Wang and Song *Advances in Difference Equations* 2013, 2013:367
http://www.advancesindifferenceequations.com/content/2013/1/367

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH **Open Access**

Periodic solutions for the p-Laplacian neutral functional differential system

Zhenyou Wang and Changxiu Song*

*Correspondence: scx168@sohu.com
School of Applied Mathematics, Guangdong University of Technology, Guangzhou, 510006, China

Abstract

By using the generalized Borsuk theorem in coincidence degree theory, we prove the existence of periodic solutions for the p-Laplacian neutral functional differential system.

MSC: 34C25

Keywords: p-Laplacian; periodic solutions; coincidence degree

1 Introduction

In recent years, the existence of periodic solutions for the Rayleigh equation and the Liénard equation has been studied (see [1–9]). By using topological degree theory, some results on the existence of periodic solutions are obtained.

Motivated by the works in [1–9], we consider the existence of periodic solutions of the following system:

$$\frac{d}{dt}\phi_p\big[\big(x(t)-Cx(t-\tau)\big)'\big]+\frac{d}{dt}\operatorname{grad}F\big(x(t)\big)+\operatorname{grad}G\big(x(t)\big)=e(t), \tag{1.1}$$

where $F \in C^2(R^n, R)$, $G \in C^1(R^n, R)$, $e \in C(R, R^n)$ are periodic functions with period T; $C = [c_{ij}]_{n\times n}$ is an $n \times n$ symmetric matrix of constants, $\tau \in R$ is a constant. $\phi_p : R^n \to R^n$ is given by

$$\phi_p(u)=\phi_p(u_1,\ldots,u_n):=\big(|u_1|^{p-2}u_1,\ldots,|u_n|^{p-2}u_n\big)^T,\quad 1<p<\infty.$$

The ϕ_p is a homeomorphism of R^n with the inverse ϕ_q. By using the theory of coincidence degree, we obtain some results to guarantee the existence of periodic solutions. Even for $p = 2$, the results in this paper are also new.

In what follows, we use $\langle\cdot,\cdot\rangle$ to denote the Euclidean inner product in R^n and $|\cdot|_p$ to denote the l^p-norm in R^n, *i.e.*, $|x|_p = (\sum_{i=1}^n |x_i|^p)^{1/p}$.

The norm in $R^{n\times n}$ is defined by $\|A\|_p = \sup_{|x|_p=1, x\in R^n} |Ax|_p$.

The corresponding L^p-norm in $L^p([0,T], R^n)$ is defined by

$$\|x\|_p=\left(\sum_{i=1}^n\int_0^T\big|x_i(t)\big|^p\,dt\right)^{\frac{1}{p}}=\left(\int_0^T\big|x(t)\big|_p^p\,dt\right)^{\frac{1}{p}},$$

and the L^∞-norm in $L^\infty([0,T],R^n)$ is

$$\|x\|_\infty = \max_{1\le i\le n} \|x_i\|_\infty,$$

where $\|x_i\|_\infty = \sup_{t\in[0,\omega]} |x_i(t)|$ $(i = 1,\dots,n)$.

Let $W = W^{1,p}([0,T],R]$ be the Sobolev space.

Lemma 1.1 (See [8]) *Suppose $u \in W$ and $u(0) = u(T) = 0$, then*

$$\|u\|_p \le \left(\frac{T}{\pi_p}\right)\|u'\|_p,$$

where

$$\pi_p = 2\int_0^{(p-1)^{1/p}} \frac{ds}{(1-\frac{s^p}{p-1})^{1/p}} = \frac{2\pi(p-1)^{1/p}}{p\sin(\frac{\pi}{p})}.$$

In order to use coincidence degree theory to study the existence of T-periodic solutions for (1.1), we rewrite (1.1) in the following form:

$$\begin{cases} (x(t) - Cx(t-\tau))'(t) = \phi_q(y(t)), \\ y'(t) = \frac{d}{dt}\operatorname{grad} F(x(t)) - \operatorname{grad} G(x(t)) + e(t). \end{cases} \tag{1.2}$$

If $z(t) = \binom{x(t)}{y(t)}$ is a T-periodic solution of (1.2), $x(t)$ must be a T-periodic solution of (1.1). Thus, the problem of finding a T-periodic solution for (1.1) reduces to finding one for (1.2).

