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A switching rule for exponential stability of switched recurrent neural networks with interval time-varying delay

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Abstract

This paper studies the problem for exponential stability of switched recurrent neural networks with interval time-varying delay. The time delay is a continuous function belonging to a given interval, but not necessarily differentiable. By constructing a set of argumented Lyapunov-Krasovskii functionals combined with the Newton-Leibniz formula, a switching rule for exponential stability of switched recurrent neural networks with interval time-varying delay is designed via linear matrix inequalities, and new sufficient conditions for the exponential stability of switched recurrent neural networks with interval time-varying delay via linear matrix inequalities (LMIs) are derived. A numerical example is given to illustrate the effectiveness of the obtained result.

Keywords: neural networks; switching design; exponential stability; interval time-varying delays; Lyapunov function; linear matrix inequalities

1 Introduction

In recent years, neural networks (especially recurrent neural networks, Hopfield neural networks, and cellular neural networks) have been successfully applied in many areas such as signal processing, image processing, pattern recognition, fault diagnosis, associative memory, and combinatorial optimization; see, for example, [1–6]. One of the best important works in these applications is to study the stability of the equilibrium point of neural networks. A major purpose is to find stability conditions *i.e.*, the conditions for the stability of the equilibrium point of neural networks. The stability and control of recurrent neural networks with time delay have attracted considerable attention in recent years [1–10]. In many practical systems, it is desirable to design neural networks which are not only asymptotically or exponentially stable but can also guarantee an adequate level of system performance. In the area of control, signal processing, pattern recognition, and image processing, delayed neural networks have many useful applications. Some of these applications require the equilibrium points of the designed network to be stable. In both biological and artificial neural systems, time delays due to integration and communication are ubiquitous and often become a source of instability. The time delays in electronic neural networks are usually time-varying and sometimes vary violently with respect to time due to the finite switching speed of amplifiers and faults in the electrical circuitry. The Lyapunov-Krasovskii functional technique has been among the popular and effective tools in the design of guaranteed cost controls for neural networks with time delay. Nevertheless, despite such diversity of results available, most existing works either assumed that the time delays are constant or differentiable [9–14]. To the best of our knowledge, a switching rule and exponential stability for switched recurrent neural networks with interval time-varying delay, non-differentiable time-varying delays have not been fully studied yet (see, *e.g.*, [4–9, 13–25] and the references therein), and they are important in both theories and applications. This motivates our research.

In this paper, we investigate the exponential stability for a switched recurrent neural networks problem. The novel features here are that the delayed neural network under consideration is with various globally Lipschitz continuous activation functions, and the time-varying delay function is interval, non-differentiable. Based on constructing a set of augmented Lyapunov-Krasovskii functionals combined with Newton-Leibniz formula, a switching rule for exponential stability of switched recurrent neural networks with interval time-varying delay and new delay-dependent exponential stability criteria for switched recurrent neural networks with interval time-varying delay are established in terms of LMIs, which allow simultaneous computation of two bounds that characterize the exponential stability rate of the solution and can be easily determined by utilizing MATLABs LMI control toolbox.

The outline of the paper is as follows. Section 2 presents definitions and some well-known technical propositions needed for the proof of the main result. LMI delay-dependent exponential stability criteria for switched recurrent neural networks with interval time-varying delay criteria, a switching rule for exponential stability of switched recurrent neural networks with interval time-varying delay, and a numerical example showing the effectiveness of the result are presented in Section 3. The paper ends with conclusions and cited references.

