Advances in Difference Equations
a SpringerOpen Journal

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Eigenvalue of boundary value problem for nonlinear singular third-order q-difference equations

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Abstract
In this paper, we establish the existence of positive solutions of a boundary value problem for nonlinear singular third-order q-difference equations $D_q^3 u(t) + \lambda a(t)f(u(t)) = 0$, $t \in I_q$, $u(0) = 0$, $D_q u(0) = 0$, $\alpha D_q u(1) + \beta D_q^2 u(1) = 0$, by using Krasnoselskii's fixed-point theorem on a cone, where λ is a positive parameter. Finally, we give an example to demonstrate the use of the main result of this paper. The conclusions in this paper essentially extend and improve known results.

Keywords: q-difference equations; positive solutions; singular boundary value problem; Krasnoselskii's fixed-point theorem

1 Introduction

The q-difference equations initiated in the beginning of the 20th century [1–4], is a very interesting field in difference equations. In the last few decades, it has evolved into a multidisciplinary subject and plays an important role in several fields of physics, such as cosmic strings and black holes [5], conformal quantum mechanics [6], and nuclear and high-energy physics [7]. For some recent work on q-difference equations, we refer the reader to [8–12]. However, the theory of boundary value problems (BVPs) for nonlinear q-difference equations is still in an early stage and many aspects of this theory need to be explored. To the best of our knowledge, for the BVPs of nonlinear third-order q-difference equations, a few works were done, see [13, 14] and the references therein.

Recently, in [15], El-Shahed has studied the existence of positive solutions for the following nonlinear singular third-order BVP:

$$\begin{cases} u'''(t) + \lambda a(t) f(u(t)) = 0, \quad 0 \le t \le 1, \\ u(0) = u'(0) = 0, \qquad \alpha u'(1) + \beta u''(1) = 0, \end{cases}$$

by Krasnoselskii's fixed-point theorem on a cone.

More recently, in [13] Ahmad has studied the existence of positive solutions for the following nonlinear BVP of third-order q-difference equations:

$$\begin{cases} D_q^3 u(t) = f(t, u(t)), \quad 0 \le t \le 1, \\ u(0) = 0, \qquad D_q u(0) = 0, \qquad u(1) = 0, \end{cases}$$

by Leray-Schauder degree theory and some standard fixed-point theorems.

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Motivated by the work above, in this paper, we will study the following BVP of nonlinear singular third-order q-difference equations:

$$\begin{cases} D_q^3 u(t) + \lambda a(t) f(u(t)) = 0, & t \in I_q, \\ u(0) = 0, \quad D_q u(0) = 0, \quad \alpha D_q u(1) + \beta D_q^2 u(1) = 0, \end{cases} \tag{1.1}$$

where $\lambda > 0$ is a positive parameter, $a : (0,1) \to [0,\infty)$ is continuous and $0 < \int_0^1 a(t)\,d_q t < \infty$, f is a continuous function, $I_q = \{q^n : n \in N\} \cup \{0,1\}$, $q \in (0,1)$ is a fixed constant, and $\alpha, \beta \geq 0$, $\alpha + \beta > 0$.

Obviously, when $q \to 1^-$, BVP (1.1) reduces to the standard BVP in [15].

Throughout this paper, we always suppose the following conditions to hold:

(C_1) $f \in C([0,1],[0,+\infty))$;
(C_2) $\alpha, \beta \geq 0$, $\alpha + \beta > 0$ and $\frac{\alpha - \beta}{\alpha + \beta} \leq q$.

2 Preliminary results

In this section, firstly, let us recall some basic concepts of q-calculus [16, 17].

Definition 2.1 For $0 < q < 1$, we define the q-derivative of a real-value function f as

$$D_q f(t) = \frac{f(t) - f(qt)}{(1-q)t}, \quad t \in I_q - \{0\}, \qquad D_q f(0) = \lim_{t \to 0} D_q f(t).$$

Note that $\lim_{q \to 1^-} D_q f(t) = f'(t)$.

