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Traveling wave solutions in a Lotka-Volterra type competition recursion

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Abstract

This paper is concerned with the traveling wave solutions of a competitive integrodifference system with Lotka-Volterra type nonlinearity. The existence of traveling wave solutions is proved by constructing generalized upper and lower solutions. The asymptotic behavior of traveling wave solutions is established by combining the theory of asymptotic spreading with the idea of contracting rectangles. The nonexistence of monotone traveling wave solutions is also confirmed by the theory of asymptotic spreading.

Keywords: competitive interaction; spreading speed; invasion and coexistence

1 Introduction

In this paper, we study the traveling wave solutions of the following integrodifference system with Lotka-Volterra type nonlinearity:

$$\begin{cases} X_{n+1}(x) = \int_{\mathbb{R}} r_1 X_n(y) [1 - X_n(y) - a_1 Y_n(y)] g_1(x - y) dy, \\ Y_{n+1}(x) = \int_{\mathbb{R}} r_2 Y_n(y) [1 - Y_n(y) - a_2 X_n(y)] g_2(x - y) dy, \end{cases} \quad (1.1)$$

in which $n \in \mathbb{N} \cup \{0\}$, and $r_i > 0$, $a_i \geq 0$, $i = 1, 2$, are constants. For $i = 1, 2$, g_i is the probability function describing the random walk of individuals under consideration, and it is the so-called kernel function. In particular, we take the following conditions in this paper:

(A1) g_i is Lebesgue measurable and integrable on \mathbb{R} and $\int_{\mathbb{R}} g_i(y) dy = 1$;

(A2) $g_i(y) = g_i(-y) \geq 0$, $y \in \mathbb{R}$, and for each $\lambda \in \mathbb{R}$, $\int_{\mathbb{R}} g_i(y) e^{\lambda y} dy < \infty$.

For the parameters in (1.1), we also give the following assumptions:

(A3) $r_1, r_2 \in (1, 4)$;

(A4) $1 - l_1 - a_1 l_2 \geq 0$, $1 - l_2 - a_2 l_1 \geq 0$, where

$$l_i = \begin{cases} \frac{r_i - 1}{r_i}, & r_i \in (1, 2), \\ \frac{r_i}{4}, & r_i \in [2, 4) \end{cases}$$

for $i = 1, 2$;

(A5) $r_1(1 - a_1 l_2) > 1$, $r_2(1 - a_2 l_1) > 1$;

(A6) $1 - 1/r_1 > a_1(1 - 1/r_2)$, $1 - 1/r_2 > a_2(1 - 1/r_1)$;

(A7) $1 - 2l_1 - a_1 l_2 \geq 0$, $1 - 2l_2 - a_2 l_1 \geq 0$.

We now give some illustration of (A3)-(A7) from the viewpoint of both the literature and the population dynamics of the corresponding difference system

$$X_{n+1} = r_1 X_n [1 - X_n - a_1 Y_n], \quad Y_{n+1} = r_2 Y_n [1 - Y_n - a_2 X_n]. \tag{1.2}$$

If the inter-specific vanishes ($a_1 = a_2 = 0$) and (A3) holds in (1.2), then $X_{n+1} = r_1 X_n [1 - X_n]$ or $Y_{n+1} = r_2 Y_n [1 - Y_n] : [0, 1] \rightarrow [0, 1]$ and is persistent in population dynamics (see Murray [1, Section 2.3] for the dynamics). Condition (A4) provides a positive invariant region of the difference system (1.2), namely, $[0, l_1] \times [0, l_2]$. (A5) indicates that the inter-specific is weak, namely, even if the competitor X_n (Y_n) takes the maximal value l_1 (l_2), the species Y_n (X_n) still persists. The coexistence steady state of (1.2) exists if (A6) holds. (A7) leads to a comparison principle appealing to the difference system (1.2) in $[0, l_1] \times [0, l_2]$. Finally, if (A3)-(A7) hold, then (1.2) has a stable steady state. In particular, there are different parameters such that part or all of (A3)-(A7) hold. For example, if $r_1 = 1.5$, $r_2 = 1.6$, $a_1 = a_2 = 0.1$, then (1.2) satisfies (A3)-(A7); if $r_1 = 1.5$, $r_2 = 1.8$, $a_1 = a_2 = 0.5$, then (1.2) satisfies (A3)-(A6), if $r_1 = 1.2$, $r_2 = 1.8$, $a_1 = a_2 = 0.5$, then (A3) and (A4) hold.