2 Preliminaries

The following notations will be used in this paper. $\mathbb{R}^+$ denotes the set of all real non-negative numbers; $\mathbb{R}^n$ denotes the n-dimensional space with the scalar product $\langle x,y\rangle$ or x^Ty of two vectors x, y and the vector norm $\|\cdot\|$; $M^{n\times r}$ denotes the space of all matrices of $(n\times r)$-dimensions. A^T denotes the transpose of matrix A; A is symmetric if $A=A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A)=\max\{\operatorname{Re}\lambda;\lambda\in\lambda(A)\}$, $\lambda_{\min}(A)=\min\{\operatorname{Re}\lambda;\lambda\in\lambda(A)\}$. $x_t:=\{x(t+s):s\in[-h_1,0]\}$, $\|x_t\|=\sup_{s\in[-h_1,0]}\|x(t+s)\|$; $C^1([0,t],\mathbb{R}^n)$ denotes the set of all $\mathbb{R}^n$-valued continuously differentiable functions on $[0,t]$; $L_2([0,t],\mathbb{R}^m)$ denotes the set of all the $\mathbb{R}^m$-valued square integrable functions on $[0,t]$.

Matrix A is called semi-positive definite ($A\geq 0$) if $\langle Ax,x\rangle\geq 0$ for all $x\in\mathbb{R}^n$; A is positive definite ($A>0$) if $\langle Ax,x\rangle>0$ for all $x\neq 0$; $A>B$ means $A-B>0$. The notation $\operatorname{diag}\{\cdots\}$ stands for a block-diagonal matrix. The symmetric term in a matrix is denoted by $*$.

Consider the following switched recurrent neural networks with interval time-varying delay:

$$\begin{aligned}
&\dot{x}(t)=-A_{\gamma(x(t))}x(t)+W_{0\gamma(x(t))}f\big(x(t)\big)+W_{1\gamma(x(t))}g\big(x\big(t-h(t)\big)\big),\quad t\geq 0,\\
&x(t)=\phi(t),\quad t\in[-h_1,0],
\end{aligned}\tag{2.1}$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in \mathbb{R}^n$ is the state of the neural, n is the number of neurals, and

$$f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \dots, f_n(x_n(t))]^T,$$
$$g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \dots, g_n(x_n(t))]^T$$

are the activation functions; $\gamma(\cdot) : R^n \to \mathcal{N} := \{1, 2, \dots, N\}$ is the switching rule, which is a function depending on the state at each time and will be designed. A switching function is a rule which determines a switching sequence for a given switching system. Moreover, $\gamma(x(t)) = j$ implies that the system realization is chosen as the jth system, $j = 1, 2, \dots, N$. It is seen that the system (2.1) can be viewed as an autonomous switched system in which the effective subsystem changes when the state $x(t)$ hits predefined boundaries.

$A_j = \mathrm{diag}(\overline{a}_{1j}, \overline{a}_{2j}, \dots, \overline{a}_{nj})$, $\overline{a}_{ij} > 0$ represents the self-feedback term; W_{0j}, W_{1j} denote the connection weights, the discretely delayed connection weights, and the distributively delayed connection weight, respectively. The time-varying delay function $h(t)$ satisfies the condition

$$0 \le h_0 \le h(t) \le h_1.$$

The initial functions $\phi(t) \in C^1([-h_1, 0], R^n)$ with the norm

$$\|\phi\| = \sup_{t \in [-h_1, 0]} \sqrt{\|\phi(t)\|^2 + \|\dot{\phi}(t)\|^2}.$$

In this paper, we consider various activation functions and assume that the activation functions $f(\cdot)$, $g(\cdot)$ are Lipschitzian with the Lipschitz constants $f_i, e_i > 0$:

$$\begin{aligned} |f_i(\xi_1) - f_i(\xi_2)| &\le f_i|\xi_1 - \xi_2|, \quad i = 1, 2, \dots, n, \forall \xi_1, \xi_2 \in \mathbb{R}, \\ |g_i(\xi_1) - g_i(\xi_2)| &\le e_i|\xi_1 - \xi_2|, \quad i = 1, 2, \dots, n, \forall \xi_1, \xi_2 \in \mathbb{R}. \end{aligned} \tag{2.2}$$

Definition 2.1 The zero solution of switched recurrent neural networks with interval time-varying delay (2.1) is α-exponentially stable if there exist two positive numbers $\alpha > 0$, $N > 0$ such that every solution $x(t, \phi)$ satisfies the following condition:

$$\|x(t, \phi)\| \le N e^{-\alpha t} \|\phi\|, \quad \forall t \ge 0.$$

We introduce the following technical well-known propositions, which will be used in the proof of our results.

