Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

Dark soliton and periodic wave solutions of nonlinear evolution equations

Özkan Güner[1], Ahmet Bekir[1] and Adem Cengiz Cevikel[2*]

*Correspondence: acevikel@yildiz.edu.tr
[2]Mathematics Education Department, Education Faculty, Yildiz Technical University, Istanbul, Turkey
Full list of author information is available at the end of the article

Abstract

This paper studies the Kudryashov-Sinelshchikov and Jimbo-Miwa equations. Subsequently, we formally derive the dark (topological) soliton solutions for these equations. By using the sine-cosine method, some additional periodic solutions are derived. The physical parameters in the soliton solutions of the *ansatz* method, amplitude, inverse width and velocity, are obtained as functions of the dependent model coefficients.

PACS Codes: 02.30.Jr; 02.70.Wz; 05.45.Yv; 94.05.Fg

Keywords: dark soliton; sine-cosine method; *ansatz* method; Kudryashov-Sinelshchikov equation; Jimbo-Miwa equation

1 Introduction

In recent years, many powerful methods to construct exact solutions of nonlinear partial differential equations have been established and developed, which led to one of the most exciting advances of nonlinear science and theoretical physics. Particularly, the existence of soliton-type solutions for nonlinear models is of great importance because of their potential application in many physics areas such as nonlinear optics, plasmas, fluid mechanics, condensed matter and many more. Remarkably, the interest in dark and bright solitons has been growing steadily in recent years [1–3]. In fact, many kinds of exact soliton solutions have been obtained by using, for example, the tanh-sech method [4–6], extended tanh method [7–9], homogeneous balance method [10, 11], first integral method [12, 13], Jacobi elliptic function method [14, 15], $(\frac{G'}{G})$-expansion method [16, 17], F-expansion method [18, 19], Hirota bilinear method [20, 21], multiple exp-function method [22] and transformed rational function method [23] and so on.

In 2010, Kudryashov and Sinelshchikov [24] obtained a more common nonlinear partial differential equation for describing the pressure waves in a mixture liquid and gas bubbles taking into consideration the viscosity of the liquid and the heat transfer. The equation reads as follows [25]:

$$u_t + \alpha u u_x + u_{xxx} - (u u_{xx})_x - b u_x u_{xx} = 0. \tag{1.1}$$

In this equation, u is a density and α, b are real parameters. Ryabov [26] obtained some exact solutions for $b = -3$ and $b = -4$ using a modification of the truncated expansion method. Solutions are derived in a more straightforward manner and cast into a simpler form, and some new types of solutions which contain solitary wave and periodic wave solutions are presented in [27].

On the other hand, there is the Jimbo-Miwa equation (JM)

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0. \tag{1.2}$$

This equation was first introduced by Jimbo and Miwa [28] and it is known that this model is not Painlevé integrable. For many years, many scientists have researched it and certain explicit solutions have been obtained [29–31]. Exact three-wave solutions including periodic cross-kink wave solutions, doubly periodic solitary wave solutions and breather type of two-solitary wave solutions for the Jimbo-Miwa equation have been obtained using the generalized three-wave method in [32].

The layout of this paper is organized as follows. In Section 2, we give the description of the sine-cosine method and we apply this method to the Kudryashov-Sinelshchikov (KS) and Jimbo-Miwa equations. We apply the *ansatz* method to the KS and JM equations in Section 3. Conclusions are given in the last section.

2 Sine-cosine method

In this section, the sine-cosine method will be first described and then subsequently applied to solve the Kudryashov-Sinelshchikov and Jimbo-Miwa equations.

2.1 Brief of the method

The sine-cosine method was first proposed by Wazwaz in 2004 [33]. This method has been applied to various kinds of nonlinear problems arising in the applied sciences [34–37].

1. We introduce the wave variable $\xi = x + y - ct$ into the PDE

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{yy}, u_{xt}, u_{xy}, u_{ty}, \ldots) = 0, \tag{2.1}$$

where $u(x, y, t)$ is a traveling wave solution. This enables us to use the following changes:

$$\begin{aligned} &\frac{\partial}{\partial t} = -c\frac{\partial}{\partial \xi}, \qquad \frac{\partial^2}{\partial t^2} = c^2\frac{\partial^2}{\partial \xi^2}, \qquad \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}, \\ &\frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial \xi^2}, \qquad \frac{\partial}{\partial y} = \frac{\partial}{\partial \xi}, \qquad \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial \xi^2}, \qquad \ldots. \end{aligned} \tag{2.2}$$

One can immediately reduce nonlinear PDE (2.1) into the nonlinear ODE

$$Q(u, u_\xi, u_{\xi\xi}, u_{\xi\xi\xi}, \ldots) = 0. \tag{2.3}$$

Ordinary differential equation (2.3) is then integrated as long as all terms contain derivatives, where we neglect integration constants.