Definition 2.2 The higher-order q-derivatives are defined inductively as

$$D_q^0 f(t) = f(t), \qquad D_q^n f(t) = D_q D_q^{n-1} f(t), \quad n \in N.$$

For example, $D_q(t^k) = [k]_q t^{k-1}$, where k is a positive integer and the bracket $[k]_q = (q^k - 1)/(q-1)$. In particular, $D_q(t^2) = (1+q)t$.

Definition 2.3 The q-integral of a function f defined in the interval $[a,b]$ is given by

$$\int_a^x f(t)\,d_q t := \sum_{n=0}^{\infty} x(1-q)q^n f(xq^n) - af(aq^n), \quad x \in [a,b],$$

and for $a = 0$, we denote

$$I_q f(x) = \int_0^x f(t)\,d_q t = \sum_{n=0}^{\infty} x(1-q)q^n f(xq^n),$$

then

$$\int_a^b f(t)\,d_q t = \int_0^b f(t)\,d_q t - \int_0^a f(t)\,d_q t.$$

Similarly, we have

$$I_q^0 f(t) = f(t), \qquad I_q^n f(t) = I_q I_q^{n-1} f(t), \quad n \in N.$$

Observe that

$$D_qI_qf(x) = f(x),$$

and if f is continuous at $x = 0$, then $I_qD_qf(x) = f(x) - f(0)$.

In q-calculus, the product rule and integration by parts formula are

$$D_q(gh)(t) = D_qg(t)h(t) + g(qt)D_qh(t), \tag{2.1}$$

$$\int_0^x f(t)D_qg(t)\,d_qt = \bigl[f(t)g(t)\bigr]_0^x - \int_0^x D_qf(t)g(qt)\,d_qt. \tag{2.2}$$

Remark 2.1 In the limit $q \to 1^-$, the above results correspond to their counterparts in standard calculus.

Definition 2.4 Let E be a real Banach space. A nonempty closed convex set $P \subset E$ is called a cone if it satisfies the following two conditions:

(i) $x \in P$, $\lambda \geq 0$ implies $\lambda x \in P$;
(ii) $x \in P$, $-x \in P$ implies $x = 0$.

Theorem 2.1 (Krasnoselskii) [18] *Let E be a Banach space and let $K \in E$ be a cone in E. Assume that Ω_1 and Ω_2 are open subsets of E with $0 \in \Omega_1$ and $\overline{\Omega}_1 \subset \Omega_2$. Let $T : K \cap (\overline{\Omega}_2 \backslash \Omega_1) \to K$ be a completely continuous operator. In addition, suppose either*

(H_1) $\|Tu\| \leq \|u\|$, $\forall u \in K \cap \partial\Omega_1$ *and* $\|Tu\| \geq \|u\|$, $\forall u \in K \cap \partial\Omega_2$ *or*
(H_2) $\|Tu\| \leq \|u\|$, $\forall u \in K \cap \partial\Omega_2$ *and* $\|Tu\| \geq \|u\|$, $\forall u \in K \cap \partial\Omega_1$

holds. Then T has a fixed point in $K \cap (\overline{\Omega}_2 \backslash \Omega_1)$.

Lemma 2.1 *Let $y \in C[0,1]$, then the BVP*

$$\begin{cases} D_q^3u(t) + y(t) = 0, \quad t \in I_q, \\ u(0) = 0, \qquad D_qu(0) = 0, \qquad \alpha D_qu(1) + \beta D_q^2u(1) = 0, \end{cases} \tag{2.3}$$

has a unique solution

$$u(t) = \int_0^1 G(t,s;q)y(s)\,d_qs,$$

where

$$G(t,s;q) = \frac{1}{(1+q)(\alpha+\beta)} \begin{cases} \alpha t^2(1-qs) + \beta t^2 - (t-qs)(t-q^2s)(\alpha+\beta), & 0 \leq s \leq t \leq 1, \\ \alpha t^2(1-qs) + \beta t^2, & 0 \leq t \leq s \leq 1. \end{cases}$$