Proposition 2.1 (Schur complement lemma [26]) *Given constant matrices X, Y, Z with appropriate dimensions satisfying $X = X^T$, $Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0.$$

Proposition 2.2 (Integral matrix inequality [27]) *For any symmetric positive definite matrix $M > 0$, scalar $\sigma > 0$ and vector function $\omega : [0,\sigma] \to \mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds*:

$$\left(\int_0^\sigma \omega(s)\,ds\right)^T M\left(\int_0^\sigma \omega(s)\,ds\right) \le \sigma\left(\int_0^\sigma \omega^T(s)M\omega(s)\,ds\right).$$

3 Main results

Let us set

$$\begin{aligned}
w_{11} &= -[P+\alpha I]A_j - A_j^T[P+\alpha I] + \sum_{i=0}^{1} G_i - PA_j - A_j^T P \\
&\quad - \sum_{i=0}^{1} e^{-2\alpha h_i} H_i + 4PFD_0^{-1}FP, \\
w_{12} &= P + A_jP, \qquad w_{13} = e^{-2\alpha h_0}H_0 + A_jP, \\
w_{14} &= 2e^{-2\alpha h_1}H_1 + A_jP, \qquad w_{15} = P + A_jP, \\
w_{22} &= \sum_{i=0}^{1} W_{ij}D_iW_{ij}^T + \sum_{i=0}^{1} h_i^2 H_i + (h_1 - h_0)U - 2P, \\
w_{23} &= P, \qquad w_{24} = P, \qquad w_{25} = P, \\
w_{33} &= -e^{-2\alpha h_0}G_0 - e^{-2\alpha h_0}H_0 - e^{-2\alpha h_1}U + \sum_{i=0}^{1} W_{ij}D_iW_{ij}^T, \\
w_{34} &= 0, \qquad w_{35} = -2\alpha h_1 U, \\
w_{44} &= \sum_{i=0}^{1} W_{ij}D_iW_{ij}^T - e^{-2\alpha h_1}U - e^{-2\alpha h_1}G_1 - e^{-2\alpha h_1}H_1, \\
w_{45} &= e^{-2\alpha h_1}U, \\
w_{55} &= \sum_{i=0}^{1} W_{ij}D_iW_{ij}^T - e^{-2\alpha h_1}U - e^{-2\alpha h_2}U + 4PED_1^{-1}EP, \\
E &= \operatorname{diag}\{e_i, i = 1,\ldots,n\}, \qquad F = \operatorname{diag}\{f_i, i = 1,\ldots,n\}, \\
\lambda_1 &= \lambda_{\min}\left(P^{-1}\right), \\
\lambda_2 &= \lambda_{\max}\left(P^{-1}\right) + h_0\lambda_{\max}\left[P^{-1}\left(\sum_{i=0}^{1} G_i\right)P^{-1}\right] \\
&\quad + h_1^2\lambda_{\max}\left[P^{-1}\left(\sum_{i=0}^{1} H_i\right)P^{-1}\right] + (h_1 - h_0)\lambda_{\max}\left(P^{-1}UP^{-1}\right).
\end{aligned}$$

Theorem 3.1 *The zero solution of the switched recurrent neural networks with interval time-varying delay* (2.1) *is α-exponentially stable if there exist a positive number $\alpha > 0$, symmetric positive definite matrices P, U, G_0, G_1, H_0, H_1, and diagonal positive definite* *matrices D_i, $i = 0,1$ satisfying the following LMIs*:

$$\begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} & w_{15} \\ * & w_{22} & w_{23} & w_{24} & w_{25} \\ * & * & w_{33} & w_{34} & w_{35} \\ * & * & * & w_{44} & w_{45} \\ * & * & * & * & w_{55} \end{bmatrix} < 0, \quad j = 1,2,\dots,N, \tag{3.1}$$

the switching rule is chosen as $\gamma(x(t)) = j$. Moreover, the solution $x(t,\phi)$ of the system satisfies

$$\|x(t,\phi)\| \le \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t}\|\phi\|, \quad \forall t \ge 0.$$