2. The solutions of many nonlinear equations can be expressed in the form [34]

$$u(x,t) = \begin{cases} \lambda \cos^\beta(\mu\xi), & |\xi| \le \frac{\pi}{2\mu}, \\ 0, & \text{otherwise}, \end{cases} \tag{2.4}$$

or in the form

$$u(x,t) = \begin{cases} \lambda \sin^\beta(\mu\xi), & |\xi| \le \frac{\pi}{\mu}, \\ 0, & \text{otherwise}, \end{cases} \tag{2.5}$$

where λ, μ and $\beta \neq 0$ are parameters that will be determined, μ and c are the wave number and the wave speed respectively. We use

$$\begin{aligned} &u(\xi) = \lambda \cos^{\beta}(\mu\xi), \\ &u^n(\xi) = \lambda^n \cos^{n\beta}(\mu\xi), \\ &(u^n)_{\xi} = -n\mu\beta\lambda^n \sin(\mu\xi)\cos^{n\beta-1}(\mu\xi), \\ &(u^n)_{\xi\xi} = -n^2\mu^2\beta^2\lambda^n \cos^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\cos^{n\beta-2}(\mu\xi), \end{aligned} \tag{2.6}$$

and the derivatives of (2.5) become

$$\begin{aligned} &u(\xi) = \lambda \sin^{\beta}(\mu\xi), \\ &u^n(\xi) = \lambda^n \sin^{n\beta}(\mu\xi), \\ &(u^n)_{\xi} = n\mu\beta\lambda^n \cos(\mu\xi)\sin^{n\beta-1}(\mu\xi), \\ &(u^n)_{\xi\xi} = -n^2\mu^2\beta^2\lambda^n \sin^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\sin^{n\beta-2}(\mu\xi), \end{aligned} \tag{2.7}$$

and so on for other derivatives.

3. We substitute (2.6) or (2.7) into the reduced equation obtained above in (2.3), balance the terms of the cosine functions when (2.7) is used, or balance the terms of the sine functions when (2.6) is used, and solve the resulting system of algebraic equations by using the computerized symbolic calculations. We next collect all terms with the same power in $\cos^k(\mu\xi)$ or $\sin^k(\mu\xi)$ and set to zero their coefficients to get a system of algebraic equations among the unknowns μ, β and λ. We obtained all possible values of the parameters μ, β and λ [33].

2.2 Application of the sine-cosine method to the Kudryashov-Sinelshchikov equation

We begin first with Eq. (1.1). Using the wave variable $\xi = x - vt$, Eq. (1.1) is carried to the ODE

$$-(uv)' + \alpha\left(\frac{u^2}{2}\right)' + u''' - (uu'')' - \frac{b}{2}\left[(u')^2\right]' = 0, \tag{2.8}$$

where by integrating once we obtain

$$-uv + \alpha\frac{u^2}{2} + u'' - uu'' - \frac{b}{2}(u')^2 + k = 0, \tag{2.9}$$

where k is the integration constant.

Substituting (2.6) into (2.9) gives

$$\begin{aligned} &-v\lambda\cos^{\beta}(\mu\xi) + \frac{1}{2}\alpha\lambda^2\cos^{2\beta}(\mu\xi) + \lambda\mu^2\beta^2\cos^{\beta-2}(\mu\xi) \\ &\quad - \lambda\mu^2\beta^2\cos^{\beta}(\mu\xi) - \lambda\beta\mu^2\cos^{\beta-2}(\mu\xi) - \lambda^2\mu^2\beta^2\cos^{2\beta-2}(\mu\xi) \\ &\quad + \lambda^2\mu^2\beta^2\cos^{2\beta}(\mu\xi) + \lambda^2\mu^2\beta\cos^{2\beta-2}(\mu\xi) \\ &\quad + \frac{1}{2}b\lambda^2\mu^2\beta^2\cos^{2\beta}(\mu\xi) - \frac{1}{2}b\lambda^2\mu^2\beta^2\cos^{2\beta-2}(\mu\xi) + k = 0. \end{aligned} \tag{2.10}$$

