $e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T$.

Then, taking the derivative of $V(t)$ with respect to time t along the solution of Eq. (4), we have

$$\begin{aligned}
\dot{V}(t) &\le \sum_{i=1}^{N} 2\xi_i e_i^T(t)P\left\{(-\alpha I_n + \Delta)e_i(t) + c\sum_{j=1}^{N} g_{ij}^{\sigma}\Gamma e_j(t-\tau)\right\} \\
&= \sum_{i=1}^{N} \xi_i e_i^T(t)P\bigl(-(2\alpha+\beta)I_n + 2\Delta\bigr)e_i(t) + 2c\sum_{i=1}^{N}\sum_{j=1}^{N} \xi_i g_{ij}^{\sigma} e_i^T(t)P\Gamma e_j(t-\tau) \\
&\quad + \sum_{i=1}^{N} \beta\xi_i e_i^T(t)Pe_i(t) - \sum_{i=1}^{N} \xi_i e_i^T(t-\tau)Qe_i(t-\tau) + \sum_{i=1}^{N} \xi_i e_i^T(t-\tau)Qe_i(t-\tau) \\
&\le \xi e^T(t)\bigl[I_N \otimes \bigl(2P\Delta - (2\alpha+\beta)P\bigr)\bigr]e(t) + 2ce^T(t)\bigl(\Xi G^{\sigma} \otimes P\Gamma\bigr)e(t-\tau) \\
&\quad + \beta e^T(t)(\Xi \otimes P)e(t) - \xi e^T(t-\tau)(I_N \otimes Q)e(t-\tau) \\
&\quad + e^T(t-\tau)(\Xi \otimes Q)e(t-\tau),
\end{aligned} \tag{11}$$

where $\xi = \min\{\xi_i | i = 1,2,\ldots,N\}$.

Consider the properties of a symmetric matrix. There exists an orthogonal matrix $U_\sigma = (u_1^\sigma, u_2^\sigma, \ldots, u_n^\sigma) \in R^{N\times N}$ such that $U_\sigma^T(\Xi G^\sigma)U_\sigma = \mathrm{diag}(\lambda_{\sigma 1}, \lambda_{\sigma 2}, \ldots, \lambda_{\sigma N}) = \Lambda_\sigma$ and $\sigma \in \aleph$. Let $Z_\sigma(t) = ((U_\sigma \otimes I_n)e(t))$. According to Eq. (11) and the properties of the Kronecker product, we can get

$$\begin{aligned}
\dot{V}(t) &\le \xi Z_\sigma^T(t)\bigl[I_N \otimes \bigl(2P\Delta - (2\alpha+\beta)P\bigr)\bigr]Z_\sigma(t) + 2cZ_\sigma^T(t)(\Lambda_\sigma \otimes P\Gamma)Z_\sigma(t-\tau) \\
&\quad + \beta e^T(t)(\Xi \otimes P)e(t) - \xi Z_\sigma^T(t-\tau)(I_N \otimes Q)Z_\sigma(t-\tau) + e^T(t-\tau)(\Xi \otimes Q)e(t-\tau) \\
&= \begin{bmatrix} Z_\sigma^T(t) & Z_\sigma^T(t-\tau) \end{bmatrix} \begin{bmatrix} \xi I_N \otimes (2P\Delta - (2\alpha+\beta)P) & c\Lambda_\sigma \otimes P\Gamma \\ c\Lambda_\sigma \otimes P\Gamma & -\xi I_N \otimes Q \end{bmatrix} \begin{bmatrix} Z_\sigma(t) \\ Z_\sigma(t-\tau) \end{bmatrix} \\
&\quad + \beta e^T(t)(\Xi \otimes P)e(t) + e^T(t-\tau)(\Xi \otimes Q)e(t-\tau).
\end{aligned}$$

Basing on Condition (2) of Theorem 2 and $\lambda = \max\{|\lambda_i(\Xi G^\sigma)|| i = 1,2,\ldots,n; \sigma \in \aleph\}$, where $\lambda_i(\Xi G^\sigma)$ are the eigenvalues of matrices ΞG^σ, for all $i = 1,2,\ldots,n$ and $\sigma \in \aleph$, we have

$$\xi\bigl[2P\Delta - (2\alpha+\beta)P\bigr] + \frac{(c\lambda_{\sigma i})^2}{\xi}P\Gamma Q^{-1}\Gamma P \le 0.$$