Hereafter, a traveling wave solution of (1.1) is a special solution having the form

$$X_n(x) = \phi(t), \quad Y_n(x) = \psi(t), \quad t = x + cn,$$

in which $c > 0$ is the wave speed and (ϕ, ψ) describes the wave profile that propagates in the media \mathbb{R} . Then (ϕ, ψ) and c must satisfy

$$\begin{cases} \phi(t+c) = \int_{\mathbb{R}} r_1 \phi(y) (1 - \phi(y) - a_1 \psi(y)) g_1(t-y) dy, \\ \psi(t+c) = \int_{\mathbb{R}} r_2 \psi(y) (1 - \psi(y) - a_2 \phi(y)) g_2(t-y) dy \end{cases} \tag{1.3}$$

for $t \in \mathbb{R}$. To describe determinative transition processes between different states, proper asymptotic behavior satisfied by the traveling wave solutions is often necessary. In this paper, we require the following asymptotic boundary conditions:

$$\lim_{t \rightarrow -\infty} (\phi(t), \psi(t)) = (0, 0), \quad \liminf_{t \rightarrow \infty} \phi(t) > 0, \quad \liminf_{t \rightarrow \infty} \psi(t) > 0. \tag{1.4}$$

Furthermore, we also consider the following stronger ones:

$$\lim_{t \rightarrow -\infty} (\phi(t), \psi(t)) = (0, 0), \quad \lim_{t \rightarrow \infty} \phi(t) = k_1, \quad \lim_{t \rightarrow \infty} \psi(t) = k_2, \tag{1.5}$$

in which

$$k_1 = \frac{1 - \frac{1}{r_1} - a_1(1 - \frac{1}{r_2})}{1 - a_1 a_2} > 0, \quad k_2 = \frac{1 - \frac{1}{r_2} - a_2(1 - \frac{1}{r_1})}{1 - a_1 a_2} > 0,$$

provided that (A6) is true. In population dynamics, (1.3) with (1.4) or (1.3) with (1.5) could formulate the successful invasion of two competitors.

To study the existence of traveling wave solutions of competitive recursions of two competitors, Lin *et al.* [2] established an abstract scheme and the existence of traveling wave solutions was reduced to the existence of upper and lower solutions. Since the competitive system in [2] does not generate monotone semiflows when the synchronous invasion of two competitors is concerned, the asymptotic behavior of traveling wave solutions cannot

be confirmed by the monotonicity of them (see [3–15] for monostable traveling wave solutions of (local) monotone recursions). In [2], the asymptotic behavior of traveling wave solutions was obtained by that of upper and lower solutions. The method was also applied to several competitive systems; see [16–18] and the references cited therein. Without the requirements of upper and lower solutions, it is difficult to obtain the asymptotic behavior of traveling wave solutions [19]. In fact, for coupled systems with general kernels, it is not an easy job to construct proper upper and lower solutions satisfying the asymptotic behavior in [2], and [16–18] just obtained the existence of traveling wave solutions of some systems with special kernels. Although the kernel functions in [2, 16–18] satisfy some special conditions, the verification of upper and lower solutions is still very complex, and the nonexistence of nontrivial traveling wave solutions of models in [2, 16–18] remains open.

To simplify the construction of upper and lower solutions and provide a more general result of the existence of traveling wave solutions of recursions, Lin [20] further considered the traveling wave solutions of recursions and gave some simpler conditions. By the theory in [20], the existence of traveling wave solutions can be obtained by the existence of upper and lower solutions which are easy to construct. Moreover, by the properties of the corresponding difference systems, the asymptotic behavior of traveling wave solutions was also studied. Moreover, for the model investigated by [2, 17], Lin [20] also obtained the nonexistence of nontrivial traveling wave solutions by the theory of asymptotic spreading.