Proof Let $Y = P^{-1}$, $y(t) = Yx(t)$. We consider the following Lyapunov-Krasovskii functional:

$$\begin{aligned}
&V(t,x_t) = \sum_{i=1}^{6} V_i(t,x_t),\\
&V_1 = x^T(t)Yx(t),\\
&V_2 = \int_{t-h_0}^{t} e^{2\alpha(s-t)} x^T(s)YG_0Yx(s)\,ds,\\
&V_3 = \int_{t-h_1}^{t} e^{2\alpha(s-t)} x^T(s)YG_1Yx(s)\,ds,\\
&V_4 = h_0\int_{-h_0}^{0}\int_{t+s}^{t} e^{2\alpha(\tau-t)}\dot{x}^T(\tau)YH_0Y\dot{x}(\tau)\,d\tau\,ds,\\
&V_5 = h_1\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha(\tau-t)}\dot{x}^T(\tau)YH_1Y\dot{x}(\tau)\,d\tau\,ds,\\
&V_6 = (h_1-h_0)\int_{t-h_1}^{t-h_0}\int_{t+s}^{t} e^{2\alpha(\tau-t)}\dot{x}^T(\tau)YUY\dot{x}(\tau)\,d\tau\,ds.
\end{aligned}$$

It is easy to check that

$$\lambda_1\|x(t)\|^2 \le V(t,x_t) \le \lambda_2\|x_t\|^2, \quad \forall t \ge 0. \tag{3.2}$$

Taking the derivative of V_1, we have

$$\begin{aligned}
\dot{V}_1 &= 2x^T(t)Y\dot{x}(t)\\
&= y^T(t)\bigl[-PA_j^T - A_jP\bigr]y(t) + 2y^T(t)W_{0j}f(\cdot)Py(t) + 2y^T(t)W_{1j}g(\cdot)Py(t);\\
\dot{V}_2 &= y^T(t)G_0y(t) - e^{-2\alpha h_0}y^T(t-h_0)G_0y(t-h_0) - 2\alpha V_2;\\
\dot{V}_3 &= y^T(t)G_1y(t) - e^{-2\alpha h_1}y^T(t-h_1)G_1y(t-h_1) - 2\alpha V_3;\\
\dot{V}_4 &= h_0^2\dot{y}^T(t)H_0\dot{y}(t) - h_1e^{-2\alpha h_0}\int_{t-h_0}^{t}\dot{x}^T(s)H_0\dot{x}(s)\,ds - 2\alpha V_4;
\end{aligned}$$

$$\dot{V}_5 = h_1^2\dot{y}^T(t)H_1\dot{y}(t) - h_1e^{-2\alpha h_1}\int_{t-h_1}^{t}\dot{y}^T(s)H_1\dot{y}(s)\,ds - 2\alpha V_5;$$

$$\dot{V}_6 = (h_1-h_0)^2\dot{y}^T(t)U\dot{y}(t) - (h_1-h_0)e^{-2\alpha h_1}\int_{t-h_1}^{t-h_0}\dot{y}^T(s)U\dot{y}(s)\,ds - 2\alpha V_6.$$