By the linear matrix inequality, for all $i = 1,2,\ldots,n$, $\sigma \in \aleph$, one gets

$$\begin{bmatrix} \xi[2P\Delta - (2\alpha+\beta)P] & c\lambda_{\sigma i}P\Gamma \\ c\lambda_{\sigma i}\Gamma P & -\xi Q \end{bmatrix} \le 0.$$

Then, applying Lemma 2, we obtain

$$\begin{bmatrix} \xi I_N \otimes (2P\Delta - (2\alpha+\beta)P) & c\Lambda_\sigma \otimes P\Gamma \\ c\Lambda_\sigma \otimes P\Gamma & \xi I_N \otimes Q \end{bmatrix} \le 0.$$

Hence,

$$\dot{V}(t) \leq \beta e^T(t)(\Xi \otimes P)e(t) + e^T(t-\tau)(\Xi \otimes Q)e(t-\tau) \leq \beta V(t) + aV(t-\tau), \qquad (12)$$

where $a = \{\frac{\max q_i}{\min p_j} | i, j = 1, 2, \ldots, N\}$.

According to Eqs. (8)-(10) and (12), for any $t \in [t_k, t_{k+1})$, we have

$$V(t) \leq \Big(\sup_{t_0-\tau \leq s \leq t_0} V(s)\Big) e^{[(\beta+a)(1+\frac{\tau}{T_{\min}})+2\frac{\ln|1+b|}{T_{\max}}](t-t_0)}.$$

Using Condition (3) of Theorem 2, we get

$$V(t) \leq \Big(\sup_{t_0-\tau \leq s \leq t_0} V(s)\Big) e^{-\eta(t-t_0)}.$$

It is clear that $\xi_i e_i^T(t) P e_i(t) \leq V(t)$.

So, $\|e_i(t)\| \leq (\frac{(\sup_{t_0-\tau\leq s\leq t_0} V(s))}{p\xi_i})^{\frac{1}{2}} e^{-\frac{\eta}{2}(t-t_0)}$, where $p = \min\{p_j | j = 1, 2, \ldots, n\}$.

The proof of Theorem 2 is completed. □

Let impulsive gains $b_k = b$, and choose the synchronization state $s(t) = \frac{1}{N}\sum_{i=1}^N x_i(t)$. By the proof of Theorem 2, we can derive the exponential synchronization criteria of the network (1) with the fixed impulsive gain, which is given as follows.

Corollary 2 *The network* (1) *with the fixed impulsive gain is exponential synchronization if there exist positive definite diagonal matrices* $P = \operatorname{diag}(p_1, p_2, \ldots, p_n)$ *and* $Q = \operatorname{diag}(q_1, q_2, \ldots, q_n)$, *and positive constants* β *and* η *such that*

(1) $f(x) \in \mathrm{QUAD}(\Delta, P)$,

(2) $2P\Delta - (2\alpha + \beta)P + (c\lambda)^2 P\Gamma Q^{-1}\Gamma P \leq 0$,

(3) $(\beta + a)\left(1 + \dfrac{\tau}{T_{\min}}\right) + 2\dfrac{\ln|1+b|}{T_{\max}} < -\eta$,

where $a = \{\frac{\max q_i}{\min p_j} | i, j = 1, 2, \ldots, N\}$, $T_{\min} = \min\{t_k - t_{k-1} | k \in Z^+\}$, $T_{\max} = \max\{t_k - t_{k-1} | k \in Z^+\}$, $\lambda = \max\{|\lambda_i(G^\sigma)| | i = 1, 2, \ldots, n; \sigma \in \aleph\}$ *and* $\lambda_i(G^\sigma)$ *are the eigenvalues of matrices* G^σ.

4 Numerical simulation

In this section, we give two numerical simulations to illustrate the feasibility and effectiveness of the theoretical results presented in the previous sections.

Consider a three-order Chua's circuit [16] (see Figure 1) described as follows:

$$\dot{x}(t) = f\big(x(t)\big),$$

where $x(t) = (x_1(t), x_2(t), x_3(t))^T$ and the function $f(x(t))$ was chosen as follows:

$$f\big(x(t)\big) = \begin{bmatrix} m[x_2 - h(x_1)] \\ x_1 - x_2 + x_3 \\ -nx_2 \end{bmatrix},$$

where $h(x_1) = \frac{2}{7}x_1 - \frac{3}{14}[|x_1 + 1| - |x_1 - 1|]$, $m = 9$ and $n = 14\frac{2}{7}$.