In this paper, we shall establish the existence and nonexistence of (1.3) with (1.4) or (1.3) with (1.5), and we present the corresponding mathematical results by the idea in [20]. In particular, we shall not take special general kernels g_1, g_2 , and just add conditions (A1)-(A2) in what follows.

The rest of this paper is organized as follows. In Section 2, we investigate the existence of traveling wave solutions by constructing upper and lower solutions and applying Schauder's fixed point theorem in a functional space equipped with the decay norm. In Section 3, the asymptotic boundary conditions (1.4) and (1.5) will be considered by combining the theory of asymptotic spreading with the idea of contracting rectangles in [20]. Finally, the nonexistence of monotone traveling wave solutions is proved, which indicates that the threshold in the paper is the minimal wave speed of monotone traveling wave solutions of (1.1).

2 Existence of traveling wave solutions

In this paper, we shall utilize standard partial ordering in \mathbb{R}^2 . Also let X be the set of uniformly continuous and bounded functions from \mathbb{R} to \mathbb{R}^2 . Moreover, we denote

$$X_{[0,1]} = \{ \Phi = (\phi, \psi) \in X : 0 \leq \psi(t) \leq l_1, 0 \leq \phi(t) \leq l_2, t \in \mathbb{R} \}.$$

Let $|\cdot|$ be the supremum norm in \mathbb{R}^2 and $\mu > 0$ be a constant. We define

$$B_\mu(\mathbb{R}, \mathbb{R}^2) = \left\{ \Phi \in X : \sup_{x \in \mathbb{R}} |\Phi(x)| e^{-\mu|x|} < \infty \right\},$$

and the decay norm

$$|\Phi|_\mu = \sup_{x \in \mathbb{R}} |\Phi(x)| e^{-\mu|x|}, \quad \Phi \in B_\mu(\mathbb{R}, \mathbb{R}^2).$$

Then $(B_\mu(\mathbb{R}, \mathbb{R}^2), |\cdot|_\mu)$ is a Banach space.

Before discussing (1.1), we first consider the corresponding difference equation

$$\begin{cases} X_{n+1} = r_1 X_n [1 - X_n - a_1 Y_n], & X_0 = x_0, \\ Y_{n+1} = r_2 Y_n [1 - Y_n - a_2 X_n], & Y_0 = y_0, \quad n + 1 \in \mathbb{N}. \end{cases} \quad (2.1)$$

Lemma 2.1 For (2.1), we have the following conclusions:

- (D1) if (A3)-(A4) hold, then $[0, l_1] \times [0, l_2]$ is invariant, namely, if $x_0 \in [0, l_1], y_0 \in [0, l_2]$, then $X_n \in [0, l_1], Y_n \in [0, l_2], n \in \mathbb{N}$;
- (D2) if (A3), (A4), and (A7) are true and $x_0 \in [0, l_1], y_0 \in [0, l_2]$, then for each $n \in \mathbb{N}$, $X_n (Y_n)$ is monotone increasing in $X_{n-1} (Y_{n-1})$ and monotone decreasing in $Y_{n-1} (X_{n-1})$.

We now introduce some constants. Denote

$$\Lambda_i(\lambda, c) = r_i \int_{\mathbb{R}} e^{\lambda y - \lambda c} g_i(y) dy, \quad i = 1, 2$$

for $\lambda \in \mathbb{R}$ and $c \geq 0$. Then the following result holds.

Lemma 2.2 There exists a positive constant $c^* > 0$ such that $c < c^*$ implies that $\Lambda_1(\lambda, c) > 1$ for any $\lambda \geq 0$ or $\Lambda_2(\lambda, c) > 1$ for any $\lambda \geq 0$. If $c > c^*$, then $\Lambda_i(\lambda, c) = 1$ has at least one positive root $\lambda_i(c)$ such that $\Lambda_i(\lambda_i(c), c) = 1$ and $\Lambda_i(\lambda, c) > 1$ for $\lambda \in (0, \lambda_i(c)), i = 1, 2$. Moreover, there exists $\gamma = \gamma(c) \in (1, 2)$ such that $\Lambda_i(\lambda'_i(c), c) < 1$ for all $\lambda'_i(c) \in (\lambda_i(c), \gamma \lambda_i(c)), i = 1, 2$.

