Wang et al. *Advances in Difference Equations* 2013, **2013**:225
http://www.advancesindifferenceequations.com/content/2013/1/225

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH **Open Access**

The lag projective (anti-)synchronization of chaotic systems with bounded nonlinearity via an adaptive control scheme

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Abstract

This paper investigates the lag projective synchronization and anti-synchronization problems for a general master-slave chaotic system with bounded nonlinearity. An adaptive controller is designed to guarantee that the slave system synchronizes with the master system by using Lyapunov stability theory and the idea of bang-bang control. Different from some existing master-slave models, the nonlinear terms in the considered chaotic system only need to satisfy bounded conditions. Furthermore, the structure of the master system does not need to match that of the slave system. Finally, three numerical examples are given to illustrate the main results.

1 Introduction

In the past few decades, the synchronization of master-slave chaotic systems has become a hot research topic in the nonlinear science field. One of the most important reasons is its potential applications in the real world such as the secure communications [1, 2], telecommunication [3–6], chemical reactions and biological systems [7, 8], *etc.* For a master-slave system, the synchronization means that the state of a slave system follows the state of a master system asymptotically by using the output of the master system to control the slave system. It is well known that the state trajectories of chaotic systems closely depend on the initial conditions, then it is difficult to drive a master-slave chaotic system with different initial conditions to achieve synchronization. In particular, for a master-slave chaotic system with different state equations, this problem becomes much more difficult.

To date, many papers on the study of the synchronization control of chaotic systems have appeared. For example, Su *et al.* investigated the synchronization of a discrete master-slave chaotic system with impulsive effect in [9], and Qi *et al.* investigated the H_∞ synchronization of a discrete master-slave chaotic system with external disturbance in [10], respectively. In these two papers, the master system is required to match well the slave system; moreover, the nonlinear parts in the master-slave system must satisfy the Lipschitz condition. For continuous cases, there are also many related results. For instance, in [11] and [12], Yang *et al.* and Xu *et al.* studied the lag synchronization of the chaotic system with time delay. The anti-synchronization of chaotic systems is investigated in [13, 14] by using the adaptive control method. The projective synchronization of chaotic systems is studied by utilizing adaptive back-stepping control, state feedback control and impulsive control in [15–17], respectively. Similar results can be found in [18–28] and the references therein. In these papers, the chaotic synchronization problems mainly include synchronization, projective synchronization, anti-synchronization and lag synchronization. The used synchronization control methods mostly include the state feedback control, pinning control, adaptive control and impulsive control.

Although there have been a lot of literature works studying the synchronization problem of chaotic systems, we find the following deficiencies: (1) Most of them require the nonlinear parts in the chaotic systems to satisfy the Lipschitz condition. (2) Each paper usually only could solve a single synchronization problem, which leads to the lack of a unified method to solve all the synchronization problems for the same master-slave model. Motivated by these factors, we aim in this paper to find a kind of effective method to deal with all the synchronization problems for a general master-salve chaotic system. The contributions of this paper are as follows: (i) The lag projective synchronization which can include synchronization, projective synchronization, anti-synchronization and lag synchronization at the same time is investigated. (ii) The considered master-slave system model is different from the systems in the literature, in which the nonlinearities only need to satisfy a bounded condition. Moreover, the state equations of the master system and the slave system are non-identical. (iii) The presented results are very concise and it is easy to adjust the synchronization rate by the control gains.

The rest of this paper is organized as follows. In Section 2, the investigated master-slave chaotic system is presented and an adaptive synchronization controller is designed. In Section 3, three numerical examples are provided to illustrate the effectiveness of the obtained method. Conclusions are drawn in Section 4.

Notations

R^n and $R^{n\times m}$ denote the n-dimensional Euclidean space and the set of $n\times m$ real matrices, respectively. R_+ stands for the set of positive real numbers. For an n-dimensional vector $v=(v_1,v_2,\ldots,v_n)^T\in R^n$, whose norms are defined as $\|v\|_1=\sum_{i=1}^n|v_i|$, $\|v\|_2=\sqrt{\sum_{i=1}^n v_i^2}$ and $\|v\|_\infty=\max_{1\le i\le n}\{|v_i|\}$, $\operatorname{sign}(v)=(\operatorname{sign}(v_1),\operatorname{sign}(v_2),\ldots,\operatorname{sign}(v_n))^T$. A^T denotes the transpose of matrix A, the notation $X\ge Y$ (respectively, $X>Y$) means that $X-Y$ is a symmetric semi-definite matrix (respectively, positive definite matrix), where X, Y are symmetric matrices. I_n is the $n\times n$ identical matrix.