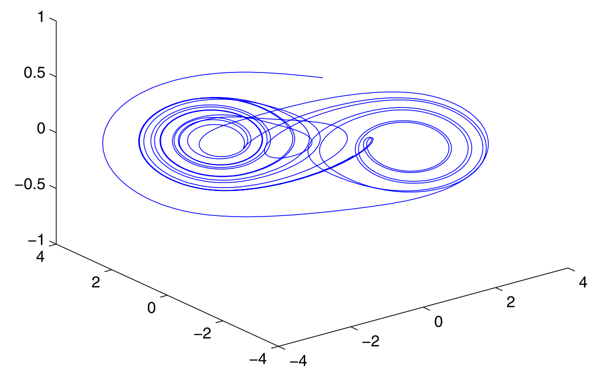

Figure 1 Chaotic behavior of Chua's circuit.

Example 1 Consider a network model consisting of five nodes and three connective topology. Each node in the network is three-order Chua's circuit described by

$$\begin{cases} \dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{5} g_{ij}^{\sigma(t)}\Gamma x_j(t-\tau), \quad t \neq t_k, \\ \Delta x_i(t_k) = x_i(t_k) - x_i(t_k^-) = b_k x_i(t_k^-), \quad t = t_k, \\ \quad i = 1, 2, \ldots, 5, \end{cases} \tag{13}$$

where $c = 0.1$, $\tau = 0.05$ and $\Gamma = 0.8I_3$.

If the coupling matrices are selected as follows and $b_k = 0$ (without impulsive controller), then the network (13) is not synchronized (see Figure 2).

$$G^1 = \begin{bmatrix} -2 & 2 & 0 & 0 & 0 \\ 0 & -2 & 2 & 0 & 0 \\ 0 & 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & -2 & 2 \\ 2 & 0 & 0 & 0 & -2 \end{bmatrix}, \qquad G^2 = \begin{bmatrix} -2 & 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 1 & 0 \\ 0 & 0 & -2 & 1 & 1 \\ 1 & 0 & 0 & -2 & 1 \\ 1 & 1 & 0 & 0 & -2 \end{bmatrix},$$

and

$$G^3 = \begin{bmatrix} -4 & 1 & 1 & 1 & 1 \\ 1 & -4 & 1 & 1 & 1 \\ 1 & 1 & -4 & 1 & 1 \\ 1 & 1 & 1 & -4 & 1 \\ 1 & 1 & 1 & 1 & -4 \end{bmatrix}.$$

If we choose $P = I_3$ and $\Delta = 10I_3$, then the function $f(x)$ satisfies the condition of the function class QUAD(Δ, P), where $\alpha = 0.6218$. The switch time is $t = 0.2s$. Let $\beta = 18.9$, $Q = 4.1I_3$, $|1 + b| = 0.16$, $T_{\max} = T_{\min} = 0.1$ and let the synchronization state be $s(t) = 0.2x_1 + 0.3x_2 + 0.1x_3 + 0.3x_4 + 0.1x_5$, then all the conditions in Theorem 1 are satisfied, and $\eta = -2.1516$, $a = 4.1$, so the asymmetric coupled network (13) can achieve exponential synchronization. The simulation results are given in Figures 3-5. It can be seen clearly from Figures 3-5 that all states of the asymmetric coupled network (13) tend to the synchronization state $s(t)$.

