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Global existence and blow-up analysis to a cooperating model with self-diffusion

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Abstract

In this paper, a two-species cooperating model with free diffusion and self-diffusion is investigated. The existence of the global solution is first proved by using lower and upper solution method. Then the sufficient conditions are given for the solution to blow up in a finite time. Our results show that the solution is global if the intra-specific competition is strong, while if the intra-specific competition is weak and the self-diffusion rate is small, blow-up occurs provided that the initial value is large enough or the free diffusion rate is small. Numerical simulations are also given to illustrate the blow-up results.

MSC: 35K57; 92D25

Keywords: global solution; blow-up; self-diffusion; cooperating model

1 Introduction

In this paper, we are concerned with the following nonlinear reaction-diffusion system:

$$\begin{cases} u_{1t} - \Delta[(d_1 + \alpha_1 u_1)u_1] = u_1(a_1 - b_1 u_1 + c_1 u_2) & \text{in } \Omega \times (0,T), \\ u_{2t} - \Delta[(d_2 + \alpha_2 u_2)u_2] = u_2(a_2 + b_2 u_1 - c_2 u_2) & \text{in } \Omega \times (0,T), \\ u_1(x,t) = u_2(x,t) = 0 & \text{on } \partial\Omega \times (0,T), \\ u_1(x,0) = \eta_1(x), \qquad u_2(x,0) = \eta_2(x) & \text{in } \Omega, \end{cases} \tag{1.1}$$

where $\Delta = \sum_{i=1}^{N} \partial^2/\partial x_i^2$ is the Laplace operator, Ω is a bounded domain in $\mathbf{R}^N$ with smooth boundary $\partial\Omega$. a_i, b_i, c_i, d_i, and α_i $(i = 1, 2)$ are positive constants. System (1.1) is usually referred as the cooperating two-species Lotka-Volterra model describing the interaction of diffusive biological species. The spatial density of the ith species at time t is represented by $u_i(x,t)$ and its respective free diffusion rate is denoted by d_i. α_i is the self-diffusion rate. The real number a_i is the net birth rate of the ith species and b_1, c_2 are the crowding-effect coefficients. The parameters c_1 and b_2 measure cooperations between the species. The known $\eta_i(x)$ is a smooth function satisfying the compatibility condition $\eta_i(x) = 0$ for $x \in \partial\Omega$. The boundary conditions in (1.1) imply that the habitat is surrounded by a totally hostile environment.

Recently, the global existence or blow-up problem for parabolic equations describing the ecological models have been considered by many authors, *e.g.* [1–11]. In [10], Pao studied the following cooperating model with free diffusion:

$$\begin{cases} u_{1t} - d_1 \Delta u_1 = u_1(a_1 - b_1 u_1 + c_1 u_2) & \text{in } \Omega \times (0, T), \\ u_{2t} - d_2 \Delta u_2 = u_2(a_2 + b_2 u_1 - c_2 u_2) & \text{in } \Omega \times (0, T), \\ u_1(x,t) = u_2(x,t) = 0 & \text{on } \partial\Omega \times (0, T), \\ u_1(x,0) = \eta_1(x), \qquad u_2(x,0) = \eta_2(x) & \text{in } \Omega, \end{cases} \tag{1.2}$$

here all parameters are positive constants except a_1 and a_2 which can be chosen positive, zero or negative. He proved that a unique solution of (1.2) exists and is uniformly bounded in $\bar{\Omega} \times [0, +\infty)$ if $b_1 c_2 > b_2 c_1$, while if $b_2 c_1 > b_1 c_2$ the solution blows up in a finite time for big a_i with any nontrivial nonnegative initial data or for any a_i with big initial data. His results imply that the solution is global if the intra-specific competition is strong, while the solution may blow up if the intra-specific competition is weak. Lou *et al.* [7] considered the problem (1.2) with homogeneous Neumann boundary conditions and studied the effect of diffusion on the blow-up. They gave a sufficient condition on the initial data for the solution to blow up in a finite time. Wang *et al.* [11] studied a reaction-diffusion system with nonlinear absorption terms and boundary flux. Some sufficient conditions for global existence and finite time blow-up of the solutions are given.

On the other hand, Shigesada *et al.* [12] in 1979 proposed a generalization of Lotka-Volterra competing model in order to describe spatial segregation of interacting population species in one space dimension. More and more attention has been given to the SKT model with other types of reaction terms. For example, the two-species prey-predator model is in [13, 14], two-species cooperating model is in [15], three-species cooperating model is in [16, 17]. These works concentrate on the existence of time-dependent solution or uniform boundedness and stability of global solutions. In this paper we are interested in studying the blow-up properties of the solution and we will consider the effect of self-diffusion coefficients α_1 and α_2 on the long time behaviors of the solution.

The content of this paper is organized as the follows: In Section 2, the existence and uniqueness of global solution are given by using upper and lower solutions and their associated monotone iterations as in [18]. Section 3 is devoted to a sufficient condition for the solution to blow up. Numerical illustrations are performed in Section 4 and a brief discussion is also given in Section 5.

2 Existence of global solution

This section is devoted to the global existence of (1.1). First we give the definition of ordered upper and lower solutions of (1.1), then a pair of ordered upper and lower solutions is constructed.

Definition 2.1 A pair of functions $\tilde{\mathbf{u}} = (\tilde{u}_1, \tilde{u}_2)$, $\hat{\mathbf{u}} = (\hat{u}_1, \hat{u}_2)$ in $C([0,T] \times \bar{\Omega}) \cap C^2((0,T] \times \Omega)$ are called ordered upper and lower solutions of the problem (1.1), if $\tilde{\mathbf{u}} \geq \hat{\mathbf{u}}$ and if $\tilde{\mathbf{u}}$ satisfies the relations

$$\begin{cases} \tilde{u}_{1t} - \Delta[(d_1 + \alpha_1 \tilde{u}_1)\tilde{u}_1] \geq \tilde{u}_1(a_1 - b_1 \tilde{u}_1 + c_1 \tilde{u}_2) & \text{in } \Omega \times (0, T), \\ \tilde{u}_{2t} - \Delta[(d_2 + \alpha_2 \tilde{u}_2)\tilde{u}_2] \geq \tilde{u}_2(a_2 + b_2 \tilde{u}_1 - c_2 \tilde{u}_2) & \text{in } \Omega \times (0, T), \\ \tilde{u}_i(x,t) \geq 0 & \text{on } \partial\Omega \times (0, T), \\ \tilde{u}_i(x,0) \geq \tilde{u}_{i,0}(x) & \text{in } \Omega \end{cases} \tag{2.1}$$

and $\hat{\mathbf{u}}$ satisfies the above inequalities in reversed order.

