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Statistical convergence through de la Vallée-Poussin mean in locally solid Riesz spaces

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Abstract
The notion of statistical convergence was defined by Fast (Colloq. Math. 2:241-244, 1951) and over the years was further studied by many authors in different setups. In this paper, we define and study statistical τ-convergence, statistically τ-Cauchy and $S^*(\tau)$-convergence through de la Vallée-Poussin mean in a locally solid Riesz space.
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1 Introduction and preliminaries

Since 1951, when Steinhaus [1] and Fast [2] defined statistical convergence for sequences of real numbers, several generalizations and applications of this notion have been investigated. For more detail and related concepts, we refer to [3–29] and references therein. Quite recently, Di Maio and Kŏcinac [30] studied this notion in topological and uniform spaces and Albayrak and Pehlivan [31], and Mohiuddine and Alghamdi [32] for real and lacunary sequences, respectively, in locally solid Riesz spaces. Afterward, the idea was extended to double sequences by Mohiuddine *et al.* [33] in the framework of locally solid Riesz spaces.

Let K be a subset of $\mathbb{N}$, the set of natural numbers. Then the *asymptotic density* of K denoted by $\delta(K)$ is defined as

$$\delta(K) = \lim_{n\to\infty} \frac{1}{n}\left|\{k \leq n : k \in K\}\right|,$$

where the vertical bars denote the cardinality of the enclosed set.

The number sequence $x = (x_j)$ is said to be *statistically convergent* to the number ℓ if for each $\epsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{n}\left|\left\{j \leq n : |x_j - \ell| \geq \epsilon\right\}\right| = 0.$$

In this case, we write $st\text{-}\lim x_j = \ell$.

Remark 1.1 It is well known that every statistically convergent sequence is convergent, but the converse is not true. For example, suppose that the sequence $x = (x_n)$ is defined as

$$x = (x_n) = \begin{cases} \sqrt{n} & \text{if } n \text{ is a square,} \\ 0 & \text{otherwise.} \end{cases}$$

It is clear that the sequence $x = (x_n)$ is statistically convergent to 0, but it is not convergent.

Now we recall some definitions related to the notion of a locally solid Riesz space. Let X be a real vector space and $\leq$ be a partial order on this space. Then X is said to be an *ordered vector space* if it satisfies the following properties:

(i) If $x, y \in X$ and $y \leq x$, then $y + z \leq x + z$ for each $z \in X$.

(ii) If $x, y \in X$ and $y \leq x$, then $\lambda y \leq \lambda x$ for each $\lambda \geq 0$.

If in addition X is a lattice with respect to the partial order $\leq$, then X is said to be a *Riesz space* (or a *vector lattice*) [34].

For an element x of a Riesz space X, the positive part of x is defined by $x^+ = x \vee \theta = \sup\{x, \theta\}$, the negative part of x by $x^- = (-x) \vee \theta$ and the absolute value of x by $|x| = x \vee (-x)$, where θ is the zero element of X.

A subset S of a Riesz space X is said to be *solid* if $y \in S$ and $|x| \leq |y|$ imply $x \in S$.

A *topological vector space* (X, τ) is a vector space X which has a (linear) topology τ such that the algebraic operations of addition and scalar multiplication in X are continuous. The continuity of addition means that the function $f : X \times X \to X$ defined by $f(x, y) = x + y$ is continuous on $X \times X$, and the continuity of scalar multiplication means that the function $f : \mathbb{R} \times X \to X$ defined by $f(\lambda, x) = \lambda x$ is continuous on $\mathbb{R} \times X$.

Every linear topology τ on a vector space X has a base $\mathcal{N}$ for the neighborhoods of θ satisfying the following properties:

(C_1) Each $Y \in \mathcal{N}$ is a *balanced set*, that is, $\lambda x \in Y$ holds for all $x \in Y$ and every $\lambda \in \mathbb{R}$ with $|\lambda| \leq 1$.

(C_2) Each $Y \in \mathcal{N}$ is an *absorbing set*, that is, for every $x \in X$, there exists $\lambda > 0$ such that $\lambda x \in Y$.

(C_3) For each $Y \in \mathcal{N}$, there exists some $E \in \mathcal{N}$ with $E + E \subseteq Y$.