Equating the exponents and the coefficients of each pair of the cosine functions, we find the following system of algebraic equations:

$$\begin{aligned}
&(\beta - 1) \neq 0, \\
&\beta - 2 = 2\beta, \\
&\frac{1}{2}\alpha\lambda^2 + 4\lambda^2\mu^2 + 2b\lambda^2\mu^2 = 0, \\
&-\nu\lambda - 4\lambda\mu^2 - 2\lambda^2\mu^2 - 2b\lambda^2\mu^2 = 0, \\
&2\lambda\mu^2 + k = 0.
\end{aligned} \tag{2.11}$$

Solving the system (2.11) yields

$$\begin{aligned}
&\beta = -2, \\
&\mu = \sqrt{-\frac{\alpha}{8+4b}}, \\
&\lambda = \frac{2k(b+2)}{\alpha}, \\
&\nu = \frac{a + 2k + 3kb + b^2k}{b+2}.
\end{aligned} \tag{2.12}$$

The result (2.11) can be easily obtained if we also use the sine method (2.5). Consequently, for $\frac{\alpha}{8+4b} < 0$, the following periodic solutions can be obtained:

$$u(x,t) = \left(\frac{2k(b+2)}{\alpha}\right)\sec^2\left[\sqrt{-\frac{\alpha}{8+4b}}(x - \nu t)\right], \tag{2.13}$$

where $|\sqrt{-\frac{\alpha}{8+4b}}(x-\nu t)| < \frac{\pi}{2}$, and

$$u(x,t) = \left(\frac{2k(b+2)}{\alpha}\right)\csc^2\left[\sqrt{-\frac{\alpha}{8+4b}}(x - \nu t)\right], \tag{2.14}$$

where $0 < \sqrt{-\frac{\alpha}{8+4b}}(x-\nu t) < \pi$.

However, for $\frac{\alpha}{8+4b} > 0$, we obtain the soliton solutions

$$u(x,t) = \left(\frac{2k(b+2)}{\alpha}\right)\operatorname{sech}^2\left[\sqrt{-\frac{\alpha}{8+4b}}(x - \nu t)\right] \tag{2.15}$$

and

$$u(x,t) = \left(-\frac{2k(b+2)}{\alpha}\right)\operatorname{csch}^2\left[\sqrt{\frac{\alpha}{8+4b}}(x - \nu t)\right]. \tag{2.16}$$

All the solutions reported in this paper have been verified with Maple by putting them back into original Eq. (1.1), which cannot be obtained by the methods [25–27]. To the best of our knowledge, these solutions are new and have not been reported yet.

2.3 Application of the sine-cosine method to the Jimbo-Miwa equation

We begin second with Eq. (1.2). Using the wave variable $\xi = x + y + z - \nu t$, Eq. (1.2) is carried to the ODE

$$u'''' + 3\left[\left(u'\right)^2\right]' - (2\nu + 3)u'' = 0, \tag{2.17}$$

where by integrating once we obtain

$$u''' + 3\left(u'\right)^2 - (2\nu + 3)u' = 0, \tag{2.18}$$

which is obtained upon setting the constant of integration to zero. Setting $u' = \rho$, Eq. (2.18) becomes

$$\rho'' + 3\rho^2 - (2\nu + 3)\rho = 0. \tag{2.19}$$

Substituting (2.4) into (2.19) gives

$$\begin{aligned} &-\lambda\beta^2\mu^2\cos^\beta(\mu\xi) + \lambda\beta^2\mu^2\cos^{\beta-2}(\mu\xi) - \lambda\beta\mu^2\cos^{\beta-2}(\mu\xi) \\ &\quad + 3\lambda^2\cos^{2\beta}(\mu\xi) - (2\nu+3)\lambda\cos^\beta(\mu\xi) = 0. \end{aligned} \tag{2.20}$$

Equating the exponents and the coefficients of each pair of the cosine functions, we find the following system of algebraic equations:

$$\begin{aligned} &(\beta - 1) \neq 0, \\ &\beta - 2 = 2\beta, \\ &6\lambda\mu^2 + 3\lambda^2 = 0, \\ &-4\lambda\mu^2 - 2\lambda\nu - 3\lambda = 0. \end{aligned} \tag{2.21}$$

