Xu and Wu *Advances in Difference Equations* 2014, 2014:264
http://www.advancesindifferenceequations.com/content/2014/1/264

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

Dynamics of a nonautonomous Lotka-Volterra predator-prey dispersal system with impulsive effects

Lijun Xu[1] and Wenquan Wu[2*]

*Correspondence: mathwqwu@163.com
[2]Department of Mathematics, Aba Teachers College, Wenchuan, Sichuan 623002, China
Full list of author information is available at the end of the article

Abstract

By applying the comparison theorem, Lyapunov functional, and almost periodic functional hull theory of the impulsive differential equations, this paper gives some new sufficient conditions for the uniform persistence, global asymptotical stability, and almost periodic solution to a nonautonomous Lotka-Volterra predator-prey dispersal system with impulsive effects. The main results of this paper extend some corresponding results obtained in recent years. The method used in this paper provides a possible method to study the uniform persistence, global asymptotical stability, and almost periodic solution of the models with impulsive perturbations in biological populations.

MSC: 34K14; 34K20; 34K45; 92D25

Keywords: uniform persistence; diffusion; comparison theorem; predator-prey; impulse

1 Introduction

Because of the ecological effects of human activities and industry, more and more habitats are broken into patches and some of them are polluted. Negative feedback crowding or the effect of the past life history of the species on its present birth rate are common examples illustrating the biological meaning of time delays and justifying their use in these systems. Recently, diffusions have been introduced into Lotka-Volterra type systems. The effect of an environment change in the growth and diffusion of a species in a heterogeneous habitat is a subject of considerable interest in the ecological literature [1–7].

As was pointed out by Berryman [8], the dynamic relationship between predators and their prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. In recent years, the predator-prey system has been extensively studied by many scholars, many excellent results were obtained concerned with the persistent property and positive periodic solution of the system; see [9–15] and the references cited therein.

Considering the effect of almost periodically varying environment is an important selective forces on systems in a fluctuating environment, Meng and Chen [16] studied the case of combined effects: dispersion, time delays, almost periodicity of the environment. Namely, they investigated the following general nonautonomous Lotka-Volterra type predator-prey dispersal system:

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1(t) - a_1(t)x_1(t) - b_1(t)x_1(t-\tau_1(t)) \\ \qquad - \int_{-\sigma_1}^0 k_1(t,s)x_1(t+s)\,\mathrm{d}s - \frac{c(t)y(t)}{1+\alpha(t)x_1(t)}] + \sum_{i=2}^n D_{i1}(t)[x_i(t) - x_1(t)], \\ \dot{x}_i(t) = x_i(t)[r_i(t) - a_i(t)x_i(t) - b_i(t)x_i(t-\tau_i(t)) \\ \qquad - \int_{-\sigma_i}^0 k_i(t,s)x_i(t+s)\,\mathrm{d}s] + \sum_{j=1}^n D_{ji}(t)[x_j(t) - x_i(t)], \quad i = 2,3,\ldots,n, \\ \dot{y}(t) = y(t)[-r_{n+1}(t) + \frac{f(t)x_1(t)}{1+\alpha(t)x_1(t)} - a_{n+1}(t)y(t) - b_{n+1}(t)y(t-\tau_{n+1}(t)) \\ \qquad - \int_{-\sigma_{n+1}}^0 k_{n+1}(t,s)y(t+s)\,\mathrm{d}s]. \end{cases} \tag{1.1}$$

By using the comparison theorem and functional hull theory of almost periodic system, the authors [16] obtained some sufficient conditions for the uniform persistence, global asymptotical stability, and almost periodic solution to system (1.1).

However, the ecological system is often deeply perturbed by human exploitation activities such as planting and harvesting and so on, which makes it unsuitable to be considered continually. To obtain a more accurate description of such systems, we need to consider impulsive differential equations. In recent years, the impulsive differential equations have been intensively investigated (see [17–29] for more details). To the best of the authors' knowledge, in the literature, there are few papers concerning the permanence, global asymptotical stability, and almost periodic solution to the Lotka-Volterra type predator-prey dispersal system with impulsive effects. Therefore, we consider the following Lotka-Volterra type predator-prey dispersal system with impulsive effects:

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1(t) - a_1(t)x_1(t) - b_1(t)x_1(t-\tau_1(t)) \\ \qquad - \int_{-\sigma_1}^0 k_1(t,s)x_1(t+s)\,\mathrm{d}s - \frac{c(t)y(t)}{1+\alpha(t)x_1(t)}] + \sum_{i=2}^n D_{i1}(t)[x_i(t) - x_1(t)], \\ \dot{x}_i(t) = x_i(t)[r_i(t) - a_i(t)x_i(t) - b_i(t)x_i(t-\tau_i(t)) \\ \qquad - \int_{-\sigma_i}^0 k_i(t,s)x_i(t+s)\,\mathrm{d}s] + \sum_{j=1}^n D_{ji}(t)[x_j(t) - x_i(t)], \quad i = 2,3,\ldots,n, \\ \dot{y}(t) = y(t)[-r_{n+1}(t) + \frac{f(t)x_1(t)}{1+\alpha(t)x_1(t)} - a_{n+1}(t)y(t) - b_{n+1}(t)y(t-\tau_{n+1}(t)) \\ \qquad - \int_{-\sigma_{n+1}}^0 k_{n+1}(t,s)y(t+s)\,\mathrm{d}s], \quad t \neq t_k, \\ \Delta x_j(t_k) = h_{jk}x_j(t_k), \quad j = 1,2,\ldots,n, \\ \Delta y(t_k) = h_{n+1,k}y(t_k), \quad k \in \mathbb{Z}, \end{cases} \tag{1.2}$$

where x_1 and y are population density of prey species x and predator species y in patch 1, and x_i is density of prey species x in patch i; predator species y is confined to patch 1, while the prey species x can disperse among n patches; $D_{ij}(t)$ is the dispersion rate of the species from patch j to patch i, the terms $b_i(t)x_i(t-\tau_i(t))$ $(i = 1,2,\ldots,n)$, $b_{n+1}(t)y(t-\tau_{n+1}(t))$, $\int_{-\sigma_i}^0 k_i(t,s)x_i(t+s)\,\mathrm{d}s$ $(i = 1,2,\ldots,n)$ and $\int_{-\sigma_{n+1}}^0 k_{n+1}(t,s)y(t+s)\,\mathrm{d}s$ represent the negative feedback crowding and the effect of all the past life history of the species on its present birth rate, respectively; $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$, $x_i(t_k^+)$ and $x_i(t_k^-)$ represent the right and the left limit of $x_i(t_k)$, $x_i(t_k^-) = x_i(t_k)$, $k \in \mathbb{Z}$, $i = 1,2,\ldots,n$. Related to a continuous function f, we use the following notations: $f^l = \inf_{s\in\mathbb{R}} f(s)$, $f^u = \sup_{s\in\mathbb{R}} f(s)$.

In system (1.2), we always assume that for all $i = 1,2,\ldots,n+1$, $j = 1,2,\ldots,n$:

- (H_1) $r_i(t)$, $a_i(t)$, $b_i(t)$, $c(t)$, $f(t)$, $\alpha(t)$ and $D_{ij}(t)$ ($D_{ii}(t) = 0$) are nonnegative and continuous almost periodic functions for all $t \in \mathbb{R}$, and $a_i^l + b_i^l > 0$.
- (H_2) $k_i(t,s)$ are defined on $\mathbb{R} \times (-\infty, 0]$ and nonnegative and continuous almost periodic functions with respect to $t \in \mathbb{R}$ and integrable with respect to s on $(-\infty, 0]$ such that

$\int_{-\sigma_i}^{0} k_i(t,s)\,\mathrm{d}s$ is continuous and bounded with respect to $t \in \mathbb{R}$, $0 < \int_{-\sigma_i}^{0}(-s)k_i^u(s)\,\mathrm{d}s < +\infty$.

(H_3) $\tau_i(t)$ is continuous and differentiable bounded almost periodic functions on $\mathbb{R}$, and $\inf_{t\in\mathbb{R}}\{1 - \dot{\tau}_i(t)\} > 0$.

(H_4) The sequences $\{h_{ik}\}$ are almost periodic and $h_{ik} > -1$.

(H_5) The set of sequences $\{t_k^j\}$, $t_k^j = t_{k+j} - t_k$, $k \in \mathbb{Z}$, $j \in \mathbb{Z}$ is uniformly almost periodic and $\theta := \inf_{k\in\mathbb{Z}} t_k^1 > 0$.