A linear topology τ on a Riesz space X is said to be *locally solid* (*cf.* [35, 36]) if τ has a base at zero consisting of solid sets. A *locally solid Riesz space* (X, τ) is a Riesz space equipped with a locally solid topology τ.

In this paper, we define and study statistical τ-convergence, statistically τ-Cauchy and $S^*(\tau)$-convergence through de la Vallée-Poussin mean in a locally solid Riesz space.

2 Generalized statistical τ-convergence

Throughout the text, we write $\mathcal{N}_{\text{sol}}$ for any base at zero consisting of solid sets and satisfying the conditions (C_1), (C_2) and (C_3) in a locally solid topology. The following idea of λ-statistical convergence was introduced in [37] and further studied in [38–40].

Let $\lambda = (\lambda_n)$ be a non-decreasing sequence of positive numbers tending to ∞ such that

$$\lambda_{n+1} \leq \lambda_n + 1, \qquad \lambda_1 = 0.$$

The generalized de la Vallée-Poussin mean is defined by

$$t_n(x) =: \frac{1}{\lambda_n} \sum_{j \in I_n} x_j,$$

where $I_n = [n - \lambda_n + 1, n]$.

A sequence $x = (x_j)$ is said to be (V, λ)-*summable* to a number ℓ if

$$t_n(x) \to \ell \quad \text{as } n \to \infty.$$

A sequence $x = (x_j)$ is said to be *strongly* (V, λ)-*summable* to a number ℓ if

$$\frac{1}{\lambda_n} \sum_{j \in I_n} |x_j - \ell| \to 0 \quad \text{as } n \to \infty.$$

We denote it by $x_j \to \ell[V, \lambda]$ as $j \to \infty$.

Let $K \subseteq \mathbb{N}$ be a set of positive integers, then

$$\delta_\lambda(K) = \lim_{n\to\infty} \frac{1}{\lambda_n} \left|\{n - \lambda_n + 1 \le j \le n : j \in K\}\right|$$

is said to be the λ-*density* of K.

In case $\lambda_n = n$, the λ-density reduces to the natural density.

The number sequence $x = (x_j)$ is said to be λ-*statistically convergent* to the number ℓ if for each $\epsilon > 0$, $\delta_\lambda(K_\epsilon) = 0$, where $K_\epsilon = \{j \in \mathbb{N} : |x_j - \ell| > \epsilon\}$, *i.e.*,

$$\lim_{n\to\infty} \frac{1}{\lambda_n} \left|\left\{j \in I_n : |x_j - \ell| > \epsilon\right\}\right| = 0.$$

In this case, we write st_λ-$\lim_j x_j = \ell$ and we denote the set of all λ-statistically convergent sequences by S_λ. This notion was extended to double sequences in [41, 42].

Remark 2.1 As in Remark 1.1, we observe that if a sequence is (V, λ)-summable to a number ℓ, then it is also λ-statistically convergent to the same number ℓ, but the converse need not be true. For example, let the sequence $z = (z_k)$ be defined by

$$z_k = \begin{cases} k & \text{if } n - [\sqrt{\lambda_n}] + 1 \le k \le n, \\ 0 & \text{otherwise,} \end{cases}$$

where $[a]$ denotes the integer part of $a \in \mathbb{R}$. Then x is λ-statistically convergent to 0 but not (V, λ)-summable.

Definition 2.1 Let (X, τ) be a locally solid Riesz space. Then a sequence $x = (x_j)$ in X is said to be *generalized statistically* τ-*convergent* (*or* $S_\lambda(\tau)$-*convergent*) to the number $\xi \in X$ if for every τ-neighborhood U of zero,

$$\lim_{n\to\infty} \frac{1}{\lambda_n} \left|\{j \in I_n : x_j - \xi \notin U\}\right| = 0.$$

In this case, we write $S_\lambda(\tau)$-$\lim x = \xi$ or $x_j \xrightarrow{S_\lambda(\tau)} \xi$.

