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On the existence of a mild solution for impulsive hybrid fractional differential equations

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Abstract

This paper is motivated by some recent contributions on the existence of solution of impulsive fractional differential equations and the theory of fractional hybrid differential equations by Agarwal, Ahmad, Baleanu, Benchohra, Fečkan, Nieto, Sun, Bai, Zhou, Zhang and Wang. Here, we derive new existence results of a mild solution of impulsive hybrid fractional differential equations. Finally, an example is given to illustrate the result.

Keywords: Caputo fractional derivative; impulsive hybrid differential equations; existence

1 Introduction

The study of differential equations of fractional order is motivated by the intensive development of the theory of fractional calculus itself (see [1–7]) and the application of fractional differential equations in the modeling of many physical phenomena. There have been many works on the theory of fractional calculus and applications of it. Fractional differential equations, including Riemann-Liouville fractional derivative or Caputo fractional derivative, have received more and more attention (see [8–18]).

In recent years, hybrid differential equations have attracted much attention. The theory of hybrid differential equations has been developed, and we refer the readers to the articles [19–22]. The authors [23] discussed the following fractional hybrid differential equations involving Riemann-Liouville differential operators:

$$\begin{cases} {}^{L}D^{q}_{0,t}\left[\frac{u(t)}{f(t,u(t))}\right] = g(t,u(t)), & \text{a.e. } t \in J := [0,T], \\ u(0) = 0, \end{cases} \tag{1}$$

where ${}^{L}D^{q}_{0,t}$ is the Riemann-Liouville fractional derivative of order $q \in (0,1)$ with the lower limit zero, $f \in C(J \times \mathbb{R}, \mathbb{R} \setminus \{0\})$ and $g \in C(J \times \mathbb{R}, \mathbb{R})$. They developed existence of solutions under mixed Lipschitz and Carathéodory conditions. Moreover, they have established some fundamental fractional differential inequalities and the comparison principle. Some recent papers have treated the problem of the existence of solutions for impulsive fractional differential equations.

The authors [24] considered the following basic impulsive Cauchy problems:

$$\begin{cases} {}^cD^q_{0,t}u(t) = f(t,u(t)), & t \in J' := J \setminus \{t_1,\ldots,t_m\}, \\ \Delta u(t_k) = I_k(u(t_k^-)), & k = 1,2,\ldots,m, \\ u(0) = u_0, \end{cases} \tag{2}$$

where ${}^cD^q_{0,t}$ is the generalized Caputo fractional derivative of order $q \in (0,1)$ with the lower limit zero and $I_k : \mathbb{R} \to \mathbb{R}$ and t_k satisfy $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = T$, $u(t_k^+) = \lim_{\varepsilon \to 0^+} u(t_k + \varepsilon)$ and $u(t_k^-) = \lim_{\varepsilon \to 0^-} u(t_k + \varepsilon)$ represent the right and left limits of $u(t)$ at $t = t_k$. They established some sufficient conditions for the existence of solutions.

In the recent paper [25], Herzallah obtained the existence of a mild solution for the fractional order hybrid differential equations:

$$\begin{cases} {}^cD^q_{0,t}[\frac{u(t)}{f(t,u(t))}] = g(t,u(t)), & \text{a.e. } t \in J, \\ u(0) = u_0 \in \mathbb{R}. \end{cases} \tag{3}$$

In the present paper, we study the following impulsive hybrid fractional differential equations (IHFDE):

$$\begin{cases} {}^cD^q_{0,t}[\frac{u(t)}{f(t,u(t))}] = g(t,u(t)), & t \in J' := J \setminus \{t_1,\ldots,t_m\}, \\ u(t_k^+) = u(t_k^-) + I_k(u(t_k^-)), & k = 1,2,\ldots,m, \\ u(0) = u_0, \end{cases} \tag{4}$$

where ${}^cD^q_{0,t}$ is the generalized Caputo fractional derivative of order $q \in (0,1)$ with the lower limit zero, $f \in C(J \times \mathbb{R}, \mathbb{R} \setminus \{0\})$ and $g \in C(J \times \mathbb{R}, \mathbb{R})$.

This paper is arranged as follows. In Section 2, we recall some concepts and some fractional calculation law and establish preparation results. In Section 3, we give the main results based on the Dhage fixed point theorem. In Section 4, we give an example to demonstrate the application of our main result.

2 Preliminaries

In this section, we recall some basic definitions and properties of the fractional calculus theory and preparation results. Throughout this paper, J_1 denotes the interval $[t_0, t_1]$ and J_{k+1} denotes the interval $(t_k, t_{k+1}]$, $k = 1,2,\ldots,m$. Let $C(J,\mathbb{R})$ be the Banach algebra of all continuous functions from J into $\mathbb{R}$ with the norm $\|u\|_c = \sup\{|u(t)| : t \in J\}$ for $u \in C(J,\mathbb{R})$ and with multiplication $(uv)(s) = u(s)v(s)$ for $u, v \in C(J,\mathbb{R})$. Define $PC(J,\mathbb{R}) = \{u : J \to \mathbb{R} : u \in C((t_k, t_{k+1}], \mathbb{R}), k = 0,1,\ldots,m$ and there exist $u(t_k^-)$ and $u(t_k^+)$, $k = 1,2,\ldots,m$, with $u(t_k^-) = u(t_k)\}$ with the norm $\|u\|_{PC} := \sup\{\|u(t)\| : t \in J\}$ that is a Banach space.

We introduce the following known definitions. For more details, one can see [2].