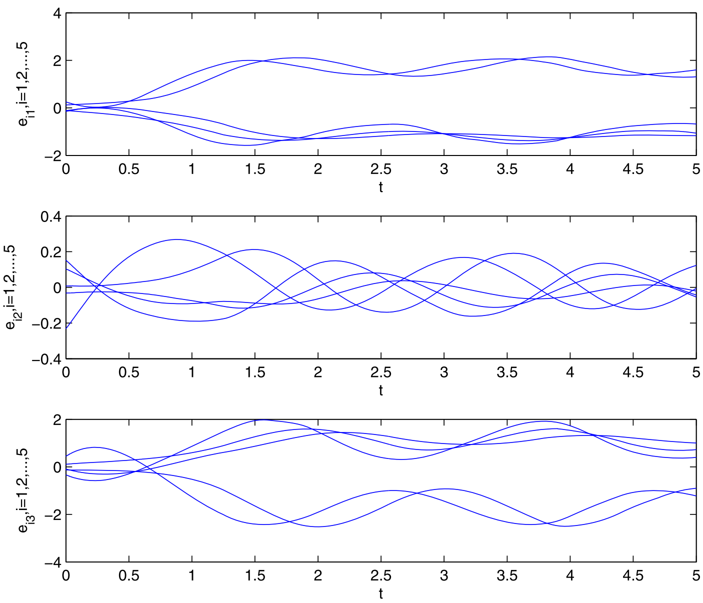

Figure 2 Time evolution of errors $e_{ij}(t)$, $i = 1, 2, \ldots, 5$, $j = 1, 2, 3$ of the asymmetric coupled network (13) without impulsive controller.

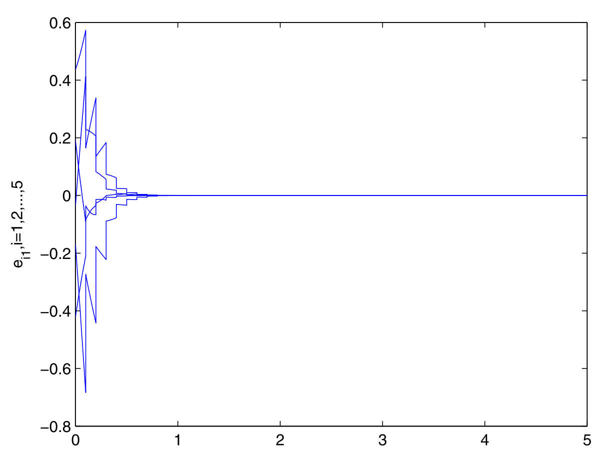

Figure 3 Time evolution of synchronization errors $e_{i1}(t)$, $i = 1, 2, \ldots, 5$ of the asymmetric coupled network (13) with random initial values.

Example 2 The network model is the same as Example 1. If the coupling matrices are chosen as follows and $b_k = 0$ (without impulsive controller), then the network (13) is not synchronized (see Figure 6).

$$G^1 = \begin{bmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 1 & 0 & 0 & 1 & -2 \end{bmatrix}, \qquad G^2 = \begin{bmatrix} -4 & 1 & 1 & 1 & 1 \\ 1 & -2 & 0 & 1 & 0 \\ 1 & 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & -3 & 0 \\ 1 & 0 & 1 & 0 & -2 \end{bmatrix},$$

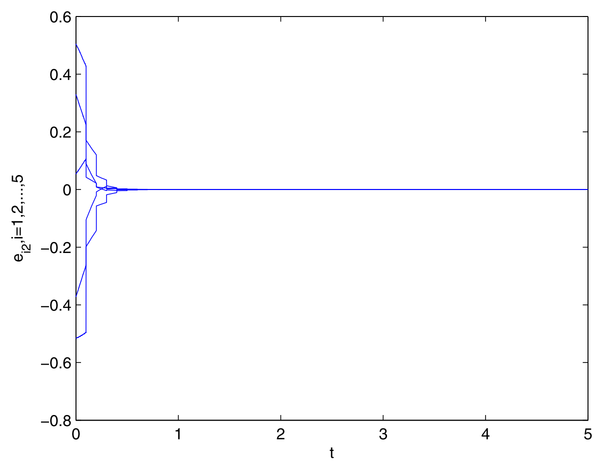

Figure 4 Time evolution of synchronization errors $e_{i2}(t)$, $i = 1, 2, \ldots, 5$ of the asymmetric coupled network (13) with random initial values.

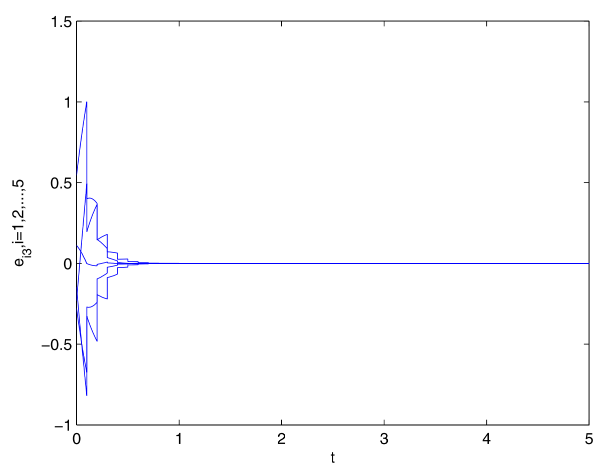

Figure 5 Time evolution of synchronization errors $e_{i3}(t)$, $i = 1, 2, \ldots, 5$ of the asymmetric coupled network (13) with random initial values.