The main purpose of this paper is to establish some new sufficient conditions which guarantee the uniform persistence, global asymptotical stability, and almost periodic solution of system (1.2) by using the comparison theorem, the Lyapunov functional, and almost periodic functional hull theory of the impulsive differential equations [17, 18] (see Theorem 3.1, Theorem 4.1, and Theorem 5.1 in Sections 3-5).

The organization of this paper is as follows. In Section 2, we give some basic definitions and necessary lemmas which will be used in later sections. In Section 3, by using the comparison theorem of the impulsive differential equations, we give the permanence of system (1.2). In Section 4, we study the global asymptotical stability of system (1.2) by constructing a suitable Lyapunov functional. In Section 5, some new sufficient conditions are obtained for the existence, uniqueness, and global asymptotical stability of the positive almost periodic solution of system (1.2).

2 Preliminaries

Now, let us state the following definitions and lemmas, which will be useful in proving our main result.

Let $\mathbb{R}^n$ be the n-dimensional Euclidean space with norm $\|x\| = \sum_{i=1}^n |x_i|$. By $\mathbb{I}$, $\mathbb{I} = \{\{t_k\} \in \mathbb{R} : t_k < t_{k+1}, k \in \mathbb{Z}, \lim_{k\to\pm\infty} t_k = \pm\infty\}$, we denote the set of all sequences that are unbounded and strictly increasing with distance $\rho(\{t_k^{(1)}\}, \{t_k^{(2)}\})$. Let $\Omega \subset \mathbb{R}$, $\Omega \neq \emptyset$, $\tau := \sup_{t\in\mathbb{R}}\{\tau_i(t) : i = 1,2,\dots,n\}$, $\xi_0 \in \mathbb{R}$, introduce the following notations:

$PC(\xi_0)$ is the space of all functions $\phi : [\xi_0 - \tau, \xi_0] \to \Omega$ having points of discontinuity at $\mu_1, \mu_2, \ldots \in [\xi_0 - \tau, \xi_0]$ of the first kind and being left continuous at these points.

For $J \subset \mathbb{R}$, $PC(J, \mathbb{R})$ is the space of all piecewise continuous functions from J to $\mathbb{R}$ with points of discontinuity of the first kind t_k, at which it is left continuous.

Let $\phi_i, \varphi \in PC(0)$. Denote by $x_i(t) = x_i(t; 0, \phi_i)$, $y(t) = y(t; 0, \varphi)$, $x_i, y \in \Omega$, $i = 1,2,\dots,n$ the solution of system (1.2) satisfying the initial conditions

$$0 \le x_i(s; 0, \phi_i) = \phi_i(s) < +\infty, \quad s \in [-\tau, 0], \qquad x_i(0+0; 0, \phi_i) = \phi_i(0) > 0;$$
$$0 \le y(s; 0, \varphi) = \varphi(s) < +\infty, \quad s \in [-\tau, 0], \qquad y(0+0; 0, \varphi) = \varphi(0) > 0.$$

Remark 2.1 The problems of existence, uniqueness, and continuity of the solutions of impulsive differential equations have been investigated by many authors. Efficient sufficient conditions which guarantee the existence of the solutions of such systems are given in [17, 18].

Since the solution of system (1.2) is a piecewise continuous function with points of discontinuity of the first kind t_k, $k \in \mathbb{Z}$ we adopt the following definitions for almost periodicity.

Let $T, P \in \mathbb{I}$, $s(T \cup P) : \mathbb{I} \to \mathbb{I}$ be a map such that the set $s(T \cup P)$ forms a strictly increasing sequence and if $D \subset \mathbb{R}$ and $\epsilon > 0$, $\theta_\epsilon(D) = \{t + \epsilon : t \in D\}$, $F_\epsilon(D) = \bigcap\{\theta_\epsilon(D) : \epsilon > 0\}$.

By $\phi = (\varphi(t), T)$ we denote the element from the space $PC \times \mathbb{I}$, and for every sequence of real numbers $\{\alpha_n\}$ we let $\theta_{\alpha_n}\phi$ denote the sets $\{\varphi(t - \alpha_n), T - \alpha_n\} \subset PC \times \mathbb{I}$, where $T - \alpha_n = \{t_k - \alpha_n : k \in \mathbb{Z}, n = 1, 2, \ldots\}$.

Definition 2.1 ([18]) The set of sequences $\{t_k^j\}$, $t_k^j = t_{k+j} - t_k$, $k \in \mathbb{Z}$, $j \in \mathbb{Z}$, $\{t_k\} \in \mathbb{I}$ is said to be uniformly almost periodic if for arbitrary $\epsilon > 0$ there exists a relatively dense set of ϵ-almost periods common for any sequences.

Definition 2.2 ([18]) The function $\varphi \in PC(\mathbb{R}, \mathbb{R})$ is said to be almost periodic, if the following hold:

(1) The set of sequences $\{t_k^j\}$, $t_k^j = t_{k+j} - t_k$, $k \in \mathbb{Z}$, $j \in \mathbb{Z}$, $\{t_k\} \in \mathbb{I}$ is uniformly almost periodic.

(2) For any $\epsilon > 0$ there exists a real number $\delta > 0$ such that if the points t' and t'' belong to one and the same interval of continuity of $\varphi(t)$ and satisfy the inequality $|t' - t''| < \delta$, then $|\varphi(t') - \varphi(t'')| < \epsilon$.

(3) For any $\epsilon > 0$ there exists a relatively dense set T such that if $\eta \in T$, then $|\varphi(t + \eta) - \varphi(t)| < \epsilon$ for all $t \in \mathbb{R}$ satisfying the condition $|t - t_k| > \epsilon$, $k \in \mathbb{Z}$. The elements of T are called ϵ-almost periods.

Lemma 2.1 ([18]) *The set of sequences* $\{t_k^j\}$, $t_k^j = t_{k+j} - t_k$, $k \in \mathbb{Z}$, $j \in \mathbb{Z}$, $\{t_k\} \in \mathbb{I}$ *is uniformly almost periodic if and only if from each infinite sequence of shifts* $\{t_k - \alpha_n\}$, $k \in \mathbb{Z}$, $n = 1, 2, \ldots$, $\alpha_n \in \mathbb{R}$, *we can choose a subsequence which is convergent in* $\mathbb{I}$.

Definition 2.3 ([18]) The sequence ϕ_n, $\phi_n = (\varphi_n(t), T_n) \in PC \times \mathbb{I}$ is uniformly convergent to ϕ, $\phi = (\varphi(t), T) \in PC \times \mathbb{I}$ if and only if for any $\epsilon > 0$ there exists $n_0 > 0$ such that

$$\rho(T, T_n) < \epsilon, \qquad \left\|\varphi_n(t) - \varphi(t)\right\| < \epsilon$$

hold uniformly for $n \geq n_0$ and $t \in \mathbb{R} \setminus F_\epsilon(s(T_n \cup T))$.

Definition 2.4 ([18]) The function $\phi \in PC$ is said to be an almost periodic piecewise continuous function with points of discontinuity of the first kind from the set T if for every sequence of real numbers $\{\alpha'_m\}$ there exists a subsequence $\{\alpha_n\}$ such that $\theta_{\alpha_n}\phi$ is compact in $PC \times \mathbb{I}$.

Lemma 2.2 ([18]) *Let* $\{t_k\} \in \mathbb{I}$. *Then there exists a positive integer* A *such that on each interval of length* 1, *we have no more than* A *elements of the sequence* $\{t_k\}$, *i.e.*,

$$i(s,t) \leq A(t - s) + A,$$

where $i(s,t)$ *is the number of the points* t_k *in the interval* (s,t).

Lemma 2.3 *Let* $\{t_k\} \in \mathbb{I}$. *Then*

$$i(s,t) \geq \frac{t-s}{\theta} - 1,$$

where $i(s,t)$ *is the number of the points* t_k *in the interval* (s,t).

Proof The proof of this lemma is easy and we omit it. This completes the proof. □

3 Uniform persistence

In this section, we establish a uniform persistence result for system (1.2).

Lemma 3.1 ([17]) *Assume that $x \in PC(\mathbb{R})$ with points of discontinuity at $t = t_k$ and is left continuous at $t = t_k$ for $k \in \mathbb{Z}^+$, and*

$$\begin{cases} \dot{x}(t) \le f(t, x(t)), & t \ne t_k, \\ x(t_k^+) \le I_k(x(t_k)), & k \in \mathbb{Z}^+, \end{cases} \tag{3.1}$$

where $f \in C(\mathbb{R} \times \mathbb{R}, \mathbb{R})$, $I_k \in C(\mathbb{R}, \mathbb{R})$ and $I_k(x)$ is nondecreasing in x for $k \in \mathbb{Z}^+$. Let $u^*(t)$ be the maximal solution of the scalar impulsive differential equation

$$\begin{cases} \dot{u}(t) = f(t, u(t)), & t \ne t_k, \\ u(t_k^+) = I_k(u(t_k)) \ge 0, & k \in \mathbb{Z}^+, \\ u(t_0^+) = u_0 \end{cases} \tag{3.2}$$

existing on $[t_0, \infty)$. Then $x(t_0^+) \le u_0$ implies $x(t) \le u^*(t)$ for $t \ge t_0$.

