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The general solutions of an auxiliary ordinary differential equation using complex method and its applications

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Abstract

In this paper, we employ the complex method to obtain all meromorphic solutions of an auxiliary ordinary differential equation at first, and then find all meromorphic general solutions of in combination the Newell-Whitehead equation, the NLS equation, and the Fisher equation with degree three. Our result shows that all rational and simply periodic exact solutions of the combined the Newell-Whitehead equation, NLS equation, and Fisher equation with degree three are solitary wave solutions, and the method is simpler than other methods.

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Keywords: differential equation; general solution; meromorphic function; elliptic function

1 Introduction and main results

Nonlinear partial differential equations (NLPDEs) are widely used as models to describe many important dynamical systems in various fields of sciences, particularly in fluid mechanics, solid state physics, plasma physics, and nonlinear optics. Exact solutions of NLPDEs of mathematical physics have attracted significant interest in the literature. Over the last years, much work has been done on the construction of exact solitary wave solutions and periodic wave solutions of nonlinear physical equations. Many methods have been developed by mathematicians and physicists to find special solutions of NLPDEs, such as the inverse scattering method [1], the Darboux transformation method [2], the Hirota bilinear method [3], the Lie group method [4], the bifurcation method of dynamic systems [5–7], the sine-cosine method [8], the tanh-function method [9, 10], the Fan-expansion method [11], and the homogeneous balance method [12]. Practically, there is no unified technique that can be employed to handle all types of nonlinear differential equations. Recently, the complex method was introduced by Yuan *et al.* [13, 14]. It is shown that the complex method provides a powerful mathematical tool for solving a great many nonlinear partial differential equations in mathematical physics.

Recently, Yuan *et al.* [15] derived all traveling wave exact solutions by using the complex method for a type of ordinary differential equations (ODEs):

$$Aw'' + Bw + Cw^3 + D = 0, \tag{1}$$

where A, B, C, and D are arbitrary constants.

In order to state this result, we need some concepts and notations.

A meromorphic function $w(z)$ means that $w(z)$ is holomorphic in the complex plane $\mathbb{C}$ except for poles. α, b, c, c_i and c_{ij} are constants, which may be different from each other in different places. We say that a meromorphic function f belongs to the class W if f is an elliptic function, or a rational function of $e^{\alpha z}$, $\alpha \in \mathbb{C}$, or a rational function of z.

Theorem 1.1 [15] *Suppose that $AC \neq 0$, then all meromorphic solutions w of an Eq.* (1) *belong to the class W. Furthermore, Eq.* (1) *has the following three forms of solutions*:

(I) *The elliptic function solutions*

$$w_{2d}(z) = \pm\frac{1}{2}\sqrt{-\frac{2A}{C}}\frac{(-\wp + c)(4\wp c^2 + 4\wp^2 c + 2\wp' d - \wp g_2 - cg_2)}{((12c^2 - g_2)\wp + 4c^3 - 3cg_2)\wp' + (4\wp^3 + 12c\wp^2 - 3g_2\wp - cg_2)d}.$$

Here $B = 0$, $D = 0$, $g_3 = 0$, $d^2 = 4c^3 - g_2 c$, g_2, and c are arbitrary.

(II) *The simply periodic solutions*

$$w_{2s,1}(z) = \alpha\sqrt{-\frac{A}{2C}}\left(\coth\frac{\alpha}{2}(z - z_0) - \coth\frac{\alpha}{2}(z - z_0 - z_1) - \coth\frac{\alpha}{2}z_1\right)$$

and

$$w_{2s,2}(z) = \alpha\sqrt{-\frac{A}{2C}}\tanh\frac{\alpha}{2}(z - z_0),$$

where $z_0 \in \mathbb{C}$, $B = A\alpha^2(\frac{1}{2} + \frac{3}{2\sinh^2\frac{\alpha}{2}z_1})$, $D = \sqrt{-\frac{A}{2C}}\frac{\tanh\frac{\alpha}{2}z_1}{\sinh^2\frac{\alpha}{2}z_1}$, $z_1 \neq 0$ in the former formula, or $B = \frac{A\alpha^2}{2}$, $D = 0$.