and

$$G^3 = \begin{bmatrix} -4 & 1 & 1 & 1 & 1 \\ 1 & -4 & 1 & 1 & 1 \\ 1 & 1 & -4 & 1 & 1 \\ 1 & 1 & 1 & -4 & 1 \\ 1 & 1 & 1 & 1 & -4 \end{bmatrix}.$$

Hence, $\lambda = \{|\lambda_i(G^\sigma)||i = 1, \ldots, 5, \sigma = 1, 2, 3\} = 5$. Choose the synchronization state $s(t) = 0.2\sum_{i=1}^{5} x_i(t)$ and switch time $t = 0.2s$. If $\beta = 18.7996$, $Q = 3.7I_3$, $|1 + b| = 0.18$, and $T_{\max} = T_{\min} = 0.1$, then all the conditions in Theorem 2 are satisfied, and $\eta = -0.5466$, $a = 3.7$, so the symmetric coupled network (13) can achieve exponential synchronization. The simu-

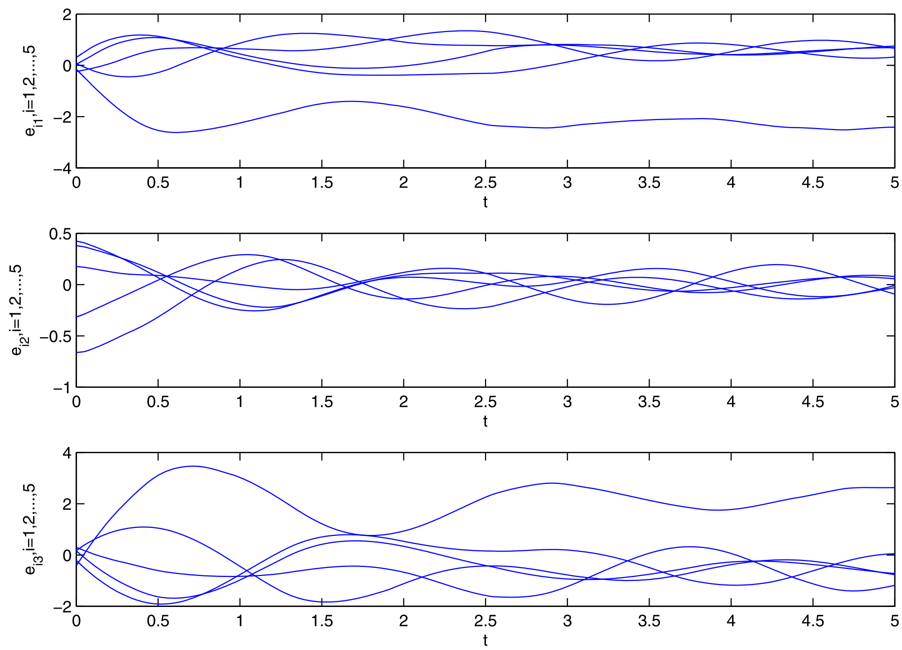

Figure 6 Time evolution of errors $e_{ij}(t)$, $i = 1, 2, \ldots, 5$, $j = 1, 2, 3$ of the symmetric coupled network (13) without impulsive controller.

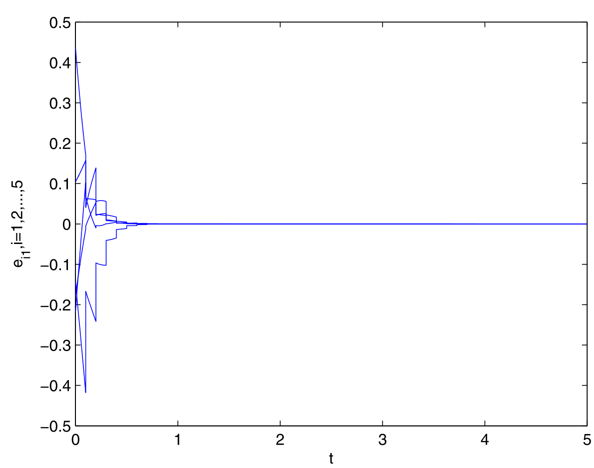

Figure 7 Time evolution of synchronization errors $e_{i1}(t)$, $i = 1, 2, \ldots, 5$ of the symmetric coupled network (13) with random initial values.

lation results are given in Figures 7-9. It can be seen clearly from Figures 7-9 that all states of the symmetric coupled network (13) tend to the synchronization state $s(t)$.

