Manuel et al. *Advances in Difference Equations* 2014, 2014:109
http://www.advancesindifferenceequations.com/content/2014/1/109

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

Oscillation of solutions of some generalized nonlinear α-difference equations

Mariasebastin Maria Susai Manuel[1], Adem Kılıçman[2*], K Srinivasan[3] and G Dominic Babu[4]

*Correspondence: akilic@upm.edu.my
[2]Department of Mathematics and Institute for Mathematical Research, University Putra Malaysia (UPM), Serdang, Selangor 43400, Malaysia
Full list of author information is available at the end of the article

Abstract

In this paper, the authors discuss the oscillation of solutions of some generalized nonlinear α-difference equation

$$\Delta_{\alpha(\ell)}(p(k)\Delta_{\alpha(\ell)}u(k)) + q(k)f(u(k-\tau(k))) = 0, \qquad (1)$$

$k \in [a,\infty)$, where the functions p, q, f and τ are defined in their domain of definition and $\alpha > 1$, ℓ is a positive real. Further, $uf(u) > 0$ for $u \neq 0$, $p(k) > 0$ and $\lim_{k\to\infty}(k-\tau(k)) = \infty$, where $R_k = \sum_{r=0}^{[\frac{k-\ell}{\ell}]} \frac{1}{\alpha^r p(r\ell)} \to \infty$ as $k \to \infty$ and $u(k)$ is defined for $k \geq \min_{i\geq 0}(i-\tau(i))$ for all $k \in [a,\infty)$ for some $a \in [0,\infty)$.

MSC: 39A12

Keywords: generalized α-difference equation; generalized α-difference operator; oscillation and nonoscillation

1 Introduction

The basic theory of difference equations is based on the operator Δ defined as $\Delta u(k) = u(k+1) - u(k)$, $k \in \mathbb{N} = \{0,1,2,3,\ldots\}$. Even though many authors [1–4] have suggested the definition of Δ as

$$\Delta u(k) = u(k+\ell) - u(k), \quad k \in \mathbb{R}, \ell \in \mathbb{R} - \{0\}, \qquad (2)$$

there was no significant progress in this area. But recently, [5] considered the definition of Δ as given in (2) and developed the theory of difference equations in a different direction. For convenience, the operator Δ defined by (2) is labeled as Δ_ℓ, and by defining its inverse Δ_ℓ^{-1}, many interesting results and applications in number theory (see [5–7]) were obtained. By extending the study related to the sequences of complex numbers and ℓ being real, some new qualitative properties of the solutions like rotatory, expanding, shrinking, spiral and weblike were obtained for difference equation involving Δ_ℓ. The results obtained using Δ_ℓ can be found in [5–7]. Popenda and Szmanda [8, 9] defined Δ as

$$\Delta_\alpha u(k) = u(k+1) - \alpha u(k), \qquad (3)$$

and based on this definition, they studied the qualitative properties of a particular difference equation, and no one else has handled this operator. Recently Manuel *et al.* [6, 10]

considered the definition of Δ_ℓ as given in (3), and by defining its inverse, some interesting results on number theory were obtained.

In [11], Szafranski and Szmanda obtained sufficient conditions for the oscillation of a similar difference equation involving Δ. In this paper the theory is extended from Δ to $\Delta_{\alpha(\ell)}$ for all real $k \in [a,\infty)$, and we discuss the oscillatory behavior of solutions of generalized nonlinear α-difference equation (1).

Throughout this paper, we make use of the following assumptions.

(a) $\mathbb{N} = \{0,1,2,3,\ldots\}$, $\mathbb{N}(a) = \{a,a+1,a+2,\ldots\}$;
(b) $\mathbb{N}_\ell(a) = \{a,a+\ell,a+2\ell,\ldots\}$;
(c) $\lceil x\rceil$ and $[x]$ denote upper integer and integer part of x, respectively;
(d) $j = k - k_i - [\frac{k-k_i}{\ell}]\ell$, $k_i \in [0,\infty)$.

2 Preliminaries

In this section, we present some preliminaries which will be useful for future discussion.

Definition 2.1 [12] The inverse of the generalized α-difference operator denoted by $\Delta^{-1}_{\alpha(\ell)}$ on $u(k)$ is defined as follows. If $\Delta_{\alpha(\ell)}v(k) = u(k)$, then

$$\Delta^{-1}_{\alpha(\ell)}u(k) = v(k) - \alpha^{\lceil\frac{k}{\ell}\rceil}v(j), \tag{4}$$

where $k \in \mathbb{N}_\ell(j)$, $j = k - [\frac{k}{\ell}]\ell$.