2 Synchronization control scheme

Consider a general master-slave chaotic dynamical system that is described as follows:

$$\begin{cases}\text{Master: } \dot{x}(t)=Ax(t)+f(x(t),t),\\ \text{Slave: } \dot{y}(t)=By(t)+g(y(t),t)+u(t),\end{cases}\tag{1}$$

where $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))^T\in R^n$ and $y(t)=(y_1(t),y_2(t),\ldots,y_n(t))^T\in R^n$ are the state vectors of the master system and the slave system, respectively. $A\in R^{n\times n}$ and $B\in R^{n\times n}$ are two known real matrices, $f(\cdot):R^n\times R_+\to R^n$ and $g(\cdot):R^n\times R_+\to R^n$ are two bounded nonlinear vector functions. $u(t)\in R^n$ is the control input to be determined.

For given scalars $\tau\ge 0$ and $\mu\ne 0$, define the error state as $e(t)=y(t)+\mu x(t-\tau)$, then one can obtain the following error system:

$$\dot{e}(t)=Ae(t)+g\big(y(t),t\big)+\mu f\big(x(t-\tau),t-\tau\big)+(B-A)y(t)+u(t).\tag{2}$$

Definition 1 For given constants τ and μ, master-salve chaotic system (1) is said to be lag projective synchronization if

$$\lim_{t\to+\infty}\|e(t)\|_2 = 0 \tag{3}$$

for any initial conditions.

Remark 1 The aim of this paper is to design a suitable controller $u(t)$ such that the controlled slave system synchronizes with the master system. Thus, by Definition 1, one only needs to design a controller such that error system (2) is stable.

Remark 2 Especially, master-salve chaotic system (1) is said to be lag anti-synchronization if $\mu = 1$; master-salve chaotic system (1) is said to be lag synchronization if $\mu = -1$. When time delay $\tau = 0$, master-salve chaotic system (1) is said to be anti-synchronization if $\mu = 1$; master-salve chaotic system (1) is said to be synchronization if $\mu = -1$; master-salve chaotic system (1) is said to be projective synchronization if $\mu \neq \pm 1$. Then Definition 1 unites several kinds of synchronization definitions together and is very general.

In order to study the lag projective synchronization of master-slave chaotic system (1), the following assumption is firstly given.

Assumption 1 Assume that the nonlinear functions in master-slave system (1) are bounded, *i.e.*, there exist two constants $M_f \geq 0$ and $M_g \geq 0$ such that

$$\|f(x(t),t)\|_\infty \leq M_f, \qquad \|g(y(t),t)\|_\infty \leq M_g$$

hold for any $x(t), y(t) \in R^n$ and $t > 0$.

Remark 3 There exist a lot of papers that study the synchronization problems of master-slave chaotic systems. Different from them, the nonlinear functions $f(x(t),t)$ and $g(y(t),t)$ in system (1) need not satisfy the Lipschitz condition. In fact, the bounded functions could satisfy the Lipschitz condition. For example, $f(z) = \sin\frac{1}{z}$ is bounded but does not satisfy the Lipschitz condition in the neighborhood of zero. Conversely, the functions satisfying the Lipschitz condition may not be bounded. For instance, $f(z) = z$ satisfies the Lipschitz condition but it is not bounded. Therefore, the Lipschitz condition and bounded condition are different. To the best of our knowledge, the results on this topic are few.

In this paper, the following controller and updated laws are designed

$$\begin{cases} u(t) = -(B-A)y(t) - L_1(t)e(t) - L_2(t)\cdot \operatorname{sign}(Pe(t)), \\ \dot{L}_1(t) = \alpha e^T(t)Pe(t), \\ \dot{L}_2(t) = \beta\|Pe(t)\|_1 \end{cases} \tag{4}$$

to promote master-slave system (1) achieving synchronization, where α and β are any given positive constants, $P \in R^{n\times n}$ is a positive definite symmetric matrix to be determined.

In what follows, the main result is obtained.