Remark 3.1 If the inequalities (3.1) in Lemma 3.1 is reversed and $u_*(t)$ is the minimal solution of system (3.2) existing on $[t_0, \infty)$, then $x(t_0^+) \ge u_0$ implies $x(t) \ge u_*(t)$ for $t \ge t_0$.

Lemma 3.2 *Assume that $a\theta > \xi^l$, $b > 0$, $h_k > -1$, and $x(t) > 0$ is a solution of the following impulsive logistic equation*:

$$\begin{cases} \dot{x}(t) = x(t)[a - bx(t)], & t \ne t_k, \\ \Delta x(t_k) = h_k x(t_k), & k \in \mathbb{Z}, \end{cases} \tag{3.3}$$

then

$$\limsup_{t \to +\infty} x(t) \le \frac{e^{\xi^l}(a\theta - \xi^l)}{b\theta},$$

where $\xi^l := \ln \inf_{k \in \mathbb{Z}} \frac{1}{1+h_k}$.

Proof Let $u = \frac{1}{x}$, then system (3.3) changes to

$$\begin{cases} \frac{\mathrm{d}u(t)}{\mathrm{d}t} = -au(t) + b, & t \ne t_k, \\ u(t_k^+) = \frac{u(t_k)}{1+h_k}, & k \in \mathbb{Z}. \end{cases}$$

Similar to the proof in [18], we can obtain from Lemma 2.3

$$\begin{aligned} u(t) &= W(t,0)u(0) + b\int_0^t W(t,s)\,\mathrm{d}s \\ &= \prod_{t_k \in [0,t]} \frac{1}{1+h_k} e^{-at} u(0) + b\int_0^t \prod_{t_k \in [s,t]} \frac{1}{1+h_k} e^{-a(t-s)}\,\mathrm{d}s \\ &= \left[\frac{1}{1+h_k}\right]^{\frac{t}{\theta}-1} e^{-at} u(0) + b\int_0^t \left[\frac{1}{1+h_k}\right]^{\frac{t-s}{\theta}-1} e^{-a(t-s)}\,\mathrm{d}s \end{aligned}$$

$$\begin{aligned} &\geq e^{-\xi^l} e^{-(a-\frac{\xi^l}{\theta})t} u(0) + b \int_0^t e^{-\xi^l} e^{-(a-\frac{\xi^l}{\theta})(t-s)}\, \mathrm{d}s \\ &= e^{-\xi^l} e^{-(a-\frac{\xi^l}{\theta})t} u(0) + \frac{b e^{-\xi^l}[1 - e^{-(a-\frac{\xi^l}{\theta})t}]}{a - \frac{\xi^l}{\theta}}, \end{aligned} \tag{3.4}$$

where

$$W(t,s) = \begin{cases} e^{-a(t-s)}, & t_{k-1} < s < t < t_k; \\ \prod_{j=m}^{k+1} \frac{1}{1+h_j} e^{-a(t-s)}, & t_{m-1} < s \leq t_m < t_k < t \leq t_{k+1}. \end{cases}$$

Then

$$\limsup_{t \to +\infty} x(t) = \limsup_{t \to +\infty} \big[u(t)\big]^{-1} \leq \frac{e^{\xi^l}(a\theta - \xi^l)}{b\theta}.$$

This completes the proof. □

Lemma 3.3 *Assume that* $a > \xi^u A$, $b > 0$, $h_k > -1$ *and* $x(t) > 0$ *is a solution of the following impulsive logistic equation*:

$$\begin{cases} \dot{x}(t) = x(t)[a - bx(t)], & t \neq t_k, \\ \Delta x(t_k) = h_k x(t_k), & k \in \mathbb{Z}, \end{cases} \tag{3.5}$$

then

$$\liminf_{t \to +\infty} x(t) \geq \frac{a - \xi^u A}{b e^{\xi^u A}},$$

where A *is defined as that in Lemma* 2.2, $\xi^u := \ln \sup_{k \in \mathbb{Z}} \frac{1}{1+h_k}$.

Proof Let $u = \frac{1}{x}$, then system (3.5) changes to

$$\begin{cases} \frac{\mathrm{d}u(t)}{\mathrm{d}t} = -au(t) + b, & t \neq t_k, \\ u(t_k^+) = \frac{u(t_k)}{1+h_k}, & k \in \mathbb{Z}. \end{cases}$$

Similar to the proof as that in (3.4), we can obtain from Lemma 2.2

$$\begin{aligned} u(t) &= W(t,0)u(0) + b \int_0^t W(t,s)\, \mathrm{d}s \\ &\leq \prod_{t_k \in [0,t]} \frac{1}{1+h_k} e^{-at} u(0) + b \int_0^t \prod_{t_k \in [s,t]} \frac{1}{1+h_k} e^{-a(t-s)}\, \mathrm{d}s \\ &\leq \left[\frac{1}{1+h_k}\right]^{At+A} e^{-at} u(0) + b \int_0^t \left[\frac{1}{1+h_k}\right]^{A(t-s)+A} e^{-a(t-s)}\, \mathrm{d}s \\ &\leq e^{\xi^u A} e^{-(a-\xi^u A)t} u(0) + b \int_0^t e^{\xi^u A} e^{-(a-\xi^u A)(t-s)}\, \mathrm{d}s \\ &= e^{\xi^u A} e^{-(a-\xi^u A)t} u(0) + \frac{b e^{\xi^u A}[1 - e^{-(a-\xi^u A)t}]}{a - \xi^u A}, \end{aligned}$$

which implies that

$$\liminf_{t\to+\infty} x(t) = \liminf_{t\to+\infty} \bigl[u(t)\bigr]^{-1} \geq \frac{a-\xi^u A}{be^{\xi^u A}}.$$

This completes the proof. □

Lemma 3.4 *Assume that $a\theta > \xi^l$ and for $x(t) > 0$, we have*

$$\begin{cases} \dot{x}(t) \leq x(t)[a - b_0 x(t) - b_1 x(t-\tau(t))], & t \neq t_k, \\ \Delta x(t_k) \leq h_k x(t_k), & k \in \mathbb{Z}, \end{cases} \tag{3.6}$$

where

$$a > 0, \qquad b_0, b_1 \geq 0, \qquad b_0 + b_1 > 0.$$

Then there exists a positive constant M such that

$$\limsup_{t\to+\infty} x(t) \leq \frac{e^{\xi^l}(a\theta - \xi^l)}{B\theta} := M,$$

where $B = b_0 + \inf_{t\in\mathbb{R}} b_1 \prod_{t_k\in[t-\tau(t),t)} (1+h_k)^{-1} e^{-a\tau(t)}$.

Proof From system (3.6), we have

$$\begin{cases} \dot{x}(t) \leq ax(t), & t \neq t_k, \\ \Delta x(t_k) \leq h_k x(t_k), & k \in \mathbb{Z}, \end{cases}$$

is equivalent to

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t}[x(t)e^{-at}] \leq 0, & t \neq t_k, \\ \Delta x(t_k) \leq h_k x(t_k), & k \in \mathbb{Z}^+. \end{cases} \tag{3.7}$$

For some $t \in [0,+\infty)$ and $t \neq t_k$, $k \in \mathbb{Z}^+$, consider interval $[t-\tau(t), t)$. Assume that $t_1 < t_2 < \cdots < t_j$ are the impulse points in $[t-\tau(t), t)$. Integrating the first inequality of system (3.7) from $t-\tau(t)$ to t_1 leads to

$$x(t_1)e^{-at_1} \leq x\bigl(t-\tau(t)\bigr)e^{-a(t-\tau(t))}.$$

Integrating the first inequality of system (3.7) from t_1 to t_2 leads to

$$x(t_2)e^{-at_2} \leq x\bigl(t_1^+\bigr)e^{-at_1} \leq (1+h_1)x(t_1)e^{-at_1} \leq (1+h_1)x\bigl(t-\tau(t)\bigr)e^{-a(t-\tau(t))}.$$