Define

$$w_i = d_i u_i + \alpha_i u_i^2 = I(u_i). \tag{2.2}$$

Since we only consider the positive solution, then we have the inverse

$$u_i = \frac{-d_i + \sqrt{d_i^2 + 4\alpha_i w_i}}{2\alpha_i} = q_i(w_i),$$

which is an increasing function of $w > 0$. In view of $w_{it} = (d_i + 2\alpha_i u_i)u_{it}$, we may write the problem (1.1) in the equivalent form

$$\begin{cases} (d_1 + 2\alpha_1 u_1)^{-1} w_{1t} - \Delta w_1 = u_1(a_1 - b_1 u_1 + c_1 u_2) & \text{in } \Omega \times (0,T), \\ (d_2 + 2\alpha_2 u_2)^{-1} w_{2t} - \Delta w_2 = u_2(a_2 + b_2 u_1 - c_2 u_2) & \text{in } \Omega \times (0,T), \\ w_1(x,t) = w_2(x,t) = 0 & \text{on } \partial\Omega \times (0,T), \\ w_i(x,0) = w_{i,0}(x) & \text{in } \Omega, \\ u_i = q_i(w_i) & \text{in } \bar{\Omega} \times (0,T). \end{cases} \tag{2.3}$$

Let $\tilde{w}_i = I_i(\tilde{u}_i)$, $\hat{w}_i = I_i(\hat{u}_i)$, it is easy to see $(\tilde{u}_1, \tilde{u}_2, \tilde{w}_1, \tilde{w}_2)$ and $(\hat{u}_1, \hat{u}_2, \hat{w}_1, \hat{w}_2)$ are ordered upper and lower solutions of (2.3).

Define $Lw_i = \Delta w_i - \mu w_i$ and

$$F_1(u_1, u_2) = \mu(d_1 + \alpha_1 u_1)u_1 + (a_1 - b_1 u_1 + c_1 u_2)u_1,$$

$$F_2(u_1, u_2) = \mu(d_2 + \alpha_2 u_2)u_2 + (a_2 + b_2 u_1 - c_2 u_2)u_2,$$

where μ is a positive constant such that

$$\mu(d_1 + \alpha_1 u_1) + a_1 - 2b_1 u_1 + c_1 u_2 \geq 0, \qquad \mu(d_2 + \alpha_2 u_2) + a_2 + b_2 u_1 - 2c_2 u_2 \geq 0$$

for any $u_1 \geq 0$ and $u_2 \geq 0$. The problem (2.3) becomes

$$\begin{cases} (d_1 + 2\alpha_1 u_1)^{-1} w_{1t} - Lw_1 = F_1(u_1, u_2) & \text{in } \Omega \times (0,T), \\ (d_2 + 2\alpha_2 u_2)^{-1} w_{2t} - Lw_2 = F_2(u_1, u_2) & \text{in } \Omega \times (0,T), \\ w_1(x,t) = w_2(x,t) = 0 & \text{on } \partial\Omega \times (0,T), \\ w_i(x,0) = w_{i,0}(x) & \text{in } \Omega, \\ u_i = q_i(w_i) & \text{in } \bar{\Omega} \times (0,T). \end{cases} \tag{2.4}$$

We denote by $C(\bar{\Omega} \times [0,T])$ the space of all bounded and continuous functions in $\bar{\Omega} \times [0,T]$, the vector-value functions are denoted by $\mathcal{C}(\bar{\Omega} \times [0,T])$. Set

$$S_i \equiv \{u_i \in C(\bar{\Omega} \times [0,T]) : \hat{u}_i \leq u_i \leq \tilde{u}_i\}, \qquad S = \{\mathbf{u} \in \mathcal{C}(\bar{\Omega} \times [0,T]) : \hat{\mathbf{u}} \leq \mathbf{u} \leq \tilde{\mathbf{u}}\},$$

$$S \times S \equiv \{(\mathbf{u}, \mathbf{w}) \in [\mathcal{C}(\bar{\Omega} \times [0,T])]^2 : (\hat{\mathbf{u}}, \hat{\mathbf{w}}) \leq (\mathbf{u}, \mathbf{w}) \leq (\tilde{\mathbf{u}}, \tilde{\mathbf{w}})\},$$

where $\mathbf{u} = (u_1, u_2)$ and $\mathbf{w} = (w_1, w_2)$. It is easy to see, for any $(u_1, u_2), (v_1, v_2) \in S$, if $(u_1, u_2) \leq (v_1, v_2)$ then

$$F_i(u_1, u_2) \leq F_i(v_1, v_2). \tag{2.5}$$

Before constructing monotone sequences, we present the following positivity lemma, which will be used in the proof of the monotone property of the sequences.

Lemma 2.1 (Lemma 2.1 of [18]) *Let $\sigma(x,t) > 0$ in $\Omega \times (0,T]$, $\beta \geq 0$ on $\partial\Omega \times (0,T]$, and let either* (i) *$e(x,t) > 0$ in $\Omega \times (0,T]$ or* (ii) *$(-e/\sigma(x,t))$ be bounded in $\bar{\Omega} \times [0,T]$. If $z \in C^{2,1}(\Omega \times (0,T]) \cap C(\bar{\Omega} \times [0,T])$ satisfies the following inequalities*:

$$\begin{cases} \sigma(x,t)z_t - \Delta z + e(x,t)z \geq 0 & \text{in } \Omega \times (0,T], \\ \frac{\partial z}{\partial \nu} + \beta(x,t)z \geq 0 & \text{on } \partial\Omega \times (0,T], \\ z(x,0) \geq 0 & \text{in } \Omega, \end{cases}$$

then $z \geq 0$ in $\Omega \times [0,T]$.

By using either $u_i^{(0)} = \hat{u}_i$ or $u_i^{(0)} = \tilde{u}_i$ as the initial iteration we can construct a sequence $\{\mathbf{u}^{(m)}, \mathbf{w}^{(m)}\}$ from the nonlinear iteration process

$$\begin{cases} (d_i + 2\alpha_i u_i^{(m)})^{-1} w_{it}^{(m)} - Lw_i^{(m)} = F_i(u_1^{(m-1)}, u_2^{(m-1)}) & \text{in } \Omega \times (0,T], \\ w_i^{(m)}(x,t) = 0 & \text{on } \partial\Omega \times (0,T], \\ w_i^{(m)}(x,0) = w_{i,0}^{(m-1)}(x) & \text{in } \Omega, \\ u_i^{(m)} = q_i(w_i^{(m)}) & \text{in } \bar{\Omega} \times (0,T] \end{cases} \tag{2.6}$$

for any m and $i = 1, 2$.