Let $C_T = \{x \in C(R,R^n) : x(t+T) \equiv x(t)\}$ with the norm $\|x\|_\infty = \max_{1\le i\le n} \|x_i\|_\infty$, $X = Z = \{z = \binom{x(\cdot)}{y(\cdot)} \in C(R,R^{2n}) : z(t+T) \equiv z(t)\}$ with the norm $\|z\| = \max\{\|x\|_\infty, \|y\|_\infty\}$. Clearly, X and Z are Banach spaces.

Denote the operator A by

$$A : C_T \to C_T, \qquad (Ax)(t) = x(t) - Cx(t-\tau).$$

Meanwhile, let

$$L : \operatorname{Dom} L \subset X \to Z, \qquad (Lz)(t) = z'(t) = \begin{pmatrix} (Ax)'(t) \\ y'(t) \end{pmatrix},$$

$$N : X \to Z,$$

$$(Nz)(t) = \begin{pmatrix} \phi_q(y(t)) \\ -\frac{d}{dt}\operatorname{grad} F(x(t)) - \operatorname{grad} G(x(t)) + e(t) \end{pmatrix} := H(z,t).$$

It is easy to see that $\operatorname{Ker} L = R^{2n}$, $\operatorname{Im} L = \{z \in Z : \int_0^T z(s)\,ds = 0\}$. So, L is a Fredholm operator with index zero. Let $P : X \to \operatorname{Ker} L$ and $Q : Z \to \operatorname{Im} Q$ be defined by

$$Pu = \frac{1}{T}\int_0^T u(s)\,ds, \quad u \in X;$$

$$Qv = \frac{1}{T}\int_0^T v(s)\,ds, \quad v \in Z,$$

and let K_p denote the inverse of $L|_{\operatorname{Ker} P \cap \operatorname{Dom} L}$.

Obviously, $\operatorname{Ker} L = \operatorname{Im} Q = R^{2n}$ and

$$(K_p z)(t) = \begin{pmatrix} (A^{-1}Fx)(t) \\ (Fy)(t) \end{pmatrix}, \tag{1.3}$$

where $z = (x^T(\cdot), y^T(\cdot))^T \in Z$, $(Fh)(t) = \int_0^t h(s)\,ds - \frac{1}{T}\int_0^T \int_0^t h(s)\,ds\,dt$, $h \in C_T$.

From (1.3), one can easily see that N is L-compact on $\bar{\Omega}$, where Ω is an open bounded subset of X.

Lemma 1.2 (See [9]) *Suppose that $\lambda_1, \lambda_2, \ldots, \lambda_n$ are eigenvalues of the matrix C. If $|\lambda_i| \neq 1$, $\forall i \in \{1, 2, \ldots, n\}$, then A has a continuous bounded inverse A^{-1} with the following relationships*:

(1) $\|A^{-1}u\|_\infty \leq (\sum_{i=1}^n \frac{1}{|1-|\lambda_i||})\|u\|\infty$, $\forall u \in C_T$;

(2) $\|A^{-1}u\|_p^p\,dt \leq \sigma \|u\|_p^p\,dt$, $\forall u \in C_T$, $p \geq 1$, *where*

$$\sigma = \begin{cases} \max_{i\in\{1,2,\ldots,n\}}\{\frac{1}{|1-|\lambda_i||^2}\}, & p = 2, \\ (\sum_{i=1}^n \frac{1}{|1-|\lambda_i||^{\frac{2p}{2-p}}})^{(2-p)/2}, & p \in [1, 2), \\ (\sum_{i=1}^n \frac{1}{|1-|\lambda_i||^q})^{p/q}, & p \in (2, +\infty), \end{cases}$$

where q is a constant with $1/p + 1/q = 1$;

(3) $Ax' = (Ax)'$, $\forall x \in C_T'$.