Definition 2.1 The fractional integral of order γ with the lower limit zero for a function $f : [0,\infty) \to \mathbb{R}$ is defined as $I_t^\gamma f(t) = \frac{1}{\Gamma(\gamma)} \int_0^t \frac{f(s)}{(t-s)^{1-\gamma}}\,dx$, $t > 0$, $\gamma > 0$ provided the right-hand side is point-wise defined on $[0,\infty)$, where $\Gamma(\cdot)$ is the gamma function.

Definition 2.2 The Riemann-Liouville derivative of order γ with the lower limit zero for a function $f : [0,\infty) \to \mathbb{R}$ can be written as ${}^LD^\gamma_{0,t}f(t) = \frac{1}{\Gamma(n-\gamma)} \frac{d^n}{dt^n} \int_0^t \frac{f(s)}{(t-s)^{\gamma+1-n}}\,ds$, $t > 0$, $n - 1 < \gamma < n$.

Definition 2.3 The Caputo derivative of order γ for a function $f : [0,\infty) \to \mathbb{R}$ can be written as

$$^{c}D_{0,t}^{\gamma}f(t) = {}^{L}D_{0,t}^{\gamma}\left[f(t) - \sum_{k=0}^{n-1}\frac{t^k}{k!}f^{(k)}(0)\right], \quad t > 0, n-1 < \gamma < n.$$

Remark (i) We have to explain that we follow the ideas from the recent contributions on impulsive fractional differential equations by Fečkan *et al.* [26, 27].

(ii) In Definition 2.3, the integrable function f can be discontinuous.

(iii) For more details and explanations on such interesting problems, one can refer to [26, Discussions I-V, p.4214] and [27, Remark 2.21(iii)].

Lemma 2.4 ([28]) *Let S be a non-empty, closed convex and bounded subset of the Banach algebra X, and let $A : X \to X$ and $B : S \to X$ be two operators such that*

(a) *A is Lipschitzian with a Lipschitz constant α*;
(b) *B is completely continuous*;
(c) *$u_1 = Au_1Bu_2 \Rightarrow u_1 \in S$ for all $u_2 \in S$*;
(d) *$\alpha M < 1$, where $M = \|B(S)\| = \sup\{\|B(u)\| : u \in S\}$.*

Then the operator equation $AuBu=u$ has a solution in S.

Lemma 2.5 ([24]) *Let $q \in (0,1)$ and $h : [0,T_0] \mapsto \mathbb{R}$ be continuous. A function $u \in C([0,T_0],\mathbb{R})$ is a solution of the fractional integral equation*

$$u(t) = u_0 - \frac{1}{\Gamma(q)}\int_0^a (a-s)^{q-1}h(s)\,ds + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds,$$

if and only if u is a solution of the following fractional Cauchy problems:

$$\begin{cases} ^{c}D_{0,t}^{q}u(t) = h(t), & t \in [0,T_0], \\ u(a) = u_0, & a > 0. \end{cases}$$

We introduce the following hypotheses in what follows.

(H1) The function $u \mapsto \frac{u}{f(t,u)}$ is increasing in $\mathbb{R}$ for every $t \in J_1$.
(H2) There exists a constant $L_k > 0$ such that $|f(t,u_1) - f(t,u_2)| \le L_k|u_1 - u_2|$ for all $t \in J_k$ $(k = 1,2,\ldots,m+1)$ and $u_1,u_2 \in \mathbb{R}$.
(H3) There exists a function $h_k \in L^\infty(J_k)$ $(k = 1,2,\ldots,m+1)$ such that $|g(t,u(t))| \le h_k(t)$, $t \in J_k$ $(k = 1,2,\ldots,m+1)$ for all $u(t) \in \mathbb{R}$.
(H4) $f(t,u(t))$ is continuous on J_k $(k = 1,2,\ldots,m+1)$ for every $u \in PC(J,\mathbb{R})$.

Lemma 2.6 *Assume that hypotheses* (H1) *and* (H4) *hold. Let $q \in (0,1)$ and $h : J \to \mathbb{R}$ be continuous. A function u is a solution of the fractional integral equation*

$$u(t) = \begin{cases} f(t,u(t))[\frac{u_0}{f(0,u_0)} + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds], & t \in J_1, \\ f(t,u(t))[\frac{u_0}{f(0,u_0)} + \frac{I_1(u(t_1^-))}{f(t_1,u(t_1))} + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds], & t \in J_2, \\ f(t,u(t))[\frac{u_0}{f(0,u_0)} + \frac{I_1(u(t_1^-))}{f(t_1,u(t_1))} + \frac{I_2(u(t_2^-))}{f(t_2,u(t_2))} & \\ \quad + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds], & t \in J_3, \\ \vdots & \\ f(t,u(t))[\frac{u_0}{f(0,u_0)} + \sum_{i=1}^{m}\frac{I_i(u(t_i^-))}{f(t_i,u(t_i))} + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds], & t \in J_{m+1}, \end{cases} \tag{5}$$

if and only if u is a solution of the following impulsive problem:

$$\begin{cases} {}^cD^q_{0,t}[\frac{u(t)}{f(t,u(t))}] = h(t), & t \in J', \\ u(t_k^+) = u(t_k^-) + I_k(u(t_k^-)), & k = 1, 2, \dots, m, \\ u(0) = u_0. \end{cases} \tag{6}$$

Proof Assume that u satisfies (6). If $t \in J_1$, then

$${}^cD^q_{0,t}\left[\frac{u(t)}{f(t,u(t))}\right] = h(t), \quad t \in J_1, \tag{7}$$

$$u(0) = u_0. \tag{8}$$

Operating by $I^q_{0,t}$ on both sides of (7), one can obtain

$$\begin{aligned} \frac{u(t)}{f(t,u(t))} &= \frac{u(0)}{f(0,u(0))} + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds \\ &= \frac{u_0}{f(0,u_0)} + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds, \end{aligned}$$

i.e.,

$$u(t) = f\bigl(t,u(t)\bigr)\left[\frac{u_0}{f(0,u_0)} + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds\right].$$