Solving the system (2.21) yields

$$\begin{aligned} &\beta = -2, \\ &\mu = \frac{1}{2}\sqrt{-2\nu - 3}, \\ &\lambda = \nu + \frac{3}{2}. \end{aligned} \tag{2.22}$$

The result (2.21) can be easily obtained if we also use the sine method (2.5). Consequently, for $2\nu + 3 < 0$, the following periodic solutions can be obtained:

$$u(x,y,z,t) = \left(\nu + \frac{3}{2}\right)\sec^2\left[\frac{1}{2}\sqrt{-2\nu - 3}(x + y + z - \nu t)\right], \tag{2.23}$$

where $|\frac{1}{2}\sqrt{-2\nu - 3}(x + y + z - \nu t)| < \frac{\pi}{2}$, and

$$u(x,y,z,t) = \left(\nu + \frac{3}{2}\right)\csc^2\left[\frac{1}{2}\sqrt{-2\nu - 3}(x + y + z - \nu t)\right], \tag{2.24}$$

where $0 < \frac{1}{2}\sqrt{-2\nu - 3}(x + y + z - \nu t) < \pi$.

However, for $2\nu + 3 > 0$, we obtain the soliton solutions

$$u(x,y,z,t) = \left(\nu + \frac{3}{2}\right) \operatorname{sech}^2\left[\frac{1}{2}\sqrt{-2\nu - 3}(x + y + z - \nu t)\right] \tag{2.25}$$

and

$$u(x,y,z,t) = \left(-\nu - \frac{3}{2}\right) \operatorname{csch}^2\left[\frac{1}{2}\sqrt{2\nu + 3}(x + y + z - \nu t)\right]. \tag{2.26}$$

Comparing the above results with the relevant ones in [28–31], it can be seen that some of the obtained results are new and the rest of solutions are the same.

3 *Ansatz* method

In this section, the *ansatz* method will be used to carry out the integration of the Kudryashov-Sinelshchikov and Jimbo-Miwa equations. The search is going to be for a topological 1-soliton solution which is also known as a kink solution or a shock wave solution. This will be demonstrated in the following two subsections. For both equations, arbitrary constant coefficients will be considered. There are many applications of this method [38–44].

3.1 Application of the *ansatz* method to the Kudryashov-Sinelshchikov equation

In this section the search is going to be for a topological 1-soliton solution to the Kudryashov-Sinelshchikov equation given by (1.1). To start off, the hypothesis is given by [45, 46]

$$u(x,t) = A \tanh^p \tau, \quad \text{where} \tag{3.1}$$

$$\tau = B(x - \nu t). \tag{3.2}$$

Here, A and B are free parameters and ν is the velocity of the wave in (3.1) and (3.2). The exponent p is unknown at this point and its values will fall out in the process of deriving the solution of this equation. Thus from (3.1) we get

$$u_t = p\nu AB\left\{\tanh^{p+1}\tau - \tanh^{p-1}\tau\right\}, \tag{3.3}$$

$$u_x = pAB\left\{\tanh^{p-1}\tau - \tanh^{p+1}\tau\right\}, \tag{3.4}$$

$$u_{xx} = pAB^2\left\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\right\}, \tag{3.5}$$

$$\begin{aligned} u_{xxx} = pAB^3\big\{&(p-1)(p-2)\tanh^{p-3}\tau - \left[2p^2 + (p-1)(p-2)\right]\tanh^{p-1}\tau \\ &+ \left[2p^2 + (p+1)(p+2)\right]\tanh^{p+1}\tau - (p+1)(p+2)\tanh^{p+3}\tau\big\}, \end{aligned} \tag{3.6}$$

$$\begin{aligned} u_x u_{xx} = p^2A^2B^3\big\{&(p-1)\tanh^{2p-3}\tau - (3p-1)\tanh^{2p-1}\tau \\ &+ (3p+1)\tanh^{2p+1}\tau - (p+1)\tanh^{2p+3}\tau\big\}. \end{aligned} \tag{3.7}$$