Proof Integrate the q-difference equation from 0 to t, we get

$$D_q^2u(t) = -\int_0^t y(s)\,d_qs + a_2. \tag{2.4}$$

Integrate (2.4) from 0 to t, and change the order of integration, we have

$$D_qu(t) = -\int_0^t (t-qs)y(s)\,d_qs + a_2t + a_1. \tag{2.5}$$

Integrating (2.5) from 0 to t, and changing the order of integration, we obtain

$$u(t) = -\int_0^t \left(\frac{t^2 + q^3 s^2}{1+q} - qts\right) y(s)\, d_q s + \frac{a_2}{1+q} t^2 + a_1 t + a_0, \tag{2.6}$$

where a_2, a_1, a_0 are arbitrary constants. Using the boundary conditions $u(0) = 0$, $D_q u(0) = 0$, $\alpha D_q u(1) + \beta D_q^2 u(1) = 0$ in (2.6), we find that $a_0 = a_1 = 0$, and

$$a_2 = \frac{1}{\alpha + \beta}\left(\alpha \int_0^1 (1 - qs) y(s)\, d_q s + \beta \int_0^1 y(s)\, d_q s\right).$$

Substituting the values of a_2, a_1, and a_0 in (2.6), we obtain

$$\begin{aligned} u(t) &= -\int_0^t \left(\frac{t^2 + q^3 s^2}{1+q} - qts\right) y(s)\, d_q s \\ &\quad + \frac{t^2}{(1+q)(\alpha+\beta)}\left(\alpha \int_0^1 (1 - qs) y(s)\, d_q s + \beta \int_0^1 y(s)\, d_q s\right) \\ &= \int_0^1 G(t,s;q) y(s)\, d_q s, \end{aligned}$$

where

$$G(t,s;q) = \frac{1}{(1+q)(\alpha+\beta)} \begin{cases} \alpha t^2(1-qs) + \beta t^2 - (t-qs)(t-q^2 s)(\alpha+\beta), & 0 \le s \le t \le 1, \\ \alpha t^2(1-qs) + \beta t^2, & 0 \le t \le s \le 1. \end{cases}$$

This completes the proof. □

Remark 2.2 For $q \to 1$, equation (2.6) takes the form

$$u(t) = -\frac{1}{2}\int_0^t (t-s)^2 y(s)\, d_q s + \frac{a_2}{2} t^2 + a_1 t + a_0,$$

which is the solution of a classical third-order ordinary differential equation $u'''(t) + y(t) = 0$ and the associated form of Green's function for the classical case is

$$G(t,s) = \frac{1}{2(\alpha+\beta)} \begin{cases} \alpha t^2(1-s) + \beta t^2 - (t-s)^2(\alpha+\beta), & 0 \le s \le t \le 1, \\ \alpha t^2(1-s) + \beta t^2, & 0 \le t \le s \le 1. \end{cases}$$

It is obvious that, when (C_2) holds, $G(t,s;q) \ge 0$, and $G(t,s;q) \le G(1,s;q)$, $0 \le t,s \le 1$.

Lemma 2.2 *Let* (C_2) *hold, then* $G(t,s;q) \ge g(t) G(1,s;q)$ *for* $0 \le t,s \le 1$, *where* $g(t) = \frac{4\beta}{5(\alpha+\beta)} t^2$.

Proof If $t \le s$, then

$$\begin{aligned} \frac{G(t,s;q)}{G(1,s;q)} &= \frac{\frac{\alpha t^2(1-qs)}{(1+q)(\alpha+\beta)} + \frac{\beta t^2}{(1+q)(\alpha+\beta)}}{\frac{\alpha(1-qs)}{(1+q)(\alpha+\beta)} + \frac{\beta}{(1+q)(\alpha+\beta)}} = \frac{t^2 - \frac{\alpha qs}{\alpha+\beta} t^2}{1 - \frac{\alpha qs}{\alpha+\beta}} \\ &\ge t^2 - \frac{\alpha}{\alpha+\beta} t^2 = \frac{\beta}{\alpha+\beta} t^2 \ge \frac{4\beta}{5(\alpha+\beta)} t^2. \end{aligned}$$