In the proof of our results on the existence of periodic solutions, we use the following generalized Borsuk theorem in coincidence degree theory of Gaines and Mawhin [10].

Lemma 1.3 *Let X and Z be real normed vector spaces. Let L be a Fredholm mapping of index zero. Ω is an open bounded subset of X and Ω is symmetric with respect to the origin and contains it. Let $\tilde{N} : \bar{\Omega} \times [0, 1] \to Z$ be L-compact and such that*

(a) $\tilde{N}(-x, 0) = -\tilde{N}(x, 0)$, $\forall x \in \bar{\Omega}$,

(b) $Lx \neq \tilde{N}(x, \lambda)$, $\forall x \in \operatorname{Dom} L \cap \partial\Omega$.

Then, for every $\lambda \in [0, 1]$, the equation $Lx = \tilde{N}(x, \lambda)$ has at least one solution in Ω.

2 Main results

Theorem 2.1 *Suppose that the matrix C satisfies the conditions of Lemma 1.2 and that there exist constants $a > 0$, $b > 0$, $c \geq 0$ and $\alpha > 1$ such that*

(H1) $y^T \frac{\partial^2 F(x)}{\partial x^2} y \geq a|y|_2^2$ *or* $y^T \frac{\partial^2 F(x)}{\partial x^2} y \leq -a|y|_2^2$, $\forall x, y \in R^n$;

(H2) $\langle y, \operatorname{grad} G(x)\rangle \geq b|y|_\alpha^\alpha - c$, $\forall x, y \in R^n$.

Then equation (1.1) has at least one T-periodic solution for $1 < p \leq 2$.

Proof For any $\lambda \in [0, 1]$, let

$$\tilde{N}(z, \lambda)(t) = \frac{1+\lambda}{2} H(z, t) - \frac{1-\lambda}{2} H(-z, t).$$

Consider the following parameter equation:

$$(Lz)(t) = \tilde{N}(z, \lambda)(t), \quad \lambda \in [0, 1]. \tag{2.1}$$

Let $z(t) = \binom{x(t)}{y(t)}$ be a possible T-periodic solution of (2.3) for some $\lambda \in [0,1]$, then $x = x(t)$ is a T-periodic solution of the following system:

$$\begin{aligned}&\big(\phi_p\big((Ax')(t)\big)\big)' + \frac{1+\lambda}{2}\frac{d}{dt}\operatorname{grad}F\big(x(t)\big) - \frac{1-\lambda}{2}\frac{d}{dt}\operatorname{grad}F\big(-x(t)\big)\\&\quad + \frac{1+\lambda}{2}\operatorname{grad}G\big(x(t)\big) - \frac{1-\lambda}{2}\operatorname{grad}G\big(-x(t)\big) = \lambda e(t).\end{aligned}\tag{2.2}$$

Noticing that $x(t)$ is a T-periodic solution, we have

$$-\|Ax'\|_p^p = \int_0^T \big\langle Ax, \big(\phi_p(Ax')\big)'\big\rangle\,dt.\tag{2.3}$$

Multiplying the two sides of (2.2) by $(Ax)(t)$ and integrating them on the interval $[0,T]$, by (2.3) and (H1)-(H2), we obtain

$$-\|Ax'\|_p^p + a\|Ax\|_2^2 + b\|Ax\|_\alpha^\alpha - cT \le \|e\|_\beta\|Ax\|_\alpha, \quad \text{where } \frac{1}{\alpha} + \frac{1}{\beta} = 1.\tag{2.4}$$

On the other hand,

$$\int_0^T \big\langle Ax', \big[\phi_p(Ax')\big]'\big\rangle\,dt = 0.$$

So, multiplying the two sides of (2.2) by $(Ax')(t)$ and integrating them on the interval $[0,T]$, by (H1)-(H2), we get

$$a\|Ax'\|_2^2 - cT \le a\|Ax\|_2^2 + b\|Ax\|_\alpha^\alpha - cT \le \|e\|_2\|Ax'\|_2.$$