Repeating the above process, integrating the first inequality of system (3.7) from t_j to t leads to

$$x(t)e^{-at} \leq x\bigl(t_j^+\bigr)e^{-at_j} \leq (1+h_j)x(t_j)e^{-at_j} \leq \prod_{t_k\in[t-\tau(t),t)} (1+h_k)x\bigl(t-\tau(t)\bigr)e^{-a(t-\tau(t))}.$$

Then

$$x(t-\tau(t)) \geq \prod_{t_k \in [t-\tau(t),t)} (1+h_k)^{-1} e^{-a\tau(t)} x(t). \tag{3.8}$$

Substituting (3.8) into system (3.7) leads to

$$\begin{cases} \dot{x}(t) \leq x(t)[a - Bx(t)], & t \neq t_k, \\ \Delta x(t_k) \leq h_k x(t_k), & k \in \mathbb{Z}. \end{cases}$$

Consider the auxiliary system

$$\begin{cases} \dot{z}(t) = z(t)[a - Bz(t)], & t \neq t_k, \\ z(t_k^+) = (1+h_k) z(t_k), & k \in \mathbb{Z}, \\ z(0^+) = x(0^+). \end{cases} \tag{3.9}$$

By Lemma 3.1, $x(t) \leq z(t)$, where $z(t)$ is the solution of system (3.9). By Lemma 3.2, we have from (3.9)

$$\limsup_{t\to+\infty} x(t) \leq \limsup_{t\to+\infty} z(t) \leq \frac{e^{\xi^l}(a\theta - \xi^l)}{B\theta}.$$

This completes the proof. □

Lemma 3.5 *Assume that $a > \xi^u A$, for $x(t) > 0$ and $\limsup_{t\to+\infty} x(t) \leq M$, we have*

$$\begin{cases} \dot{x}(t) \geq x(t)[a - b_0 x(t) - b_1 x(t-\tau(t))], & t \neq t_k, \\ \Delta x(t_k) = h_k x(t_k), \end{cases} \tag{3.10}$$

where

$$a > K + \xi^u A, \qquad b_0, b_1 \geq 0, \qquad b := b_0 + b_1 > 0, \qquad k \in \mathbb{Z}.$$

Then there exists a positive constant N such that

$$\liminf_{t\to+\infty} x(t) \geq \frac{a - \xi^u A}{D e^{\xi^u A}} := N,$$

where

$$D := b_0 + \sup_{t\in\mathbb{R}} b_1 \prod_{t_k \in [t-\tau(t),t)} (1+h_k)^{-1} e^{-[a-bM]\tau(t)}.$$

Proof According to the assumption, for $\forall \epsilon_1 > 0$, there exists $T_1 > 0$ such that

$$x(t) \leq M + \epsilon_1 \quad \text{for } t \geq T_1.$$

From system (3.10), we have

$$\begin{cases} \dot{x}(t) \geq [a - b(M+\epsilon_1)] x(t) := L_{\epsilon_1} x(t), & t \neq t_k, t \geq T_1, \\ \Delta x(t_k) = h_k x(t_k) + d_k, & k \in \mathbb{Z}, \end{cases}$$

is equivalent to

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{dt}}[x(t)e^{-L_{\epsilon_1}t}] \geq 0, & t \neq t_k, t \geq T_1, \\ \Delta x(t_k) = h_k x(t_k) + d_k, & k \in \mathbb{Z}. \end{cases} \tag{3.11}$$

Similar to the arguments in (3.8), we obtain

$$x\bigl(t - \tau(t)\bigr) \leq \prod_{t_k \in [t-\tau(t),t)} (1 + h_k)^{-1} e^{-L_{\epsilon_1}\tau(t)} x(t). \tag{3.12}$$

Let

$$D_{\epsilon_1} := b_0 + \sup_{t\in\mathbb{R}} b_1 \prod_{t_k \in [t-\tau(t),t)} (1 + h_k)^{-1} e^{-[a-b(M+\epsilon_1)]\tau(t)}.$$

Substituting (3.12) into system (3.10) leads to

$$\begin{cases} \dot{x}(t) \geq x(t)[a - D_{\epsilon_1} x(t)], & t \neq t_k, t \geq T_1, \\ \Delta x(t_k) = h_k x(t_k), & k \in \mathbb{Z}. \end{cases}$$

Consider the auxiliary system

$$\begin{cases} \dot{z}(t) = z(t)[a - D_{\epsilon_1} z(t)], & t \neq t_k, t \geq T_1, \\ z(t_k^+) = (1 + h_k) z(t_k), & k \in \mathbb{Z}, \\ z(T_1^+) = x(T_1^+). \end{cases} \tag{3.13}$$

By Remark 3.1, $x(t) \geq z(t)$, where $z(t)$ is the solution of system (3.13). By Lemma 3.3, we have from (3.13)

$$\lim_{t\to+\infty}\inf x(t) \geq \lim_{t\to+\infty}\inf z(t) \geq \frac{a - \xi^u A}{De^{\xi^u A}}.$$

This completes the proof. □

Let

$$r^u := \max_{1\leq i\leq n} r_i^u, \qquad a^l := \min_{1\leq i\leq n} a_i^l, \qquad h_k^u := \max_{1\leq i\leq n} h_{ik}, \quad k \in \mathbb{Z},$$
$$\xi^l := \ln \inf_{k\in\mathbb{Z}} \frac{1}{1 + h_k^u}, \qquad \xi_{n+1}^l := \ln \inf_{k\in\mathbb{Z}} \frac{1}{1 + h_{n+1,k}}.$$

Proposition 3.1 *Every solution* $x(t) = (x_1(t), x_2(t), \ldots, x_n(t), y(t))^T$ *of system* (1.2) *satisfies*

$$\lim_{t\to\infty}\sup x_i(t) \leq M_i := \frac{e^{\xi^l}(r^u\theta - \xi^l)}{a^l\theta},$$
$$\lim_{t\to\infty}\sup y(t) \leq M_{n+1} := \frac{e^{\xi_{n+1}^l}(r_y^u\theta - \xi_{n+1}^l)}{B_{n+1}\theta},$$

if the following condition holds:

(H_6) $r^u\theta > \xi^l$, $r_y^u\theta > \xi_{n+1}^l$, $j = 1,2,\ldots,n$,

where $B_{n+1} := a_{n+1}^l + \inf_{t\in\mathbb{R}} b_{n+1}^l \prod_{t_k\in[t-\tau_{n+1}(t),t)}(1+h_{n+1,k})^{-1}e^{-r_{n+1}^u\tau_{n+1}(t)}$, $r_y^u := \frac{f^u M_1}{1+\alpha^l M_1}$.

Proof Define $V(t) = \max\{x_1(t),x_2(t),\ldots,x_n(t)\}$ for $t \geq 0$. For any $t^0 \geq 0$ and $t^0 \neq t_k$, $k \in \mathbb{Z}$, there must exist $i \in \{1,2,\ldots,n\}$ and $\delta > t^0$ small enough such that $V(t^0) = x_i(t^0)$ and $x_j(s) \leq x_i(s)$ for $\forall s \in [t^0,\delta)$, $j \neq i$, $i,j \in \{1,2,\ldots,n\}$. Calculating the upper right derivative of $V(t_0)$ from the positive solution for system (1.2), we have

$$D^+V(t^0) = \dot{x}_i(t^0) \leq x_i(t^0)\left[r_i^u - a_i^l x_i(t^0)\right] \leq V(t^0)\left[r^u - a^l V(t^0)\right].$$

By the arbitrariness of t^0, we have

$$D^+V(t) \leq V(t)\left[r^u - a^l V(t)\right], \quad \forall t \neq t_k, k \in \mathbb{Z}. \tag{3.14}$$

Observe that $x_i(t_k^+) = (1+h_{ik})x_i(t_k)$ and $1+h_{ik} > 0$, $k \in \mathbb{Z}$. For arbitrary impulse point t_k, there exists $i_0 \in \{1,2,\ldots,n\}$ such that $V(t_k) = \max\{x_1(t_k),x_2(t_k),\ldots,x_n(t_k)\} = x_{i_0}(t_k)$, that is,

$$V(t_k^+) = x_{i_0}(t_k^+) = (1+h_{i_0k})x_{i_0}(t_k) \leq \left(1+h_k^u\right)V(t_k), \quad k \in \mathbb{Z}. \tag{3.15}$$

By Lemma 3.4, we obtain from (3.14)-(3.15)

$$\limsup_{t\to\infty} x_i(t) \leq \limsup_{t\to\infty} V(t) \leq M_i, \quad i = 1,2,\ldots,n.$$