Lemma 2.2 [13] *If the real-valued function $u(k)$ is defined for all $k \in [a,\infty)$ and $\alpha > 1$, then*

$$\Delta^{-1}_{\alpha(\ell)}u(k) = \sum_{r=0}^{[\frac{k-a-j-\ell}{\ell}]} \frac{u(a+j+r\ell)}{\alpha^{\lceil\frac{a+j+\ell-k+r\ell}{\ell}\rceil}} + \alpha^{\lceil\frac{k-a}{\ell}\rceil}u(a+j) \tag{5}$$

for all $k \in \mathbb{N}_\ell(j)$, $j = k - a - [\frac{k-a}{\ell}]\ell$.

Definition 2.3 [7] The solution $u(k)$ of (1) is called oscillatory if for any $k_1 \in [a,\infty)$ there exists $k_2 \in \mathbb{N}_\ell(k_1)$ such that $u(k_2)u(k_2+\ell) \le 0$. The difference equation itself is called oscillatory if all its solutions are oscillatory. If the solution $u(k)$ is not oscillatory, then it is said to be nonoscillatory (*i.e.*, $u(k)u(k+\ell) > 0$ for all $k \in [k_1,\infty)$).

3 Main results

In this section we present conditions for the oscillation of equation (1).

Theorem 3.1 *Assume that*

(i) $q(k) \ge 0$ *and* $\sum_{r=0}^{\infty}\alpha^r q(r\ell) = \infty$,
(ii) $\liminf_{|u(k)|\to\infty} |f(u(k))| > 0$.

Then every solution of Equation (1) *is oscillatory.*

Proof Assume that Equation (1) has a nonoscillatory solution $u(k)$, and we assume that $u(k)$ is eventually positive. Then there is a positive integer k_1 such that

$$u\big(k-\tau(k)\big) > 0 \quad \text{for } k \ge k_1. \tag{6}$$

From Equation (1) we have

$$\Delta_{\alpha(\ell)}\big(p(k)\Delta_{\alpha(\ell)}u(k)\big) = -q(k)f\big(u\big(k-\tau(k)\big)\big), \quad k \geq k_1,$$

and so $p(k)\Delta_{\alpha(\ell)}u(k)$ is eventually nonincreasing. We first show that

$$p(k)\Delta_{\alpha(\ell)}u(k) \geq 0 \quad \text{for } k \geq k_1.$$

In fact, if there is $k_2 \geq k_1$ such that $p(k_2)\Delta_{\alpha(\ell)}u(k_2) = c < 0$ and $p(k)\Delta_{\alpha(\ell)}u(k) \leq c$ for $k \geq k_2$, that is,

$$\Delta_{\alpha(\ell)}u(k) \leq \frac{c}{p(k)},$$

hence by Lemma 2.2,

$$u(k) = \alpha^{\lceil \frac{k-k_2}{\ell} \rceil} u(k_2+j) + c \sum_{r=0}^{\frac{k-k_2-\ell-j}{\ell}} \frac{1}{\alpha^{\lceil \frac{k_2+j+\ell-k+r\ell}{\ell} \rceil} p(k_2+j+\ell+r\ell)} \to -\infty \quad \text{as } k \to \infty,$$

which contradicts the fact that $u(k) > 0$ for $k \geq k_2$. Hence, $p(k)\Delta_{\alpha(\ell)}u(k) \geq 0$ for $k \geq k_1$. Therefore we obtain

$$u\big(k-\tau(k)\big) > 0, \qquad \Delta_{\alpha(\ell)}u(k) \geq 0, \qquad \Delta_{\alpha(\ell)}\big(p(k)\Delta_{\alpha(\ell)}u(k)\big) \leq 0 \quad \text{for } k \geq k_2.$$

Let $L = \lim_{k\to\infty} u(k)$.

Then $L > 0$ is finite or infinite.

Case 1. $L > 0$ is finite.