Definition 2.2 Let (X,τ) be a locally solid Riesz space. We say that a sequence $x = (x_j)$ in X is *generalized statistically* τ*-bounded* if for every τ-neighborhood U of zero, there exists some $\lambda > 0$ such that the set

$$\{j \in \mathbb{N} : \lambda x_j \notin U\}$$

has λ-density zero.

Theorem 2.1 *Let* (X,τ) *be a Hausdorff locally solid Riesz space and* $x = (x_j)$ *and* $y = (y_k)$ *be two sequences in* X. *Then the following hold*:

(i) *If* $S_\lambda(\tau)\text{-}\lim_j x_j = \xi_1$ *and* $S_\lambda(\tau)\text{-}\lim_j x_j = \xi_2$, *then* $\xi_1 = \xi_2$.

(ii) *If* $S_\lambda(\tau)\text{-}\lim_j x_j = \xi$, *then* $S_\lambda(\tau)\text{-}\lim_j \alpha x_j = \alpha\xi$, $\alpha \in \mathbb{R}$.

(iii) *If* $S_\lambda(\tau)\text{-}\lim_j x_j = \xi$ *and* $S_\lambda(\tau)\text{-}\lim_j y_j = \eta$, *then* $S_\lambda(\tau)\text{-}\lim_j (x_j + y_j) = \xi + \eta$.

Proof (i) Suppose that $S_\lambda(\tau)\text{-}\lim_j x_j = \xi_1$ and $S_\lambda(\tau)\text{-}\lim_j x_j = \xi_2$. Let U be any τ-neighborhood of zero. Then there exists $Y \in \mathcal{N}_{\mathrm{sol}}$ such that $Y \subseteq U$. Choose any $E \in \mathcal{N}_{\mathrm{sol}}$ such that $E + E \subseteq Y$. We define the following sets:

$$K_1 = \{j \in \mathbb{N} : x_j - \xi_1 \in E\},$$
$$K_2 = \{j \in \mathbb{N} : x_j - \xi_2 \in E\}.$$

Since $S_\lambda(\tau)\text{-}\lim_j x_j = \xi_1$ and $S_\lambda(\tau)\text{-}\lim_j x_j = \xi_2$, we have $\delta_\lambda(K_1) = \delta_\lambda(K_2) = 1$. Thus $\delta(K_1 \cap K_2) = 1$ and, in particular, $K_1 \cap K_2 \neq \emptyset$. Now, let $j \in K_1 \cap K_2$. Then

$$\xi_1 - \xi_2 = \xi_1 - x_j + x_j - \xi_2 \in E + E \subseteq Y \subseteq U.$$

Hence, for every τ-neighborhood U of zero, we have $\xi_1 - \xi_2 \in U$. Since (X,τ) is Hausdorff, the intersection of all τ-neighborhoods U of zero is the singleton set $\{\theta\}$. Thus, we get $\xi_1 - \xi_2 = \theta$, *i.e.*, $\xi_1 = \xi_2$.

(ii) Let U be an arbitrary τ-neighborhood of zero and $S_\lambda(\tau)\text{-}\lim_j x_j = \xi$. Then there exists $Y \in \mathcal{N}_{\mathrm{sol}}$ such that $Y \subseteq U$ and also

$$\lim_{n\to\infty} \frac{1}{\lambda_n} \bigl|\{j \in I_n : x_j - \xi \in Y\}\bigr| = 1.$$

Since Y is balanced, $x_j - \xi \in Y$ implies $\alpha(x_j - \xi) \in Y$ for every $\alpha \in \mathbb{R}$ with $|\alpha| \leq 1$. Hence, for every $n \in \mathbb{N}$, we get

$$\begin{aligned}\{j \in I_n : x_j - \xi \in Y\} &\subseteq \{j \in I_n : \alpha x_j - \alpha\xi \in Y\} \\ &\subseteq \{j \in I_n : \alpha x_j - \alpha\xi \in U\}.\end{aligned}$$