If $t \geq s$, then

$$\begin{aligned}\frac{G(t,s;q)}{G(1,s;q)} &= \frac{\frac{\alpha t^2(1-qs)}{(1+q)(\alpha+\beta)} + \frac{\beta t^2}{(1+q)(\alpha+\beta)} - \frac{t^2+q^3s^2}{1+q} + qts}{\frac{\alpha(1-qs)}{(1+q)(\alpha+\beta)} + \frac{\beta}{(1+q)(\alpha+\beta)} - \frac{1+q^3s^2}{1+q} + qs} = \frac{-\frac{\alpha qs}{\alpha+\beta}t^2 - q^3s^2 + (1+q)qts}{-\frac{\alpha qs}{\alpha+\beta} - q^3s^2 + (1+q)qs} \\ &\geq \frac{(1+q)qt^2 - \frac{\alpha qs}{\alpha+\beta}t^2 - q^3s^2}{(1+q)qs - q^3s^2} = \frac{(1+q)t^2 - \frac{\alpha s}{\alpha+\beta}t^2 - q^2s^2}{(1+q)s - q^2s^2} \\ &\geq \frac{t^2 - \frac{\alpha}{\alpha+\beta}t^2}{1+q-q^2} \geq \frac{4\beta}{5(\alpha+\beta)}t^2.\end{aligned}$$

The proof is complete. □

We consider the Banach space $C_q = C(I_q, R)$ equipped with standard norm $\|u\| = \sup\{|u(t)|, t \in I_q\}$, $u \in C_q$. Define a cone P by

$$P = \{u \in C_q | u(t) \geq 0, u(t) \geq g(t)\|u\|, t \in I_q\}.$$

It is easy to see that if $u \in P$, then $\|u\| = u(1)$.

Define an integral operator $T : P \to C_q$ by

$$Tu(t) = \lambda \int_0^1 G(t,s;q)a(s)f\bigl(u(s)\bigr)\,d_qs, \quad t \in I_q, u \in P. \tag{2.7}$$

Obviously, T is well defined and $u \in P$ is a solution of BVP (1.1) if and only if u is a fixed point of T.

Remark 2.3 By Lemma 2.2, we obtain, for $u \in P$, $Tu(t) \geq 0$ on I_q and

$$\begin{aligned}Tu(t) &= \lambda \int_0^1 G(t,s;q)a(s)f\bigl(u(s)\bigr)\,d_qs \geq \lambda g(t) \int_0^1 G(1,s;q)a(s)f\bigl(u(s)\bigr)\,d_qs \\ &\geq \lambda g(t) \sup_{t\in I_q} \int_0^1 G(t,s;q)a(s)f\bigl(u(s)\bigr)\,d_qs = g(t)\|Tu\|.\end{aligned}$$

Thus $T(P) \subset P$.

We adopt the following assumption:

(C_3) $a(t) \in C((0,1), R^+)$ may be singular at $t = 0,1$, $0 < \int_0^1 a(t)\,d_qt < +\infty$, and $0 < \int_0^1 G(1, s;q)a(t)\,d_qt < +\infty$.

Lemma 2.3 *Assume* (C_1), (C_2), *and* (C_3) *hold, then* $T : P \to P$ *is completely continuous.*

Proof Define the functions $a_n(t)$ for $n \geq 2$ by

$$a_n(t) = \begin{cases} \inf\{a(t), a(\frac{1}{n})\}, & 0 \leq t \leq \frac{1}{n}, \\ a(t), & \frac{1}{n} \leq t \leq 1 - \frac{1}{n}, \\ \inf\{a(t), a(1-\frac{1}{n})\}, & 1 - \frac{1}{n} \leq t \leq 1. \end{cases}$$

Next, for $n \geq 2$, we define the operator $T_n : P \to P$ by

$$T_n u(t) = \lambda \int_0^1 G(t,s;q) a_n(s) f\big(u(s)\big)\, d_q s, \quad t \in I_q, u \in P.$$