(III) *The rational function solutions*

$$w_{2r,1}(z) = \pm\sqrt{-\frac{2A}{C}}\frac{1}{z - z_0}$$

and

$$w_{2r,2}(z) = \pm\sqrt{-\frac{2A}{Cz_1^2}}\left(\frac{z_1}{z - z_0} - \frac{z_1}{z - z_0 - z_1} - 1\right),$$

where $z_0 \in \mathbb{C}$, $B = 0$, $D = 0$ in the former case, or given by $z_1 \neq 0$, $B = \frac{6A}{z_1^2}$, $D = \mp 2C(\frac{-2A}{Cz_1^2})^{3/2}$.

Equation (1) is an important auxiliary equation, because many nonlinear evolution equations can be converted to Eq. (1) using the traveling wave reduction. For instance, the modified ZK equation, the modified KdV equation, the nonlinear Klein-Gordon equation, and the modified BBM equation can be converted to Eq. (1) [15].

In this paper, we employ the complex method to obtain first all meromorphic solutions of Eq. (2) below,

$$Aw'' + Bw' + Cw + Dw^3 = 0, \tag{2}$$

where A, B, C, D are arbitrary constants.

Our main result is the following theorem.

Theorem 1.2 *Suppose that $AD \neq 0$, then Eq.* (2) *is integrable if and only if $B = 0, \pm\frac{3}{\sqrt{2}}\sqrt{AC}$. Furthermore, the general solutions of Eq.* (2) *are of the following form.*

(I) [15] *When $B = 0$, we have the elliptic general solutions of Eq.* (2),

$$w_{d,1}(z) = \pm\sqrt{-\frac{2A}{D}}\frac{\wp'(z-z_0:g_2,0)}{\wp(z-z_0:g_2,0)},$$

where z_0 and g_2 are arbitrary. In particular, it degenerates to the simply periodic solutions and rational solutions,

$$w_{s,1}(z) = \alpha\sqrt{-\frac{A}{2D}}\tanh\frac{\alpha}{2}(z-z_0)$$

and

$$w_r(z) = \pm\sqrt{-\frac{2A}{D}}\frac{1}{z-z_0},$$

where $C = \frac{A\alpha^2}{2}$ and $z_0 \in \mathbb{C}$.

(II) *When $B = \pm\frac{3}{\sqrt{2}}\sqrt{AC}$, we have the general solutions of Eq.* (2),

$$w_{g,1}(z) = \pm\frac{1}{2}\exp\left\{\mp\frac{1}{\sqrt{2}}\sqrt{\frac{C}{A}}z\right\}\frac{\wp'(\sqrt{-\frac{D}{C}}\exp\{\mp\frac{1}{\sqrt{2}}\sqrt{\frac{C}{A}}z\}-s_0;g_2,0)}{\wp(\sqrt{-\frac{D}{C}}\exp\{\mp\frac{1}{\sqrt{2}}\sqrt{\frac{C}{A}}z\}-s_0;g_2,0)},$$

where $\wp(s:g_2,0)$ is the Weierstrass elliptic function, and both s_0 and g_2 are arbitrary constants. In particular, $w_{g,1}(z)$ degenerates to the one parameter family of solutions,

$$w_{f,1}(z) = \pm\sqrt{-\frac{C}{D}}\frac{1}{1-\exp\{\mp\frac{1}{\sqrt{2}}\sqrt{\frac{C}{A}}(z-z_0)\}},$$

where $z_0 \in \mathbb{C}$.

This paper is organized as follows: In the next section, the preliminary lemmas and the complex method are given. The proof of Theorem 1.2 will be given in Section 3. All exact solutions of the auxiliary Eq. (2) are derived by complex method. In Section 4, we obtain all exact solutions of the Newell-Whitehead equation, the nonlinear Scrödinger equation (NLS), and the Fisher equation, which can be converted to Eq. (2) making use of the traveling wave reduction. Some conclusions and discussions are given in the final section.

2 Preliminary lemmas and the complex method

In order to give our complex method and the proof of Theorem 1.1, we need some notations and results.

Set $m \in \mathbb{N} := \{1,2,3,\ldots\}$, $r_j \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $r = (r_0, r_1, \ldots, r_m)$, $j = 0,1,\ldots,m$. We define a differential monomial denoted by

$$M_r[w](z) := \left[w(z)\right]^{r_0}\left[w'(z)\right]^{r_1}\left[w''(z)\right]^{r_2}\cdots\left[w^{(m)}(z)\right]^{r_m}.$$

$p(r) := r_0 + r_1 + \cdots + r_m$ is called the degree of $M_r[w]$. A differential polynomial is defined by

$$P(w, w', \dots, w^{(m)}) := \sum_{r \in I} a_r M_r[w],$$

where a_r are constants, and I is a finite index set. The total degree is defined by $\deg P(w, w', \dots, w^{(m)}) := \max_{r \in I}\{p(r)\}$.