By Liang and Zhao [21, Lemma 3.8], Lemma 2.2 is clear and c^* is defined by

$$c^* = \max \left\{ \inf_{\lambda > 0} \frac{\ln(r_1 \int_{\mathbb{R}} e^{\lambda y} g_1(y) dy)}{\lambda}, \inf_{\lambda > 0} \frac{\ln(r_2 \int_{\mathbb{R}} e^{\lambda y} g_2(y) dy)}{\lambda} \right\}.$$

In what follows, we suppose that $c > c^*$ is fixed. If $(\phi, \psi) \in X_{[0, L]}$, then we define $P = (P_1, P_2) : X_{[0, L]} \rightarrow X$ by

$$\begin{aligned} P_1(\phi, \psi)(t) &= \int_{\mathbb{R}} r_1 \phi(y) (1 - \phi(y) - a_1 \psi(y)) g_1(t - c - y) dy, \\ P_2(\phi, \psi)(t) &= \int_{\mathbb{R}} r_2 \psi(y) (1 - \psi(y) - a_2 \phi(y)) g_2(t - c - y) dy. \end{aligned}$$

In particular, P also admits the following properties.

Lemma 2.3 $P : X_{[0, L]} \rightarrow X_{[0, L]}$.

Lemma 2.3 is clear by Lemma 2.1 and (A1)-(A2), and we omit the proof here. Clearly, a fixed point of P in $X_{[0, L]}$ is a solution to (1.3). Therefore, it suffices to prove the existence of the fixed points of P by Schauder's fixed point theorem, and we first construct a potential set of wave profiles.

For the purpose, we define continuous functions

$$\bar{\phi}(t) = \min\{e^{\lambda_1 t}, l_1\}, \quad \bar{\psi}(t) = \min\{e^{\lambda_2 t}, l_2\}$$

and

$$\underline{\phi}(t) = \max\{e^{\lambda_1 t} - Me^{\eta\lambda_1 t}, 0\}, \quad \underline{\psi}(t) = \max\{e^{\lambda_2 t} - Me^{\eta\lambda_2 t}, 0\},$$

where $M > 1$ is a positive constant clarified later and $\eta \in (1, 2)$ such that

$$\eta\lambda_1 < \lambda_1 + \lambda_2, \quad \eta\lambda_2 < \lambda_1 + \lambda_2, \quad \Lambda_1(\eta\lambda_1, c) < 1, \quad \Lambda_2(\eta\lambda_2, c) < 1.$$

With these notations, we define the following set of potential wave profiles:

$$\Gamma = \{(\phi, \psi) \in X, (\underline{\phi}, \underline{\psi}) \leq (\phi, \psi) \leq (\overline{\phi}, \overline{\psi})\},$$

which satisfies the following nice properties.

Lemma 2.4 *If $M > 1$ is large such that*

$$(\underline{\phi}, \underline{\psi}) \leq (\overline{\phi}, \overline{\psi}),$$

then Γ is convex and nonempty. Moreover, it is closed and bounded with respect to the decay norm $|\cdot|_\mu$.

Lemma 2.5 *Assume that (A3)-(A4) hold. If $M > 1$ is large, then $P: \Gamma \rightarrow \Gamma$.*

Proof By Lemma 2.1, it suffices to verify that

$$(P_1(\phi, \psi)(t), P_2(\phi, \psi)(t)) \leq (\overline{\phi}(t), \overline{\psi}(t)), \quad t \in \mathbb{R}$$

and

$$(P_1(\phi, \psi)(t), P_2(\phi, \psi)(t)) \geq (\underline{\phi}(t), \underline{\psi}(t)), \quad t \in \mathbb{R}$$

for $(\phi, \psi) \in \Gamma$.