Theorem 1 *Suppose master-slave chaotic system* (1) *satisfies Assumption* 1. *If there exist scalars* $L_1^* > 0$, $L_2^* > 0$ *and a positive definite symmetric matrix* $P \in R^{n\times n}$ *such that*

$$PA + A^T P - 2L_1^* P < 0 \tag{5}$$

and

$$M_f|\mu| + M_g < L_2^* \tag{6}$$

hold, then master-slave chaotic system (1) *synchronizes well under the action of adaptive controller* (4), *where* α *and* β *are any given positive constants.*

Proof Choose the following Lyapunov function:

$$V\big(e(t)\big) = e^T(t)Pe(t) + \frac{1}{\alpha}\big(L_1(t) - L_1^*\big)^2 + \frac{1}{\beta}\big(L_2(t) - L_2^*\big)^2, \tag{7}$$

where L_1^* and L_2^* are sufficiently large positive constants to be determined in the sequel. The derivative of $V(e(t))$ along the state trajectories of system (2) is

$$\begin{aligned}
\dot V\big(e(t)\big) &= \dot e^T(t)Pe(t) + e^T(t)P\dot e(t) + 2\alpha^{-1}\big(L_1(t) - L_1^*\big)\dot L_1(t) + 2\beta^{-1}\big(L_2(t) - L_2^*\big)\dot L_2(t) \\
&= \big[Ae(t) + g\big(y(t),t\big) + \mu f\big(x(t-\tau),t-\tau\big) \\
&\quad + (B-A)y(t) + u(t)\big]^T Pe(t) + e^T(t)P\big[Ae(t) + g\big(y(t),t\big) \\
&\quad + \mu f\big(x(t-\tau),t-\tau\big) + (B-A)y(t) + u(t)\big] \\
&\quad + 2\big(L_1(t) - L_1^*\big)e^T(t)Pe(t) + 2\big(L_2(t) - L_2^*\big)\big\|Pe(t)\big\|_1 \\
&= e^T(t)\big(PA + A^TP\big)e(t) + g^T\big(y(t),t\big)Pe(t) + e^T(t)Pg\big(y(t),t\big) \\
&\quad + \mu\big[f^T\big(x(t-\tau),t-\tau\big)Pe(t) + e^T(t)Pf\big(x(t-\tau),t-\tau\big)\big] \\
&\quad - 2e^T(t)P\big[L_1(t)e(t) + L_2(t)\cdot \operatorname{sign}\big(Pe(t)\big)\big] \\
&\quad + 2\big(L_1(t) - L_1^*\big)e^T(t)Pe(t) + 2\big(L_2(t) - L_2^*\big)\big\|Pe(t)\big\|_1 \\
&= e^T(t)\big(PA + A^TP\big)e(t) + g^T\big(y(t),t\big)Pe(t) + e^T(t)Pg\big(y(t),t\big) \\
&\quad + \mu\big[f^T\big(x(t-\tau),t-\tau\big)Pe(t) + e^T(t)Pf\big(x(t-\tau),t-\tau\big)\big] \\
&\quad - 2L_1^*e^T(t)Pe(t) - 2L_2^*\big\|Pe(t)\big\|_1.
\end{aligned} \tag{8}$$

From Assumption 1, one gets

$$g^T\big(y(t),t\big)Pe(t) + e^T(t)Pg\big(y(t),t\big) \le 2\big\|g\big(y(t),t\big)\big\|_\infty \cdot \big\|Pe(t)\big\|_1 \le 2M_g\big\|Pe(t)\big\|_1 \tag{9}$$

and

$$\begin{aligned}
&\mu\big[f^T\big(x(t-\tau),t-\tau\big)Pe(t) + e^T(t)Pf\big(x(t-\tau),t-\tau\big)\big] \\
&\quad \le 2|\mu|\cdot\big\|f\big(x(t-\tau),t-\tau\big)\big\|_\infty \cdot \big\|Pe(t)\big\|_1 \\
&\quad \le 2M_f|\mu|\cdot\big\|Pe(t)\big\|_1.
\end{aligned} \tag{10}$$

Inequalities (9) and (10) yield that

$$\dot{V}(e(t)) \le e^T(t)(PA + A^T P - 2L_1^* P)e(t) + 2(M_f|\mu| + M_g - L_2^*)\|Pe(t)\|_1. \tag{11}$$

From inequalities (5) and (6), we have

$$\dot{V}(e(t)) < 0.$$

By the Lasalle invariance principle of differential equation, it is known that all the state trajectories of system (1) converge to the set $S = \{e(t)|e(t) = 0\}$, which implies that $\lim_{t\to+\infty}\|e(t)\|_2 = 0$ and the slave system synchronizes with the master system. This completes the proof. □

Remark 4 Adaptive controller (4) is divided into three parts. The first and second parts can be thought of as the state feedback control, and the last part can be thought of as a suitable complement for the bounded nonlinearities in the chaotic system by using the idea of the bang-bang control.

Remark 5 It is easy to see that adaptive controller (4) can always drive the slave system to synchronize with the master system since inequalities (5) and (6) always have feasible solutions. Different from the methods in [28, 29], our method does not need to do some complex computation. Furthermore, the synchronization rate can be altered through adjusting the control gains α and β.