If $t \in J_2$, then

$${}^cD^q_{0,t}\left[\frac{u(t)}{f(t,u(t))}\right] = h(t), \quad t \in J_2, \tag{9}$$

$$u\bigl(t_1^+\bigr) = u\bigl(t_1^-\bigr) + I_1\bigl(u\bigl(t_1^-\bigr)\bigr). \tag{10}$$

According to Lemma 2.5 and the continuity of $f(t,u(t))$, we have

$$\begin{aligned} \frac{u(t)}{f(t,u(t))} &= \frac{u(t_1^+)}{f(t_1,u(t_1))} - \frac{1}{\Gamma(q)}\int_0^{t_1} (t_1-s)^{q-1}h(s)\,ds + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds \\ &= \frac{[u(t_1^-) + I_1(u(t_1^-))]}{f(t_1,u(t_1))} - \frac{1}{\Gamma(q)}\int_0^{t_1} (t_1-s)^{q-1}h(s)\,ds + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds. \end{aligned}$$

Since

$$u\bigl(t_1^-\bigr) = f\bigl(t_1,u(t_1)\bigr)\left[\frac{u_0}{f(0,u_0)} + \frac{1}{\Gamma(q)}\int_0^{t_1} (t_1-s)^{q-1}h(s)\,ds\right],$$

there exists

$$\begin{aligned} \frac{u(t)}{f(t,u(t))} &= \left[\frac{u_0}{f(0,u_0)} + \frac{1}{\Gamma(q)}\int_0^{t_1} (t_1-s)^{q-1}h(s)\,ds\right] + \frac{I_1(u(t_1^-))}{f(t_1,u(t_1))} \\ &\quad - \frac{1}{\Gamma(q)}\int_0^{t_1} (t_1-s)^{q-1}h(s)\,ds + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds \\ &= \frac{u_0}{f(0,u_0)} + \frac{I_1(u(t_1^-))}{f(t_1,u(t_1))} + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds, \end{aligned}$$

i.e.,

$$u(t) = f\bigl(t, u(t)\bigr)\left[\frac{u_0}{f(0,u_0)} + \frac{I_1(u(t_1^-))}{f(t,u(t_1))} + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds\right].$$

If $t \in J_3$, we have

$$\begin{aligned}\frac{u(t)}{f(t,u(t))} &= \frac{u(t_2^+)}{f(t_2,u(t_2))} - \frac{1}{\Gamma(q)}\int_0^{t_2}(t_2-s)^{q-1}h(s)\,ds + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds \\ &= \frac{u(t_2^-) + I_2(u(t_2^-))}{f(t_2,u(t_2))} - \frac{1}{\Gamma(q)}\int_0^{t_2}(t_2-s)^{q-1}h(s)\,ds + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds.\end{aligned}$$

For

$$u\bigl(t_2^-\bigr) = f\bigl(t_2, u(t_2)\bigr)\left[\frac{u_0}{f(0,u_0)} + \frac{I_1(u(t_1^-))}{f(t_1,u(t_1))} + \frac{1}{\Gamma(q)}\int_0^{t_2} (t_2-s)^{q-1}h(s)\,ds\right],$$

we have that

$$\begin{aligned}\frac{u(t)}{f(t,u(t))} &= \frac{u_0}{f(0,u_0)} + \frac{I_1(u(t_1^-))}{f(t_1,u(t_1))} + \frac{1}{\Gamma(q)}\int_0^{t_2}(t_2-s)^{q-1}h(s)\,ds \\ &\quad + \frac{I_2(u(t_2^-))}{f(t_2,u(t_2))} - \frac{1}{\Gamma(q)}\int_0^{t_2}(t_2-s)^{q-1}h(s)\,ds + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds \\ &= \frac{u_0}{f(0,u_0)} + \frac{I_1(u(t_1^-))}{f(t_1,u(t_1))} + \frac{I_2(u(t_2^-))}{f(t_2,u(t_2))} + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds,\end{aligned}$$

i.e.,

$$u(t) = f\bigl(t, u(t)\bigr)\left[\frac{u_0}{f(0,u_0)} + \sum_{i=1}^{2}\frac{I_i(u(t_i^-))}{f(t_i,u(t_i))} + \frac{I_2(u(t_2^-))}{f(t_2,u(t_2))} + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds\right].$$

If $t \in J_{k+1}$ ($k = 3, 4, \ldots, m$), using the same method, we get

$$u(t) = f\bigl(t, u(t)\bigr)\left[\frac{u_0}{f(0,u_0)} + \sum_{i=1}^{k}\frac{I_i(u(t_i^-))}{f(t_i,u(t_i))} + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds\right]. \tag{11}$$

Conversely, assume that u satisfies (5). If $t \in J_1$, then we have

$$u(t) = f\bigl(t, u(t)\bigr)\left[\frac{u_0}{f(0,u_0)} + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,dx\right]. \tag{12}$$

Then, dividing by $f(t,u(t))$ and applying ${}^cD_{0,t}^q$ on both sides of (12), (7) is satisfied. Again, substituting $t = 0$ in (12), we have $\frac{u(0)}{f(0,u(0))} = \frac{u_0}{f(0,u_0)}$. Since $u \mapsto \frac{u}{f(t,u)}$ is increasing in $\mathbb{R}$ for $t \in J_1$, the map $u \mapsto \frac{u}{f(0,u)}$ is injective in $\mathbb{R}$. Then we get (8).