Substituting Eqs. (3.3)-(3.7) into (1.1), we have

$$\begin{aligned} &p\nu AB\left\{\tanh^{p+1}\tau - \tanh^{p-1}\tau\right\} + \alpha pA^2B\left\{\tanh^{2p-1}\tau - \tanh^{2p+1}\tau\right\} \\ &\quad + AB^3\big\{p(p-1)(p-2)\tanh^{p-3}\tau - \left[2p^3 + p(p-1)(p-2)\right]\tanh^{p-1}\tau \end{aligned}$$

$$\begin{aligned}
&+\left[2p^3+p(p+1)(p+2)\right]\tanh^{p+1}\tau-p(p+1)(p+2)\tanh^{p+3}\tau\bigr\} \\
&-A^2B^3\bigl\{p(p-1)(p-2)\tanh^{2p-3}\tau+\left[2p^3+p(p-1)(p-2)\right]\tanh^{2p-1}\tau \\
&-\left[2p^3+p(p+1)(p+2)\right]\tanh^{2p+1}\tau+p(p+1)(p+2)\tanh^{2p+3}\tau\bigr\} \\
&-(1+b)p^2A^2B^3\bigl\{(p-1)\tanh^{2p-3}\tau-(3p-1)\tanh^{2p-1}\tau \\
&+(3p+1)\tanh^{2p+1}\tau-(p+1)\tanh^{2p+3}\tau\bigr\}=0.
\end{aligned}\tag{3.8}$$

From (3.8), equating the exponents $2p-1$ and $p+1$ gives

$$2p-1=p+1$$

so that

$$p=2.\tag{3.9}$$

It should be noted that the same value of p is yielded when the exponent pairs $2p-3$ and $p-1$, $2p+1$ and $p+3$ are equated with each other, respectively.

$$AB^3p(p+1)(p+2)+(1+b)p^2A^2B^3(p+1)=0,\tag{3.10}$$

$$\begin{aligned}
&-pvAB-AB^3\left[2p^3+p(p-1)(p-2)\right]-A^2B^3p(p-1)(p-2) \\
&\quad-(1+b)p^2A^2B^3(p-1)=0,
\end{aligned}\tag{3.11}$$

$$\begin{aligned}
&pvAB+p\alpha A^2B+AB^3\left[2p^3+p(p+1)(p+2)\right] \\
&\quad+A^2B^3\left[2p^3+p(p-1)(p-2)\right]+(1+b)p^2A^2B^3(3p-1)=0,
\end{aligned}\tag{3.12}$$

$$\begin{aligned}
&-p\alpha A^2B-AB^3p(p+1)(p+2)-A^2B^3\left[2p^3+p(p+1)(p+2)\right] \\
&\quad-(1+b)p^2A^2B^3(3p+1)=0.
\end{aligned}\tag{3.13}$$

If we put $p=2$ in (3.10)-(3.13), the system reduces to

$$-16AB^3-4A^2B^3b-2ABv-4A^2B^3=0,\tag{3.14}$$

$$36A^2B^3+40AB^3+2ABv+20A^2B^3b+2\alpha A^2B=0,\tag{3.15}$$

$$-68A^2B^3-24AB^3-2\alpha A^2B-28A^2B^3b=0.\tag{3.16}$$

Solving the above equations yields

$$b=-3,\tag{3.17}$$

$$A=\frac{12B^2}{8B^2-\alpha},\tag{3.18}$$

$$v=12AB^2-20B^2-\alpha A.\tag{3.19}$$

Hence, finally, the 1-soliton solution to (1.1) is respectively given by

$$u(x,t)=A\tanh^2\bigl[B(x-vt)\bigr],\tag{3.20}$$

where the free parameter A is given by (3.18), the velocity of the solitons v is given in (3.19).

3.2 Application of the *ansatz* method to the Jimbo-Miwa equation

In this section the search is going to be for a topological 1-soliton solution to the (3 + 1)-dimensional Jimbo-Miwa equation. Without any loss of generality, it is assumed that the dark soliton solution to (1.2) is given by

$$u(x,y,z,t) = A\tanh^p \tau, \tag{3.21}$$

where

$$\tau = B(x+y+z-vt). \tag{3.22}$$

Here, A and B are free parameters and v is the velocity of the wave in (3.21) and (3.22). The exponent p is unknown at this point and its values will fall out in the process of deriving the solution of this equation. Thus from (3.21) we get

$$u_x = pAB\{\tanh^{p-1}\tau - \tanh^{p+1}\tau\}, \tag{3.23}$$

$$u_y = pAB\{\tanh^{p-1}\tau - \tanh^{p+1}\tau\}, \tag{3.24}$$

$$u_{xx} = pAB^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\}, \tag{3.25}$$

$$u_{xz} = pAB^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\}, \tag{3.26}$$