Remark 6 From Theorem 1, it is also easy to design the synchronization controller, anti-synchronization controller, lag synchronization controller and projective synchronization controller for chaotic system (1).

3 Numerical simulations

In this section, three examples are given to show the effectiveness of the proposed method.

Example 1 Consider the known second-order non-autonomous chaotic system [30]

$$\ddot{y}(t) + \delta[d\cos 2y(t) - d + 1]\dot{y}(t) + (c^2 - \cos y(t))\sin y(t) = \delta\Gamma\cos wt, \tag{12}$$

where c, d, δ, w, Γ are positive constants.

Let $y(t) = x_1(t)$ and $\dot{y}(t) = x_2(t)$, then system (12) can be transformed to

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = -\delta[d\cos 2x_1(t) - d + 1]x_2(t) - (c^2 - \cos x_1(t))\sin x_1(t) + \delta\Gamma\cos wt \end{cases} \tag{13}$$

or

$$\dot{x}(t) = Ax(t) + f(x(t), t),$$

where $x(t) = (x_1(t), x_2(t))^T$, $A = \begin{bmatrix} 0 & 1 \\ 0 & \delta(d-1) \end{bmatrix}$, $f(x(t), t) = (0, -\delta d\cos 2x_1(t) \cdot x_2(t) - (c^2 - \cos x_1(t))\sin x_1(t) + \delta\Gamma\cos wt)^T$. When choosing $c = 0.07338$, $d = 0.8$, $\delta = 0.1$ and $w = \Gamma = 1$,

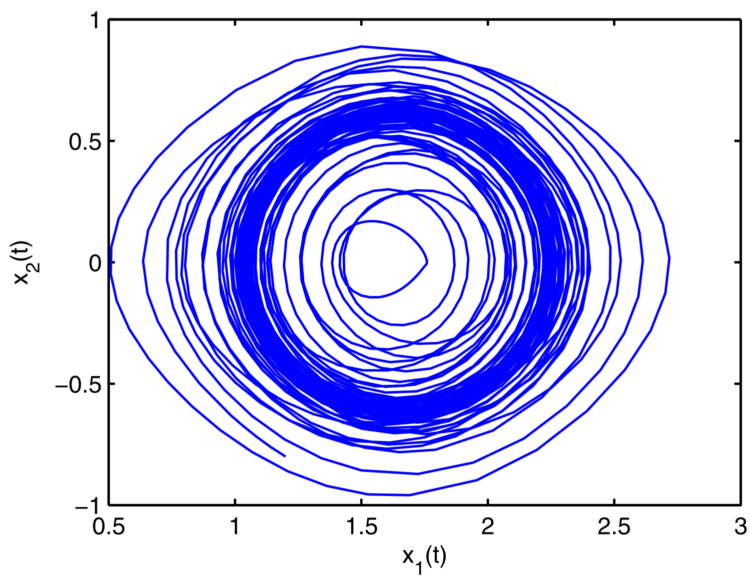

Figure 1 Phase diagram of system (13) with initial condition $x(0) = (1.2, -0.8)^T$.

the phase diagram of system (13) is shown in Figure 1. This figure shows that the state trajectories are bounded for $t > 0$, then $f(x(t), t)$ is bounded for all the $x(t)$ and Assumption 1 is satisfied. Taking two non-autonomous systems defined by (12) to constitute a master-slave system, setting $\mu = -1$ and $\tau = 2$, and computing by the LMI's Toolbox in the Matlab, the following feasible solutions

$$P = \operatorname{diag}\{1, 1\}, \qquad L_1^* = 0.1, \qquad L_2^* = 0.5$$

are obtained. Under the action of adaptive controller (4), taking the initial conditions as $x(0) = (1.2, -0.8)^T$ and $y(0) = (-12, 8)^T$, $\alpha = \beta = 0.1$, we show the state trajectories of error system (2) and updated laws in adaptive controller (4) in Figures 2-4, respectively. These figures show that the master-slave system is lag synchronization.

Remark 7 In [30], Wu *et al.* investigated a synchronization problem of the master-slave system composed of system (12). Using their methods, one needs to do some complex computation and discussion. Hence, our method is more concise than theirs.