We will consider the following complex ordinary differential equations:

$$P(w, w', \dots, w^{(m)}) = bw^n + c, \tag{3}$$

where $b \neq 0$, c are constants, $n \in \mathbb{N}$.

Let $p, q \in \mathbb{N}$. Suppose that Eq. (3) has a meromorphic solution w with at least one pole, we say that Eq. (3) satisfies the weak $\langle p, q \rangle$ condition if substituting the Laurent series

$$w(z) = \sum_{k=-q}^{\infty} c_k z^k, \quad q > 0, c_{-q} \neq 0 \tag{4}$$

into Eq. (3), we can determine p distinct Laurent singular parts as below,

$$\sum_{k=-q}^{-1} c_k z^k.$$

In order to give the representations of elliptic solutions, we need some notations and results concerning elliptic functions [16].

Let ω_1, ω_2 be two given complex numbers such that $\operatorname{Im} \frac{\omega_1}{\omega_2} > 0$, $L = L[2\omega_1, 2\omega_2]$ be discrete subset $L[2\omega_1, 2\omega_2] = \{\omega \mid \omega = 2n\omega_1 + 2m\omega_2, n, m \in \mathbb{Z}\}$, which is isomorphic to $\mathbb{Z} \times \mathbb{Z}$. The discriminant is $\Delta = \Delta(c_1, c_2) := c_1^3 - 27c_2^2$ and we have

$$s_n = s_n(L) := \sum_{\omega \in L \setminus \{0\}} \frac{1}{\omega^n}.$$

The Weierstrass elliptic function $\wp(z) := \wp(z, g_2, g_3)$ is a meromorphic function with double periods $2\omega_1$, $2\omega_2$, satisfying the equation

$$\left(\wp'(z)\right)^2 = 4\wp(z)^3 - g_2\wp(z) - g_3, \tag{5}$$

where $g_2 = 60s_4$, $g_3 = 140s_6$, and $\Delta(g_2, g_3) \neq 0$.

Lemma 2.1 [16, 17] *The Weierstrass elliptic functions $\wp(z) := \wp(z, g_2, g_3)$ have two successive degeneracies and we have the addition formula*:

(I) *Degeneracy to simply periodic functions* (*i.e., rational functions of one exponential e^{kz}*) *according to*

$$\wp(z, 3d^2, -d^3) = 2d - \frac{3d}{2} \coth^2 \sqrt{\frac{3d}{2}} z \tag{6}$$

if one root e_j is double ($\Delta(g_2, g_3) = 0$).

(II) *Degeneracy to rational functions of z according to*

$$\wp(z,0,0)=\frac{1}{z^2}$$

if one root e_j *is triple* ($g_2=g_3=0$).

(III) *We have the addition formula*

$$\wp(z-z_0)=-\wp(z)-\wp(z_0)+\frac{1}{4}\left[\frac{\wp'(z)+\wp'(z_0)}{\wp(z)-\wp(z_0)}\right]^2. \tag{7}$$

In the proof of our main result, the following lemmas are very useful, which can be deduced by Theorem 1 in [15].

Lemma 2.2 [15] *The differential equation*

$$Aw''+Bw+Cw^3=0$$

has elliptic solutions, a simply periodic solution, and a rational solution with pole at $z=0$,

$$w(z)=\pm\frac{1}{2}\sqrt{-\frac{2A}{C}}\frac{\wp'(z:g_2,0)}{\wp(z:g_2,0)},$$

$$w(z)=\alpha\sqrt{-\frac{A}{2C}}\tanh\frac{\alpha}{2}z$$

and

$$w(z)=\pm\sqrt{-\frac{2A}{C}}\frac{1}{z},$$

respectively, where $\wp(z:g_2,0)$ *is the Weierstrass elliptic function with* $g_3=0$ *and* g_2 *arbitrary and* $B=\frac{A\alpha^2}{2}$.

By the above lemmas and results, we can give a new method below, let us call it the complex method, to find exact solutions of some PDEs.

Step 1. Substituting the transform $T:u(x,t)\to w(z)$, $(x,t)\to z$ into a given PDE gives a nonlinear ordinary differential equation (3).