Furthermore, we have

$$\|Ax'\|_2 \le \sqrt{\frac{cT}{a} + \frac{\|e\|_2^2}{4a^2}} + \frac{\|e\|_2}{2a} := R_1.$$

It is obvious that there exist $c_1 > 0$ and $c_2 > 0$ such that

$$c_1|x|_2 \le |x|_p \le c_2|x|_2, \quad x \in R^n.$$

Thus,

$$\begin{aligned}\|Ax'\|_p^p &= \int_0^T \big|(Ax')(t)\big|_p^p\,dt\\&\le c_2^p\left(\int_0^T \big|Ax'(t)\big|_2^2\,dt\right)^{p/2} T^{(2-p)/2}\\&\le (c_2R_1)^p T^{(2-p)/2} := R_2,\end{aligned}\tag{2.5}$$

where $1 < p \le 2$.

From (2.4) and (2.5), we can see

$$b\|Ax\|_\alpha^\alpha - \|e\|_\beta\|Ax\|_\alpha - cT \le R_2 - a\|Ax'\|_2^2 \le R_2,$$

from which it follows that there exists a positive number R_3 such that

$$\|Ax\|_\alpha \le R_3.$$

By using Lemma 1.2, we get

$$\begin{aligned}\|x\|_\alpha &= \left(\int_0^T \left|x(t)\right|_\alpha^\alpha \, dt\right)^{1/\alpha} \\ &= \left(\int_0^T \left|A^{-1}(Ax)(t)\right|_\alpha^\alpha \, dt\right)^{1/\alpha} \\ &\le \sigma^{1/\alpha}\left(\int_0^T \left|(Ax)(t)\right|_\alpha^\alpha \, dt\right)^{1/\alpha} \\ &\le \sigma^{1/\alpha} R_3 := R_4. \end{aligned} \tag{2.6}$$

From (2.6), there exists $t_0 \in [0,T)$ such that $|x(t_0)|_\alpha \le R_4 T^{-1/\alpha}$, and

$$\begin{aligned}\left|x_i(t)\right| &= \left|x_i(t_0) + \int_{t_0}^t x_i'(s)\, ds\right| \\ &\le R_4 T^{-1/\alpha} + \sqrt{T}\left(\int_0^T \left(x_i'(s)\right)^2 ds\right)^{1/2} \\ &\le R_4 T^{-1/\alpha} + \sqrt{T} R_1 := R_5. \end{aligned}$$

Therefore $\|x\|_\infty \le R_5$ and $|x(t)|_p \le n^{1/p} R_5$.

Since $F \in C^2(R^n, R)$, $G \in C^1(R^n, R)$, there exist R_6 and R_7 such that

$$\left\|\frac{\partial^2 F(x)}{\partial x^2}\right\|_p \le R_6, \qquad \left|\operatorname{grad} G(x)\right|_p \le R_7 \quad \text{for } |x|_p \le n^{1/p} R_5.$$

From (2.4), we have

$$\begin{aligned}\int_0^T \left|\left(\phi_p\left(Ax'\right)\right)'\right|_p dt &\le R_6 \int_0^T \left|x'\right|_p dt + R_7 T + \int_0^T \left|e(t)\right|_p dt \\ &\le R_6 T^{1/q} \left\|x'\right\|_p + R_7 T + \int_0^T \left|e(t)\right|_p dt \\ &\le R_6 T^{1/q} R_2^{1/p} + R_7 T + \int_0^T \left|e(t)\right|_p dt := R_8. \end{aligned}$$

Clearly, for each $i = 1, \ldots, n$, there exists $t_i \in (0,T)$ such that $x_i'(t_i) = 0$. Thus, for any $t \in [0,T]$, we have

$$\begin{aligned}\left|y_i(t)\right| &= \left|\phi_p\left((Ax_i)'(t)\right)\right| \\ &= \left|\phi_p\left((Ax_i)'(t)\right) - \phi_p\left((Ax_i)'(t_i)\right)\right| \\ &= \left|\int_{t_i}^t \left(\phi_p\left((Ax_i)'(s)\right)\right)' ds\right| \\ &\le R_8. \end{aligned}$$

Therefore $\|y\|_\infty \leq R_8$.