Example 2 Consider the following Chua's circuit [29]:

$$\begin{cases} \dot{x}_1(t) = a[x_2(t) - m_1 x_1(t)] - ah(x_1(t)), \\ \dot{x}_2(t) = x_1(t) - x_2(t) + x_3(t), \\ \dot{x}_3(t) = -bx_2(t), \end{cases} \tag{14}$$

where $h(x_1(t)) = \frac{1}{2}(m_0 - m_1)(|x_1(t) + c| - |x_1(t) - c|)$, $a = 9$, $b = 14.28$, $c = 1$, $m_0 = -1/7$ and $m_1 = 2/7$. System (14) can be represented by

$$\dot{x}(t) = Ax(t) + f\big(x(t), t\big),$$

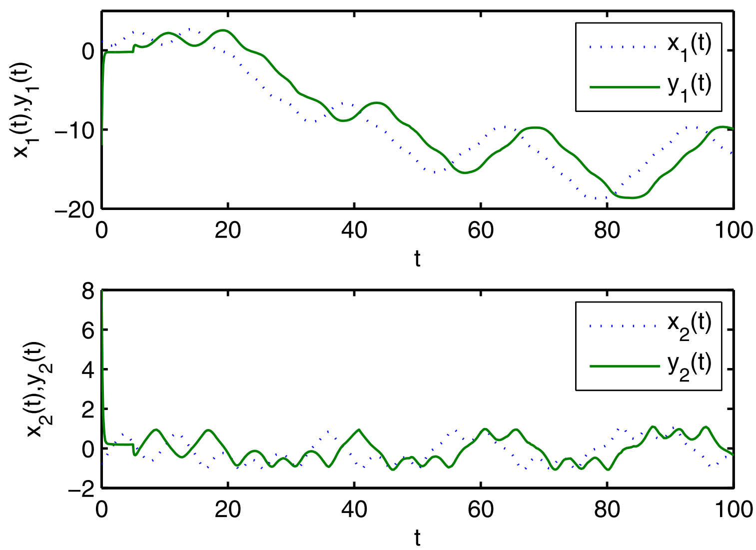

Figure 2 State trajectories of the master-slave system with initial conditions $x(0) = (1.2, -0.8)^T$ and $y(0) = (-12, 8)^T$.

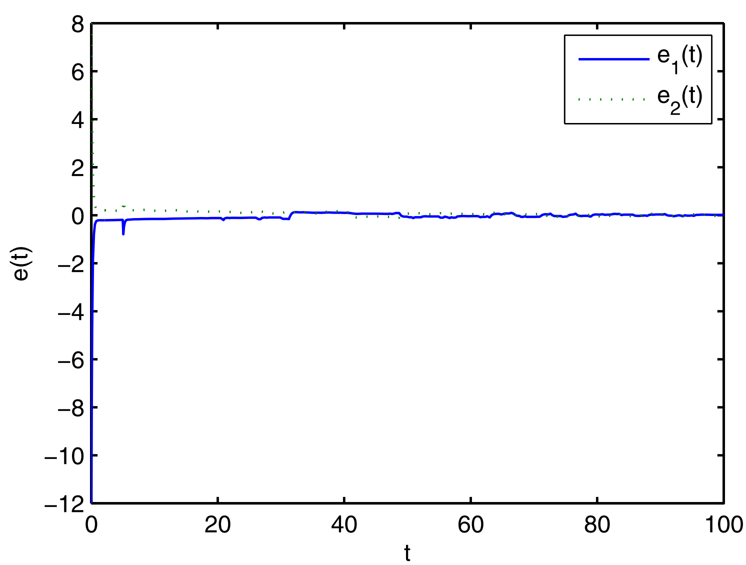

Figure 3 Synchronization error trajectories.

where $A = \begin{bmatrix} -am_1 & a & 0 \\ 1 & -1 & 1 \\ 0 & -b & 0 \end{bmatrix}$, $f(x(t), t) = (-ah(x_1(t)), 0, 0)^T$. Taking this system as the master system, the corresponding slave system is

$$\dot{y}(t) = Ay(t) + g(y(t), t),$$

where $g(y(t), t) = (0, h(y_2(t)), 0)^T$. Since

$$|x_1(t) + c| - |x_1(t) - c| \le |(x_1(t) + c) - (x_1(t) - c)| = 2c,$$

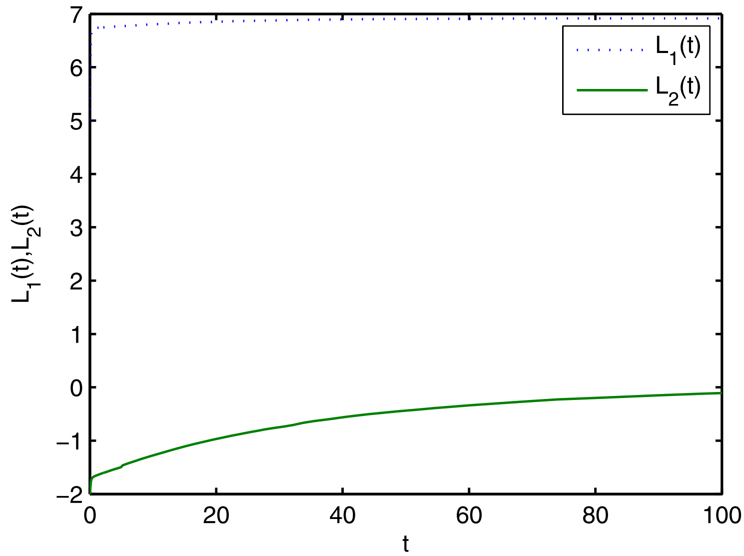

Figure 4 State trajectories of updated laws.