Since the equation in (2.6) is equivalent to

$$u_{it}^{(m)} - \Delta\left[\left(d_i + \alpha_i u_i^{(m)}\right)u_i^{(m)}\right] + \mu\left(d_i + \alpha_i u_i^{(m)}\right)u_i^{(m)} = F_i\left(u_1^{(m-1)}, u_2^{(m-1)}\right)$$

under the same boundary and initial conditions, the existence of the sequence $u_i^{(m)}$ is ensured by [19] (Chapter V, Section 7). Denote the sequence by $\{\bar{\mathbf{u}}^{(m)}, \bar{\mathbf{w}}^{(m)}\}$ if $\mathbf{u}^{(0)} = \tilde{\mathbf{u}}$, and by $\{\underline{\mathbf{u}}^{(m)}, \underline{\mathbf{w}}^{(m)}\}$ if $\mathbf{u}^{(0)} = \hat{\mathbf{u}}$, and refer to them as maximal and minimal sequences, respectively. The following lemma shows that the sequences are monotone.

Lemma 2.2 *The sequences $\{\bar{\mathbf{u}}^{(m)}, \bar{\mathbf{w}}^{(m)}\}$, $\{\underline{\mathbf{u}}^{(m)}, \underline{\mathbf{w}}^{(m)}\}$ defined by* (2.6) *possess the monotone property*

$$\begin{aligned} (\hat{\mathbf{u}}, \hat{\mathbf{w}}) &\leq \left(\underline{\mathbf{u}}^{(m)}, \underline{\mathbf{w}}^{(m)}\right) \leq \left(\underline{\mathbf{u}}^{(m+1)}, \underline{\mathbf{w}}^{(m+1)}\right) \leq \left(\bar{\mathbf{u}}^{(m+1)}, \bar{\mathbf{w}}^{(m+1)}\right) \\ &\leq \left(\bar{\mathbf{u}}^{(m)}, \bar{\mathbf{w}}^{(m)}\right) \leq (\tilde{\mathbf{u}}, \tilde{\mathbf{w}}), \end{aligned} \tag{2.7}$$

$m = 1, 2, \ldots$, where $\bar{\mathbf{w}}^{(m)} = I(\bar{\mathbf{u}}^{(m)})$ and $\underline{\mathbf{w}}^{(m)} = I(\underline{\mathbf{u}}^{(m)})$.

Proof Let $\underline{z}_i^{(1)} = \underline{w}_i^{(1)} - \underline{w}_i^{(0)} \equiv \underline{w}_i^{(1)} - \hat{w}_i$, combining (2.6) with the definition of the lower solution yields

$$\begin{aligned} \left(d_i + 2\alpha_i \underline{u}_i^{(1)}\right)^{-1} \underline{z}_{it}^{(1)} - L_i \underline{z}_i^{(1)} &= F_i\left(\underline{u}_1^{(0)}, \underline{u}_2^{(0)}\right) - \left[\left(d_i + 2\alpha_i \underline{u}_i^{(1)}\right)^{-1} \underline{w}_{it}^{(0)} - L\underline{w}_i^{(0)}\right] \\ &= F_i\left(\underline{u}_1^{(0)}, \underline{u}_2^{(0)}\right) - \left[\left(d_i + 2\alpha_i \underline{u}_i^{(0)}\right)^{-1} \underline{w}_{it}^{(0)} - L\underline{w}_i^{(0)}\right] \\ &\quad - \left(\left(d_i + 2\alpha_i \underline{u}_i^{(1)}\right)^{-1} - \left(d_i + 2\alpha_i \underline{u}_i^{(0)}\right)^{-1}\right)\underline{w}_{it}^{(0)} \\ &\geq -\left(\left(d_i + 2\alpha_i \underline{u}_i^{(1)}\right)^{-1} - \left(d_i + 2\alpha_i \underline{u}_i^{(0)}\right)^{-1}\right)\underline{w}_{it}^{(0)}. \end{aligned}$$

Moreover, by the mean value theorem, we have

$$\left(d_i + 2\alpha_i \underline{u}_i^{(1)}\right)^{-1} - \left(d_i + 2\alpha_i \underline{u}_i^{(0)}\right)^{-1} = -\frac{2\alpha_i}{(d_i + 2\alpha_i \xi_i^{(0)})^3}\left(\underline{w}_i^{(1)} - \underline{w}_i^{(0)}\right)$$

for some intermediate value $\xi_i^{(0)} \equiv \xi_i^{(0)}(x,t)$ between $\underline{u}_i^{(0)}$ and $\underline{u}_i^{(1)}$. Then we have

$$\left(d_i + 2\alpha_i \underline{u}_i^{(1)}\right)^{-1} \underline{z}_{it}^{(1)} - L\underline{z}_i^{(1)} + \gamma_i^{(0)} \underline{z}_i^{(1)} \geq 0 \quad \text{in } \Omega \times (0,T], \tag{2.8}$$

where

$$\gamma_i^{(0)} = -\frac{2\alpha_i}{(d_i + 2\alpha_i \xi_i^{(0)})^3} \underline{w}_{it}^{(0)}. \tag{2.9}$$

On the other hand,

$$\begin{aligned} &\underline{z}_i^{(1)}(x,t) = \underline{w}_i^{(1)}(x,t) - \underline{w}_i^{(0)}(x,t) = 0 \quad \text{on } \partial\Omega \times (0,T], \\ &\underline{z}_i^{(1)}(x,0) = \underline{w}_i^{(1)}(x,0) - \hat{w}_i(x,0) \geq 0 \quad \text{in } \Omega. \end{aligned}$$

It follows from Lemma 2.1 that $\underline{z}_i^{(1)} \geq 0$, which leads to $\underline{w}_i^{(1)} \geq \underline{w}_i^{(0)}$, and thus $\underline{u}_i^{(1)} \geq \underline{u}_i^{(0)}$. A similar argument gives $\bar{w}_i^{(1)} \leq \bar{w}_i^{(0)}$ and $\bar{u}_i^{(1)} \leq \bar{u}_i^{(0)}$. Moreover, based on (2.5) and (2.6) we know that $\phi_i^{(1)} := \bar{w}_i^{(1)} - \underline{w}_i^{(1)}$ satisfies

$$\begin{cases} (d_i + 2\alpha_i \bar{u}_i^{(1)})^{-1} \phi_{it}^{(1)} - L\phi_i^{(1)} + \gamma_i^{(1)} \phi_i^{(1)} = F_i(\bar{u}_1^{(0)}, \bar{u}_2^{(0)}) - F_i(\underline{u}_1^{(0)}, \underline{u}_2^{(0)}) \geq 0, \\ \phi_i^{(1)}(x,t) = 0, \\ \phi_i^{(1)}(x,0) = 0, \end{cases}$$

where $\gamma_i^{(1)} = -\frac{2\alpha_i}{(d_i + 2\alpha_i \xi_i^{(1)})^3} \underline{w}_{it}^{(1)}$, $\xi_i^{(1)} \equiv \xi_i^{(1)}(x,t)$ is between $\underline{u}_i^{(1)}$ and $\underline{u}_i^{(1)}$ for $i = 1, 2$.