Step 2. Substitute (4) into Eq. (3) to determine that the weak $\langle p,q\rangle$ condition holds, and pass the Painlevé test for Eq. (3).

Step 3. Find the meromorphic solutions $w(z)$ of Eq. (3) with a pole at $z=0$, which have $m-1$ integral constants.

Step 4. By the addition formula of Lemma 2.1 we obtain the general meromorphic solutions $w(z-z_0)$.

Step 5. Substituting the inverse transform T^{-1} into these meromorphic solutions $w(z-z_0)$, we get all exact solutions $u(x,t)$ of the original given PDE.

3 Proof of Theorem 1.2

Proof Substituting (4) into Eq. (2) we have $q=1$, $p=2$, $c_{-1}=\pm\sqrt{\frac{-2A}{E}}$, $c_0=\mp\frac{1}{6}\frac{\sqrt{-2}B}{AE}$, $c_1=\pm\frac{\sqrt{-2}}{36}\frac{6AC-B^2}{A\sqrt{AE}}$, $c_2=\pm\frac{\sqrt{-2}}{108}\frac{9AC-2B^2}{A^2\sqrt{AE}}$, and

$$0\times c_3+B^2\left(B-\frac{3}{\sqrt{2}}\sqrt{AC}\right)\left(B+\frac{3}{\sqrt{2}}\sqrt{AC}\right)=0.$$

For the Laurent expansion (4) to be valid B satisfies this equation and c_3 is an arbitrary constant. Therefore, $B = 0, \pm\frac{3}{\sqrt{2}}\sqrt{AC}$. For other B it would be necessary to add logarithmic terms to the expansion, thus giving a branch point rather than a pole.

For $B = 0$, Eq. (2) is completely integrable by standard techniques and the solutions are expressible in terms of elliptic functions (*cf.* [15]), *i.e.*, by Lemmas 2.1 and 2.2, the elliptic general solutions of Eq. (2)

$$w_{d,1}(z) = \pm\sqrt{-\frac{2A}{D}}\frac{\wp'(z-z_0:g_2,0)}{\wp(z-z_0:g_2,0)},$$

where z_0 and g_2 are arbitrary. In particular, it degenerates to the simply periodic solutions and rational solutions,

$$w_{s,1}(z) = \alpha\sqrt{-\frac{A}{2D}}\tanh\frac{\alpha}{2}(z-z_0)$$

and

$$w_r(z) = \pm\sqrt{-\frac{2A}{D}}\frac{1}{z-z_0},$$

where $C = \frac{A\alpha^2}{2}$ and $z_0 \in \mathbb{C}$.

For $B = \pm\frac{3}{\sqrt{2}}\sqrt{AC}$, we transform Eq. (2) into the second Painlevé type equation. In this way we find the general solutions.

Setting $w(z) = f(z)u(s)$, $s = g(z)$, and substituting in Eq. (2), we find that the equation for $u(s)$ is

$$-\frac{A}{D}(g')^2u'' = \frac{u'g'}{D}\left\{2A\frac{f'}{f} + A\frac{g''}{g'} + B\right\} + \frac{u}{D}\left\{A\frac{f''}{f} + B\frac{f'}{f} + C\right\} + f^2u^3. \tag{8}$$

If we take f and g such that

$$A\frac{f''}{f} + B\frac{f'}{f} + C = 0, \qquad 2A\frac{f'}{f} + A\frac{g''}{g'} + B = 0, \tag{9}$$

then Eq. (8) for u is integrable. By (9), one takes $f(z) = \exp\{\alpha z\}$ and

$$g(z) = \beta\exp\left\{-\left(\frac{B}{A} + 2\alpha\right)z\right\},$$

where $\alpha = \mp\frac{1}{\sqrt{2}}\sqrt{\frac{C}{A}}$, $\beta^2 = -\frac{D}{C}$.

Thus Eq. (8) reduces to

$$u'' = 2u^3. \tag{10}$$

Both Lemmas 2.1 and 2.2 show that the general solutions of Eq. (10) are of the form

$$u(s) = \pm\frac{1}{2}\frac{\wp'(s-s_0;g_2,0)}{\wp(s-s_0;g_2,0)},$$

where $\wp(s)$ is the Weierstrass elliptic function, s_0 and g_2 are two arbitrary constants.