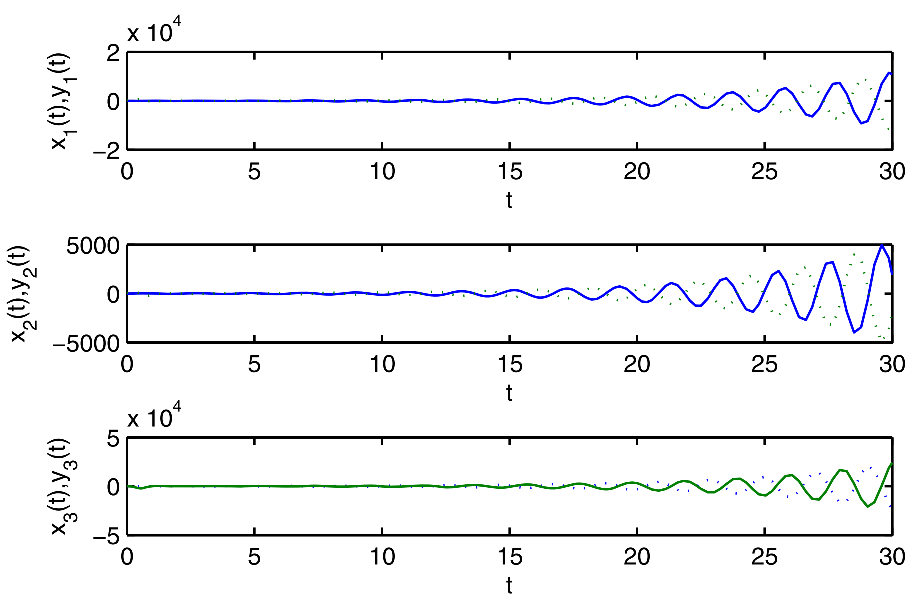

Figure 5 State trajectories of the master-slave system with initial conditions $x(0) = (20, -20, 50)^T$ and $y(0) = (40, 70, -50)^T$.

then $\|f(x(t), t)\|_\infty = \|g(y(t), t)\|_\infty \leq ac|m_0 - m_1|$, which implies that Assumption 1 holds. Considering the anti-synchronization of the master-slave system composed of system (14), choosing $\mu = 1$, $\tau = 0$, and computing by the LMI's Toolbox in the Matlab, one can obtain the feasible solutions of inequalities (5) and (6) as follows:

$$P = \operatorname{diag}\{1, 1, 1\}, \qquad L_1^* = 7.5, \qquad L_2^* = 8.$$

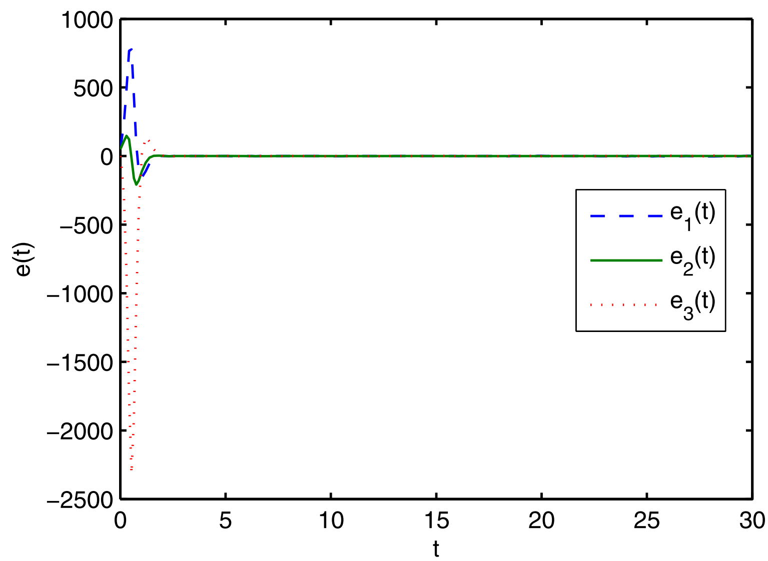

Figure 6 Anti-synchronization error trajectories.