Thus, we obtain

$$\lim_{n\to\infty} \frac{1}{\lambda_n} \bigl|\{j \in I_n : \alpha x_j - \alpha\xi \in U\}\bigr| = 1$$

for each τ-neighborhood U of zero. Now let $|\alpha| > 1$ and $[|\alpha|]$ be the smallest integer greater than or equal to $|\alpha|$. There exists $E \in \mathcal{N}_{\mathrm{sol}}$ such that $[|\alpha|]E \subseteq Y$. Since $S_\lambda(\tau)\text{-}\lim_j x_j = \xi$, the set

$$K = \{j \in \mathbb{N} : x_j - \xi \in E\}$$

has λ-density zero. Therefore, for all $n \in \mathbb{N}$ and $j \in K \cap I_n$, we have

$$|\alpha\xi - \alpha x_j| = |\alpha||\xi - x_j| \leq \bigl[|\alpha|\bigr]|\xi - x_j| \in \bigl[|\alpha|\bigr]E \subseteq Y \subseteq U.$$

Since the set Y is solid, we have $\alpha\xi - \alpha x_j \in Y$. This implies that $\alpha\xi - \alpha x_j \in U$. Thus,

$$\lim_{n\to\infty} \frac{1}{\lambda_n}\bigl|\{j \in I_n : \alpha x_j - \alpha\xi \in U\}\bigr| = 1$$

for each τ-neighborhood U of zero. Hence $S_\lambda(\tau)\text{-}\lim_j \alpha x_j = \alpha\xi$.

(iii) Let U be an arbitrary τ-neighborhood of zero. Then there exists $Y \in \mathcal{N}_{\text{sol}}$ such that $Y \subseteq U$. Choose E in $\mathcal{N}_{\text{sol}}$ such that $E + E \subseteq Y$. Since $S_\lambda(\tau)\text{-}\lim_j x_j = \xi$ and $S_\lambda(\tau)\text{-}\lim_j y_j = \eta$, we have $\delta_\lambda(H_1) = 1 = \delta_\lambda(H_2)$, where

$$H_1 = \{j \in \mathbb{N} : x_j - \xi \in E\},$$
$$H_2 = \{j \in \mathbb{N} : y_j - \eta \in E\}.$$

Let $H = H_1 \cap H_2$. Hence, we have $\delta_\lambda(H) = 1$. For all $n \in \mathbb{N}$ and $j \in H \cap I_n$, we get

$$(x_j + y_j) - (\xi + \eta) = (x_j - \xi) + (y_j - \eta) \in E + E \subseteq Y \subseteq U.$$

Therefore,

$$\lim_{n\to\infty} \frac{1}{\lambda_n}\bigl|\bigl\{j \in I_n : (x_j + y_j) - (\xi + \eta) \in U\bigr\}\bigr| = 1.$$

Since U is arbitrary, we have $S_\lambda(\tau)\text{-}\lim_j(x_j + y_j) = \xi + \eta$. □

Theorem 2.2 *Let (X, τ) be a locally solid Riesz space. If a sequence $x = (x_j)$ is generalized statistically τ-convergent, then it is generalized statistically τ-bounded.*

Proof Suppose $x = (x_j)$ is generalized statistically τ-convergent to the point $\xi \in X$ and let U be an arbitrary τ-neighborhood of zero. Then there exists $Y \in \mathcal{N}_{\text{sol}}$ such that $Y \subseteq U$. Let us choose $E \in \mathcal{N}_{\text{sol}}$ such that $E + E \subseteq Y$. Since $S_\lambda(\tau)\text{-}\lim_{j\to\infty} x_j = \xi$, the set

$$K = \{j \in \mathbb{N} : x_j - \xi \notin E\}$$

has λ-density zero. Since E is absorbing, there exists $\lambda > 0$ such that $\lambda\xi \in E$. Let $\alpha \in (0, \min\{1, \lambda\})$. Since E is solid and $|\alpha\xi| \leq |\lambda x|$, we have $\alpha\xi \in E$. Since E is balanced, $x_j - \xi \in E$ implies $\alpha(x_j - \xi) \in E$. Then, for each $n \in \mathbb{N}$ and $j \in (\mathbb{N} \setminus K) \cap I_n$, we have

$$\alpha x_j = \alpha(x_j - \xi) + \alpha\xi \in E + E \subseteq Y \subseteq U.$$

Thus

$$\lim_{n\to\infty}\frac{1}{\lambda_n}\left|\{j\in I_n:\alpha x_j\notin U\}\right|=0.$$

Hence, (x_j) is generalized statistically τ-bounded. □

Theorem 2.3 *Let (X,τ) be a locally solid Riesz space. If (x_j), (y_j) and (z_j) are three sequences such that*

(i) $x_j \le y_j \le z_j$ *for all* $j\in\mathbb{N}$,

(ii) $S_\lambda(\tau)\text{-}\lim_j x_j = \xi = S_\lambda(\tau)\text{-}\lim_j z_j$,

then $S_\lambda(\tau)\text{-}\lim_j y_j = \xi$.