Obviously, T_n is completely continuous on P for any $n \geq 2$ by an application of the Ascoli-Arzelá theorem. Denote $B_K = \{u \in P : \|u\| \leq K\}$. Then T_n converges uniformly to T as $n \to \infty$. In fact, for any $t \in I_q$, for each fixed $K > 0$ and $u \in B_K$, from (C_1), we obtain

$$\begin{aligned}
\big|T_n u(t) - Tu(t)\big| &= \left|\lambda \int_0^1 G(t,s;q)\big[a(s) - a_n(s)\big] f\big(u(s)\big)\, d_q s\right| \\
&\leq \lambda \int_0^{\frac{1}{n}} G(1,s;q)\big|a(s) - a_n(s)\big| f\big(u(s)\big)\, d_q s \\
&\quad + \lambda \int_{\frac{1}{n}}^{1-\frac{1}{n}} G(1,s;q)\big|a(s) - a_n(s)\big| f\big(u(s)\big)\, d_q s \\
&\quad + \lambda \int_{1-\frac{1}{n}}^{1} G(1,s;q)\big|a(s) - a_n(s)\big| f\big(u(s)\big)\, d_q s \to 0 \quad (n \to \infty),
\end{aligned}$$

where we have used the fact that $G(t,s;q) \geq 0$, and $G(t,s;q) \leq G(1,s;q)$, $0 \leq t,s \leq 1$. Hence, T_n converges uniformly to T as $n \to \infty$, and therefore T is completely continuous also. This completes the proof. □

3 Main results

In this section, we will apply Krasnoselskii's fixed-point theorem to the eigenvalue problem (1.1). First, we define some important constants:

$$\begin{aligned}
&A_q = \int_0^1 G(1,s;q) a(s) g(s)\, d_q s, \qquad B_q = \int_0^1 G(1,s;q) a(s)\, d_q s, \\
&F_0 = \lim_{u \to 0^+} \sup \frac{f(u)}{u}, \qquad f_0 = \lim_{u \to 0^+} \inf \frac{f(u)}{u}, \\
&F_\infty = \lim_{u \to +\infty} \sup \frac{f(u)}{u}, \qquad f_\infty = \lim_{u \to +\infty} \inf \frac{f(u)}{u}.
\end{aligned}$$

Here we assume that $\frac{1}{A_q f_\infty} = 0$ if $f_\infty = \infty$ and $\frac{1}{B_q F_0} = \infty$ if $F_0 = 0$ and $\frac{1}{A_q f_0} = 0$ if $f_0 = \infty$ and $\frac{1}{B_q F_\infty} = \infty$ if $F_\infty = 0$.

The main result of this paper is the following.

Theorem 3.1 *Suppose that* (C_1), (C_2) *and* (C_3) *hold and* $A_q f_\infty > B_q F_0$. *Then for each* $\lambda \in (\frac{1}{A_q f_\infty}, \frac{1}{B_q F_0})$, *BVP* (1.1) *has at least one positive solution.*

Proof By the definition of F_0, we see that there exists an $l_1 > 0$, such that $f(u) \leq (F_0 + \varepsilon)u$ for $0 \leq u \leq l_1$. If $u \in P$ with $\|u\| = l_1$, we have

$$\|Tu\| = Tu(1) = \lambda \int_0^1 G(1,s;q) a(s) f\big(u(s)\big)\, d_q s \leq \lambda(F_0 + \varepsilon)\|u\| B_q.$$

Choose $\varepsilon > 0$ sufficiently small such that $\lambda(F_0 + \varepsilon)B_q \leq 1$. Then we obtain $\|Tu\| \leq \|u\|$. Thus if we let $\Omega_1 = \{u \in C_q \mid \|u\| < l_1\}$, then $\|Tu\| \leq \|u\|$ for $u \in P \cap \partial\Omega_1$.