5 Conclusions

In this paper, by establishing an impulsive delay differential inequality, the exponential synchronization of the coupling delay switching complex networks has been investigated. Based on Lyapunov stability theory, some simple yet generic criteria for exponential synchronization have been derived. It shows that criteria can provide an effective impulsive

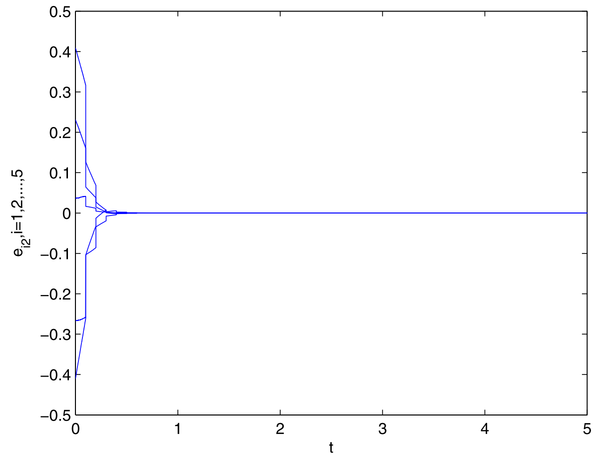

Figure 8 Time evolution of synchronization errors $e_{i2}(t)$, $i = 1, 2, \dots, 5$ of the symmetric coupled network (13) with random initial values.

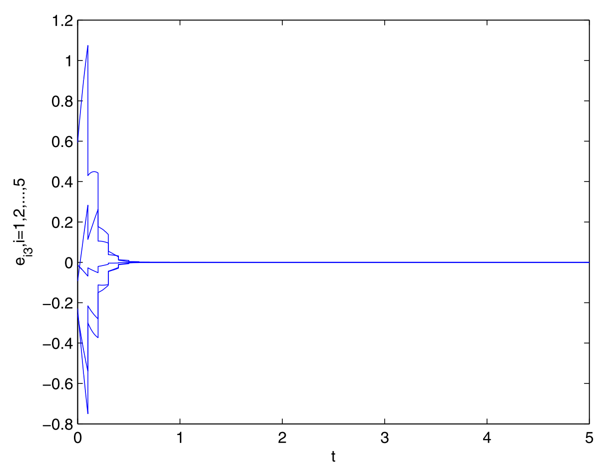

Figure 9 Time evolution of synchronization errors $e_{i3}(t)$, $i = 1, 2, \dots, 5$ of the symmetric coupled network (13) with random initial values.

control scheme to synchronize for an arbitrary given switch topology. Furthermore, the effectiveness of the presented method has been verified by numerical simulations.

Appendix

Proof of Lemma 1 For $t \in [t_k, t_{k+1})$, integrating both sides of equation (5) from t_k to t, we can get

$$\begin{aligned} u(t) - u(t_k) &\le \int_{t_k}^{t} pu(s) + qu(s-\tau)\,ds \\ &= \int_{t_k}^{t} pu(s)\,ds + \int_{t_k}^{t} qu(s-\tau)\,ds. \end{aligned}$$

It is easy to obtain that

$$\begin{aligned} u(t) &\leq u(t_k) + \int_{t_k-\tau}^{t} pu(s)\,ds + \int_{t_k-\tau}^{t} qu(s)\,ds \\ &= u(t_k) + \int_{t_k-\tau}^{t} (p+q)u(s)\,ds. \end{aligned} \tag{14}$$

Now, we begin to prove that

$$u(t) \leq \left(\prod_{i=1}^{k} \alpha_i\right) e^{(p+q)(t-t_0+k\tau)} \Bigl(\sup_{t_0-\tau\leq s\leq t_0} \phi(s)\Bigr), \quad t \in [t_k, t_{k+1}), k \in Z^+. \tag{15}$$

We shall show this by induction.