Using Lemma 2.1 again, we have $\phi_i^{(1)} \geq 0$, which leads to $\bar{w}_i^{(1)} \geq \underline{w}_i^{(1)}$ and therefore $\bar{u}_i^{(1)} \geq \underline{u}_i^{(1)}$. Moreover,

$$\left(\underline{u}_i^{(0)}, \underline{w}_i^{(0)}\right) \leq \left(\underline{u}_i^{(1)}, \underline{w}_i^{(1)}\right) \leq \left(\bar{u}_i^{(1)}, \bar{w}_i^{(1)}\right) \leq \left(\bar{u}_i^{(0)}, \bar{w}_i^{(0)}\right).$$

Assume by induction that if

$$\left(\underline{u}_i^{(m-1)}, \underline{w}_i^{(m-1)}\right) \leq \left(\underline{u}_i^{(m)}, \underline{w}_i^{(m)}\right) \leq \left(\bar{u}_i^{(m)}, \bar{w}_i^{(m)}\right) \leq \left(\bar{u}_i^{(m-1)}, \bar{w}_i^{(m-1)}\right)$$

holds for some $m > 1$. Then $\underline{z}_i^{(m+1)} := \underline{w}_i^{(m+1)} - \underline{w}_i^{(m)}$ satisfies

$$\begin{cases} (d_i + 2\alpha_i \underline{u}_i^{(m+1)})^{-1} \underline{z}_{it}^{(m+1)} + ((d_i + 2\alpha_i \underline{u}_i^{(m+1)})^{-1} - (d_i + 2\alpha_i \underline{u}_i^{(m)})^{-1}) \underline{w}_{it}^{(m)} - L\underline{z}_i^{(m+1)} \\ \quad = F_1(\underline{u}_1^{(m)}, \underline{u}_2^{(m)}) - F_1(\underline{u}_1^{(m-1)}, \underline{u}_2^{(m-1)}) \geq 0, \\ \underline{z}_i^{(m+1)}(x,t) = 0, \\ \underline{z}_i^{(m+1)}(x,0) = 0. \end{cases}$$

By the mean value theorem, we obtain

$$\left(d_i + 2\alpha_i \underline{u}_i^{(m+1)}\right)^{-1} - \left(d_i + 2\alpha_i \underline{u}_i^{(m)}\right)^{-1} = \frac{-2\alpha_i}{(d_i + 2\alpha_i \xi_i^{(m)})^3}\left(\underline{w}_i^{(m+1)} - \underline{w}_i^{(m)}\right)$$

for some intermediate value $\xi^{(m)}$ between $\underline{u}^{(m)}$ and $\underline{u}^{(m+1)}$. It is easy to see that

$$\left(d_i + 2\alpha_i u_i^{(m+1)}\right)^{-1} \underline{z}_{it}^{(m+1)} - L\underline{z}_i^{(m+1)} + \gamma_i^{(m)} \underline{z}_i^{(m+1)} \geq 0,$$

where $\gamma_i^{(m)} = -\frac{2\alpha_i}{(d_i+2\alpha_i\xi_i^{(m)})^3} \underline{w}_{it}^{(m)}$.

By Lemma 2.1, we have $\underline{z}_i^{(m+1)} \geq 0$, which leads to $\underline{w}_i^{(m+1)} \geq \underline{w}_i^{(m)}$ and thus $\underline{u}_i^{(m+1)} \geq \underline{u}_i^{(m)}$. Similarly, we have $\bar{w}_i^{(m)} \geq \bar{w}_i^{(m+1)} \geq \underline{w}_i^{(m+1)}$ and $\bar{u}_i^{(m)} \geq \bar{u}_i^{(m+1)} \geq \underline{u}_i^{(m+1)}$. The conclusion of the lemma follows from the induction principle. □

From the proof of Lemma 2.2, we know that the following comparison principle holds.

Lemma 2.3 (Comparison Principle) *Let* $(\tilde{u}_1, \tilde{u}_2)$ *and* $(\hat{u}_1, \hat{u}_2)$ *in* $C([0,T] \times \bar{\Omega}) \cap C^2((0,T] \times \Omega)$ *be the ordered upper and lower solutions of the problem* (1.1), *respectively. Then we have*

$$(\tilde{u}_1, \tilde{u}_2) \geq (\hat{u}_1, \hat{u}_2) \quad \text{in } [0,T] \times \bar{\Omega}.$$

In view of Lemma 2.2, the pointwise limits

$$\lim_{m\to\infty} \left(\bar{\mathbf{u}}^{(m)}, \bar{\mathbf{w}}^{(m)}\right) = (\bar{\mathbf{u}}, \bar{\mathbf{w}}) \quad \text{and} \quad \lim_{m\to\infty} \left(\underline{\mathbf{u}}^{(m)}, \underline{\mathbf{w}}^{(m)}\right) = (\underline{\mathbf{u}}, \underline{\mathbf{w}})$$

exist and satisfy $(\bar{\mathbf{u}}, \bar{\mathbf{w}}) \geq (\underline{\mathbf{u}}, \underline{\mathbf{w}})$.

Now we show that $(\bar{\mathbf{u}}, \bar{\mathbf{w}}) = (\underline{\mathbf{u}}, \underline{\mathbf{w}})$ $(:= (\mathbf{u}^*, \mathbf{w}^*))$ and $\mathbf{u}^*$ is the unique solution of (1.1).

Theorem 2.1 *Let* $(\tilde{u}_1, \tilde{u}_2)$, $(\hat{u}_1, \hat{u}_2)$ *be a pair of ordered upper and lower solutions of problem* (1.1). *Then the sequences* $\{\bar{\mathbf{u}}^{(m)}, \bar{\mathbf{w}}^{(m)}\}$, $\{\underline{\mathbf{u}}^{(m)}, \underline{\mathbf{w}}^{(m)}\}$ *obtained from* (2.6) *converge monotonically to the unique solution* $(\mathbf{u}^*, \mathbf{w}^*) \in S \times \bar{S}$ *of problem* (2.3), *and they satisfy the relation*

$$\begin{aligned}(\hat{\mathbf{u}}, \hat{\mathbf{w}}) &\leq \left(\underline{\mathbf{u}}^{(m)}, \underline{\mathbf{w}}^{(m)}\right) \leq \left(\underline{\mathbf{u}}^{(m+1)}, \underline{\mathbf{w}}^{(m+1)}\right) \leq \left(\mathbf{u}^*, \mathbf{w}^*\right) \\ &\leq \left(\bar{\mathbf{u}}^{(m+1)}, \bar{\mathbf{w}}^{(m+1)}\right) \leq \left(\bar{\mathbf{u}}^{(m)}, \bar{\mathbf{w}}^{(m)}\right) \leq (\tilde{\mathbf{u}}, \tilde{\mathbf{w}}), \quad m = 1, 2, \ldots.\end{aligned}$$

Moreover, $\mathbf{u}^*$ *is the unique solution of problem* (1.1).