From the function $f(k)$ defined in its domain of definition, we have

$$\lim_{k\to\infty} f\big(u\big(k-\tau(k)\big)\big) = f(L) > 0.$$

Thus, we may choose a positive integer k_4 ($\geq k_1$) such that

$$f\big(u\big(k-\tau(k)\big)\big) > \frac{1}{2}f(L), \quad k \geq k_4. \tag{7}$$

By substituting (7) in Equation (1) we obtain

$$\Delta_{\alpha(\ell)}\big(p(k)\Delta_{\alpha(\ell)}u(k)\big) + \frac{1}{2}f(L)q(k) \leq 0, \quad k \geq k_4. \tag{8}$$

By Lemma 2.2, we obtain

$$p(k+\ell)\Delta_{\alpha(\ell)}u(k+\ell) - \alpha^{\lceil \frac{k-k_4}{\ell} \rceil} p(k_4+j)\Delta_{\alpha(\ell)}u(k_4+j) + \frac{1}{2}f(L) \sum_{r=0}^{\frac{k-k_4-\ell-j}{\ell}} \frac{p(k_4+j+r\ell)}{\alpha^{\lceil \frac{k_4+j+\ell-k+r\ell}{\ell} \rceil}} \leq 0,$$

and so

$$\frac{1}{2}f(L)\sum_{r=0}^{\frac{k-k_4-\ell-j}{\ell}}\frac{p(k_4+j+r\ell)}{\alpha^{\lceil\frac{k_4+j+\ell-k+r\ell}{\ell}\rceil}}\leq\alpha^{\lceil\frac{k-k_4}{\ell}\rceil}p(k_4+j)\Delta_{\alpha(\ell)}u(k_4+j),\quad k\geq k_4,$$

which contradicts (i).

Case 2. $L=\infty$. For this case, from condition (ii) we have

$$\liminf_{k\to\infty}f\bigl(u\bigl(k-\tau(k)\bigr)\bigr)>0,$$

and so we may choose a positive constant c and a positive integer k_5 sufficiently large such that

$$f\bigl(u\bigl(k-\tau(k)\bigr)\bigr)\geq c\quad\text{for } k\geq k_5. \tag{9}$$

Substituting (9) into Equation (1) we have

$$\Delta_{\alpha(\ell)}\bigl(p(k)\Delta_{\alpha(\ell)}u(k)\bigr)+cq(k)\leq 0,\quad k\leq k_5.$$

Using a similar argument as in Case 1, we obtain a contradiction to condition (i). This completes the proof. □

Example 3.2 For the generalized α-difference equation

$$\Delta_{\alpha(\ell)}\left(\frac{1}{k}\Delta_{\alpha(\ell)}u(k)\right)=\left(\frac{4k^2+6k\ell+\ell^2}{(k+\ell)_\ell^{(2)}}\right)(-\alpha)^{\lceil\frac{k+2\ell}{\ell}\rceil},$$

all the conditions of Theorem 3.1 hold and hence all the solutions are oscillatory. In fact $u(k)=(-\alpha)^{\lceil\frac{k}{\ell}\rceil}k$ is one such solution.

Theorem 3.3 *Assume that*

(iii) $q(k)\geq 0$ *and* $\sum_{r=0}^{\infty}\alpha^r R(r\ell)q(r\ell)=\infty$.

Then every bounded solution of (1) *is oscillatory.*

Proof Proceeding as in the proof of Theorem 3.1, with the assumption that $u(k)$ is a bounded nonoscillatory solution of (1), we get inequality (8), and so we obtain

$$R(k)\Delta_{\alpha(\ell)}\bigl(p(k)\Delta_{\alpha(\ell)}u(k)\bigr)+\frac{1}{2}f(L)R(k)q(k)\leq 0,\quad k\geq k_4. \tag{10}$$

It is easy to see that

$$\begin{aligned}&R(k)\Delta_{\alpha(\ell)}\bigl(p(k)\Delta_{\alpha(\ell)}u(k)\bigr)\\&\quad\geq\Delta_{\alpha(\ell)}\bigl(R(k)p(k)\Delta_{\alpha(\ell)}u(k)\bigr)-\alpha p(k)\Delta_{\alpha(\ell)}u(k)\Delta_{\alpha(\ell)}R(k).\end{aligned} \tag{11}$$