$$u_{xy} = pAB^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\}, \tag{3.27}$$

$$u_{yt} = -pvAB^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\}, \tag{3.28}$$

$$\begin{aligned} u_{xxxy} = pAB^4\{&(p-1)(p-2)(p-3)\tanh^{p-4}\tau - 4(p-1)(p^2-2p+2)\tanh^{p-2}\tau \\ &+ 2p(3p^2+5)\tanh^p\tau - 4(p+1)(p^2+2p+2)\tanh^{p+2}\tau \\ &+ (p+1)(p+2)(p+3)\tanh^{p+4}\tau\}, \end{aligned} \tag{3.29}$$

$$\begin{aligned} u_y u_{xx} = p^2A^2B^3\{&(p-1)\tanh^{2p-3}\tau - (3p-1)\tanh^{2p-1}\tau \\ &+ (3p+1)\tanh^{2p+1}\tau - (p+1)\tanh^{2p+3}\tau\}, \end{aligned} \tag{3.30}$$

$$\begin{aligned} u_x u_{xy} = p^2A^2B^3\{&(p-1)\tanh^{2p-3}\tau - (3p-1)\tanh^{2p-1}\tau \\ &+ (3p+1)\tanh^{2p+1}\tau - (p+1)\tanh^{2p+3}\tau\}. \end{aligned} \tag{3.31}$$

Substituting Eqs. (3.23)-(3.31) into (1.2), we have

$$\begin{aligned} &pAB^4\{(p-1)(p-2)(p-3)\tanh^{p-4}\tau \\ &\quad - 4(p-1)(p^2-2p+2)\tanh^{p-2}\tau + 2p(3p^2+5)\tanh^p\tau \\ &\quad - 4(p+1)(p^2+2p+2)\tanh^{p+2}\tau + (p+1)(p+2)(p+3)\tanh^{p+4}\tau\} \\ &\quad + 6p^2A^2B^3\{(p-1)\tanh^{2p-3}\tau - (3p-1)\tanh^{2p-1}\tau \\ &\quad + (3p+1)\tanh^{2p+1}\tau - (p+1)\tanh^{2p+3}\tau\} \\ &\quad - 2pvAB^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\} \\ &\quad - 3pAB^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\} = 0. \end{aligned} \tag{3.32}$$

From (3.32), equating the exponents $2p + 3$ and $p + 4$ gives

$$2p + 3 = p + 4$$

so that

$$p = 1.$$

It should be noted that the same value of p is yielded when the exponent pairs $2p - 3$ and $p - 2$, $2p + 1$ and $p + 2$, $2p - 1$ and p are equated with each other, respectively.

$$AB^4 p(p+1)(p+2)(p+3) - 6p^2(p+1)A^2B^3 = 0, \tag{3.33}$$

$$-4p(p+1)\left(p^2 + 2p + 2\right)AB^4 + 6p^2(3p+1)A^2B^3 - 2p(p+1)vAB^2 - 3p(p+1)AB^2 = 0, \tag{3.34}$$

$$2p^2AB^4\left(3p^2 + 5\right) - 6p^2(3p-1)A^2B^3 + 4p^2vAB^2 + 6p^2AB^2 = 0. \tag{3.35}$$

Solving the above system for $p = 1$ gives

$$\begin{aligned} A &= 2B, \\ v &= 2B^2 - \frac{3}{2}. \end{aligned} \tag{3.36}$$

Thus, finally, the 1-soliton solution to (1.2) is respectively given by

$$u(x, y, z, t) = A \tanh\left[B(x + y + z - vt)\right], \tag{3.37}$$

where the free parameter A is given by (3.36) and the velocity of the solitons v is given in (3.36).

4 Conclusion

In this paper, the KS and JM equations are solved by the sine-cosine method as well as by the solitary wave *ansatz* method. There are several solutions that are obtained by the first method. The solitary wave *ansatz* method is used to carry out the integration of these equations. The obtained solutions may be useful for understanding of the mechanism of complicated nonlinear physical phenomena in wave interaction. In addition, we note that the solitary wave *ansatz* method is an efficient method for constructing exact soliton solutions for nonlinear wave equations. These results are going to be very useful for conducting research in future.

Competing interests
The authors declare that there is no conflict of interest among the authors.

Authors' contributions
AÇ carried out the theoretical studies. AB carried out the computational studies and wrote the paper according to the rules of journal. ÖG found out the problem in literature, and carried out the computational studies. All authors read and approved the final manuscript.