For any positive constant $\epsilon_2 > 0$, there exists $T_2 > 0$ such that

$$x_i(t) \leq M_i + \epsilon_2 \quad \text{for } t \geq T_2, i = 1,2,\ldots,n.$$

In view of system (1.2), it follows that

$$\begin{cases} \dot{y}(t) \leq y(t)\left[\frac{f^u(M_1+\epsilon_2)}{1+\alpha^l(M_1+\epsilon_2)} - a_{n+1}^l y(t) - b_{n+1}^l y(t-\tau_{n+1}(t))\right], & t \neq t_k, \\ \Delta y(t_k) = h_{n+1,k}y(t_k), & k \in \mathbb{Z}, \end{cases}$$

which implies from Lemma 3.4 that

$$\limsup_{t\to\infty} y(t) \leq M_{n+1}.$$

This completes the proof. □

Define

$$\xi_i^u := \ln\sup_{k\in\mathbb{Z}} \frac{1}{1+h_{ik}}, \quad i = 1,2,\ldots,n,n+1.$$

Proposition 3.2 *Assume that the following condition* (H_7) *holds*:

$$p_1 := r_1^l - \sum_{i=2}^n D_{i1}^u - \int_{-\sigma_1}^0 k_1^u(s)\,ds M_1 - c^u M_{n+1} \geq \xi_1^u A,$$

$$p_i := r_i^l - \sum_{j=1}^{n} D_{ji}^u - \int_{-\sigma_i}^{0} k_i^u(s)\,\mathrm{d}sM_i \geq \xi_i^u A, \quad i = 2,\dots,n,$$

$$p_{n+1} := -r_{n+1}^u + \frac{f^l N_1}{1+\alpha^u N_1} - \int_{-\sigma_{n+1}}^{0} k_{n+1}^u(s)\,\mathrm{d}sM_{n+1} \geq \xi_{n+1}^u A,$$

then every solution $x(t) = (x_1(t), x_2(t),\dots,x_n(t),y(t))^T$ *of system* (1.2) *satisfies*

$$\liminf_{t\to+\infty} x_i(t) \geq N_i := \frac{p_i - \xi_i^u A}{Q_i e^{\xi_i^u A}},$$

$$\liminf_{t\to+\infty} y(t) \geq N_{n+1} := \frac{p_{n+1} - \xi_{n+1}^u A}{Q_{n+1} e^{\xi_{n+1}^u A}},$$

where

$$Q_i := a_i^u + \sup_{t\in\mathbb{R}} b_i^u \prod_{t_k\in[t-\tau_i(t),t)} (1+h_{ik})^{-1} e^{-[p_i-(a_i^u+b_i^u)M_i]\tau_i(t)}, \quad i = 1,2,\dots,n+1.$$

Proof For $\forall\epsilon_3 > 0$, there exists $T_3 > 0$ such that

$$x_i(t) \leq M_i + \epsilon_3, \qquad y(t) \leq M_{n+1} + \epsilon_3 \quad \text{for } t \geq T_3, i = 1,2,\dots,n.$$

From system (1.2), for $t \geq T_3$, we have

$$\begin{cases} \dot{x}_1(t) \geq x_1(t)[r_1^l - \sum_{i=2}^n D_{i1}^u - a_1^u x_1(t) - b_1^u x_1(t-\tau_1(t)) \\ \qquad - \int_{-\sigma_1}^0 k_1^u(s)\,\mathrm{d}s(M_1+\epsilon_2) - c^u(M_{n+1}+\epsilon_2)], \\ \dot{x}_i(t) \geq x_i(t)[r_i^l - \sum_{j=1}^n D_{ji}^u - a_i^u x_i(t) - b_i^u x_i(t-\tau_i(t)) \\ \qquad - \int_{-\sigma_i}^0 k_i^u(s)\,\mathrm{d}s(M_i+\epsilon_2)], \quad i = 2,3,\dots,n, t \neq t_k, \\ \Delta x_i(t_k) = h_{ik} x_i(t_k), \quad k \in \mathbb{Z}. \end{cases}$$

By Lemma 3.5 and the arbitrariness of ϵ_3, we have

$$\liminf_{t\to+\infty} x_i(t) \geq N_i, \quad i = 1,2,\dots,n.$$

Then for $\forall\epsilon_4 > 0$, there exists $T_4 > 0$ such that

$$x_1(t) \geq N_1 - \epsilon_4, \qquad y(t) \leq M_{n+1} + \epsilon_4 \quad \text{for } t \geq T_4.$$

From system (1.2), for $t \geq T_4$, we have

$$\begin{cases} \dot{y}(t) \geq y(t)[-r_{n+1}^u + \frac{f(t)(N_1-\epsilon_4)}{1+\alpha(t)(N_1-\epsilon_4)} - a_{n+1}^u y(t) - b_{n+1}^u y(t-\tau_{n+1}(t)) \\ \qquad - \int_{-\sigma_{n+1}}^0 k_{n+1}^u(s)\,\mathrm{d}s(M_{n+1}+\epsilon_4)], \quad t \neq t_k, \\ \Delta y(t_k) = h_{n+1,k} y(t_k), \quad k \in \mathbb{Z}. \end{cases}$$

By Lemma 3.5 and the arbitrariness of ϵ_4, we have

$$\liminf_{t\to+\infty} y(t) \geq N_{n+1}.$$

This completes the proof. □

Remark 3.2 When h_{ik} $(i = 1, 2, \dots, n + 1, k \in \mathbb{Z}) \equiv 0$ in system (1.2), then Propositions 3.1 and 3.2 improve the corresponding results in [16]. So Propositions 3.1 and 3.2 extend and improve the corresponding results in [16].

Remark 3.3 In view of Propositions 3.1 and 3.2, the distance θ between impulse points, the values of impulse coefficients h_{ik} $(i = 1, 2, \dots, n + 1,\ k \in \mathbb{Z})$ and the number A of the impulse points in each interval of length 1 have negative effect on the uniform persistence of system (1.2).

By Propositions 3.1 and 3.2, we have:

Theorem 3.1 *Assume that* (H_1)-(H_7) *hold, then system* (1.2) *is uniformly persistent.*

Remark 3.4 Theorem 3.1 gives the sufficient conditions for the uniform persistence of system (1.2). Therefore, Theorem 3.1 provides a possible method to study the permanence of the models with almost periodic impulsive perturbations in biological populations.

4 Global asymptotical stability

The main result of this section concerns the global asymptotical stability of positive solution of system (1.2).

Theorem 4.1 *Assume that* (H_1)-(H_7) *hold. Suppose further that*

(H_8) *there exist positive constants* λ_i *such that*

$$\begin{aligned}
&\inf_{t\in\mathbb{R}}\Bigg[\lambda_1 a_1(t) - \frac{\lambda_1 b_1(\delta_1^{-1}(t))}{1 - \dot{\tau}_1(\delta_1^{-1}(t))} - \lambda_1 \int_{-\sigma_1}^{0} k_1(t-s,s)\,\mathrm{d}s \\
&\quad - \frac{\alpha(t)c(t)M_{n+1}}{[1+\alpha(t)N_1]^2} - \sum_{j=1}^{n} \frac{\lambda_1 D_{j1}(t)}{N_1} - \frac{\lambda_{n+1} f(t)}{1+\alpha(t)N_1}\Bigg] > 0, \\
&\inf_{t\in\mathbb{R}}\Bigg[\lambda_i a_i(t) - \frac{\lambda_i b_i(\delta_i^{-1}(t))}{1 - \dot{\tau}_i(\delta_i^{-1}(t))} - \lambda_i \int_{-\sigma_i}^{0} k_i(t-s,s)\,\mathrm{d}s \\
&\quad - \sum_{j=1}^{n} \frac{\lambda_j D_{ij}(t)}{N_j} - \frac{\lambda_{n+1} f(t)}{1+\alpha(t)N_1}\Bigg] > 0, \\
&\inf_{t\in\mathbb{R}}\Bigg[\lambda_{n+1} a_{n+1}(t) - \frac{\lambda_{n+1} b_{n+1}(\delta_{n+1}^{-1}(t))}{1 - \dot{\tau}_{n+1}(\delta_{n+1}^{-1}(t))} \\
&\quad - \lambda_{n+1} \int_{-\sigma_{n+1}}^{0} k_{n+1}(t-s,s)\,\mathrm{d}s - \frac{c(t)}{1+\alpha(t)N_1}\Bigg] > 0,
\end{aligned}$$

where δ_j^{-1} *is an inverse function of* τ_j, $i = 2, \dots, n$, $j = 1, 2, \dots, n + 1$.