The proof of this theorem is similar to Theorem 3.1 in [18], so we omit it here.

The existence and uniqueness of the solution to problem (1.1) can be ensured by constructing a pair of ordered upper and lower solutions of (1.1). In fact, we only need to construct a bounded positive upper solution since that we can take $(0,0)$ as lower solution. We have the following theorem.

Theorem 2.2 *If* $b_1c_2 > b_2c_1$, *then for any nonnegative initial data, problem* (1.1) *admits a unique global solution* (u_1, u_2), *which is uniformly bounded in* $\bar{\Omega} \times [0,\infty)$.

Proof Let (η_1, η_2) be a positive solution of the following problem:

$$\begin{cases} +b_1 x - c_1 y > 0, \\ -b_2 x + c_2 y > 0 \end{cases} \tag{2.10}$$

and let $(\tilde{u}_1, \tilde{u}_2) = (\rho\eta_1, \rho\eta_2)$, then $(\tilde{w}_1, \tilde{w}_2) = (d_1\rho\eta_1 + \alpha_1\rho^2\eta_1^2, d_2\rho\eta_2 + \alpha_2\rho^2\eta_2^2)$, where $\rho > 1$ is a constant such that $(u_1(x,0), u_2(x,0)) \le (\rho\eta_1, \rho\eta_2)$ and

$$\begin{cases} a_1 - b_1\rho\eta_1 + c_1\rho\eta_2 \le 0, \\ a_2 + b_2\rho\eta_1 - c_2\rho\eta_2 \le 0. \end{cases} \tag{2.11}$$

It is easily to check that $(\tilde{u}_1, \tilde{u}_2, \tilde{w}_1, \tilde{w}_2) = (\rho\eta_1, \rho\eta_2, d_1\rho\eta_1 + \alpha_1\rho^2\eta_1^2, d_2\rho\eta_2 + \alpha_2\rho^2\eta_2^2)$ is the upper solution of problem (1.1). Further, $(\tilde{u}_1, \tilde{u}_2)$ is global and uniformly bounded. The desired results can obtain from Theorem 2.1. □

Remark 2.1 For the problem (1.1) with the Neumann boundary condition instead of the Dirichlet boundary condition, it can be discussed similarly. For the one-dimensional case, we can also obtain the existence and uniform boundedness of the global solution by using Gagliardo-Nirenberg-type inequalities; see [17] for details.

3 Blow-up of the solution

In this section, we consider the existence of the blow-up solution. Here we say the solution (u_1, u_2) blows up in a finite time $T > 0$ if

$$\lim_{t\to T} \max_{\bar{\Omega}} \big(|u_1(\cdot,t)| + |u_2(\cdot,t)|\big) = +\infty.$$

To get the blow-up results for the system (1.1), we first consider the scalar problem:

$$\begin{cases} kz_t - \Delta[(D+\alpha z)z] = z(A+bz) & \text{in } \Omega \times (0,T), \\ z(x,t) = 0 & \text{on } \partial\Omega \times (0,T), \\ z(0,x) \ge 0 & \text{in } \Omega, \end{cases} \tag{3.1}$$

where k, D, b, and α are positive constants.

Lemma 3.1 *The problem*

$$\begin{cases} \Delta[(D+\alpha\psi(x))\psi(x)] + \psi(x)(A+b\psi(x)) = 0 & \text{in } \Omega, \\ \psi(x) = 0 & \text{on } \partial\Omega \end{cases} \tag{3.2}$$

has a nontrivial nonnegative solution if $b > \alpha\lambda$, where λ is the first eigenvalue of Laplace operator subject to the homogeneous Dirichlet boundary condition.

The proof of Lemma 3.1 is similar to that of Lemma 11.7.1 of [10], where $\alpha = 0$, and we omit it here.

Lemma 3.2 *Let $z(x,t)$ be a nontrivial nonnegative solution of problem* (3.1). *If*

$$\Delta\big[\big(D+\alpha z(x,0)\big)z(x,0)\big] + z(x,0)\big(A+bz(x,0)\big) \ge 0,$$

then $z_t(x,t) \ge 0$ in $\Omega \times (0,T)$.

Proof Under the transform $w = (D + \alpha z)z$, the problem (3.1) becomes

$$\begin{cases} k(D+2\alpha z)^{-1}w_t - \Delta w = z(A+bz) & \text{in } \Omega \times (0,T), \\ w(x,t) = 0 & \text{on } \partial\Omega \times (0,T), \\ w(x,0) \geq 0 & \text{in } \Omega, \end{cases} \tag{3.3}$$

where $z = \frac{-D+\sqrt{D^2+4\alpha w}}{2\alpha}$.

Using the comparison principle and assumptions on $z(x,0)$, we have $z(x,t) \geq z(x,0)$ in $\Omega \times (0,T)$. So $w(x,t) \geq w(x,0)$ in $\Omega \times (0,T)$. Then using again the comparison principle yields $w(x,t+\varepsilon) \geq w(x,t)$ in $\Omega \times (0, T-\varepsilon)$ for an arbitrarily small $\varepsilon > 0$. Hence $z_t(x,t) = (D+2\alpha z)^{-1}w_t(x,t) \geq 0$ in $\Omega \times (0,T)$. □

Lemma 3.3 *Let λ be the first eigenvalue of Laplace operator subject to the homogeneous Dirichlet boundary condition. If $b > \alpha\lambda$ and one of the following conditions holds*:

(i) $A \geq D\lambda$;

(ii) *the initial data is large enough*,

then all nontrivial nonnegative solutions of problem (3.1) *blow up*.

Proof Let $\Phi > 0$ be the corresponding eigenfunction of the eigenvalue λ, which is chosen to satisfy $\int_\Omega \Phi(x)\,dx = 1$. Define

$$F(t) = \int_\Omega \Phi(x)z(x,t)\,dx.$$

Then using the first equation in (3.1) and integrating by parts yield

$$\begin{aligned} kF_t(t) &= k\int_\Omega \Phi(x)z_t(x,t)\,dx \\ &= \int_\Omega \Phi(x)\bigl[\Delta\bigl[\bigl(D+\alpha z(x,t)\bigr)z(x,t)\bigr] + z(x,t)\bigl(A+bz(x,t)\bigr)\bigr]\,dx \\ &= \int_\Omega \Delta\Phi(x)\bigl(D+\alpha z(x,t)\bigr)z(x,t)\,dx + \int_\Omega \Phi(x)z(x,t)\bigl(A+bz(x,t)\bigr)\,dx \\ &= -\lambda DF(t) + \int_\Omega \Phi(x)z(x,t)\bigl(A+bz(x,t)-\alpha\lambda z(x,t)\bigr)\,dx \\ &= (A-\lambda D)F(t) + (b-\alpha\lambda)\int_\Omega \Phi(x)z^2(x,t)\,dx. \end{aligned}$$

(i) If $b > \alpha\lambda$ and $A \geq \lambda D$, we get

$$kF_t(t) \geq (b-\alpha\lambda)F^2(t),$$

obviously $F(t)$ blows up in a fine time, and so does $z(x,t)$.