If $t \in J_2$, then we have

$$u(t) = f\bigl(t, u(t)\bigr)\left[\frac{u_0}{f(0,u_0)} + \frac{I_1(u(t_1^-))}{f(t_1,u(t_1))} + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,dx\right]. \tag{13}$$

Then, dividing by $f(t,u(t))$ and applying ${}^cD_t^q$ on both sides of (13), (9) is satisfied. Again, by (H4), substituting $t = t_1$ in (12) and taking the limit of (13), then (13) minus (12) gives (10).

If $t \in J_{k+1}$ ($k = 2, 4, \dots, m$), similarly we get

$$\begin{cases} {}^cD_{0,t}^q[\frac{u(t)}{f(t,u(t))}] = h(t), & t \in (t_k, t_{k+1}], \\ u(t_k^+) = u(t_k^-) + I_k(u(t_k^-)). \end{cases} \tag{14}$$

This completes the proof. □

Now we give the following definition.

Definition 2.7 If a function $u \in PC(J, \mathbb{R})$ satisfies the fractional integral equation

$$u(t) = \begin{cases} f(t,u(t))[\frac{u_0}{f(0,u_0)} + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1} g(s,u(s))\,ds], & t \in J_1, \\ f(t,u(t))[\frac{u_0}{f(0,u_0)} + \frac{I_1(u(t_1^-))}{f(t_1,u(t_1))} + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1} g(s,u(s))\,ds], & t \in J_2, \\ f(t,u(t))[\frac{u_0}{f(0,u_0)} + \frac{I_1(u(t_1^-))}{f(t_1,u(t_1))} + \frac{I_2(u(t_2^-))}{f(t_2,u(t_2))} & \\ \quad + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1} g(s,u(s))\,ds], & t \in J_3, \\ \vdots & \\ f(t,u(t))[\frac{u_0}{f(0,u_0)} + \sum_{i=1}^m \frac{I_i(u(t_i^-))}{f(t_i,u(t_i))} & \\ \quad + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1} g(s,u(s))\,ds], & t \in J_{m+1}, \end{cases} \tag{15}$$

it is said to be a mild solution of (4).

3 Main results

In this section, we prove the existence of a mild solution for IHFDE (4) by Lemma 2.4 and Lemma 2.6.

Theorem 3.1 *Assume that hypotheses* (H1)-(H4) *hold. Further, if*

$$L_k\left[\left|\frac{u_0}{f(0,u_0)} + \sum_{i=1}^{k-1} \frac{I_i(u_i(t_i^-))}{f(t_i,u_i(t_i))}\right| + \frac{1}{\Gamma(q+1)} \sum_{j=0}^{k} (t_k - t_j)^q \|h_{j+1}\|_{L^\infty}\right] < 1,$$
$$k = 1, 2, \dots, m, m+1$$

for all $u_i(t) \in C(J_i, \mathbb{R})$ and $u_i(t_i)$ is bounded, then IHFDE (4) *has a mild solution defined on J.*

Proof By Lemma 2.6, IHFDE (4) is equivalent to the fractional integral equation (15).

If $t \in J_1$, we have

$$u(t) = f\big(t,u(t)\big)\left[\frac{u_0}{f(0,u_0)} + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1} g\big(s,u(s)\big)\,ds\right]. \tag{16}$$

Set $X_1 = C(J_1, \mathbb{R})$ and

$$S_1 = \{u \in X_1 \mid \|u\| \le N_1\}, \tag{17}$$

where

$$N_1 = \frac{F_0(|u_0|\Gamma(q+1) + t_1^q |f(0,t_0)| \|h_1\|_{L^\infty})}{\Gamma(q+1)|f(0,u_0)| - L_1(|u_0|\Gamma(q+1) + t_1^q |f(0,t_0)| \|h_1\|_{L^\infty})}, \tag{18}$$

and $F_0 = \sup_{t\in J} |f(t,0)|$.

Define two operators $A_1 : X_1 \mapsto X_1$ and $B_1 : S_1 \mapsto X_1$ by

$$A_1 u(t) = f\bigl(t, u(t)\bigr), \quad t \in J_1 \tag{19}$$

and

$$B_1 u(t) = \frac{u_0}{f(0,u_0)} + \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} g\bigl(s, u(s)\bigr)\, ds, \quad t \in J_1. \tag{20}$$

We will show that the operators A_1 and B_1 satisfy all the conditions of Lemma 2.4.

First, we will show that A_1 is a Lipschitz operator on X_1 with the Lipschitz constant L_1. Set $u_1, u_2 \in X_1$. Then, by hypotheses (H2) and (H3), we have

$$\begin{aligned} \bigl|A_1 u_1(t) - A_1 u_2(t)\bigr| &= \bigl|f\bigl(t, u_1(t)\bigr) - f\bigl(t, u_2(t)\bigr)\bigr| \\ &\le L_1 \bigl|u_1(t) - u_2(t)\bigr| \le L_1 \|u_1 - u_2\| \end{aligned} \tag{21}$$

for all $t \in J_1$. Taking supremum over t, we have

$$\|A_1 u_1 - A_1 u_2\| \le L_1 \|u_1 - u_2\| \tag{22}$$

for all $u_1, u_2 \in X_1$.