Applying Proposition 2.2 and the Leibniz-Newton formula

$$\int_s^t \dot{y}(\tau)\,d\tau = y(t) - y(s),$$

we have, for $i = 0,1$,

$$\begin{aligned} -h_i\int_{t-h_i}^{t}\dot{y}^T(s)H_i\dot{y}(s)\,ds &\le -\left[\int_{t-h_i}^{t}\dot{y}(s)\,ds\right]^T H_i\left[\int_{t-h_i}^{t}\dot{y}(s)\,ds\right] \\ &\le -\bigl[y(t) - y\bigl(t-h(t)\bigr)\bigr]^T H_i\bigl[y(t) - y\bigl(t-h(t)\bigr)\bigr] \\ &= -y^T(t)H_iy(t) + 2y^T(t)H_iy\bigl(t-h(t)\bigr) \\ &\quad - y^T(t-h_i)H_iy(t-h_i). \end{aligned} \tag{3.3}$$

Note that

$$\int_{t-h_1}^{t-h_0}\dot{y}^T(s)U\dot{y}(s)\,ds = \int_{t-h_1}^{t-h(t)}\dot{y}^T(s)U\dot{y}(s)\,ds + \int_{t-h(t)}^{t-h_0}\dot{y}^T(s)U\dot{y}(s)\,ds.$$

Applying Proposition 2.2 gives

$$\begin{aligned} \bigl[h_1 - h(t)\bigr]\int_{t-h_1}^{t-h(t)}\dot{y}^T(s)U\dot{y}(s)\,ds &\ge \left[\int_{t-h_1}^{t-h(t)}\dot{y}(s)\,ds\right]^T U\left[\int_{t-h_1}^{t-h(t)}\dot{y}(s)\,ds\right] \\ &\ge \bigl[y\bigl(t-h(t)\bigr) - y(t-h_1)\bigr]^T U\bigl[y\bigl(t-h(t)\bigr) - y(t-h_1)\bigr]. \end{aligned}$$

Since $h_1 - h(t) \le h_1 - h_0$, we have

$$[h_1 - h_0]\int_{t-h_1}^{t-h(t)}\dot{y}^T(s)U\dot{y}(s)\,ds \ge \bigl[y\bigl(t-h(t)\bigr) - y(t-h_1)\bigr]^T U\bigl[y\bigl(t-h(t)\bigr) - y(t-h_1)\bigr],$$

then

$$-[h_1 - h_0]\int_{t-h_1}^{t-h(t)}\dot{y}^T(s)U\dot{y}(s)\,ds \le -\bigl[y\bigl(t-h(t)\bigr) - y(t-h_1)\bigr]^T U\bigl[y\bigl(t-h(t)\bigr) - y(t-h_1)\bigr].$$

Similarly, we have

$$-(h_1 - h_0)\int_{t-h(t)}^{t-h_0}\dot{y}^T(s)U\dot{y}(s)\,ds \le -\bigl[y(t-h_0) - y\bigl(t-h(t)\bigr)\bigr]^T U\bigl[y(t-h_0) - y\bigl(t-h(t)\bigr)\bigr].$$

Then we have

$$\begin{aligned} \dot{V}(\cdot) + 2\alpha V(\cdot) &\le y^T(t)\bigl[-PA_j^T - A_jP\bigr]y(t) + 2y^T(t)W_{0j}f(\cdot)Py(t) \\ &\quad + 2y^T(t)W_{1j}g(\cdot)Py(t) + y^T(t)\left(\sum_{i=0}^{1}G_i\right)y(t) + 2\alpha\bigl\langle Py(t), y(t)\bigr\rangle \end{aligned}$$

$$
\begin{aligned}
&+ \dot{y}^T(t)\left(\sum_{i=0}^{1} h_i^2 H_i\right)\dot{y}(t) + (h_1 - h_0)\dot{y}^T(t)U\dot{y}(t) \\
&- \sum_{i=0}^{1} e^{-2\alpha h_i} y^T(t-h_i)G_i y(t-h_i) \\
&- e^{-2\alpha h_0}\big[y(t) - y(t-h_0)\big]^T H_0\big[y(t) - y(t-h_0)\big] \\
&- e^{-2\alpha h_1}\big[y(t) - y(t-h_1)\big]^T H_1\big[y(t) - y(t-h_1)\big] \\
&- e^{-2\alpha h_1}\big[y\big(t-h(t)\big) - y(t-h_1)\big]^T U\big[y\big(t-h(t)\big) - y(t-h_1)\big] \\
&- e^{-2\alpha h_1}\big[y(t-h_0) - y\big(t-h(t)\big)\big]^T U\big[y(t-h_0) - y\big(t-h(t)\big)\big]. \qquad (3.4)
\end{aligned}
$$