Using (10) in (11), (11) reduces to

$$\begin{aligned} &R(k)p(k)\Delta_{\alpha(\ell)}u(k) - \alpha^{\lceil\frac{k-k_2}{\ell}\rceil}R(k_2+j)p(k_2+j)\Delta_{\alpha(\ell)}u(k_2+j) \\ &\quad - \alpha \sum_{r=0}^{\frac{k-k_4-j-\ell}{\ell}} \frac{p(k_2+j+r\ell)}{\alpha^{\lceil\frac{k_2-k+j+\ell+r\ell}{\ell}\rceil}} \Delta_{\alpha(\ell)}u(k_2+j+r\ell)\Delta_{\alpha(\ell)}R(k_2+j+r\ell) \\ &\quad + \frac{1}{2}f(L) \sum_{r=0}^{\frac{k-k_4-j-\ell}{\ell}} \frac{R(k_2+j+r\ell)}{\alpha^{\lceil\frac{k_2-k+j+\ell+r\ell}{\ell}\rceil}} q(k_2+j+r\ell) \le 0, \end{aligned}$$

which implies

$$\begin{aligned} &\frac{1}{2}f(L) \sum_{r=0}^{\frac{k-k_4-j-\ell}{\ell}} \frac{R(k_2+j+r\ell)}{\alpha^{\lceil\frac{k_2-k+j+\ell+r\ell}{\ell}\rceil}} q(k_2+j+r\ell) \\ &\quad \le u(k+\ell) + \alpha^{\lceil\frac{k-k_4}{\ell}\rceil}R(k_4+j)p(k_4+j)\Delta_{\alpha(\ell)}u(k_4+j+r\ell) - \alpha^{\lceil\frac{k-k_4}{\ell}\rceil}u(k_4+j), \quad k \ge k_4. \end{aligned}$$

Hence, there exists a constant c such that

$$\sum_{r=0}^{\frac{k-k_4-j-\ell}{\ell}} \frac{R(k_4+j+r\ell)}{\alpha^{\lceil\frac{k_2-k+j+\ell+r\ell}{\ell}\rceil}} q(k_4+j+r\ell) \le c \quad \text{for all } k \ge k_4,$$

which is a contradiction to condition (iii) which completes the proof. □

Example 3.4 For the generalized α-difference equation

$$\Delta_{\alpha(\ell)}\bigl(k\Delta_{\alpha(\ell)}u(k)\bigr) = \left(\frac{(\alpha 2^{\ell}+1)(\alpha k + k + \ell)}{2^{k+2\ell}}\right)(-1)^{\lceil\frac{k+2\ell}{\ell}\rceil},$$

all the conditions of Theorem 3.3 hold and hence all the solutions are oscillatory. In fact $u(k) = \frac{(-1)^{\lceil\frac{k}{\ell}\rceil}}{2^k}$ is one such solution.

Theorem 3.5 *Assume that*

(iv) $(k-\tau(k))$ *is nondecreasing, where* $\tau(k) \in [0,\infty)$,

(v) *there exists* $p(k_n)$ *such that* $p(k_n) \le 1$ *for* $k_n \in [0,\infty)$,

(vi) $\sum_{r=0}^{\infty} \alpha^r q(r\ell) = \infty$,

(vii) f *is nondecreasing and there is a nonnegative constant* M *such that*

$$\limsup_{s\to 0} \frac{s}{f(s)} = M. \tag{12}$$

Then the difference $\Delta_{\alpha(\ell)}u(k)$ *of every solution* $u(k)$ *of Equation* (1) *oscillates.*

Proof If not, then Equation (1) has a solution $u(k)$ such that its difference $\Delta_{\alpha(\ell)}u(k)$ is nonoscillatory. Assume first that the sequence $\Delta_{\alpha(\ell)}u(k)$ is eventually negative. Then there is a positive integer k_1 such that

$$\Delta_{\alpha(\ell)}u(k) < 0, \quad k > k_1,$$

and so $u(k)$ is decreasing for $k \geq k_1$, which implies that $u(k)$ is also nonoscillatory. Set

$$w(k) = \frac{p(k)\Delta_{\alpha(\ell)}u(k)}{f(u(k-\tau(k)))} k \geq k_2 \geq k_1. \tag{13}$$