Choose a number $R_9 > \max(R_5, R_8)$, and let $\Omega = \{z \in X : \|z\| < R_9\}$, then $Lz \neq \tilde{N}(z, \lambda)$ for any $z \in \operatorname{Dom} L \cap \partial\Omega$, $\lambda \in [0,1]$. It is easy to see that $\tilde{N}$ is L-compact on $\bar{\Omega} \times [0,1]$, $Lz = \tilde{N}(z,1)$ is (2.1) and $\tilde{N}(-z,0) = -\tilde{N}(z,0)$. From Lemma 1.3, (2.1) has at least one T-periodic solution $\tilde{z} = \binom{\tilde{x}(t)}{\tilde{y}(t)}$, $\tilde{x}(t)$ is a T-periodic solution of (1.1). □

Theorem 2.2 *Let* $\lambda_\infty = \max\{|\lambda_1|, |\lambda_2|, \dots, |\lambda_n|\}$, *where* $\lambda_1, \lambda_2, \dots, \lambda_n$ *are eigenvalues of the matrix* C *with* $|\lambda_i| \neq 1$, $\forall i \in \{1, 2, \dots, n\}$. *Suppose that there exist constants* $b \geq 0$, $c \geq 0$ *and* $d > 0$ *such that*

(H3) *there is a constant* $r \geq 0$ *such that* $\lim_{|x| \to +\infty} \frac{|\operatorname{grad} F(x)|}{|x|^{p-1}} \leq r$;

(H4) $\langle y, \operatorname{grad} G(x)\rangle \leq b|y|_p^p + c$, $\forall x, y \in R^n$;

(H5) $\forall i \in \{1, \dots, n\}$, *either* $x_i[\frac{\partial G(x)}{\partial x_i} - \bar{e}_i] > 0$ *or* $x_i[\frac{\partial G(x)}{\partial x_i} - \bar{e}_i] < 0$ *for* $|x_i| > d$, *where* $\bar{e}_i = \frac{1}{T}\int_0^T e_i(t)\,dt$.

Then (1.1) *has at least one* T*-periodic solution for* $(\lambda_\infty r + b)\frac{T}{\pi_p} < \sigma$.

Proof Let $z(t) = \binom{x(t)}{y(t)}$ be a possible T-periodic solution of (2.1). From assumption (H3), there exists a constant $\rho > d$ such that

$$\left|\operatorname{grad} F(x)\right| < r|x|^{p-1}, \quad \forall x \in R^n \text{ with } |x_i| > \rho \text{ for } i = 1, 2, \dots, n.$$

From (H3) and (2.2), we have

$$\begin{aligned} &-\left\|Ax'\right\|_p^p + \int_0^T \left\langle Ax(t), \frac{1+\lambda}{2}\frac{d}{dt}\operatorname{grad} F\big(x(t)\big) - \frac{1-\lambda}{2}\frac{d}{dt}\operatorname{grad} F\big(-x(t)\big)\right\rangle dt + b\|x\|_p^p + cT \\ &\quad \geq \lambda \int_0^T \big\langle x(t), e(t)\big\rangle\,dt \geq -\|e\|_q\|x\|_p, \end{aligned}$$