Author details
[1]Mathematics and Computer Science Department, Art-Science Faculty, Eskisehir Osmangazi University, Eskisehir, Turkey.
[2]Mathematics Education Department, Education Faculty, Yildiz Technical University, Istanbul, Turkey.

Received: 8 October 2012 Accepted: 27 February 2013 Published: 21 March 2013

References

1. Triki, H, Wazwaz, AM: Bright and dark soliton solutions for a $K(m,n)$ equation with t-dependent coefficients. Phys. Lett. A **373**, 2162-2165 (2009)
2. Green, PD, Biswas, A: Bright and dark optical solitons with time-dependent coefficients in a non-Kerr law media. Commun. Nonlinear Sci. Numer. Simul. **15**, 3865-3873 (2010)
3. Biswas, A, Milovic, D: Bright and dark solitons of the generalized nonlinear Schrödinger's equation. Commun. Nonlinear Sci. Numer. Simul. **15**, 1473-1484 (2010)
4. Malfliet, W: Solitary wave solutions of nonlinear wave equations. Am. J. Phys. **60**, 650-654 (1992)
5. Malfliet, W, Hereman, W: The tanh method. I: exact solutions of nonlinear evolution and wave equations. Phys. Scr. **54**, 563-568 (1996)
6. Wazwaz, AM: The tanh method for travelling wave solutions of nonlinear equations. Appl. Math. Comput. **154**(3), 713-723 (2004)
7. Abdou, MA: New solitons and periodic wave solutions for nonlinear physical models. Nonlinear Dyn. **52**(1-2), 129-136 (2008)
8. Fan, E: Extended tanh-function method and its applications to nonlinear equations. Phys. Lett. A **277**, 212-218 (2000)
9. Wazwaz, AM: The extended tanh method for new soliton solutions for many forms of the fifth-order KdV equations. Appl. Math. Comput. **184**(2), 1002-1014 (2007)
10. Fan, E, Zhang, H: A note on the homogeneous balance method. Phys. Lett. A **246**, 403-406 (1998)
11. Wang, ML: Exact solutions for a compound KdV-Burgers equation. Phys. Lett. A **213**, 279-287 (1996)
12. Feng, ZS: The first integral method to study the Burgers-KdV equation. J. Phys. A, Math. Gen. **35**, 343 (2002)
13. Taghizadeh, N, Mirzazadeh, M: The first integral method to some complex nonlinear partial differential equations. J. Comput. Appl. Math. **235**(16), 4871 (2011)
14. Fan, E, Zhang, J: Applications of the Jacobi elliptic function method to special-type nonlinear equations. Phys. Lett. A **305**, 383-392 (2002)
15. Liu, S, Fu, Z, Liu, S, Zhao, Q: Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations. Phys. Lett. A **289**, 69-74 (2001)
16. Wang, ML, Li, X, Zhang, J: The $(\frac{G'}{G})$-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics. Phys. Lett. A **372**(4), 417-423 (2008)
17. Bekir, A: Application of the $(\frac{G'}{G})$-expansion method for nonlinear evolution equations. Phys. Lett. A **372**(19), 3400-3406 (2008)
18. Abdou, MA: Further improved F-expansion and new exact solutions for nonlinear evolution equations. Nonlinear Dyn. **52**(3), 277-288 (2008)
19. Zhang, JL, Wang, ML, Wang, YM, Fang, ZD: The improved F-expansion method and its applications. Phys. Lett. A **350**, 103-109 (2006)
20. Wazwaz, AM: Multiple soliton solutions for coupled KdV and coupled KP systems. Can. J. Phys. **87**(12), 1227-1232 (2010)
21. Wazwaz, AM: Extended KP equations and extended system of KP equations: multiple-soliton solutions. Can. J. Phys. **89**(7), 739-743 (2011)
22. Ma, WX, Huang, T, Zhang, Y: A multiple exp-function method for nonlinear differential equations and its application. Phys. Scr. **82**, 065003 (2010)
23. Ma, WX: Comment on the (3 + 1)-dimensional Kadomtsev-Petviashvili equations. Commun. Nonlinear Sci. Numer. Simul. **16**, 2663 (2011)
24. Kudryashov, NA, Sinelshchikov, DI: Nonlinear waves in bubbly liquids with consideration for viscosity and heat transfer. Phys. Lett. A **374**, 2011-2016 (2010)
25. He, B, Meng, Q, Zhang, J, Long, Y: Periodic loop solutions and their limit forms for the Kudryashov-Sinelshchikov equation. Math. Probl. Eng. **2012**, 320163 (2012)
26. Ryabov, PN: Exact solutions of the Kudryashov and Sinelshchikov equation. Appl. Math. Comput. **217**, 3585-3590 (2010)
27. Randrüüt, M: On the Kudryashov-Sinelshchikov equation for waves in bubbly liquids. Phys. Lett. A **375**, 3687-3692 (2011)
28. Jimbo, M, Miwa, T: Solitons and infinite dimensional Lie algebras. Publ. Res. Inst. Math. Sci. **19**, 943-1001 (1983)
29. Senthilvelan, M: On the extended applications of homogenous balance method. Appl. Math. Comput. **123**, 381-388 (2001)
30. Tang, XY, Liang, ZF: Variable separation solutions for the (3 + 1)-dimensional Jimbo-Miwa equation. Phys. Lett. A **351**, 398-402 (2006)
31. Wazwaz, AM: New solutions of distinct physical structures to high-dimensional nonlinear evolution equations. Appl. Math. Comput. **196**, 363-370 (2008)
32. Li, Z, Dai, Z, Liu, J: Exact three-wave solutions for the (3 + 1)-dimensional Jimbo-Miwa equation. Comput. Math. Appl. **61**, 2062-2066 (2011)
33. Wazwaz, AM: A sine-cosine method for handling nonlinear wave equations. Math. Comput. Model. **40**, 499-508 (2004)
34. Wazwaz, AM: The tanh method and the sine-cosine method for solving the KP-MEW equation. Int. J. Comput. Math. **82**(2), 235-246 (2005)
35. Bekir, A, Cevikel, AC: New solitons and periodic solutions for nonlinear physical models in mathematical physics. Nonlinear Anal., Real World Appl. **11**(4), 3275-3285 (2010)