Then system (1.2) *is globally asymptotically stable.*

Proof Suppose that $X(t) = (x_1(t), \dots, x_n(t), y(t))^T$ and $X^*(t) = (x_1^*(t), \dots, x_n^*(t), y^*(t))^T$ are any two solutions of system (1.2).

By Theorem 3.1 and (H_8), for $\epsilon_5 > 0$ small enough, there exist $T_5 > 0$ and $\Theta > 0$ such that

$$0 < N_i - \epsilon_5 \le x_i(t) \le M_i + \epsilon_5, \qquad 0 < N_{n+1} - \epsilon_5 \le y(t) \le M_{n+1} + \epsilon_5 \quad \text{for } t \ge T_5,$$

$$\inf_{t\in\mathbb{R}}\Bigg[\lambda_1 a_1(t) - \frac{\lambda_1 b_1(\delta_1^{-1}(t))}{1-\dot{\tau}_1(\delta_1^{-1}(t))} - \lambda_1\int_{-\sigma_1}^{0} k_1(t-s,s)\,\mathrm{d}s - \frac{\alpha(t)c(t)(M_{n+1}+\epsilon_5)}{[1+\alpha(t)(N_1-\epsilon_5)]^2} - \sum_{j=1}^{n}\frac{\lambda_1 D_{j1}(t)}{N_1-\epsilon_5} - \frac{\lambda_{n+1}f(t)}{[1+\alpha(t)(N_1-\epsilon_5)]}\Bigg] > \Theta,$$

$$\inf_{t\in\mathbb{R}}\Bigg[\lambda_i a_i(t) - \frac{\lambda_i b_i(\delta_i^{-1}(t))}{1-\dot{\tau}_i(\delta_i^{-1}(t))} - \lambda_i\int_{-\sigma_i}^{0} k_i(t-s,s)\,\mathrm{d}s - \sum_{j=1}^{n}\frac{\lambda_j D_{ij}(t)}{N_j-\epsilon_5} - \frac{\lambda_{n+1}f(t)}{[1+\alpha(t)(N_1-\epsilon_5)]}\Bigg] > \Theta,$$

$$\inf_{t\in\mathbb{R}}\Bigg[\lambda_{n+1} a_{n+1}(t) - \frac{\lambda_{n+1} b_{n+1}(\delta_{n+1}^{-1}(t))}{1-\dot{\tau}_{n+1}(\delta_{n+1}^{-1}(t))} - \lambda_{n+1}\int_{-\sigma_{n+1}}^{0} k_{n+1}(t-s,s)\,\mathrm{d}s - \frac{c(t)}{[1+\alpha(t)(N_1-\epsilon_5)]}\Bigg] > \Theta,$$

where $i = 2, 3, \ldots, n$.

Construct a Lyapunov functional as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \quad \forall t \ge T_5,$$

where

$$V_1(t) = \sum_{i=1}^{n}\lambda_i\big|\ln x_i(t) - \ln x_i^*(t)\big| + \lambda_{n+1}\big|\ln y(t) - \ln y^*(t)\big|,$$

$$V_2(t) = \sum_{i=1}^{n}\int_{t-\tau_i(t)}^{t}\frac{\lambda_i b_i(\delta_i^{-1}(s))}{1-\dot{\tau}_i(\delta_i^{-1}(s))}\big|x_i(s)-x_i^*(s)\big|\,\mathrm{d}s + \int_{t-\tau_{n+1}(t)}^{t}\frac{\lambda_{n+1} b_{n+1}(\delta_{n+1}^{-1}(s))}{1-\dot{\tau}_{n+1}(\delta_{n+1}^{-1}(s))}\big|y(s)-y^*(s)\big|\,\mathrm{d}s,$$

$$V_3(t) = \sum_{i=1}^{n}\lambda_i\int_{-\sigma_i}^{0}\int_{t+s}^{t} k_i(l-s,s)\big|x_i(l)-x_i^*(l)\big|\,\mathrm{d}l\,\mathrm{d}s + \lambda_{n+1}\int_{-\sigma_{n+1}}^{0}\int_{t+s}^{t} k_{n+1}(l-s,s)\big|y(l)-y^*(l)\big|\,\mathrm{d}l\,\mathrm{d}s.$$

For $t \ne t_k$, $k \in \mathbb{Z}$, calculating the upper right derivative of $V_1(t)$ along the solution of system (1.2), it follows that

$$\begin{aligned} D^+V_1(t) &= \sum_{i=1}^{n}\lambda_i\left[\frac{\dot{x}_i(t)}{x_i(t)} - \frac{\dot{x}_i^*(t)}{x_i^*(t)}\right]\operatorname{sgn}\big(x_i(t)-x_i^*(t)\big) \\ &\quad + \lambda_{n+1}\left[\frac{\dot{y}(t)}{y(t)} - \frac{\dot{y}^*(t)}{y^*(t)}\right]\operatorname{sgn}\big(y(t)-y^*(t)\big) \\ &\le \sum_{i=1}^{n}\lambda_i\Big[-a_i(t)\big|x_i(t)-x_i^*(t)\big| + b_i(t)\big|x_i\big(t-\tau_i(t)\big) - x_i^*\big(t-\tau_i(t)\big)\big| \end{aligned}$$

$$
\begin{aligned}
&+ \int_{-\sigma_i}^{0} k_i(t,s)\bigl|x_i(t+s) - x_i^*(t+s)\bigr|\,\mathrm{d}s\Bigr] \\
&+ \lambda_1 \operatorname{sgn}\bigl(x_1(t) - x_1^*(t)\bigr) \sum_{j=2}^{n} D_{j1}(t) \frac{[x_j(t)x_1^*(t) - x_1(t)x_j^*(t)]}{x_1(t)x_1^*(t)} \\
&+ \sum_{i=2}^{n} \lambda_i \operatorname{sgn}\bigl(x_i(t) - x_i^*(t)\bigr) \sum_{j=1}^{n} D_{ji}(t) \frac{[x_j(t)x_i^*(t) - x_i(t)x_j^*(t)]}{x_i(t)x_i^*(t)} \\
&+ \lambda_1 \operatorname{sgn}\bigl(x_1(t) - x_1^*(t)\bigr)\biggl[-\frac{c(t)y(t)}{1+\alpha(t)x_1(t)} + \frac{c(t)y^*(t)}{1+\alpha(t)x_1^*(t)}\biggr] \\
&+ \lambda_{n+1}\biggl|\frac{f(t)x_1(t)}{1+\alpha(t)x_1(t)} - \frac{f(t)x_1^*(t)}{1+\alpha(t)x_1^*(t)}\biggr| \\
&- \lambda_{n+1}a_{n+1}(t)\bigl|y(t) - y^*(t)\bigr| + \lambda_{n+1}b_{n+1}(t)\bigl|y\bigl(t-\tau_{n+1}(t)\bigr) - y^*\bigl(t-\tau_{n+1}(t)\bigr)\bigr| \\
&+ \lambda_{n+1}\int_{-\sigma_{n+1}}^{0} k_{n+1}(t,s)\bigl|y(t+s) - y^*(t+s)\bigr|\,\mathrm{d}s \\
\le{}& -\sum_{i=1}^{n} \lambda_i a_i(t)\bigl|x_i(t) - x_i^*(t)\bigr| + \sum_{i=1}^{n} \lambda_i b_i(t)\bigl|x_i\bigl(t-\tau_i(t)\bigr) - x_i^*\bigl(t-\tau_i(t)\bigr)\bigr| \\
&+ \sum_{i=1}^{n} \lambda_i \int_{-\sigma_i}^{0} k_i(t,s)\bigl|x_i(t+s) - x_i^*(t+s)\bigr|\,\mathrm{d}s \\
&+ \sum_{j=1}^{n} \frac{\lambda_1 D_{j1}(t)}{N_1 - \epsilon_5}\bigl|x_1(t) - x_1^*(t)\bigr| + \sum_{i=2}^{n}\sum_{j=1}^{n} \frac{\lambda_j D_{ij}(t)}{N_j - \epsilon_5}\bigl|x_i(t) - x_i^*(t)\bigr| \\
&+ \frac{\alpha(t)c(t)(M_{n+1}+\epsilon_5)}{[1+\alpha(t)(N_1-\epsilon_5)]^2}\bigl|x_1(t) - x_1^*(t)\bigr| + \frac{c(t)}{[1+\alpha(t)(N_1-\epsilon_5)]}\bigl|y(t) - y^*(t)\bigr| \\
&+ \frac{\lambda_{n+1}f(t)}{[1+\alpha(t)(N_1-\epsilon_5)]}\bigl|x_1(t) - x_1^*(t)\bigr| \\
&- \lambda_{n+1}a_{n+1}(t)\bigl|y(t) - y^*(t)\bigr| + \lambda_{n+1}b_{n+1}(t)\bigl|y\bigl(t-\tau_{n+1}(t)\bigr) - y^*\bigl(t-\tau_{n+1}(t)\bigr)\bigr| \\
&+ \lambda_{n+1}\int_{-\sigma_{n+1}}^{0} k_{n+1}(t,s)\bigl|y(t+s) - y^*(t+s)\bigr|\,\mathrm{d}s. \qquad (4.1)
\end{aligned}
$$