i.e.,

$$\begin{aligned} \left\|Ax'\right\|_p^p &\leq \int_0^T \left\langle Cx'(t-\tau), \frac{1+\lambda}{2}\frac{d}{dt}\operatorname{grad} F\big(x(t)\big) - \frac{1-\lambda}{2}\frac{d}{dt}\operatorname{grad} F\big(-x(t)\big)\right\rangle dt \\ &\quad + b\|x\|_p^p + \|e\|_q\|x\|_p + cT \\ &\leq \left\|Cx'\right\|_p \left\|\operatorname{grad} F\big(x(t)\big)\right\|_{\frac{p}{p-1}} + b\|x\|_p^p + \|e\|_q\|x\|_p + cT \\ &\leq \lambda_\infty \left\|x'\right\|_p \big(r\|x\|_p^{p-1} + \theta\big) + b\|x\|_p^p + \|e\|_q\|x\|_p + cT, \end{aligned} \tag{2.7}$$

where $\theta = \max_{|u| \leq \sqrt{n}\rho} |\operatorname{grad} F(u)| T^{(p-1)/p}$.

Integrating both sides of (2.2) over $[0, T]$, we get

$$\frac{1+\lambda}{2}\int_0^T \left[\frac{\partial G(x(t))}{\partial x_i} - \bar{e}_i\right] dt - \frac{1-\lambda}{2}\int_0^T \left[\frac{\partial G(-x(t))}{\partial x_i} - \bar{e}_i\right] dt = 0, \quad i = 1, \dots, n.$$

So, there exist $\tilde{t}_i \in [0, T]$ such that

$$\frac{1+\lambda}{2}\int_0^T \left[\frac{\partial G(x(\tilde{t}_i))}{\partial x_i} - \bar{e}_i\right] dt - \frac{1-\lambda}{2}\int_0^T \left[\frac{\partial G(-x(\tilde{t}_i))}{\partial x_i} - \bar{e}_i\right] dt = 0, \quad i = 1, \dots, n.$$

From (H4), one can see $|x_i(\tilde{t}_i)| \le d$. Let $\chi_i(t) = x_i(t+\tilde{t}_i) - x_i(\tilde{t}_i)$, $\chi(t) = (\chi_1(t), \dots, \chi_n(t))^T$, then $\chi(0) = \chi(T) = 0$. By Lemma 1.1, one can obtain

$$\|\chi\|_p \le \frac{T}{\pi_p}\|\chi'\|p.$$

Noticing the periodicity of $x(t)$, we have

$$\begin{aligned}
\|x_i\|_p^p &= \int_0^T |x_i(t)|^p \, dt \\
&= \int_0^T |x_i(t+\tilde{t}_i)|^p \, dt \\
&\le \int_0^T \bigl(|\chi_i(t)| + d\bigr)^p \, dt \\
&\le \bigl(\|\chi_i\|_p + T^{1/p} d\bigr)^p.
\end{aligned}$$

From Minkovski's inequality, we have

$$\begin{aligned}
\|x\|_p &= \left(\sum_{i=1}^n \|x_i\|_p^p\right)^{1/p} \\
&\le \left(\sum_{i=1}^n \|\chi_i(t)\| + T^{1/p} d\right)^p \\
&\le \|\chi\|_p + (nT)^{1/p} d \le \frac{T}{\pi_p}\|\chi'\|_p + (nT)^{1/p} d \\
&= \frac{T}{\pi_p}\|x'\|_p + (nT)^{1/p} d.
\end{aligned}$$

In view of (2.7) and Lemma 1.2, we get

$$\begin{aligned}
\sigma\|x'\|_p^p \le \lambda_\infty \|x'\|_p &\left(r\left(\frac{T}{\pi_p}\|x'\|_p + (nT)^{1/p} d\right)^{p-1} + \theta \right) \\
&+ b\left(\frac{T}{\pi_p}\|x'\|_p + (nT)^{1/p} d\right)^p + \|e\|_q \left(\frac{T}{\pi_p}\|x'\|_p + (nT)^{1/p} d\right) + cT. \qquad (2.8)
\end{aligned}$$

Since $(\lambda_\infty r + b)\frac{T}{\pi_p} < \sigma$, from (2.8), there exists a constant $R_9 > 0$ such that

$$\|x'\|_p \le R_9. \qquad (2.9)$$

Therefore,

$$\|x\|_p \le \frac{T}{\pi_p} R_9 + (nT)^{1/p} d := R_{10}. \qquad (2.10)$$

From (2.9) and (2.10), we know that the rest of the proof of the theorem is similar to that of Theorem 2.1. □

Remark 2.1 If $C \equiv \mathbf{0}_{n\times n}$, system (1.1) can be reduced to the system in [2].