Next, we will show that B_1 is a compact and continuous operator on S_1 into X_1. We show that B_1 is continuous on S_1. Let $\{u_n\} \subseteq S_1$ converge to a point $u \in S_1$. Then, by the Lebesgue dominated convergence theorem, we have

$$\begin{aligned} \lim_{n\to\infty} B_1 u_n(t) &= \frac{u_0}{f(0,u_0)} + \lim_{n\to\infty} \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} g\bigl(s, u_n(s)\bigr)\, ds \\ &= \frac{u_0}{f(0,u_0)} + \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} \lim_{n\to\infty} g\bigl(s, u_n(s)\bigr)\, ds \\ &= \frac{u_0}{f(0,u_0)} + \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} g\bigl(s, u(s)\bigr)\, ds \\ &= B_1 u(t) \end{aligned} \tag{23}$$

for all $t \in J_1$. So, we have obtained that B_1 is continuous on S_1.

Next we will show that B_1 is a compact operator on S_1. We show that $B_1(S_1)$ is a uniformly bounded and equicontinuous set in S_1. Let $u \in S_1$ be arbitrary. Then, by hypothesis (H3), we have

$$\begin{aligned} \bigl|B_1 u(t)\bigr| &\le \left|\frac{u_0}{f(0,u_0)}\right| + \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} \bigl|g\bigl(s, u(s)\bigr)\bigr|\, ds \\ &\le \left|\frac{u_0}{f(0,u_0)}\right| + \frac{1}{\Gamma(q)} \left(\int_0^t (t-s)^{q-1} h_1(s)\, ds\right) \\ &\le \left|\frac{u_0}{f(0,u_0)}\right| + \frac{t_1^q}{\Gamma(q+1)} \|h_1\|_{L^\infty} \end{aligned} \tag{24}$$

for all $t \in J_1$. Taking supremum over t, we have

$$\|B_1 u\| \le \left|\frac{u_0}{f(0,u_0)}\right| + \frac{t_1^q}{\Gamma(q+1)}\|h_1\|_{L^\infty}$$

for all $u \in S_1$. We have obtained that $B_1(S_1)$ is uniformly bounded.

Let $t_{11}, t_{12} \in J_1$ with $t_{11} < t_{12}$. Then, for any $u \in S_1$, we have

$$\begin{aligned}
&\bigl|B_1 u(t_{11}) - B_1 u(t_{12})\bigr| \\
&\quad\le \left|\frac{1}{\Gamma(q)}\int_0^{t_{11}} (t_{11}-s)^{q-1} g\bigl(s,u(s)\bigr)\,ds - \frac{1}{\Gamma(q)}\int_0^{t_{11}} (t_{12}-s)^{q-1} g\bigl(s,u(s)\bigr)\,ds\right| \\
&\qquad + \left|\frac{1}{\Gamma(q)}\int_0^{t_{11}} (t_{12}-s)^{q-1} g\bigl(s,u(s)\bigr)\,ds - \frac{1}{\Gamma(q)}\int_0^{t_{12}} (t_{12}-s)^{q-1} g\bigl(s,u(s)\bigr)\,ds\right| \\
&\quad\le \frac{\|h_1\|_{L^\infty}}{\Gamma(q+1)}\bigl[\bigl|t_{12}^q - t_{11}^q - (t_{12}-t_{11})^q\bigr| + (t_{12}-t_{11})^q\bigr].
\end{aligned} \tag{25}$$

Hence, for $\varepsilon > 0$, there exists a constant $\delta > 0$ such that

$$|t_{11} - t_{12}| < \delta \quad\Rightarrow\quad \bigl|B_1 u(t_{11}) - B_1 u(t_{12})\bigr| < \varepsilon \tag{26}$$

for all $t_{11}, t_{12} \in [0, t_1]$ and for all $u \in S_1$. This obtains that $B_1(S_1)$ is an equicontinuous set in X_1. By the Arzela-Ascoli theorem, we know that B_1 is compact. As a result B_1 is a complete continuous operator on S_1.

Next, we show that hypothesis (c) of Lemma 2.4 is satisfied. Let $u_{11} \in X_1$ and any $u_{12} \in S_1$ such that $u_{11} = A_1 u_{11} B_1 u_{12}$. Then, by assumption (H2), we have

$$\begin{aligned}
\bigl|u_{11}(t)\bigr| &\le \bigl[\bigl|f\bigl(t,u_{11}(t)\bigr) - f(t,0)\bigr| + \bigl|f(t,0)\bigr|\bigr]\left[\left|\frac{u_0}{f(0,u_0)}\right| \right. \\
&\qquad \left. + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}\bigl|g\bigl(s,u_{12}(s)\bigr)\bigr|\,ds\right] \\
&\le \bigl[L_1\bigl|u_{11}(t)\bigr| + F_0\bigr]\left[\left|\frac{u_0}{f(0,u_0)}\right| + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1} h_1(s)\,ds\right] \\
&\le \bigl[L_1\bigl|u_{11}(t)\bigr| + F_0\bigr]\left[\left|\frac{u_0}{f(0,u_0)}\right| + \frac{t_1^q}{\Gamma(q+1)}\|h_1\|_{L^\infty}\right].
\end{aligned} \tag{27}$$

Thus,

$$\bigl|u_{11}(t)\bigr| \le \frac{F_0(|u_0|\Gamma(q+1) + t_1^q|f(0,u_0)|\|h_1\|_{L^\infty})}{\Gamma(q+1)|f(0,u_0)| - L_1(|u_0|\Gamma(q+1) + t_1^q|f(0,t_0)|\|h_1\|_{L^\infty})}. \tag{28}$$

Taking supremum over t, we have

$$\|u_{11}\| \le \frac{F_0(|u_0|\Gamma(q+1) + t_1^q|f(0,t_0)|\|h_1\|_{L^\infty})}{\Gamma(q+1)|f(0,u_0)| - L_1(|u_0|\Gamma(q+1) + t_1^q|f(0,t_0)|\|h_1\|_{L^\infty})} = N_1. \tag{29}$$

Finally, we have

$$M_1 = \bigl\|B_1(S_1)\bigr\| = \sup\bigl\{\|B_1 u\| : u \in S_1\bigr\} \le \left|\frac{u_0}{f(0,u_0)}\right| + \frac{t_1^q}{\Gamma(q+1)}\|h_1\|_{L^\infty},$$

and so

$$L_1M_1 \le L_1\left[\left|\frac{u_0}{f(0,u_0)}\right| + \frac{t_1^q}{\Gamma(q+1)}\|h_1\|_{L^\infty}\right] < 1. \tag{30}$$

So, the operator equation $A_1uB_1u = u$ has a solution denoted by $u_1(t)$ in S_1.