(ii) If $b > \alpha\lambda$ and the initial data is large enough so that

$$(A-\lambda D)F(0) + (b-\alpha\lambda)F^2(0) > 0,$$

then there exists $\varepsilon > 0$ such that $(A-\lambda D)F(0) + (b-\alpha\lambda-\varepsilon)F^2(0) > 0$. Therefore $F(t) \geq F(0)$ for $t > 0$ and $kF_t(t) \geq (A-\lambda D)F(t) + (b-\alpha\lambda)F^2(t) \geq \varepsilon F^2(t) + (A-\lambda D)F(t) + (b-\alpha\lambda -$

$\varepsilon)F^2(t) \geq \varepsilon F^2(t)$. Based on the discussion, we show that if the initial value is large enough the solution of problem (3.1) must blow up in a finite time. Thereby $z(x,t)$ blows up in finite time. □

Our main result in this section can now be stated as follows.

Theorem 3.1 *Assume that α_1, α_2 are small enough and*

$$\min\left\{\frac{c_2 d_1}{b_2 d_2}, \frac{\alpha_1 c_2^2}{\alpha_2 b_2^2}\right\} < \min\left\{\frac{c_1 d_1}{b_1 d_2}, \frac{\alpha_1 c_1^2}{\alpha_2 b_1^2}\right\}. \tag{3.4}$$

(i) *If* $\max\{a_1/d_1, a_2/d_2\} \geq \lambda$, *then the solution of problem* (1.1) *with any nontrivial nonnegative initial value blows up.*
(ii) *If the initial value is large enough,*
then the solution of (1.1) *blows up.*

Proof To show this, it suffices to construct a lower solution of (2.3) and prove the lower solution blows up in a finite time with some suitable conditions.

Take $(\underline{u}_1, \underline{u}_2, \underline{w}_1, \underline{w}_2) = (q_1(\delta_1 z_1), q_2(\delta_2 z_2), \delta_1 z_1, \delta_2 z_2)$, where δ_i $(i = 1, 2)$ is positive constant to be defined later, $z_i(x,t)$ is a nonnegative function in $\bar{\Omega} \times (0, T_0)$ and vanishes on the boundary, the functions q_i are same as in Section 2. From Definition 2.1 and (2.3) we know that $(\underline{u}_1, \underline{u}_2, \underline{w}_1, \underline{w}_2)$ is the lower solution of (2.3) if $(\delta_1 z, \delta_2 z)$ satisfies the following inequalities:

$$\begin{cases} (d_1 + 2\alpha_1 u_1)^{-1} z_{1t} - \Delta z_1 \leq \frac{z_1}{d_1 + 2\alpha_1 u_1}(a_1 - b_1 \frac{\delta_1 z_1}{(d_1 + 2\alpha_1 u_1)} + c_1 \frac{\delta_2 z_2}{d_2 + 2\alpha_2 u_2}), \\ (d_2 + 2\alpha_2 u_2)^{-1} z_{2t} - \Delta z_2 \leq \frac{z_2}{d_2 + 2\alpha_2 u_2}(a_2 + b_2 \frac{\delta_1 z_1}{(d_1 + 2\alpha_1 u_1)} - c_2 \frac{\delta_2 z_2}{d_2 + 2\alpha_2 u_2}), \\ \delta_1 z(x,0) \leq \eta_1(x), \qquad \delta_2 z(x,0) \leq \eta_2(x), \end{cases}$$

which is equivalent to

$$\begin{cases} \frac{1}{\sqrt{d_1^2 + 4\alpha_1 \delta_1 z_1}} z_{1t} - \Delta z_1 \leq \frac{z_1}{\sqrt{d_1^2 + 4\alpha_1 \delta_1 z_1}}(a_1 - b_1 \frac{\delta_1 z_1}{\sqrt{d_1^2 + 4\alpha_1 \delta_1 z_1}} + c_1 \frac{\delta_2 z_2}{\sqrt{d_2^2 + 4\alpha_2 \delta_2 z_2}}), \\ \frac{1}{\sqrt{d_2^2 + 4\alpha_2 \delta_2 z_2}} z_{2t} - \Delta z_2 \leq \frac{z_2}{\sqrt{d_2^2 + 4\alpha_2 \delta_2 z_2}}(a_2 + b_2 \frac{\delta_1 z_1}{\sqrt{d_1^2 + 4\alpha_1 \delta_1 z_2}} - c_2 \frac{\delta_2 z_2}{\sqrt{d_2^2 + 4\alpha_2 \delta_2 z_2}}), \\ \delta_1 z(x,0) \leq \eta_1(x), \qquad \delta_2 z(x,0) \leq \eta_2(x). \end{cases} \tag{3.5}$$

If we choose $z_1 = z_2 = z$, then (3.5) holds when

$$\begin{cases} \frac{1}{\sqrt{d_1^2 + 4\alpha_1 \delta_1 z}} z_t - \Delta z \leq \frac{z}{\sqrt{d_1^2 + 4\alpha_1 \delta_1 z}}(a_1 - b_1 \frac{\delta_1 z}{\sqrt{d_1^2 + 4\alpha_1 \delta_1 z}} + c_1 \frac{\delta_2 z}{\sqrt{d_2^2 + 4\alpha_2 \delta_2 z}}), \\ \frac{1}{\sqrt{d_2^2 + 4\alpha_2 \delta_2 z}} z_t - \Delta z \leq \frac{z}{\sqrt{d_2^2 + 4\alpha_2 \delta_2 z}}(a_2 + b_2 \frac{\delta_1 z}{\sqrt{d_1^2 + 4\alpha_1 \delta_1 z}} - c_2 \frac{\delta_2 z}{\sqrt{d_2^2 + 4\alpha_2 \delta_2 z}}), \\ \delta_1 z(x,0) \leq \eta_1(x), \qquad \delta_2 z(x,0) \leq \eta_2(x). \end{cases} \tag{3.6}$$

Now we consider the right hand sides of the first and second inequalities in (3.6); it is easy to check that

$$c_1 \frac{\delta_2}{\sqrt{d_2^2 + 4\alpha_2 \delta_2 z}} > b_1 \frac{\delta_1}{\sqrt{d_1^2 + 4\alpha_1 \delta_1 z}}, \qquad b_2 \frac{\delta_1}{\sqrt{d_1^2 + 4\alpha_1 \delta_1 z}} > c_2 \frac{\delta_2}{\sqrt{d_2^2 + 4\alpha_2 \delta_2 z}}$$

holds, provided that

$$\frac{d_1}{b_1\delta_1} > \frac{d_2}{c_1\delta_2}, \qquad \frac{\alpha_1}{b_1^2\delta_1} > \frac{\alpha_2}{c_1^2\delta_2}, \qquad \frac{d_2}{c_2\delta_2} > \frac{d_1}{b_2\delta_1}, \qquad \frac{\alpha_2}{c_2^2\delta_2} > \frac{\alpha_1}{b_2^2\delta_1},$$

which is equivalent to

$$\frac{c_2d_1}{b_2d_2} < \frac{\delta_1}{\delta_2} < \frac{d_1c_1}{d_2b_1} \quad \text{and} \quad \frac{\alpha_1c_2^2}{\alpha_2b_2^2} < \frac{\delta_1}{\delta_2} < \frac{\alpha_1c_1^2}{\alpha_2b_1^2}.$$