36. Bekir, A: New solitons and periodic wave solutions for some nonlinear physical models by using sine-cosine method. Phys. Scr. **77**(4), 501-505 (2008)
37. Zedan, HA, Monaquel, SJ: The sine-cosine method for the Davey-Stewartson equations. Appl. Math. E-Notes **10**, 103-111 (2010)
38. Triki, H, Wazwaz, AM: A one-soliton solution of the ZK(m,n,k) equation with generalized evolution and time-dependent coefficients. Nonlinear Anal., Real World Appl. **12**, 2822-2825 (2011)
39. Triki, H, Wazwaz, AM: Bright soliton solution to a generalized Burgers-KdV equation with time-dependent coefficients. Appl. Math. Comput. **217**, 466-471 (2010)
40. Triki, H, Wazwaz, AM: Bright and dark solitons for a generalized Korteweg de-Vries-modified Korteweg de-Vries equation with higher-order nonlinear terms and time-dependent coefficients. Can. J. Phys. **89**(3), 253-259 (2011)
41. Triki, H, Wazwaz, AM: Soliton solutions for a generalized KdV and BBM equations with time-dependent coefficients. Commun. Nonlinear Sci. Numer. Simul. **16**, 1122-1126 (2011)
42. Biswas, A: 1-soliton solution of the $B(m,n)$ equation with generalized evolution. Commun. Nonlinear Sci. Numer. Simul. **14**, 3226-3229 (2009)
43. Triki, H, Ismail, MS: Soliton solutions of a BBM(m,n) equation with generalized evolution. Appl. Math. Comput. **217**(1), 48-54 (2010)
44. Esfahani, A: On the generalized Kadomtsev-Petviashvili equation with generalized evolution and variable coefficients. Phys. Lett. A **374**, 3635-3645 (2010)
45. Biswas, A, Triki, H, Labidi, M: Bright and dark solitons of the Rosenau-Kawahara equation with power law nonlinearity. Phys. Wave Phenom. **19**(1), 24-29 (2011)
46. Triki, H, Wazwaz, AM: Dark solitons for a combined potential KdV and Schwarzian KdV equations with t-dependent coefficients and forcing term. Appl. Math. Comput. **217**, 8846-8851 (2011)

doi:10.1186/1687-1847-2013-68
Cite this article as: Güner et al.: Dark soliton and periodic wave solutions of nonlinear evolution equations. *Advances in Difference Equations* 2013 **2013**:68.