Using equation (2.1),

$$
P\dot{y}(t) + A_j P y(t) - W_{0j} f(\cdot) - W_{1j} g(\cdot) = 0,
$$

and multiplying both sides by $[2y(t), -2\dot{y}(t), 2y(t-h_0), 2y(t-h_1), 2y(t-h(t))]^T$, we have

$$
\begin{aligned}
&2y^T(t)P\dot{y}(t) + 2y^T(t)A_j P y(t) - 2y^T(t)W_{0j} f(\cdot) - 2y^T(t)W_{1j} g(\cdot) = 0, \\
&-2\dot{y}^T(t)P\dot{y}(t) - 2\dot{y}^T(t)A_j P y(t) + 2\dot{y}^T(t)W_{0j} f(\cdot) + 2\dot{y}^T(t)W_{1j} g(\cdot) = 0, \\
&2y^T(t-h_0)P\dot{y}(t) + 2y^T(t-h_0)A_j P y(t) \\
&\quad - 2y^T(t-h_0)W_{0j} f(\cdot) - 2y^T(t-h_0)W_{1j} g(\cdot) = 0, \qquad (3.5) \\
&2y^T(t-h_1)P\dot{y}(t) + 2y^T(t-h_1)A_j P y(t) - 2y^T(t-h_1)W_{0j} f(\cdot) - 2y^T(t-h_1)W_{1j} g(\cdot) = 0, \\
&2y^T\big(t-h(t)\big)P\dot{y}(t) + 2y^T\big(t-h(t)\big)A_j P y(t) \\
&\quad - 2y^T\big(t-h(t)\big)W_{0j} f(\cdot) - 2y^T\big(t-h(t)\big)W_{1j} g(\cdot) = 0.
\end{aligned}
$$

Adding all the zero items of (3.5) into (3.4) for the following estimations:

$$
\begin{aligned}
&2\langle W_{0j} f(x), y\rangle \le \langle W_{0j} D_0 W_{0j}^T y, y\rangle + \langle D_0^{-1} f(x), f(x)\rangle, \\
&2\langle W_{1j} g(z), y\rangle \le \langle W_{1j} D_1 W_{1j}^T y, y\rangle + \langle D_1^{-1} g(z), g(z)\rangle, \\
&2\langle D_0^{-1} f(x), f(x)\rangle \le \langle F D_0^{-1} F x, x\rangle, \\
&2\langle D_1^{-1} g(z), g(z)\rangle \le \langle E D_1^{-1} E z, z\rangle,
\end{aligned}
$$

we obtain

$$
\dot{V}(\cdot) + 2\alpha V(\cdot) \le \zeta^T(t)\mathcal{E}_j \zeta(t), \qquad (3.6)
$$

where $\zeta(t) = [y^T(t), \dot{y}^T(t), y^T(t-h_0), y^T(t-h_1), y^T(t-h(t))]^T$, and

$$
\mathcal{E}_j = \begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} & w_{15} \\ * & w_{22} & w_{23} & w_{24} & w_{25} \\ * & * & w_{33} & w_{34} & w_{35} \\ * & * & * & w_{44} & w_{45} \\ * & * & * & * & w_{55} \end{bmatrix}, \quad j = 1, 2, \ldots, N.
$$