If $t \in J_2$, we set

$$\begin{aligned} u(t) &= f\big(t,u(t)\big)\left[\frac{u_0}{f(0,u_0)} + \frac{I_1(u_1(t_1))}{f(t_1,u_1(t_1))} + \frac{1}{\Gamma(q)}\int_0^{t_1}(t-s)^{q-1}g\big(s,u_1(s)\big)\,ds\right] \\ &\quad + f\big(t,u(t)\big)\int_{t_1}^{t}(t-s)^{q-1}g\big(s,u(s)\big)\,ds. \end{aligned}$$

Set $X_2 = C(J_2,\mathbb{R})$ and

$$S_2 = \big\{u \in X_2 \mid \|u\| \le N_2\big\}, \tag{31}$$

where

$$N_2 = \frac{F_0H_2}{1-L_2H_2}, \tag{32}$$

$$H_2 = \left|\frac{u_0}{f(0,u_0)} + \frac{I_1(u_1(t_1))}{f(t_1,u_1(t_1))}\right| + \frac{t_2^q}{\Gamma(q+1)}\|h_1\|_{L^\infty} + \frac{(t_2-t_1)^q}{\Gamma(q+1)}\|h_2\|_{L^\infty}. \tag{33}$$

Define two operators $A_2 : X_2 \mapsto X_2$ and $B_2 : S_2 \mapsto X_2$ by

$$A_2u(t) = f\big(t,u(t)\big), \quad t \in J_2 \tag{34}$$

and

$$\begin{aligned} B_2u(t) &= \frac{u_0}{f(0,u_0)} + \frac{I_1(u_1(t_1))}{f(t_1,u_1(t_1))} + \frac{1}{\Gamma(q)}\int_0^{t_1}(t-s)^{q-1}g\big(s,u_1(s)\big)\,ds \\ &\quad + \frac{1}{\Gamma(q)}\int_{t_1}^{t}(t-s)^{q-1}g\big(s,u(s)\big)\,ds, \quad t \in J_2. \end{aligned} \tag{35}$$

The operators A_2 and B_2 satisfy all the conditions of Lemma 2.4. First, we prove that A_2 is a Lipschitz operator on X_2 with the Lipschitz constant L_2. Set $u_{21}, u_{22} \in X_2$. Then, by hypothesis (H2), we have

$$\begin{aligned} \big|A_2u_{21}(t) - A_2u_{22}(t)\big| &\le L_2\big|u_{21}(t) - u_{22}(t)\big| \\ &\le L_2\|u_{21} - u_{22}\| \end{aligned} \tag{36}$$

for all $t \in J_2$. Taking supremum over t, we have

$$\|A_2u_{21} - A_2u_{22}\| \le L_2\|u_{21} - u_{22}\| \tag{37}$$

for all $u_{21}, u_{22} \in X_2$.

Next, we prove that B_2 is a compact and continuous operator on S_2 into X_2. We show that B_2 is continuous on S_2. Let $\{u_n\} \subseteq S_2$ converge to a point $u \in B_2$. Then, by the Lebesgue dominated convergence theorem, we have

$$\begin{aligned}\lim_{n\to\infty} B_2u_n(t) &= \frac{u_0}{f(0,u_0)} + \frac{I_1(u_1(t_1))}{f(t_1,u_1(t_1))} + \frac{1}{\Gamma(q)}\int_0^{t_1}(t-s)^{q-1}g\bigl(s,u_1(s)\bigr)\,ds \\ &\quad + \frac{1}{\Gamma(q)}\int_{t_1}^{t}(t-s)^{q-1}\lim_{n\to\infty} g\bigl(s,u_n(s)\bigr)\,ds \\ &= \frac{u_0}{f(0,u_0)} + \frac{I_1(u_1(t_1))}{f(t_1,u_1(t_1))} + \frac{1}{\Gamma(q)}\int_0^{t_1}(t-s)^{q-1}g\bigl(s,u_1(s)\bigr)\,ds \\ &\quad + \frac{1}{\Gamma(q)}\int_{t_1}^{t}(t-s)^{q-1}g\bigl(s,u(s)\bigr)\,ds \\ &= B_2u(t) \end{aligned} \tag{38}$$

for all $t \in J_2$. So, B_2 is continuous on S_2.