Proof Let U be an arbitrary τ-neighborhood of zero, there exists $Y\in\mathcal{N}_{\mathrm{sol}}$ such that $Y\subseteq U$. Choose $E\in\mathcal{N}_{\mathrm{sol}}$ such that $E+E\subseteq Y$. From condition (ii), we have $\delta_\lambda(A)=1=\delta_\lambda(B)$, where

$$A=\{j\in\mathbb{N}:x_j-\xi\in E\},$$

$$B=\{j\in\mathbb{N}:x_j-\xi\in E\}.$$

Also, we get $\delta_\lambda(A\cap B)=1$, and from (i) we have

$$x_j-\xi\le y_j-\xi\le z_j-\xi$$

for all $j\in\mathbb{N}$. This implies that for all $n\in\mathbb{N}$ and $j\in A\cap B\cap I_n$, we get

$$|y_j-\xi|\le|x_j-\xi|+|z_j-\xi|\in E+E\subseteq Y.$$

Since Y is solid, we have $y_j-\xi\in Y\subseteq U$. Thus,

$$\lim_{n\to\infty}\frac{1}{\lambda_n}\left|\{j\in I_n:y_j-\xi\in U\}\right|=1$$

for each τ-neighborhood U of zero. Hence $S_\lambda(\tau)\text{-}\lim_j y_j=\xi$. □

3 Generalized statistically τ-Cauchy and $S^*_\lambda(\tau)$-convergence

Definition 3.1 Let (X,τ) be a locally solid Riesz space. A sequence $x=(x_j)$ in X is *generalized statistically τ-Cauchy* if for every τ-neighborhood U of zero there exists $p\in\mathbb{N}$ such that the set

$$\{j\in\mathbb{N}:x_j-x_p\notin U\}$$

has λ-density zero.

Theorem 3.1 *Let (X,τ) be a locally solid Riesz space. If a sequence $x=(x_j)$ is generalized statistically τ-convergent, then it is generalized statistically τ-Cauchy.*

Proof Suppose that $S_\lambda(\tau)\text{-}\lim_j x_j = \xi$. Let U be an arbitrary τ-neighborhood of zero, there exists $Y \in \mathcal{N}_{\text{sol}}$ such that $Y \subseteq U$. Choose $E \in \mathcal{N}_{\text{sol}}$ such that $E + E \subseteq Y$. By generalized statistical τ-convergence to ξ, there is $p \in \mathbb{N}$ with $\xi - x_p \in E$ and

$$\lim_{n\to\infty} \frac{1}{\lambda_n} \bigl|\{j \in I_n : x_j - \xi \notin E\}\bigr| = 0.$$

Also, for all $n \in \mathbb{N}$ and $j \in (\mathbb{N} \setminus K) \cap I_n$, where

$$K = \{j \in \mathbb{N} : x_j - \xi \notin E\},$$

we have

$$x_j - x_p = x_j - \xi + \xi - x_p \in E + E \subseteq Y \subseteq U$$

and $\delta_\lambda(K) = 0$. Therefore the set

$$\{j \in \mathbb{N} : x_j - x_p \notin U\} \subseteq K \cap I_n$$

for all $n \in \mathbb{N}$. For every τ-neighborhood U of zero there exists $p \in \mathbb{N}$ such that the set $\{j \in \mathbb{N} : x_j - x_p \notin U\}$ has λ-density zero. Hence (x_j) is generalized statistically τ-Cauchy. □

Now we define another type of convergence in locally solid Riesz spaces.