Therefore, by condition (3.1), we obtain from (3.6) that

$$\dot{V}(t,x_t) \le -2\alpha V(t,x_t), \quad \forall t \ge 0. \tag{3.7}$$

Integrating both sides of (3.7) from 0 to t, we obtain

$$V(t,x_t) \le V(\phi)e^{-2\alpha t}, \quad \forall t \ge 0.$$

Furthermore, taking condition (3.2) into account, we have

$$\lambda_1 \|x(t,\phi)\|^2 \le V(x_t) \le V(\phi)e^{-2\alpha t} \le \lambda_2 e^{-2\alpha t}\|\phi\|^2,$$

then

$$\|x(t,\phi)\| \le \sqrt{\frac{\lambda_2}{\lambda_1}}e^{-\alpha t}\|\phi\|, \quad t \ge 0,$$

which concludes the exponential stability of (2.1). This completes the proof of the theorem. □

Example 3.1 Consider the switched recurrent neural networks with interval time-varying delay (2.1) for $j = 2$, where

$$A_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix},$$

$$W_{01} = \begin{bmatrix} -0.1 & 0.3 \\ 0.2 & -0.8 \end{bmatrix}, \qquad W_{02} = \begin{bmatrix} -0.7 & 0.3 \\ 0.4 & -0.9 \end{bmatrix},$$

$$W_{11} = \begin{bmatrix} -0.4 & 0.2 \\ 0.3 & -0.3 \end{bmatrix}, \qquad W_{12} = \begin{bmatrix} -0.2 & 0.3 \\ 0.1 & -0.4 \end{bmatrix},$$

$$E = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}, \qquad F = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$\begin{cases} h(t) = 0.1 + 1.2\sin^2 t & \text{if } t \in \mathcal{I} = \bigcup_{k\ge 0}[2k\pi, (2k+1)\pi], \\ h(t) = 0 & \text{if } t \in R^+ \setminus \mathcal{I}. \end{cases}$$

Note that $h(t)$ is non-differentiable, therefore, the stability criteria proposed in [5–9, 11–14, 17–25] are not applicable to this system. We choose that $\alpha = 0.4$, $h_0 = 0.1$, $h_1 = 1.3$. By using the Matlab LMI toolbox, we can solve linear matrix inequalities for P, U, G_0, G_1, D_0, D_1, H_0 and H_1 which satisfy the conditions (3.1) in Theorem 3.1. A set of solutions is as follows:

$$P = \begin{bmatrix} 1.5219 & -0.3659 \\ -0.3659 & 2.2398 \end{bmatrix}, \qquad U = \begin{bmatrix} 3.1239 & -0.2365 \\ -0.2365 & 3.0123 \end{bmatrix},$$

$$G_0 = \begin{bmatrix} 1.3225 & 0.0258 \\ 0.0258 & 1.2698 \end{bmatrix}, \qquad G_1 = \begin{bmatrix} 2.2368 & 0.0148 \\ 0.0148 & 3.1121 \end{bmatrix},$$

$$H_0 = \begin{bmatrix} 2.2189 & 0.1238 \\ 0.1238 & 1.2368 \end{bmatrix}, \qquad H_1 = \begin{bmatrix} 2.3225 & 0.0369 \\ 0.0369 & 2.1897 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 1.2398 & 0.3659 \\ 0.3659 & 1.8935 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 2.3641 & 0.0593 \\ 0.0593 & 1.2380 \end{bmatrix}.$$

By Theorem 3.1, the switched recurrent neural networks with interval time-varying delay are exponentially stable and the switching rule is chosen as $\gamma(x(t)) = j$. Moreover, the solution $x(t,\phi)$ of the system satisfies

$$\|x(t,\phi)\| \le 12.6984e^{-0.4t}\|\phi\|, \quad \forall t \ge 0.$$

4 Conclusion

In this paper, the problem of exponential stability for switched recurrent neural networks with interval non-differentiable time-varying delay has been studied. By constructing a set of time-varying Lyapunov-Krasovskii functional combined with Newton-Leibniz formula, a switching rule for exponential stability of switched recurrent neural networks with interval time-varying delay has been presented and new sufficient conditions for the exponential stability for the system have been derived in terms of LMIs.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors contributed equally and significantly in writing this paper. The authors read and approved the final manuscript.

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