Here we use the following inequality which has been proved in [16]:

$$
\operatorname{sgn}\bigl(x_i(t) - x_i^*(t)\bigr)\sum_{j=1}^{n} D_{ji}(t)\frac{[x_j(t)x_i^*(t) - x_i(t)x_j^*(t)]}{x_i(t)x_i^*(t)} \le \sum_{j=1}^{n} \frac{D_{ji}(t)}{N_i - \epsilon_3}\bigl|x_j(t) - x_j^*(t)\bigr|.
$$

Moreover, we obtain

$$
\begin{aligned}
D^+V_2(t) ={}& \sum_{i=1}^{n} \frac{\lambda_i b_i(\delta_i^{-1}(t))}{1-\dot{\tau}_i(\delta_i^{-1}(t))}\bigl|x_i(t) - x_i^*(t)\bigr| + \frac{\lambda_{n+1}b_{n+1}(\delta_{n+1}^{-1}(t))}{1-\dot{\tau}_{n+1}(\delta_{n+1}^{-1}(t))}\bigl|y(t) - y^*(t)\bigr| \\
&- \sum_{i=1}^{n} \lambda_i b_i(t)\bigl|x_i\bigl(t-\tau_i(t)\bigr) - x_i^*\bigl(t-\tau_i(t)\bigr)\bigr| \\
&- \lambda_{n+1}b_{n+1}(t)\bigl|y\bigl(t-\tau_{n+1}(t)\bigr) - y^*\bigl(t-\tau_{n+1}(t)\bigr)\bigr|, \qquad (4.2) \\
D^+V_3(t) ={}& \sum_{i=1}^{n} \lambda_i \int_{-\sigma_i}^{0} k_i(t-s,s)\bigl|x_i(t) - x_i^*(t)\bigr|\,\mathrm{d}s
\end{aligned}
$$

$$
\begin{aligned}
&+\lambda_{n+1}\int_{-\sigma_{n+1}}^{0} k_{n+1}(t-s,s)\left|y(t)-y^{*}(t)\right|\,\mathrm{d}s \\
&-\sum_{i=1}^{n}\lambda_{i}\int_{-\sigma_{i}}^{0} k_{i}(t,s)\left|x_{i}(t+s)-x_{i}^{*}(t+s)\right|\,\mathrm{d}s \\
&-\lambda_{n+1}\int_{-\sigma_{n+1}}^{0} k_{n+1}(t,s)\left|y(t+s)-y^{*}(t+s)\right|\,\mathrm{d}s.
\end{aligned} \tag{4.3}
$$

From (4.1)-(4.3), one has

$$
\begin{aligned}
D^{+}V(t) \leq{}& -\Bigg[\lambda_{1}a_{1}(t)-\frac{\lambda_{1}b_{1}(\delta_{1}^{-1}(t))}{1-\dot{\tau}_{1}(\delta_{1}^{-1}(t))}-\lambda_{1}\int_{-\sigma_{1}}^{0} k_{1}(t-s,s)\,\mathrm{d}s \\
&-\frac{\alpha(t)c(t)(M_{n+1}+\epsilon_{5})}{[1+\alpha(t)(N_{1}-\epsilon_{5})]^{2}}-\sum_{j=1}^{n}\frac{\lambda_{1}D_{j1}(t)}{N_{1}-\epsilon_{5}}-\frac{\lambda_{n+1}f(t)}{[1+\alpha(t)(N_{1}-\epsilon_{5})]}\Bigg]\left|x_{1}(t)-x_{1}^{*}(t)\right| \\
&-\sum_{i=2}^{n}\Bigg[\lambda_{i}a_{i}(t)-\frac{\lambda_{i}b_{i}(\delta_{i}^{-1}(t))}{1-\dot{\tau}_{i}(\delta_{i}^{-1}(t))}-\lambda_{i}\int_{-\sigma_{i}}^{0} k_{i}(t-s,s)\,\mathrm{d}s \\
&-\sum_{j=1}^{n}\frac{\lambda_{j}D_{ij}(t)}{N_{j}-\epsilon_{5}}-\frac{\lambda_{n+1}f(t)}{[1+\alpha(t)(N_{1}-\epsilon_{5})]}\Bigg]\left|x_{i}(t)-x_{i}^{*}(t)\right| \\
&-\Bigg[\lambda_{n+1}a_{n+1}(t)-\frac{\lambda_{n+1}b_{n+1}(\delta_{n+1}^{-1}(t))}{1-\dot{\tau}_{n+1}(\delta_{n+1}^{-1}(t))}-\lambda_{n+1}\int_{-\sigma_{n+1}}^{0} k_{n+1}(t-s,s)\,\mathrm{d}s \\
&-\frac{c(t)}{[1+\alpha(t)(N_{1}-\epsilon_{5})]}\Bigg]\left|y(t)-y^{*}(t)\right| \\
\leq{}& -\Theta\Bigg[\sum_{i=1}^{n}\left|x_{i}(t)-x_{i}^{*}(t)\right|+\left|y(t)-y^{*}(t)\right|\Bigg].
\end{aligned} \tag{4.4}
$$

For $t=t_k$, $k\in\mathbb{Z}$, we have

$$
\begin{aligned}
V(t_{k}^{+}) &= V_{1}(t_{k}^{+})+V_{2}(t_{k}^{+})+V_{3}(t_{k}^{+}) \\
&= \sum_{i=1}^{n}\lambda_{i}\left|\ln x_{i}(t_{k}^{+})-\ln x_{i}^{*}(t_{k}^{+})\right|+\lambda_{n+1}\left|\ln y(t_{k}^{+})-\ln y^{*}(t_{k}^{+})\right|+V_{2}(t_{k})+V_{3}(t_{k}) \\
&= \sum_{i=1}^{n}\lambda_{i}\left|\ln\frac{(1+h_{ik})x_{i}(t_{k})+d_{ik}}{(1+h_{ik})x_{i}^{*}(t_{k})+d_{ik}}\right| \\
&\quad+\lambda_{n+1}\left|\ln\frac{(1+h_{n+1,k})y(t_{k})+d_{n+1,k}}{(1+h_{n+1,k})y^{*}(t_{k})+d_{n+1,k}}\right|+V_{2}(t_{k})+V_{3}(t_{k}) \\
&= V_{1}(t_{k})+V_{2}(t_{k})+V_{3}(t_{k}) \\
&= V(t_{k}).
\end{aligned}
$$

Therefore, V is nonincreasing. Integrating (4.4) from T_5 to t leads to

$$
V(t)+\Theta\int_{T_{5}}^{t}\Bigg[\sum_{i=1}^{n}\left|x_{i}(s)-x_{i}^{*}(s)\right|+\left|y(s)-y^{*}(s)\right|\Bigg]\,\mathrm{d}s\leq V(T_{5})<+\infty,\quad\forall t\geq T_{5},
$$

that is,

$$\int_{T_5}^{+\infty}\left[\sum_{i=1}^{n}\left|x_i(s)-x_i^*(s)\right|+\left|y(s)-y^*(s)\right|\right]\mathrm{d}s<+\infty,$$

which implies that

$$\lim_{s\to+\infty}\left|x_i(s)-x_i^*(s)\right|=0,\qquad\lim_{s\to+\infty}\left|y(s)-y^*(s)\right|=0,\quad i=1,2,\ldots,n.$$

Thus, system (1.2) is globally asymptotically stable. This completes the proof. □

Remark 4.1 Theorem 4.1 gives a sufficient condition for the global asymptotical stability of system (1.2). Therefore, Theorem 4.1 extends the corresponding result in [16] and provides a possible method to study the global asymptotical stability of the models with impulsive perturbations in biological populations.

5 Almost periodic solution

In this section, we investigate the existence and uniqueness of a globally asymptotically stable positive almost periodic solution of system (1.2) by using almost periodic functional hull theory of impulsive differential equations.