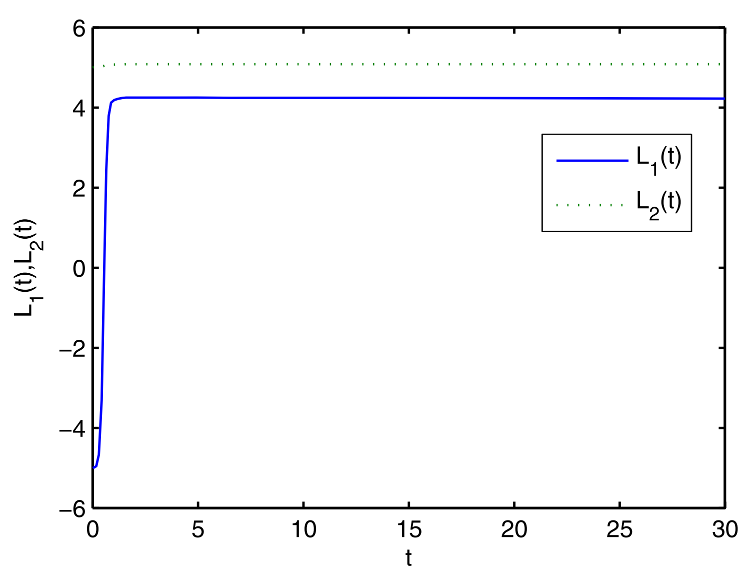

Figure 7 State trajectories of updated laws.

Taking the initial conditions as $x(0) = (20, -20, 50)^T$ and $y(0) = (40, 70, -50)^T$, $\alpha = \beta = 0.5$, under the action of adaptive controller (4), we show the illustration results in Figures 5-7, respectively. These figures show that the master-slave system is anti-synchronization.

Example 3 Consider the following dynamical system:

$$\dot{x}(t) = Ax(t) + f\big(x(t), t\big), \tag{15}$$

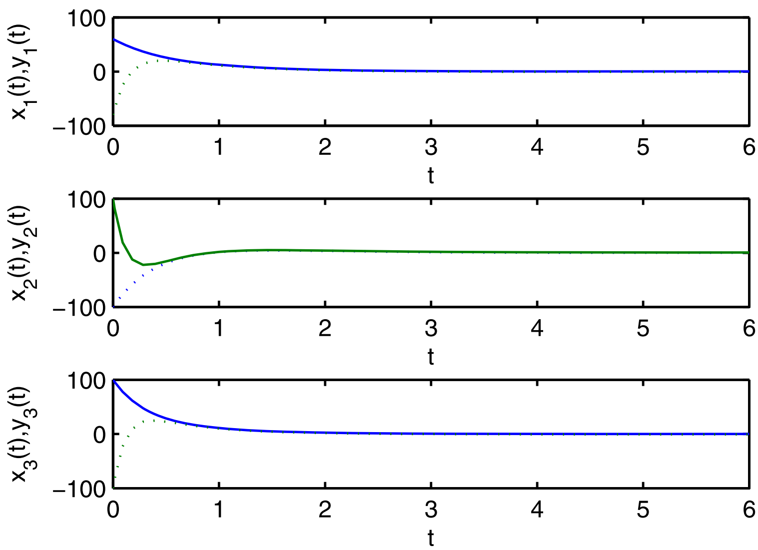

Figure 8 State trajectories of the master-slave system with initial conditions $x(0) = (60, -100, 100)^T$ and $y(0) = (-80, 100, -100)^T$.

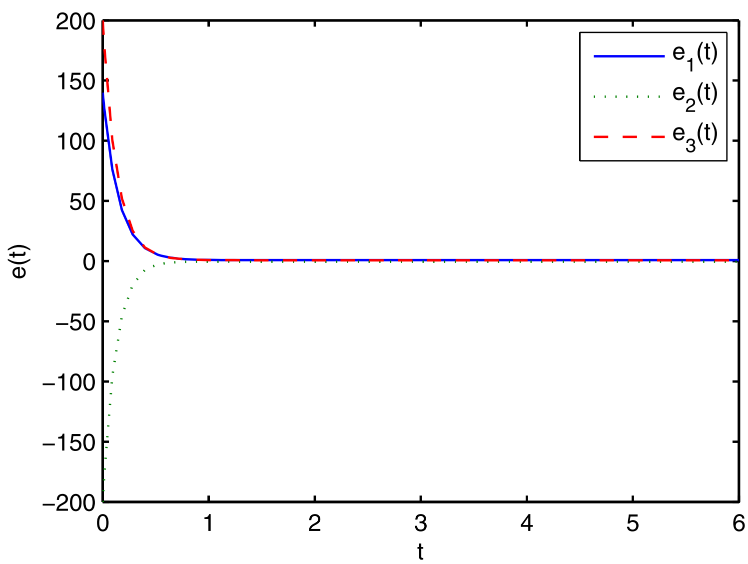

Figure 9 Synchronization error trajectories.