If $\overline{\phi}(t) = e^{\lambda_1 t}$, then

$$\begin{aligned} P_1(\phi, \psi)(t) &= \int_{\mathbb{R}} r_1 \phi(y) (1 - \phi(y) - a_1 \psi(y)) g_1(t - c - y) dy \\ &\leq \int_{\mathbb{R}} r_1 \phi(y) g_1(t - c - y) dy \\ &\leq \int_{\mathbb{R}} r_1 e^{\lambda_1 y} g_1(t - c - y) dy \\ &= e^{\lambda_1 t} = \overline{\phi}(t). \end{aligned}$$

If $\overline{\phi}(t) = l_1$, then Lemma 2.1 implies that

$$\begin{aligned} P_1(\phi, \psi)(t) &= \int_{\mathbb{R}} r_1 \phi(y) (1 - \phi(y) - a_1 \psi(y)) g_1(t - c - y) dy \\ &\leq \int_{\mathbb{R}} r_1 \phi(y) (1 - \phi(y)) g_1(t - c - y) dy \\ &\leq l_1 = \overline{\phi}(t), \end{aligned}$$

and we have verified that

$$P_1(\phi, \psi)(t) \leq \bar{\phi}(t), \quad t \in \mathbb{R}.$$

In a similar way, we can prove that

$$P_2(\phi, \psi)(t) \leq \bar{\psi}(t), \quad t \in \mathbb{R}.$$

If $\underline{\phi}(t) = 0$, then

$$\begin{aligned} 1 - \phi(y) - a_1\psi(y) &\geq 1 - \bar{\phi}(y) - a_1\bar{\psi}(y) \\ &\geq 1 - l_1 - a_1l_2 \\ &\geq 0 = \underline{\phi}(y) \end{aligned}$$

by (A4) and

$$P_1(\phi, \psi)(t) = \int_{\mathbb{R}} r_1\phi(y)(1 - \phi(y) - a_1\psi(y))g_1(t - c - y) dy \geq 0 = \underline{\phi}(t)$$

is clear. Otherwise, $(\phi, \psi) \in \Gamma$ leads to

$$\begin{aligned} r_1\phi(y)(1 - \phi(y) - a_1\psi(y)) &= r_1\phi(y) - r_1\phi^2(y) - r_1a_1\phi(y)\psi(y) \\ &\geq r_1e^{\lambda_1y} - r_1Me^{\eta\lambda_1y} - r_1e^{2\lambda_1y} - r_1a_1e^{(\lambda_1+\lambda_2)y} \end{aligned}$$

and

$$\begin{aligned} P_1(\phi, \psi)(t) &= \int_{\mathbb{R}} r_1\phi(y)(1 - \phi(y) - a_1\psi(y))g_1(t - c - y) dy \\ &\geq \int_{\mathbb{R}} [r_1e^{\lambda_1y} - r_1Me^{\eta\lambda_1y}]g_1(t - c - y) dy \\ &\quad - \int_{\mathbb{R}} [r_1e^{2\lambda_1y} + r_1a_1e^{(\lambda_1+\lambda_2)y}]g_1(t - c - y) dy \\ &= e^{\lambda_1t} - M\Lambda_1(\eta\lambda_1, c)e^{\eta\lambda_1t} - \Lambda_1(2\lambda_1, c)e^{2\lambda_1t} - a_1\Lambda_1(\lambda_1 + \lambda_2, c)e^{(\lambda_1+\lambda_2)t} \\ &\geq e^{\lambda_1t} - Me^{\eta\lambda_1t} \\ &= \underline{\phi}(t) \end{aligned}$$

provided that

$$M \geq 1 + \frac{\Lambda_1(2\lambda_1, c) + a_1\Lambda_1(\lambda_1 + \lambda_2, c)}{1 - \Lambda_1(\eta\lambda_1, c)} > 1.$$

In a similar way, when

$$M \geq 1 + \frac{\Lambda_2(2\lambda_2, c) + a_2\Lambda_2(\lambda_1 + \lambda_2, c)}{1 - \Lambda_2(\eta\lambda_1, c)} > 1$$

is true, we have

$$P_2(\phi, \psi)(t) \geq \underline{\psi}(t), \quad t \in \mathbb{R}.$$

The proof is complete. □

Lemma 2.6 *Assume that (A3)-(A4) hold. Then $P : \Gamma \rightarrow \Gamma$ is complete continuous in the sense of the decay norm $|\cdot|_\mu$.*

The proof is provided by Lin [20, Lemma 3.4] and we omit it here.