Then

$$\begin{aligned}\Delta_{\alpha(\ell)}w(k) &= \frac{p(k+\ell)\Delta_{\alpha(\ell)}u(k+\ell)}{f(u(k+\ell-\tau(k+\ell)))} - \alpha\frac{p(k)\Delta_{\alpha(\ell)}u(k)}{f(u(k-\tau(k)))} = \frac{\Delta_{\alpha(\ell)}(p(k)\Delta_{\alpha(\ell)}u(k))}{f(u(k-\tau(k)))} \\ &\quad + p(k+\ell)\Delta_{\alpha(\ell)}u(k+\ell)\frac{f(u(k-\tau(k))) - f(u(k+\ell-\tau(k+\ell)))}{f(u(k+\ell-\tau(k+\ell)))f(u(k-\tau(k)))} \\ &\leq \frac{\Delta_{\alpha(\ell)}(p(k)\Delta_{\alpha(\ell)}u(k))}{f(u(k-\tau(k)))} = -q(k), \quad k \geq k_2. \end{aligned} \tag{14}$$

By Lemma 2.2, we have

$$w(k+\ell) - \alpha^{\lceil\frac{k-k_2}{\ell}\rceil} w(k_2+j) \leq -\sum_{r=0}^{\frac{k-k_2-j}{\ell}} \frac{q(k_2+j+r\ell)}{\alpha^{\lceil\frac{k_2+j-k+r\ell}{\ell}\rceil}},$$

and by (vi) we get

$$\lim_{k\to\infty} w(k) = -\infty, \tag{15}$$

which implies that eventually

$$f\bigl(u\bigl(k-\tau(k)\bigr)\bigr) > 0 \text{ and therefore } k-\tau(k) > 0. \tag{16}$$

By (15), we can choose k_3 ($\geq k_2$) such that

$$w(k) \leq -(M+\ell), \quad k \geq k_3.$$

That is,

$$p(k)\Delta_{\alpha(\ell)}u(k) + (M+\ell)f\bigl(u\bigl(k-\tau(k)\bigr)\bigr) \leq 0, \quad k \geq k_3. \tag{17}$$

Set $\lim_{k\to\infty} u(k) = L$. Then $L \geq 0$. Now we prove that $L = 0$. If $L > 0$, then we have

$$\lim_{k\to\infty} f\bigl(u\bigl(k-\tau(k)\bigr)\bigr) = f(L) > 0$$

since $f(k)$ is defined in its domain of definition. Choosing k_4 sufficiently large such that

$$f\bigl(u\bigl(k-\tau(k)\bigr)\bigr) > \frac{1}{2}f(L), \quad k \geq k_4, \tag{18}$$

and substituting (19) into (17), we obtain

$$\Delta_{\alpha(\ell)}u(k) + \frac{1}{2p(k)}(M+\ell)f(L) \leq 0, \quad k \geq k_4. \tag{19}$$

From Lemma 2.2, we have

$$u(k+\ell) - \alpha^{\lceil \frac{k-k_4}{\ell} \rceil} u(k_4+j) + \frac{1}{2}(M+\ell)f(L) \sum_{r=0}^{\frac{k-k_4-\ell-j}{\ell}} \frac{1}{\alpha^{\lceil \frac{k_4-k+j+r\ell}{\ell} \rceil} p(k_4-k+j+r\ell)} \leq 0,$$

which implies that $\lim_{k\to\infty} u(k) = -\infty$.

This contradicts (16). Hence $\lim_{k\to\infty} u(k) = 0$.

By the assumptions we have

$$\limsup_{k\to\infty} \frac{u(k-\tau(k))}{f(u(k-\tau(k)))} \leq M.$$

From this we can choose k_4 such that

$$\frac{u(k-\tau(k))}{f(u(k-\tau(k)))} < M+\ell, \quad k \geq k_5.$$

That is, $u(k-\tau(k)) < (M+\ell)f(u(k-\tau(k)))$, $k \geq k_5$, and so from (17) we get

$$p(k)\Delta_{\alpha(\ell)} u(k) + u\bigl(k-\tau(k)\bigr) < 0, \quad k \geq k_5.$$

In particular, for a function $p(k_n)$ satisfying condition (v), we have

$$u(k_n+l) - \alpha u(k_n) + x_{k_n} - \tau(k_n) \leq p(k_n)\bigl(u(k_n+l) - \alpha u(k_n)\bigr) + u\bigl(k_n - \tau(k_n)\bigr) < 0$$

for k sufficiently large, which implies that

$$0 < u(k_n+l) + \bigl(u\bigl(k_n-\tau(k_n)\bigr) - \alpha u(k_n)\bigr) < 0$$

for all large k. This is a contradiction. The case that $\Delta_{\alpha(\ell)} u(k)$ is eventually positive can be treated in a similar fashion and this completes the proof of the theorem. □