Therefore, when $B = \pm\frac{3}{\sqrt{2}}\sqrt{AC}$, by Lemma 2.1, we have the general solutions of Eq. (2),

$$w_{g,1}(z) = \pm\frac{1}{2}\exp\left\{\mp\frac{1}{\sqrt{2}}\sqrt{\frac{C}{A}}z\right\}\frac{\wp'(\sqrt{-\frac{D}{C}}\exp\{\mp\frac{1}{\sqrt{2}}\sqrt{\frac{C}{A}}z\} - s_0; g_2, 0)}{\wp(\sqrt{-\frac{D}{C}}\exp\{\mp\frac{1}{\sqrt{2}}\sqrt{\frac{C}{A}}z\} - s_0; g_2, 0)},$$

where both s_0 and g_2 are arbitrary constants. In particular, by Lemma 2.1 and $g_2 = 0$, $w_{g,1}(z)$ degenerates to the one parameter family of solutions,

$$w_{f,1}(z) = \pm\sqrt{-\frac{C}{D}}\frac{1}{1 - \exp\{\mp\frac{1}{\sqrt{2}}\sqrt{\frac{C}{A}}(z - z_0)\}},$$

where $z_0 \in \mathbb{C}$.

This completes the proof of Theorem 1.2. □

4 Some applications of Theorem 1.2

Equation (2) include many well-known nonlinear equations that are with applied background as special examples, such as Newell-Whitehead equation, NLS equation, Fisher equation with degree three. In this section, the Newell-Whitehead equation, NLS equation, and Fisher equation with degree three are considered again and the exact solutions are derived with the aid of Eq. (2).

4.1 Newell-Whitehead equation

The Newell-Whitehead equation (Vitanov [18], Liu [19], Newell and Whitehead [20] and Wazwaz [21]) has the form

$$u_{xx} - u_t - ru^3 + su = 0, \tag{A}$$

where r, s are constants.

Substituting

$$u(x,t) = w(z), \qquad z = x + \omega t \tag{11}$$

into Eq. (A) gives

$$w'' - \omega w' + sw - rw^3 = 0. \tag{12}$$

Equation (12) is converted to Eq. (2), where

$$A = 1, \qquad B = -\omega, \qquad C = 1, \qquad D = -1.$$

4.2 NLS equation

The NLS equation [22, 23] has the form

$$iu_t + \alpha u_{xx} + \beta|u|^2u = 0, \tag{B}$$

where α, β are nonzero constants.

Substituting

$$u(x,t) = w(z)e^{kx-\omega t}, \qquad z = x + ct \tag{13}$$

into Eq. (B) gives

$$\alpha w'' + i(2\alpha k - c)w' + (\omega - \alpha k^2)w + \beta w^3 = 0. \tag{14}$$

Equation (14) is converted to Eq. (2), where

$$A = \alpha, \qquad B = i(2\alpha k - c), \qquad C = \omega - \alpha k^2, \qquad D = \beta.$$

4.3 Fisher equation with degree three

The Fisher equation with degree three [24] has the form

$$u_t = u_{xx} + u(1 - u^2). \tag{C}$$

Substituting

$$u(x,t) = w(z), \qquad z = x - ct, \tag{15}$$

into Eq. (C) gives

$$w'' + cw' + w(1 - w^2) = 0. \tag{16}$$

Equation (16) is converted to Eq. (2), where

$$A = 1, \qquad B = c, \qquad C = 1, \qquad D = -1.$$

Apparently, if we set appropriate coefficients in Eq. (2), certain well-known equations will be converted to it.

5 Conclusions

The complex method is a very important tool in finding the exact solutions of nonlinear evolution equation, and Eq. (3) is one of the most important auxiliary equations, because many nonlinear evolution equations can be converted to it. In this article, we employ the complex method to obtain all meromorphic solutions of an auxiliary ordinary differential equation at first, and then find all meromorphic exact solutions of the combined Newell-Whitehead equation, nonlinear Scrödinger equation, and Fisher equation with degree three. Our result shows that all rational and simply periodic exact solutions of the combined the Newell-Whitehead equation, nonlinear Scrödinger equation, and Fisher equation with degree three are solitary wave solutions, and the method is simpler than other methods.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions

The main idea of this paper was proposed by WY and ZH. MF and JL prepared the manuscript initially and performed all the steps of the proofs in this research. All authors read and approved the final manuscript.

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