Definition 3.2 A sequence (x_j) in a locally solid Riesz space (X, τ) is said to be $S_\lambda^*(\tau)$-*convergent* to $\xi \in X$ if there exists an index set $K = \{j_n\} \subseteq \mathbb{N}$, $n = 1, 2, \ldots$, with $\delta_\lambda(K) = 1$ such that $\lim_{n\to\infty} x_{j_n} = \xi$. In this case, we write $\xi = S_\lambda^*(\tau)\text{-}\lim x$.

Theorem 3.2 *A sequence $x = (x_j)$ in a locally solid Riesz space (X, τ) is generalized statistically τ-convergent to a number ξ if it is $S_\lambda^*(\tau)$-convergent to ξ.*

Proof Let U be an arbitrary τ-neighborhood of ξ. Since $x = (x_j)$ is $S_\lambda^*(\tau)$-convergent to ξ, there is an index set $K = \{j_n\} \subseteq \mathbb{N}$, $n = 1, 2, \ldots$, with $\delta_\lambda(K) = 1$ and $j_0 = j_0(U)$, such that $j \geq j_0$ and $j \in K$ imply $x_j - \xi \in U$. Then

$$K_U = \{j \in \mathbb{N} : x_j - \xi \notin U\} \subseteq \mathbb{N} - \{j_{N+1}, j_{N+2}, \ldots\}.$$

Therefore $\delta_\lambda(K_U) = 0$. Hence x is generalized statistically τ-convergent to ξ. □

Note that the converse holds for a first countable space.

Recall that a topological space is first countable if each point has a countable (decreasing) local base.

Theorem 3.3 *Let (X, τ) be a first countable locally solid Riesz space. If a sequence $x = (x_j)$ is generalized statistically τ-convergent to a number ξ, then it is $S_\lambda^*(\tau)$-convergent to ξ.*

Proof Let x be generalized statistically τ-convergent to a number ξ. Fix a countable local base $U_1 \supset U_2 \supset U_3 \supset \cdots$ at ξ. For each $i \in \mathbb{N}$, put

$$K_i = \{j \in \mathbb{N} : x_j - \xi \notin U_i\}.$$

By hypothesis, $\delta_\lambda(K_i) = 0$ for each i. Since the ideal $\mathcal{I}$ of all subsets of $\mathbb{N}$ having λ-density zero is a P-ideal (see, for instance, [43]), then there exists a sequence of sets $(J_i)_i$ such that the symmetric difference $K_i \Delta J_i$ is a finite set for any $i \in \mathbb{N}$ and $J := \bigcup_{i=1}^{\infty} J_i \in \mathcal{I}$.

Let $K = \mathbb{N} \setminus J$, then $\delta_\lambda(K) = 1$. In order to prove the theorem, it is enough to check that $\lim_{j \in K} x_j = \xi$.

Let $i \in \mathbb{N}$. Since $K_i \Delta J_i$ is finite, there is $j_i \in \mathbb{N}$, without loss of generality, with $j_i \in K$, $j_i > i$, such that

$$(\mathbb{N} \setminus J_i) \cap \{j \in \mathbb{N} : j \geq j_i\} = (\mathbb{N} \setminus K_i) \cap \{j \in \mathbb{N} : j \geq j_i\}. \tag{1}$$

If $j \in K$ and $j \geq j_i$, then $j \notin J_i$, and by (1), $j \notin K_i$. Thus $x_j - \xi \in U_i$. So, we have proved that for all $i \in \mathbb{N}$, there is $j_i \in K$, $j_i > i$, with $x_j - \xi \in U_i$ for every $j \geq j_i$: without loss of generality, we can suppose $j_{i+1} > j_i$ for every $i \in \mathbb{N}$. The assertion follows taking into account that the U_i's form a countable local base at ξ. □

4 Conclusion

Recently, statistical convergence has been established as a better option than ordinary convergence. It is found very interesting that some results on sequences, series and summability can be proved by replacing the ordinary convergence by statistical convergence; and further, through some examples, where some efforts are required, we can show that the results for statistical convergence happen to be stronger than those proved for ordinary convergence (*e.g.*, [44–49]). This notion has also been defined and studied in different setups. In this paper, we have studied this notion through de la Vallée-Poussin mean in a locally solid Riesz space to deal with the convergence problems in a broader sense.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

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