where $A = \begin{bmatrix} -1 & 0 & -0.5 \\ 1.5 & -2 & 0 \\ 2 & 0 & -4 \end{bmatrix}$, $f(x(t),t) = (\sin\frac{1}{|x_1(t)|+\varepsilon}, \sin\frac{1}{|x_2(t)|+\varepsilon}, \sin\frac{1}{|x_3(t)|+\varepsilon})^T$, $\varepsilon = 0.00001$. Taking system (15) as the master system, and $\dot{y}(t) = Ay(t) - g(y(t),t)$ as the slave system, where $g(y(t),t) = (\cos\frac{1}{|y_1(t)|+\varepsilon}, \cos\frac{1}{|y_2(t)|+\varepsilon}, \cos\frac{1}{|y_3(t)|+\varepsilon})^T$, it is easy to see $\|f(x(t),t)\|_\infty = \|g(y(t),t)\|_\infty \le 1$ for any $x(t), y(t) \in R^3$, and $f(x(t),t)$ and $g(y(t),t)$ do not satisfy the Lipschitz condition. Considering the synchronization of this master-slave system and choosing $\mu = -1$, $\tau = 0$, after computing by the LMI's Toolbox in the Matlab, one can get the feasible solutions of inequalities (5) and (6) as follows:

$$P = \operatorname{diag}\{1,1,1\}, \qquad L_1^* = 4, \qquad L_2^* = 1.1.$$

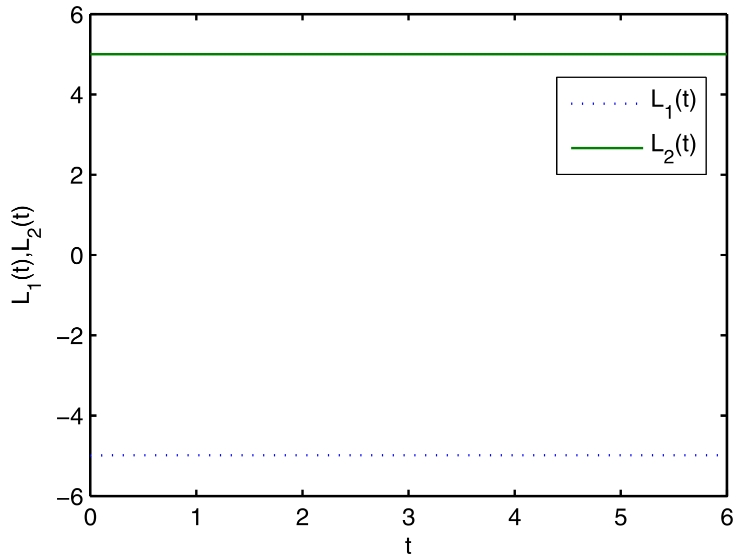

Figure 10 State trajectories of updated laws.

Taking the initial conditions as $x(0) = (60, -100, 100)^T$ and $y(0) = (-80, 100, -100)^T$, $\alpha = \beta = 0.2$, under the action of adaptive controller (4), we show the illustration results in Figures 8-10. These figures show that the master-slave system is synchronization. It should be noticed that the nonlinear function in system (15) is bounded but does not satisfy the Lipschitz condition.

4 Conclusions

In this paper, the lag projective (anti-)synchronization problems for a kind of master-slave chaotic systems by using the adaptive control method have been investigated. Based on the Lasalle invariance principle of differential equation and the idea of the bang-bang control, an adaptive controller with simple updated laws has been proposed. Three numerical examples have shown that the obtained method is effective.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
TW and SZ drafted the manuscript, WZ and WY performed the numerical illustration. All authors read and approved the final manuscript.

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Acknowledgements
This work is supported by the Connotative Construction Project of Shanghai University of Engineering Science (nhky-2012-13).

Received: 16 March 2013 Accepted: 5 July 2013 Published: 24 July 2013

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doi:10.1186/1687-1847-2013-225
Cite this article as: Wang et al.: The lag projective (anti-)synchronization of chaotic systems with bounded nonlinearity via an adaptive control scheme. *Advances in Difference Equations* 2013 **2013**:225.