Theorem 2.7 *Assume that (A3)-(A4) hold. Then for each $c > c^*$, (1.3) has a positive solution (ϕ, ψ) such that*

$$\lim_{t \rightarrow -\infty} (\phi(t), \psi(t)) = (0, 0), \quad \lim_{t \rightarrow -\infty} (\phi(t)e^{-\lambda_1 t}, \psi(t)e^{-\lambda_2 t}) = (1, 1)$$

and

$$0 < \phi(t) \leq l_1, \quad 0 < \psi(t) \leq l_2, \quad t \in \mathbb{R}.$$

The result is evident by Schauder's fixed point theorem and Lemmas 2.4-2.6, and we omit the proof here.

3 Asymptotic behavior of traveling wave solutions

In this section, we investigate the asymptotic behavior of positive solutions of (1.3). We first consider the initial value problem

$$\begin{cases} u_{n+1}(x) = \int_{\mathbb{R}} b(u_n(y))g(x-y) dy, \\ u_0(x) = u(x), \quad x \in \mathbb{R}, n+1 \in \mathbb{N}, \end{cases} \quad (3.1)$$

where $b(u)$ satisfies

- (U1) for some $U^+ > 0$, $b(u) : [0, U^+] \rightarrow [0, U^+]$ is Lipschitz continuous and monotone increasing;
- (U2) there exists $u^+ \in (0, U^+]$ such that $b(u) > u$, $u \in (0, u^+)$, $b(u^+) = u^+$; if $U^+ > u^+$, then $b(u) < u$, $u \in (u^+, U^+)$;
- (U3) $b(0) = 0$;
- (U4) $b'(0) > 1$ and $b(u) < b'(0)u$, $u \in (0, U^+]$.

In literature, (U1)-(U4) imply the comparison principle, monostability, and persistence in (3.1). More precisely, by Hsu and Zhao [4], we have the following conclusion.

Lemma 3.1 *Assume that g satisfies (A1)-(A2).*

- (1) *If $u(x)$ is bounded and uniformly continuous such that*

$$0 < u(x) \leq U^+, \quad x \in \mathbb{R},$$

then $\lim_{n \rightarrow \infty} u_n(0) = u^+$. Moreover, let $C > 0$ satisfy

$$\begin{cases} \int_{\mathbb{R}} b'(0)e^{\lambda y - \lambda c} g(y) dy = 1 \text{ has no real root } \lambda \geq 0 \text{ if } c < C, \\ \int_{\mathbb{R}} b'(0)e^{\lambda y - \lambda c} g(y) dy = 1 \text{ has positive roots if } c \geq C, \end{cases}$$

then for each $c < C$, we have

$$\liminf_{n \rightarrow \infty} \inf_{|x| < cn} u_n(x) = \limsup_{n \rightarrow \infty} \sup_{|x| < cn} u_n(x) = u^+.$$

(2) If $\bar{u}_n(x)$ is bounded and uniformly continuous for each n such that

$$0 \leq \bar{u}_n(x) \leq U^+, \quad n + 1 \in \mathbb{N}, x \in \mathbb{R}$$

and

$$\begin{cases} \bar{u}_{n+1}(x) \geq (\leq) \int_{\mathbb{R}} b(\bar{u}_n(y))g(x - y) dy, \\ \bar{u}_0(x) \geq (\leq) u(x), \quad x \in \mathbb{R}, n + 1 \in \mathbb{N}, \end{cases}$$

then $\bar{u}_n(x) \geq (\leq) u_n(x)$, $x \in \mathbb{R}$, $n \in \mathbb{N}$.

Theorem 3.2 Assume that (A3)-(A5) hold. If (ϕ, ψ) is given by Theorem 2.7, then (1.4) is true.