If $C \equiv \mathbf{0}_{n\times n}$ and $p = 2$, system (1.1) can be reduced to the system in [3].

Example 2.1 Consider the following system:

$$\frac{d}{dt}\phi_p\big[\big(x(t) - Cx(t-\tau)\big)'\big] + \frac{d}{dt}\operatorname{grad}F\big(x(t)\big) + \operatorname{grad}G\big(x(t)\big) = e(t), \tag{2.11}$$

where $F \in C^2(R^2, R)$, $G \in C^1(R^2, R)$, $e \in C(R, R^2)$ are periodic functions with period T; $C = \begin{pmatrix} -1 & -1 \\ -1 & 0 \end{pmatrix}$. Clearly, $\lambda_{1,2} = \frac{1\pm\sqrt{5}}{2} \neq \pm 1$.

Let

$$x = (x_1, x_2)^T, \qquad F(x_1, x_2) = x_1^2 + x_2^2 - \frac{x_1 x_2}{2}, \qquad G(x_1, x_2) = x_1^4 + x_1^3 - \frac{1}{4}x_1^2 x_2^2 + x_2^4,$$

then, by Theorem 2.1, (2.11) has at least one T-periodic solution for $1 < p \leq 2$.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

The first author carried out the studies, the second author participated in the studies and drafted the manuscript. All authors read and approved the final manuscript.

Acknowledgements

The authors thank the referees for helpful suggestions. This work was supported by Grants Nos. 10871052 and 109010600 from NNSF of China, and by Grant No. S2011010005029 from NSF of Guangdong.

Received: 13 May 2013 Accepted: 27 September 2013 Published: 13 Dec 2013

References

1. Din, T, Iannacci, R, Zanolin, F: Existence and multiplicity results for periodic solutions of semilinear Duffing equations. J. Differ. Equ. **105**, 364-409 (1993)
2. Peng, S, Xu, Z: On the existence of periodic solutions for a class of *p*-Laplacian system. J. Math. Anal. Appl. **325**, 166-174 (2007)
3. Ge, W: On the existence of harmonic solutions of Liénard system. Nonlinear Anal. **16**, 183-190 (1991)
4. Manásevich, R, Mawhin, J: Periodic solutions for nonlinear systems with *p*-Laplacian-like operators. J. Differ. Equ. **145**, 367-393 (1998)
5. Mawhin, J: Some boundary value problems for Hartman-type perturbations of the ordinary vector *p*-Laplacian. Nonlinear Anal. **40**, 497-503 (1998)
6. Peng, S, Zhu, S: Periodic solutions for a Rayleigh system of *p*-Laplacian type. Nonlinear Anal. **70**, 274-279 (2009)
7. Lu, S, Ge, W: Periodic solutions for a kind of Liénard equation with a deviating argument. J. Math. Anal. Appl. **289**, 231-243 (2004)
8. Zhang, MR: Nonuniform nonresonance at the first eigenvalue of the *p*-Laplacian. Nonlinear Anal. **29**, 41-51 (1997)
9. Lu, S: Periodic solutions to a second order *p*-Laplacian neutral functional differential system. Nonlinear Anal. **69**, 4215-4229 (2008)
10. Gaines, RE, Mawhin, JL: Coincidence Degree and Nonlinear Differential Equations. Springer, Berlin (1977)

10.1186/1687-1847-2013-367
Cite this article as: Wang and Song: Periodic solutions for the *p*-Laplacian neutral functional differential system. *Advances in Difference Equations* 2013, 2013:367