Example 3.6 For the generalized α-difference equation

$$\Delta_{\alpha(\ell)}\left(\frac{1}{k}\Delta_{\alpha(\ell)} u(k)\right) = \left(\frac{(2^\ell+1)((2^\ell+1)k+\ell)}{(k+\ell)_\ell^{(2)}}\right)(-\alpha)^{\lceil \frac{k+2\ell}{\ell} \rceil} 2^k,$$

all the conditions of Theorem 3.5 hold and hence all the solutions are oscillatory. In fact $u(k) = (-\alpha)^{\lceil \frac{k}{\ell} \rceil} 2^k$ is one such solution.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the manuscript and read and approved the final draft.

Author details
[1]Department of Science and Humanities, R.M.D. Engineering College, Kavaraipettai, Tamil Nadu 601 206, S. India. [2]Department of Mathematics and Institute for Mathematical Research, University Putra Malaysia (UPM), Serdang, Selangor 43400, Malaysia. [3]Department of Science and Humanities, S.K.P. Institute of Technology, Tiruvannamalai, Tamil Nadu, S. India. [4]Department of Mathematics, Sacred Heart College, Vellore District, Tirupattur, Tamil Nadu 635 601, S. India.

Acknowledgements
The authors express their sincere thanks to the referees for the careful and detailed reading of the manuscript and very helpful suggestions. Research supported by the National Board for Higher Mathematics, Department of Atomic Energy, Government of India, Mumbai. The second authors also gratefully acknowledges that part of this research was partially supported by the University Putra Malaysia under the GP-IBT Grant Scheme having project no. GP-IBT/2013/9420100.

Received: 18 October 2013 Accepted: 26 March 2014 Published: 09 Apr 2014

References

1. Agarwal, RP: Difference Equations and Inequalities. Dekker, New York (2000)
2. Mickens, RE: Difference Equations. Reinhold, New York (1990)
3. Elaydi, SN: An Introduction to Difference Equations, 2nd edn. Springer, Berlin (1999)
4. Kelley, WG, Peterson, AC: Difference Equations. An Introduction with Applications. Academic Press, New York (1991)
5. Manuel, MMS, Kılıçman, A, Xavier, GBA, Pugalarasu, R, Dilip, DS: $\ell_{2(\ell)}$ and $c_{0(\ell)}$ solutions of a second order generalized difference equation. Adv. Differ. Equ. **2012**, 105 (2012)
6. Manuel, MMS, Xavier, GBA, Dilip, DS: α-difference operator and its application on number theory. J. Mod. Methods Numer. Math. **3**(2), 79-95 (2012)
7. Manuel, MMS, Kılıçman, A, Xavier, GBA, Pugalarasu, R, Dilip, DS: An application on the second-order generalized difference equations. Adv. Differ. Equ. **2013**, 35 (2013)
8. Popenda, J, Szmanda, B: On the oscillation of solutions of certain difference equations. Demonstr. Math. **XVII**(1), 153-164 (1984)
9. Popenda, J: Oscillation and nonoscillation theorems for second-order difference equations. J. Math. Anal. Appl. **123**(1), 34-38 (1987)
10. Manuel, MMS, Xavier, GBA, Chandrasekar, V, Pugalarasu, R: On generalized difference operator of third kind and its applications to number theory. Int. J. Pure Appl. Math. **53**(1), 69-82 (2009)
11. Szafranski, Z, Szmanda, B: Oscillation of solutions of some nonlinear difference equations. Publ. Mat. **40**, 127-133 (1996)
12. Manuel, MMS, Xavier, GBA, Dilip, DS, Babu, GD: Oscillation, nonoscillation and growth of solutions of generalized second order nonlinear α-difference equations. Glob. J. Math. Sci.: Theory Pract. **4**(1), 211-225 (2012)
13. Manuel, MMS, Xavier, GBA, Thandapani, E: Theory of generalized difference operator and its applications. Far East J. Math. Sci. **20**(2), 163-171 (2006)

10.1186/1687-1847-2014-109
Cite this article as: Manuel et al.: Oscillation of solutions of some generalized nonlinear α-difference equations. *Advances in Difference Equations* 2014, 2014:109