We will prove that B_2 is a compact operator on S_2. We show that $B_2(S_2)$ is a uniformly bounded and equicontinuous set in S_2. Let $u \in S_2$ be arbitrary. Then, by hypothesis (H3), we have

$$\begin{aligned}\bigl|B_2u(t)\bigr| &\le \left|\frac{u_0}{f(0,u_0)} + \frac{I_1(u_1(t_1))}{f(t_1,u_1(t_1))}\right| + \left|\frac{1}{\Gamma(q)}\int_0^{t_1}(t-s)^{q-1}g\bigl(s,u_1(s)\bigr)\,ds\right| \\ &\quad + \left|\frac{1}{\Gamma(q)}\int_{t_1}^{t}(t-s)^{q-1}g\bigl(s,u(s)\bigr)\,ds\right| \\ &\le \left|\frac{u_0}{f(0,u_0)} + \frac{I_1(u_1(t_1))}{f(t_1,u_1(t_1))}\right| + \frac{1}{\Gamma(q)}\int_0^{t_1}(t-s)^{q-1}\bigl|g\bigl(s,u_1(s)\bigr)\bigr|\,ds \\ &\quad + \frac{1}{\Gamma(q)}\int_{t_1}^{t}(t-s)^{q-1}\bigl|g\bigl(s,u(s)\bigr)\bigr|\,ds \\ &\le \left|\frac{u_0}{f(0,u_0)} + \frac{I_1(u_1(t_1))}{f(t_1,u_1(t_1))}\right| + \frac{1}{\Gamma(q)}\int_0^{t_1}(t-s)^{q-1}h_1(s)\,ds \\ &\quad + \frac{1}{\Gamma(q)}\int_{t_1}^{t}(t-s)^{q-1}h_2(s)\,ds \\ &\le \left|\frac{u_0}{f(0,u_0)} + \frac{I_1(u_1(t_1))}{f(t_1,u_1(t_1))}\right| + \frac{t_2^q}{\Gamma(q+1)}\|h_1\|_{L^\infty} \\ &\quad + \frac{(t_2-t_1)^q}{\Gamma(q+1)}\|h_2\|_{L^\infty} \end{aligned} \tag{39}$$

for all $t \in J_2$. Taking supremum over t, we have

$$\|B_2u\| \le \left|\frac{u_0}{f(0,u_0)} + \frac{I_1(u_1(t_1))}{f(t_1,u_1(t_1))}\right| + \frac{t_2^q}{\Gamma(q+1)}\|h_1\|_{L^\infty} + \frac{(t_2-t_1)^q}{\Gamma(q+1)}\|h_2\|_{L^\infty}$$

for all $u \in S_2$. So, $B_2(S_2)$ is uniformly bounded.

Let $t_{21}, t_{22} \in J_2$ with $t_{21} < t_{22}$. Then, for any $u \in S_2$, we have

$$\begin{aligned}&\bigl|B_2u(t_{21}) - B_2u(t_{22})\bigr| \\ &\quad\le \left|\frac{1}{\Gamma(q)}\int_0^{t_1}(t_{21}-s)^{q-1}g\bigl(s,u_1(s)\bigr)\,ds - \frac{1}{\Gamma(q)}\int_0^{t_1}(t_{22}-s)^{q-1}g\bigl(s,u_1(s)\bigr)\,ds\right|\end{aligned}$$

$$
\begin{aligned}
&+\left|\frac{1}{\Gamma(q)}\int_{t_1}^{t_{21}}(t_{21}-s)^{q-1}g\big(s,u(s)\big)\,ds-\frac{1}{\Gamma(q)}\int_{t_1}^{t_{21}}(t_{22}-s)^{q-1}g\big(s,u(s)\big)\,ds\right| \\
&+\left|\frac{1}{\Gamma(q)}\int_{t_1}^{t_{21}}(t_{22}-s)^{q-1}g\big(s,u(s)\big)\,ds-\frac{1}{\Gamma(q)}\int_{t_1}^{t_{22}}(t_{22}-s)^{q-1}g\big(s,u(s)\big)\,ds\right| \\
\le{}& \frac{\|h_1\|_{L^\infty}}{\Gamma(q+1)}\left|t_{22}^q-t_{21}^q+(t_{22}-t_1)^q-(t_{21}-t_1)^q\right| \\
&+\frac{\|h_2\|_{L^\infty}}{\Gamma(q+1)}\big[\left|(t_{22}-t_1)^q-(t_{21}-t_1)^q+(t_{22}-t_{21})^q\right|+(t_{22}-t_{21})^q\big].
\end{aligned}
$$

Hence, for $\varepsilon > 0$, there exists a constant $\delta > 0$ such that

$$
|t_{21}-t_{22}|<\delta \quad\Rightarrow\quad \left|B_2u(t_{21})-B_2u(t_{22})\right|<\varepsilon \tag{40}
$$

for all $t_{21}, t_{22} \in J_2$ and for all $u \in S_2$. So, $B_2(S_2)$ is an equicontinuous set in X_2. By the Arzela-Ascoli theorem, we know that B_2 is compact. As a result, B_2 is a complete continuous operator on S_2.

Next, we show that hypothesis (c) of Lemma 2.4 is satisfied. Let $u_{21} \in X_2$ and any $u_{22} \in S_2$ such that $u_{21} = A_2u_{21}B_2u_{22}$. Then, by assumptions (H2) and (H3), we have

$$
\begin{aligned}
\left|u_{21}(t)\right| \le{}& \big[\left|f\big(t,u_{21}(t)\big)-f(t,0)\right|+\left|f(t,0)\right|\big]\left[\left|\frac{u_0}{f(0,u_0)}+\frac{I_1(u_1(t_1))}{f(t_1,u_1(t_1))}\right|\right. \\
&\left.+\frac{1}{\Gamma(q)}\int_0^{t_1}(t-s)^{q-1}\left|g\big(s,u_1(s)\big)\right|ds+\frac{1}{\Gamma(q)}\int_{t_1}^{t}(t-s)^{q-1}\left|g\big(s,u_{22}(s)\big)\right|ds\right] \\
\le{}& \big[L_2\left|u_{21}(t)\right|+F_0\big]\left[\left|\frac{u_0}{f(0,u_0)}+\frac{I_1(u_1(t_1))}{f(t_1,u_1(t_1))}\right|\right. \\
&\left.+\frac{1}{\Gamma(q)}\int_0^{t_1}(t-s)^{q-1}h_1(s)\,ds+\frac{1}{\Gamma(q)}\int_{t_1}^{t}(t-s)^{q-1}h_2(s)\,ds\right] \\
\le{}& \big[L_2\left|u_{21}(t)\right|+F_0\big]\left[\left|\frac{u_0}{f(0,u_0)}+\frac{I_1(u_1(t_1))}{f(t_1,u_1(t_1))}\right|\right. \\
&\left.+\frac{t_2^q}{\Gamma(q+1)}\|h_1\|_{L^\infty}+\frac{(t_2-t_1)^q}{\Gamma(q+1)}\|h_2\|_{L^\infty}\right].
\end{aligned}
$$