Recalling the assumption (3.4) shows that there exists a small $\varepsilon > 0$ such that

$$(c_1-\varepsilon)\frac{\delta_2}{\sqrt{d_2^2+4\alpha_2\delta_2z}} \geq \frac{b_1\delta_1}{\sqrt{d_1^2+4\alpha_1\delta_1z}},$$
$$(b_2-\varepsilon)\frac{\delta_1}{\sqrt{d_1^2+4\alpha_1\delta_1z}} \geq \frac{c_2\delta_2}{\sqrt{d_2^2+4\alpha_2\delta_2z}}.$$

Thus (3.6) holds if

$$\begin{cases} \frac{1}{\sqrt{d_1^2+4\alpha_1\delta_1z}}z_t - \Delta z \leq \frac{z}{\sqrt{d_1^2+4\alpha_1\delta_1z}}(a_1 + \varepsilon\frac{\delta_2z}{\sqrt{d_2^2+4\alpha_2\delta_2z}}), \\ \frac{1}{\sqrt{d_2^2+4\alpha_2\delta_2z}}z_t - \Delta z \leq \frac{z}{\sqrt{d_2^2+4\alpha_2\delta_2z}}(a_2 + \varepsilon\frac{\delta_1z}{\sqrt{d_1^2+4\alpha_1\delta_1z}}), \\ \delta_1z(x,0) \leq \eta_1(x), \qquad \delta_2z(x,0) \leq \eta_2(x). \end{cases} \tag{3.7}$$

If $z_t \geq 0$, take

$$d = \max\{d_1,d_2\}, \qquad d_* = \min\{d_1,d_2\}, \qquad \alpha = \min\{\alpha_1,\alpha_2\},$$
$$A = \max\{a_1/d_1, a_2/d_2\}, \qquad K = \max\{\alpha_1\delta_1/d_1^2, \alpha_2\delta_2/d_2^2\}, \qquad \delta = \min\{\delta_1,\delta_2\},$$

then (3.7) holds if

$$\begin{cases} \frac{z_t}{\sqrt{d_*^2+4\alpha\delta z}} - \Delta z \leq \frac{Az}{\sqrt{1+4Kz}} + \varepsilon\frac{\delta z^2}{d^2(1+4Kz)}, \\ \delta z(x,0) \leq \eta_1(x), \qquad \delta z(x,0) \leq \eta_2(x). \end{cases} \tag{3.8}$$

Take $k = \max\{1/d_*, \sqrt{K/(\alpha\delta)}\}$; then (3.8) can be written

$$\begin{cases} \frac{kz_t}{\sqrt{1+4Kz}} - \Delta z \leq \frac{Az}{\sqrt{1+4Kz}} + \varepsilon\frac{\delta z^2}{d^2(1+4Kz)}, \\ \delta z(x,0) \leq \eta_1(x), \qquad \delta z(x,0) \leq \eta_2(x). \end{cases} \tag{3.9}$$

Let $z = (1+Kw)w$; then (3.9) holds provided that

$$\begin{cases} kw_t - \Delta[(1+Kw)w] \leq Aw + \frac{\varepsilon\delta}{d^2}w^2, \\ w(x,0) \leq \eta_1^*(x), \qquad w(x,0) \leq \eta_2^*(x), \end{cases} \tag{3.10}$$

where $\eta_i^*(x) = \frac{-1+\sqrt{1+4K\eta_i/\delta}}{2K}$ for $i = 1,2$.

First we discuss the case when the initial value is large enough. By Lemma 3.1, problem (3.2) has a nontrivial nonnegative solution, denoted by $\psi^*(x)$. We define z^* to be the solution of (3.1) with $z^*(x,0) = \psi^*(x)$. Then it is easy to see that $z_t^* \geq 0$ from Lemma 3.2. So if $\underline{w}_1(x,0) \geq \delta_1\psi^*(x)$ and $\underline{w}_2(x,0) \geq \delta_2\psi^*(x)$, the pair $(q_1(\delta_1 z^*), q_2(\delta_2 z^*))$ is a lower solution of (1.1). On the other hand, if follows from Lemma 3.3 that the solution z^* must blow up in a finite time provided that $\int_\Omega \Phi(x)z^*(x,0)\,dx > \frac{\lambda D-A}{b-\alpha\lambda}$. So if $\underline{w}_1(x,0)$ and $\underline{w}_2(x,0)$ are sufficiently large, then the pair $(\delta_1 z^*, \delta_2 z^*)$ blows up. Therefore the solution of (1.1) blows up in a finite time.

Second, we consider the case that $\max\{a_1/d_1, a_2/d_2\} \geq \lambda$ and α_1, α_2 are small enough. For an arbitrary nontrivial nonnegative initial data $(\eta_1(x), \eta_2(x))$, the solution of (1.1) is positive for $t > 0$ by Lemma 3.2. Further, we may assume that $u_1(x,0) > 0$ and $u_2(x,0) > 0$ for $x \in \Omega$, otherwise replace the initial function $(u_1(x,0), u_2(x,0))$ by $(u_1(x,t_1), u_2(x,t_1))$ for some $t_1 > 0$.

Let $\phi(x) > 0$ be the eigenfunction corresponding to the eigenvalue λ, then there exists suitable $\varepsilon_0 > 0$, such that $\varepsilon_0\delta_1\phi(x) \leq u_1(x,0)$, $\varepsilon_0\delta_2\phi(x) \leq u_2(x,0)$. Choosing $\psi(x) = \varepsilon\phi(x)$, then $\psi(x)$ satisfies (3.2) with $D = 1$ and $b = \frac{\varepsilon\delta}{d^2}$ and define w^* be the solution of (3.1) with the initial data $\psi(x)$. Then $\Delta[(1+\alpha w^*(x,0))w^*(x,0)] + w^*(A + bw^*)(x,0) \geq 0$ and w^* is monotone nondecreasing in t by Lemma 3.2. Moreover, it follows from the comparison principle that $u_1(x,t) \geq \delta_1 w^*(x,t)$ and $u_2(x,t) \geq \delta_2 w^*(x,t)$ in $\Omega \times [0, T_0)$, where T_0 is the maximal existent time of u_1, u_2, and w^*. Hence $(\underline{u}_1, \underline{u}_2) = (q_1(\delta_1 w^*), q_2(\delta_2 w^*))$ is a lower solution of (1.1).