Proof Note that a traveling wave solution is a special invariant solution; then $\phi(t) = X_n(x)$ satisfies the following inequality:

$$\begin{cases} X_{n+1}(x) \geq \int_{\mathbb{R}} r_1 X_n(y)[1 - X_n(y) - a_1 l_2]g_1(x - y) dy, \\ X_0(x) = \phi(x) \in [0, l_1] \end{cases}$$

for all $n + 1 \in \mathbb{N}$. Define

$$\underline{b}(u) = \inf_{v \in [u, l_1]} \{r_1 v[1 - v - a_1 l_2]\}.$$

Clearly, $\underline{b}(u)$ admits the following properties:

- (b1) $\underline{b}(u) : [0, l_1] \rightarrow [0, l_1]$ is Lipschitz continuous and monotone increasing;
- (b2) $\underline{b}(0) = 0$, $\underline{b}'(0) = r_1(1 - a_1 l_2) > 1$;
- (b3) there exists $\underline{u}^+ \in (0, l_1)$ such that $\underline{b}(\underline{u}^+) = \underline{u}^+$ and

$$\underline{b}(u) > u, \quad u \in (0, \underline{u}^+);$$

- (b4) if $\underline{u}^+ < l_1$, then $\underline{b}(u) < u$, $u \in (\underline{u}^+, l_1]$.

Therefore, $\underline{b}(u)$ satisfies (U1)-(U4) and Lemma 3.1 implies that

$$\liminf_{n \rightarrow \infty} X_n(0) \geq \underline{u}^+ > 0$$

and

$$\frac{r_1}{4} \geq \liminf_{t \rightarrow \infty} \phi(t) \geq \underline{u}^+ > 0.$$

In a similar way, we have

$$\frac{r_2}{4} \geq \liminf_{t \rightarrow \infty} \psi(t) > 0.$$

The proof is complete. □

Theorem 3.3 *Assume that (A3)-(A7) hold. If (ϕ, ψ) is given by Theorem 2.7, then (1.5) is true.*

Proof By what we have done, we see that

$$\frac{r_1}{4} \geq \limsup_{t \rightarrow \infty} \phi(t) \geq \liminf_{t \rightarrow \infty} \phi(t) > 0$$

and

$$\frac{r_2}{4} \geq \limsup_{t \rightarrow \infty} \psi(t) \geq \liminf_{t \rightarrow \infty} \psi(t) > 0.$$

Define

$$\liminf_{t \rightarrow \infty} \phi(t) = \underline{u}, \quad \limsup_{t \rightarrow \infty} \phi(t) = \bar{u}$$

and

$$\liminf_{t \rightarrow \infty} \psi(t) = \underline{v}, \quad \limsup_{t \rightarrow \infty} \psi(t) = \bar{v}.$$

Applying the dominated convergence theorem in P when $t \rightarrow \infty$, then the monotone condition (A7) implies that

$$\bar{u} + a_1 \underline{v} \leq 1 - \frac{1}{r_1}, \tag{3.2}$$

$$\underline{v} + a_2 \bar{u} \geq 1 - \frac{1}{r_2}, \tag{3.3}$$

$$\underline{u} + a_1 \bar{v} \geq 1 - \frac{1}{r_1}, \tag{3.4}$$

$$\bar{v} + a_2 \underline{u} \leq 1 - \frac{1}{r_2}. \tag{3.5}$$

By (3.2) and (3.4), we have

$$\bar{u} - \underline{u} \leq a_1(\bar{v} - \underline{v}). \tag{3.6}$$

Furthermore, (3.3) and (3.5) lead to

$$\bar{v} - \underline{v} \leq a_2(\bar{u} - \underline{u}). \tag{3.7}$$

Since (A6) implies that $a_1 a_2 < 1$, then we have

$$\bar{u} = \underline{u}, \quad \bar{v} = \underline{v}$$

by (3.6) and (3.7). Again by (A6), we see that

$$\bar{u} = \underline{u} = k_1, \quad \bar{v} = \underline{v} = k_2.$$

The proof is complete. □

Remark 3.4 Although we did not construct the contracting rectangle (see [20]) in this paper, the proof of Theorem 3.3 was motivated by Lin [20, Sections 4-5]. Of course, if a model involves more unknown functions, it is difficult to obtain the asymptotic behavior of traveling wave solutions by the inequalities similar to (3.6) and (3.7).