Thus,

$$
\left|u_{21}(t)\right| \le \frac{F_0H_2}{1-L_2H_2}. \tag{41}
$$

Taking supremum over t, we have

$$
\|u_{21}\| \le \frac{F_0H_2}{1-L_2H_2} = N_2. \tag{42}
$$

Finally, we have

$$
M_2 \le \left|\frac{u_0}{f(0,u_0)}+\frac{I_1(u_1(t_1))}{f(t_1,u_1(t_1))}\right|+\frac{t_2^q}{\Gamma(q+1)}\|h_1\|_{L^\infty}+\frac{(t_2-t_1)^q}{\Gamma(q+1)}\|h_2\|_{L^\infty} \tag{43}
$$

and

$$L_2M_2 \leq L_2\left[\left|\frac{u_0}{f(0,u_0)} + \frac{I_1(u_1(t_1))}{f(t_1,u_1(t_1))}\right| + \frac{t_2^q}{\Gamma(q+1)}\|h_1\|_{L^\infty} + \frac{(t_2-t_1)^q}{\Gamma(q+1)}\|h_2\|_{L^\infty}\right] < 1. \tag{44}$$

So, the operator equation $A_2uB_2u = u$ has a solution denoted by $u_2(t)$ in S_2.

If $t \in j_{k+1}$ $(k = 2,3,\dots,m)$, repeating the same process, we obtain $u_{(k+1)}(t) \in C(J_{k+1},\mathbb{R})$ $(k = 2,3,\dots,m)$. So, we get a mild solution of IHFDE (4). The proof is completed. □

4 Example

In this section we give a simple example to illustrate the usefulness of our main result.

Example 4.1 Let us consider the impulsive hybrid fractional differential equations

$$\begin{cases} {}^cD^q_{0,t}[\frac{u(t)}{e^{\upsilon_1 t}+|u(t)|}] = \frac{e^{-\upsilon_2 t}|u(t)|}{(1+e^t)(1+|u(t)|)}, & t \in [0,1] \setminus t_1, \\ u(t_1^+) = u(t_1^-) + (-2u(t_1^-)), & t_1 \neq 0,1 \\ u(0) = 0, \end{cases} \tag{45}$$

where $\upsilon_1 > 0$, $\upsilon_2 > 0$ are constants and $0 < q < 1$.

Set

$$f\bigl(t,u(t)\bigr) = e^{\upsilon_1 t} + \bigl|u(t)\bigr|, \qquad g\bigl(t,u(t)\bigr) = \frac{e^{-\upsilon_2 t}|u(t)|}{(1+e^t)(1+|u(t)|)}$$

for all $(t,u) \in [0,1] \times \mathbb{R}$.

Obviously,

$$f\bigl(t,u(t)\bigr) \neq 0 \quad \text{and} \quad f\bigl(t_1^+,u(t_1^+)\bigr) = f\bigl(t_1^-,u(t_1^-)\bigr)$$

for all $(t,u) \in [0,1] \times \mathbb{R}$. There exist constants $L_k = 1$ $(k = 1,2)$ such that

$$\bigl|f(t,u_1) - f(t,u_2)\bigr| \leq L_k|u_1 - u_2|$$

for all $t \in J_k$ $(k = 1,2)$ and $u_1,u_2 \in \mathbb{R}$.

By

$$\frac{u(t)}{f(t,u(t))} = \begin{cases} 1 - \frac{e^{\upsilon_1 t}}{e^{\upsilon_1 t}+u(t)}, & u(t) \geq 0, \\ -1 + \frac{e^{\upsilon_1 t}}{e^{\upsilon_1 t}-u(t)}, & u(t) < 0, \end{cases} \tag{46}$$

we have that the function $u \mapsto \frac{u}{f(t,u(t))}$ is increasing in $\mathbb{R}$ for $[0,t_1]$.

For all $u \in \mathbb{R}$ and each $t \in [0,1]$, we have

$$g\bigl(t,u(t)\bigr) = \frac{e^{-\upsilon t}|u(t)|}{(1+e^t)(1+|u(t)|)} \leq \frac{e^{-\upsilon t}}{1+e^t} \leq \frac{e^{-\upsilon t}}{2}. \tag{47}$$

So, choosing some $\upsilon_1,\upsilon_2 > 0$ large enough, we have

$$L_2\left[\left|\frac{-2u(t_1^-)}{e^{\upsilon_1 t_1} + |u(t_1)|}\right| + \frac{1}{\Gamma(q+1)}\bigl((1-t_1)^q\|h_1\|_{L^1} + 1\bigr)\|h_2\|_{L^1}\right] < 1.$$

Thus all the assumptions in Theorem 3.1 are satisfied, our results can be applied to problem (45).

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors declare that the study was realized in collaboration with the same responsibility. All authors read and approved the final manuscript.

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