On the other hand, Lemma 3.3 ensures the existence of a finite T_0 such that the solution w^* exists in $\Omega \times [0, T_0)$ and is unbounded in Ω as $t \to T_0$. Thus the solution of (1.1) cannot exist beyond T_0 and is nonglobal. This finishes the proof. □

4 Numerical illustrations

In this section, we present some numerical simulations to investigate the blow-up results in Theorem 3.1. Symbolic mathematical software Matlab 7.0 is used to plot numerical graphs. For simplicity, we always take $\Omega = [0, \pi]$.

Example 4.1 (Case (i)) In system (1.1), let $a_1 = 1.5$, $a_2 = 1.0$, $b_1 = 0.5$, $b_2 = 0.8$, $c_1 = 0.6$, $c_2 = 0.3$, $d_1 = 1$, and $d_2 = 2$. Take small self-diffusion rates $\alpha_1 = 0.04$ and $\alpha_2 = 0.02$. Then it is easy to check that the condition (3.5) is satisfied. Choose the initial values of u_1 and u_2 to be $1.5 + \sin 2x$ and $2.0 - 0.1\cos 2x$, respectively, and the solution of the system blows up, as shown in Figure 1.

Example 4.2 (Case (ii)) In system (1.1), take a_i, b_i, c_i, α_i $(i = 1, 2)$ as in Example 4.1. Let $d_1 = 0.06$ and $d_2 = 0.08$. Then it is easy to check the condition (3.4) hold. We choose u_1 and u_2 to be $12 + \sin 2x$ and $15 - 0.1\cos 2x$, respectively. According to Theorem 3.1, if the initial value is large enough the solution of the system blows up. Figure 2 shows the phenomenon.

5 Discussions

In this paper, we consider a two-species cooperating model with free diffusion and self-diffusion. Our main purpose is to find sufficient conditions for the solution to blow up in a finite time. The results show that the global solution exists if $b_1c_2 > b_2c_1$, *i.e.* the inter-specific competition is strong.

If the inter-specific competition is weak, Theorem 3.1 shows that blow-up occurs provided that the initial value is large enough or the self-diffusion rate is small. The latter gives the continuity of blow-up with the self-diffusion rate since the solution without self-diffusion blows up. The former shows that the solution of the system with self-diffusion

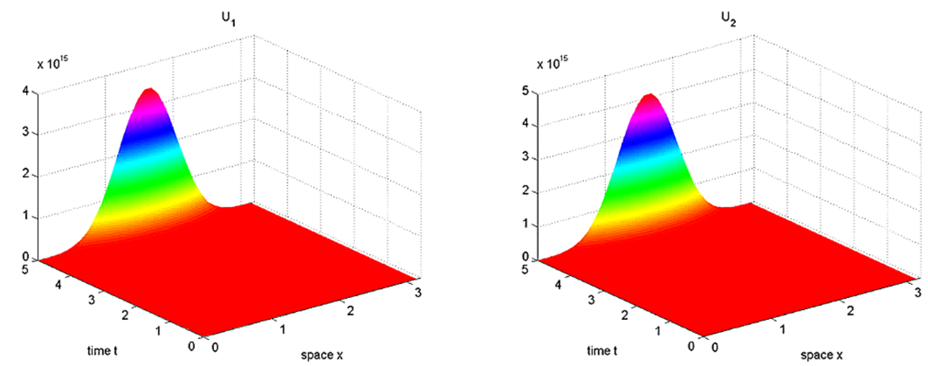

Figure 1 Blow-up solutions of Example 4.1.

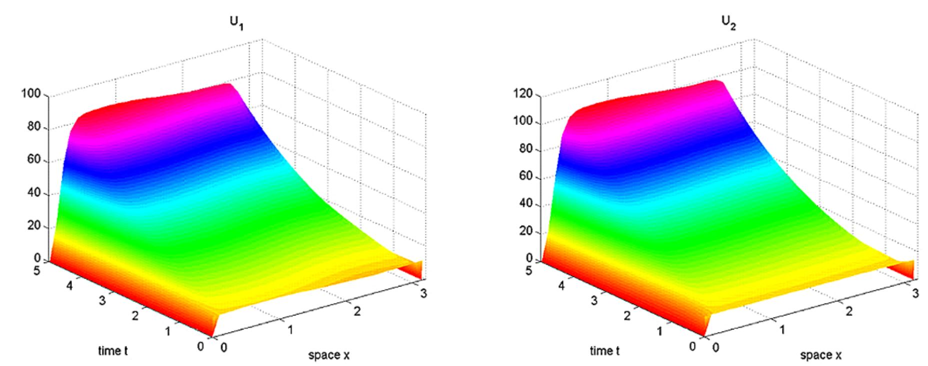

Figure 2 Blow-up solutions of Example 4.2.

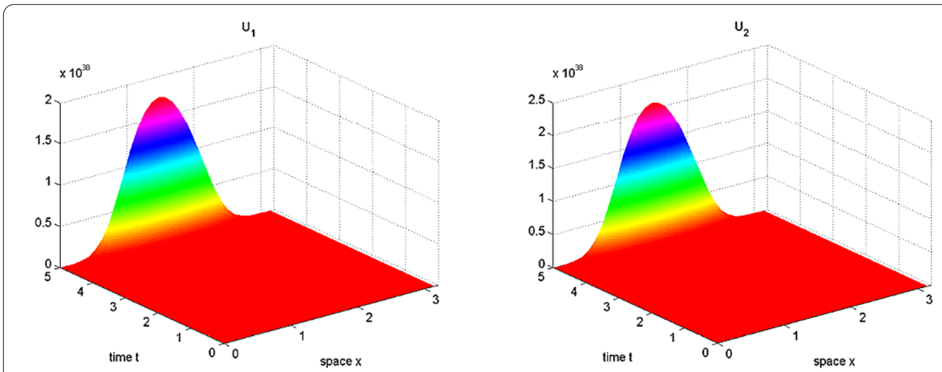

Figure 3 Blow-up solutions for large initial values and $\alpha_1 = \alpha_2 = 0$.

or without self-diffusion blows up if the initial value is large enough. A natural question is what the difference is between the solution with self-diffusion and without self-diffusion for the same big initial values. Let us take $\alpha_1 = \alpha_2 = 0$ and all other parameters and initial value the same as in Example 4.2; we then have the following simulation; see Figure 3.

Comparing Figure 2 with Figure 3, we can see that the solution without self-diffusion blows up fast, which implies the self-diffusion can 'relax' the blow-up. We still have no theoretical proof, but we feel it is worth further investigation.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors declare that the study was realized in collaboration with the same responsibility. All authors read and approved the final manuscript.

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