4 Nonexistence of monotone traveling wave solutions

In this section, we confirm that c^* is the minimal wave speed of monotone invasion traveling wave solutions by presenting the following nonexistence of monotone traveling wave solutions.

Theorem 4.1 *Assume that (A3)-(A6) hold. If $c < c^*$, then there is no strict monotone $(\phi(t), \psi(t))$ satisfying (1.3)-(1.4) and*

$$0 < \phi(t) < k_1, \quad 0 < \psi(t) < k_2. \tag{4.1}$$

Proof Without loss of generality, we assume that $a_1 > 0$ and

$$\Lambda_1(\lambda, c) > 1, \quad c \in (0, c^*), \lambda \geq 0.$$

Were the statement false, then there exists $c_1 \in (0, c^*)$ such that there is $(\phi(t), \psi(t))$ satisfying (1.3)-(1.4) and (4.1).

Since $\phi(t), \psi(t)$ are monotone increasing, then

$$t \rightarrow \phi(t), \quad t \rightarrow \psi(t)$$

are one to one and there is a continuous mapping h such that

$$h(\phi(t)) = r_1(1 - \phi(t) - a_1 \psi(t)),$$

in which $h: [0, k_1] \rightarrow [1, r_1]$ satisfying

(H1) $h(0) = r_1$;

(H2) $h: [0, k_1] \rightarrow [1, r_1]$ is continuous and strictly monotone decreasing;

(H3) $h(k_1) = 1$.

At the same time, h also leads to

$$\phi(t + c_1) = \int_{\mathbb{R}} \phi(y)h(\phi(y))g_1(t - y) dy, \quad t \in \mathbb{R}.$$

Let $-2x = (c_1 + c^*)n$, then

$$t = x + c_1 n \rightarrow -\infty, \quad n \rightarrow \infty$$

such that

$$\phi(t) \rightarrow 0, \quad t \rightarrow -\infty.$$

Note that $\phi(t) = u_n(x)$ is a special solution to

$$\begin{cases} u_{n+1}(x) = \int_{\mathbb{R}} h(u_n(y))g_1(x-y) dy, \\ u_0(x) = \phi(x) > 0, \quad x \in \mathbb{R}, n+1 \in \mathbb{N}. \end{cases} \quad (4.2)$$

Define $\underline{h}(u) = \inf_{v \in [u, k_1]} h(v)$, then $h(u)$ is monotone and continuous for $u \in [0, k_1]$. Moreover, applying Lemma 3.1 to (4.2), we see that $C = c^*$. Again by Lemma 3.1, we obtain

$$\liminf_{n \rightarrow \infty, -2x = (c_1 + c^*)n} u_n(x) > 0$$

and

$$\liminf_{t \rightarrow -\infty} \phi(t) > 0.$$

A contradiction occurs. The proof is complete. □

Before ending this paper, we make the following remark.

Remark 4.2 By what we have done, c^* is the minimal wave speed of monotone traveling wave solutions of (1.1). However, when the wave speed is c^* , the existence and nonexistence of nontrivial traveling wave solutions remain open. At the same time, if we remove the monotonicity of traveling wave solutions in Theorem 4.1, then we believe that the result still holds. Clearly, for these two problems, we cannot discuss them directly by the methods similar to those in this paper, and we shall consider these problems in our future studies.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors contributed equally to the manuscript. All authors read and approved the final manuscript.

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Acknowledgements

The first author was supported by NSF of Gansu Province of China (No. 1208RJYA004) and the Development Program for Outstanding Young Teachers in Lanzhou University of Technology (No. 1010ZCX019), the second author was supported by National Undergraduate Training Programs for Innovation and Entrepreneurship (No. 201310730086).

Received: 14 June 2013 Accepted: 4 June 2014 Published: 18 Jul 2014

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10.1186/1687-1847-2014-173

Cite this article as: Pan and Yang: Traveling wave solutions in a Lotka-Volterra type competition recursion. *Advances in Difference Equations* 2014, **2014**:173

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