From the definition of f_∞, we see that there exist an $l_3 > 0$ and $l_3 > l_1$, such that $f(u) \geq (f_\infty - \varepsilon)u$ for $u > l_2$. Let $l_2 > l_3$, if $u \in P$ with $\|u\| = l_2$ we have

$$\begin{aligned}\|Tu\| &= Tu(1) = \lambda \int_0^1 G(1,s;q)a(s)f\big(u(s)\big)\,d_qs \\ &\geq \lambda \int_0^1 G(1,s;q)a(s)g(s)f\big(u(s)\big)\,d_qs \geq \lambda(f_\infty - \varepsilon)\|u\|A_q.\end{aligned}$$

Choose $\varepsilon > 0$ sufficiently small such that $\lambda(f_\infty - \varepsilon)A_q \geq 1$. Then we have $\|Tu\| \geq \|u\|$. Let $\Omega_2 = \{u \in C_q \mid \|u\| < l_2\}$, then $\Omega_1 \subset \overline{\Omega}_2$ and $\|Tu\| \geq \|u\|$ for $u \in P \cap \partial\Omega_2$.

Condition (H_1) of Krasnoselskii's fixed-point theorem is satisfied. Hence, by Theorem 2.1, the result of Theorem 3.1 holds. This completes the proof of Theorem 3.1. □

Theorem 3.2 *Suppose that* (C_1), (C_2) *and* (C_3) *hold and* $A_qf_0 > B_qF_\infty$. *Then for each* $\lambda \in (\frac{1}{A_qf_0}, \frac{1}{B_qF_\infty})$, *BVP* (1.1) *has at least one positive solution.*

Proof It is similar to the proof of Theorem 3.1. □

Theorem 3.3 *Suppose that* (C_1), (C_2) *and* (C_3) *hold and* $\lambda B_qf(u) < u$ *for* $u \in (0, +\infty)$. *Then BVP* (1.1) *has no positive solution.*

Proof Assume to the contrary that u is a positive solution of BVP (1.1). Then

$$\begin{aligned}u(1) &= \lambda \int_0^1 G(1,s;q)a(s)f\big(u(s)\big)\,d_qs < \frac{1}{B_q}\int_0^1 G(1,s;q)a(s)u(s)\,d_qs \\ &\leq \frac{u(1)}{B_q}\int_0^1 G(1,s;q)a(s)\,d_qs = u(1).\end{aligned}$$

This is a contradiction and completes the proof. □

Theorem 3.4 *Suppose that* (C_1), (C_2) *and* (C_3) *hold and* $\lambda A_qf(u) > u$ *for* $u \in (0, +\infty)$. *Then BVP* (1.1) *has no positive solution.*

Proof It is similar to the proof of Theorem 3.3. □

4 Example

Consider the following BVP:

$$\begin{cases} D_{\frac{1}{2}}^3 u(t) + \lambda t^{-\frac{1}{2}}\frac{10u^2+u}{u+1}(5 + \sin u) = 0, \quad t \in I_q, \\ u(0) = 0, \qquad D_{\frac{1}{2}}u(0) = 0, \qquad D_{\frac{1}{2}}u(1) + 3D_{\frac{1}{2}}^2u(1) = 0. \end{cases} \tag{4.1}$$

Then $F_0 = 6$, $f_0 = 4$, $F_\infty = 60$, $f_\infty = 40$, and $4u \leq f(u) \leq 60u$. By direct calculations, we obtain $A_q = 0.110963$ and $B_q = 0.271661$. From Theorem 3.1 we see that if $\lambda \in (0.225299, 0.613510)$ then the problem (4.1) has a positive solution. From Theorem 3.3 we see that if $\lambda < 0.061351$ then the problem (4.1) has no positive solution. By Theorem 3.4 we see that if $\lambda > 2.252986$ then the problem (4.1) has no positive solution.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions

Each of the authors, CY and JW contributed to each part of this work equally and read and approved the final version of the manuscript.

Acknowledgements

This work was supported by the Natural Science Foundation of China (10901045), (11201112) and (61304106), the Natural Science Foundation of Hebei Province (A2013208147) and (A2011208012).

Received: 4 October 2013 Accepted: 26 December 2013 Published: 16 Jan 2014

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10.1186/1687-1847-2014-21

Cite this article as: Yu and Wang: Eigenvalue of boundary value problem for nonlinear singular third-order q-difference equations. *Advances in Difference Equations* 2014, 2014:21

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