For $t \in [t_0, t_1)$, by Lemma 3 in [30], we have

$$\begin{aligned} u(t) &\leq u(t_0) + \int_{t_0}^{t} \Bigl[pu(s) + q \sup_{s-\tau\leq\theta\leq s} u(\theta)\Bigr]\,ds \\ &\leq \Bigl(\sup_{t_0-\tau\leq s\leq t_0} \phi(s)\Bigr) e^{(p+q)(t-t_0)}. \end{aligned} \tag{16}$$

In view of (16), we see that (15) holds when $k = 0$. Under the inductive assumption that (15) holds for some $k \geq 0$, we shall show that (15) still holds for $k + 1$.

For $t \in [t_{k+1}, t_{k+2})$, without any loss of generality, we assume that there are l first-class intermittent points, then (14) can be rewritten as

$$\begin{aligned} u(t) \leq{}& u(t_{k+1}) + \int_{t_{k+1}-\tau}^{t_{k-l+1}} (p+q)u(s)\,ds \\ &+ \sum_{i=1}^{l} \int_{t_{k-i+1}}^{t_{k-i+2}} (p+q)u(s)\,ds + \int_{t_{k+1}}^{t} (p+q)u(s)\,ds. \end{aligned} \tag{17}$$

Noting $z(t) = \int_{t_{k+1}}^{t} (p+q)u(s)\,ds + \sum_{i=1}^{l} \int_{t_{k-i+1}}^{t_{k-i+2}} (p+q)u(s)\,ds + \int_{t_{k+1}-\tau}^{t_{k-l+1}} (p+q)u(s)\,ds$, then the derivative of $z(t)$ can be calculated as follows:

$$\begin{aligned} \dot{z}(t) &= (p+q)u(t) \\ &\leq (p+q)u(t_1) + (p+q)\left(\int_{t_{k+1}}^{t} (p+q)u(s)\,ds \right.\\ &\quad \left. + \sum_{i=1}^{l} \int_{t_{k-i+1}}^{t_{k-i+2}} (p+q)u(s)\,ds + \int_{t_{k+1}-\tau}^{t_{k-l+1}} (p+q)u(s)\,ds\right) \\ &= (p+q)u(t_{k+1}) + (p+q)z(t). \end{aligned}$$

It is not difficult to show that

$$\bigl[\dot{z}(t) - (p+q)z(t)\bigr] e^{-(p+q)(t-t_{k+1}+\tau)} \leq (p+q)u(t_{k+1}) e^{-(p+q)(t-t_{k+1}+\tau)}.$$

Clearly,

$$\frac{d}{dt}\bigl[z(t) e^{-(p+q)(t-t_{k+1}+\tau)}\bigr] \leq (p+q)u(t_{k+1}) e^{-(p+q)(t-t_{k+1}+\tau)}.$$

Hence, we have

$$z(t)e^{-(p+q)(t-t_{k+1}+\tau)} - z(t_{k+1}-\tau) \le (p+q)u(t_{k+1}) \int_{t_{k+1}-\tau}^{t} e^{-(p+q)(s-t_{k+1}+\tau)}\,ds.$$

Since $z(t_{k+1}-\tau) = 0$, one has

$$\begin{aligned} z(t) &\le e^{(p+q)(t-t_{k+1}+\tau)} \int_{t_{k+1}-\tau}^{t} (p+q)u(t_{k+1})e^{-(p+q)(s-t_{k+1}+\tau)}\,ds \\ &= u(t_{k+1})e^{(p+q)(t-t_{k+1}+\tau)} - u(t_{k+1}). \end{aligned} \tag{18}$$

Substituting (18) into (17) yields

$$\begin{aligned} u(t) &\le \alpha_{k+1}u\bigl(t_{k+1}^{-}\bigr)e^{(p+q)(t-t_{k+1}+\tau)} \\ &\le \left(\prod_{i=1}^{k+1}\alpha_i\right)\Bigl(\sup_{t_0-\tau\le s\le t_0}\phi(s)\Bigr)e^{(p+q)(t-t_0+(k+1)\tau)}. \end{aligned}$$

That is, (15) holds for $k+1$. Hence, by induction, (15) holds for all $k \ge 0$.

The proof is complete. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
AD carried out the main part of this manuscript. WZ participated in the discussion and corrected the main theorem. All authors read and approved the final manuscript.

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