Let $\{s_n\}$ be any integer valued sequence such that $s_n\to\infty$ as $n\to\infty$. Taking a subsequence if necessary, we have $r_i(t+s_n)\to r_i^*(t)$, $a_i(t+s_n)\to a_i^*(t)$, $b_i(t+s_n)\to b_i^*(t)$, $c(t+s_n)\to c^*(t)$, $f(t+s_n)\to f^*(t)$, $\alpha(t+s_n)\to\alpha^*(t)$, $D_{ij}(t+s_n)\to D_{ij}^*(t)$, $\tau_i(t+s_n)\to\tau_i^*(t)$, $k_i(t+s_n)\to k_i^*(t,s)$, as $n\to\infty$ for $t\in\mathbb{R}$, $s\in(-\infty,0]$, $i=1,2,\ldots,n+1$, $j=1,2,\ldots,n$. From Lemma 2.1 it follows that the set of sequences $\{t_k-s_n\}$, $k\in\mathbb{Z}$ is convergent to the sequence $\{t_k^s\}$ uniformly with respect to $k\in\mathbb{Z}$ as $n\to\infty$.

By $\{k_{n_i}\}$ we denote the sequence of integers such that the subsequence $\{t_{k_{n_i}}\}$ is convergent to the sequence $\{t_k^s\}$ uniformly with respect to $k\in\mathbb{Z}$ as $i\to\infty$.

From the almost periodicity of $\{h_{ik}\}$, it follows that there exists a subsequence of the sequence $\{k_{n_i}\}$ such that the sequences $\{h_{ik_{n_i}}\}$ are convergent uniformly to the limits denoted by h_{ik}^s, $i=1,2,\ldots,n+1$.

Then we get hull equations of system (1.2) as follows:

$$\begin{cases}\dot{x}_1(t)=x_1(t)[r_1^*(t)-a_1^*(t)x_1(t)-b_1^*(t)x_1(t-\tau_1^*(t))\\\qquad-\int_{-\sigma_1}^{0}k_1^*(t,s)x_1(t+s)\,\mathrm{d}s-\frac{c^*(t)y(t)}{1+\alpha^*(t)x_1(t)}]+\sum_{i=2}^{n}D_{i1}^*(t)[x_i(t)-x_1(t)],\\\dot{x}_i(t)=x_i(t)[r_i^*(t)-a_i^*(t)x_i(t)-b_i^*(t)x_i(t-\tau_i^*(t))\\\qquad-\int_{-\sigma_i}^{0}k_i^*(t,s)x_i(t+s)\,\mathrm{d}s]+\sum_{j=1}^{n}D_{ji}^*(t)[x_j(t)-x_i(t)],\quad i=2,3,\ldots,n,\\\dot{y}(t)=y(t)[-r_{n+1}^*(t)+\frac{f^*(t)x_1(t)}{1+\alpha^*(t)x_1(t)}-a_{n+1}^*(t)y(t)-b_{n+1}^*(t)y(t-\tau_{n+1}^*(t))\\\qquad-\int_{-\sigma_{n+1}}^{0}k_{n+1}^*(t,s)y(t+s)\,\mathrm{d}s],\quad t\neq t_k^s,\\\Delta x_j(t_k^s)=h_{jk}^s x_j(t_k^s),\quad j=1,2,\ldots,n,\\\Delta y(t_k^s)=h_{n+1,k}^s y(t_k^s),\quad k\in\mathbb{Z}.\end{cases}\tag{5.1}$$

By the almost periodic theory, we can conclude that if system (1.2) satisfies (H_1)-(H_8), then the hull equations (5.1) of system (1.2) also satisfy (H_1)-(H_8).

By Lemma 4.15 in [18], we can easily obtain the lemma as follows.

Lemma 5.1 *If each hull equation of system* (1.2) *has a unique strictly positive solution, then system* (1.2) *has a unique strictly positive almost periodic solution.*

By using Lemma 5.1, we obtain the following result.

Lemma 5.2 *If system* (1.2) *satisfies* (H_1)-(H_8), *then system* (1.2) *admits a unique strictly positive almost periodic solution.*

Proof By Lemma 5.1, in order to prove the existence of a unique strictly positive almost periodic solution of system (1.2), we only need to prove that each hull equation of system (1.2) has a unique strictly positive solution.

Firstly, we prove the existence of a strictly positive solution of any hull equations (5.1). According to the almost periodic hull theory of impulsive differential equations (see [9]), there exists a time sequence $\{s_n\}$ with $s_n \to \infty$ as $n \to +\infty$ such that $r_i(t+s_n) \to r_i^*(t)$, $a_i(t+s_n) \to a_i^*(t)$, $b_i(t+s_n) \to b_i^*(t)$, $c(t+s_n) \to c^*(t)$, $f(t+s_n) \to f^*(t)$, $\alpha(t+s_n) \to \alpha^*(t)$, $D_{ij}(t+s_n) \to D_{ij}^*(t)$, $\tau_i(t+s_n) \to \tau_i^*(t)$, $k_i(t+s_n) \to k_i^*(t,s)$, as $n \to \infty$ for $t \in \mathbb{R}$, $t \neq t_k$, $k \in \mathbb{Z}$, $s \in (-\infty, 0]$, $i = 1,2,\ldots,n+1$, $j = 1,2,\ldots,n$. There exists a subsequence $\{k_n\}$ of $\{n\}$, $k_n \to +\infty$, $n \to +\infty$ such that $t_{k_n} \to t_k^s$, $h_{ik_n} \to h_{ik}^s$, $i = 1,2,\ldots,n+1$. Suppose $x(t) = (x_1(t), x_2(t), \ldots, x_n(t), y(t))^T$ is any positive solution of hull equations (5.1). By the proof of Theorem 3.1, for $\forall\epsilon > 0$, there exists $T_0 > 0$ such that

$$\begin{aligned} & N_i - \epsilon \le x_i(t) \le M_i + \epsilon, \\ & N_{n+1} - \epsilon \le y(t) \le M_{n+1} + \epsilon, \quad t \ge T_0, i = 1,2,\ldots,n. \end{aligned} \tag{5.2}$$

Let $x_n(t) = x(t+s_n)$ for all $t \ge -s_n + T_0$, $n = 1,2,\ldots$, such that

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1^*(t+s_n) - a_1^*(t+s_n)x_1(t) - b_1^*(t+s_n)x_1(t-\tau_1^*(t+s_n)) \\ \qquad - \int_{-\sigma_1}^0 k_1^*(t+s_n,s)x_1(t+s)\,\mathrm{d}s - \frac{c^*(t+s_n)y(t)}{1+\alpha^*(t+s_n)x_1(t)}] \\ \qquad + \sum_{i=2}^n D_{i1}^*(t+s_n)[x_i(t) - x_1(t)], \\ \dot{x}_i(t) = x_i(t)[r_i^*(t+s_n) - a_i^*(t+s_n)x_i(t) - b_i^*(t+s_n)x_i(t-\tau_i^*(t+s_n)) \\ \qquad - \int_{-\sigma_i}^0 k_i^*(t+s_n,s)x_i(t+s)\,\mathrm{d}s] \\ \qquad + \sum_{j=1}^n D_{ji}^*(t+s_n)[x_j(t) - x_i(t)], \quad i = 2,3,\ldots,n, \\ \dot{y}(t) = y(t)[-r_{n+1}^*(t+s_n) + \frac{f^*(t+s_n)x_1(t)}{1+\alpha^*(t+s_n)x_1(t)} - a_{n+1}^*(t+s_n)y(t) \\ \qquad - b_{n+1}^*(t+s_n)y(t-\tau_{n+1}^*(t+s_n)) \\ \qquad - \int_{-\sigma_{n+1}}^0 k_{n+1}^*(t+s_n,s)y(t+s)\,\mathrm{d}s], \quad t \neq t_k^s, \\ \Delta x_j(t_k^s) = h_{jk}^s x_j(t_k^s), \quad j = 1,2,\ldots,n, \\ \Delta y(t_k^s) = h_{n+1,k}^s y(t_k^s), \quad k \in \mathbb{Z}. \end{cases} \tag{5.3}$$

From the inequality (5.2), there exists a positive constant K which is independent of n such that $|\dot{x}_n| \le K$ for all $t \ge -s_n + T_0$, $n = 1,2,\ldots$. Therefore, for any positive integer r sequence $\{x_n(t) : n \ge r\}$ is uniformly bounded and equicontinuous on $[-s_n + T_0, \infty)$. According to Ascoli-Arzela theorem, one can conclude that there exists a subsequence $\{s_m\}$ of $\{s_n\}$ such that sequence $\{x_m(t)\}$ not only converges on t on $\mathbb{R}$, but it also converges uniformly on any compact set of $\mathbb{R}$ as $m \to +\infty$. Suppose $\lim_{m\to+\infty} x_m(t) = x^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t), y(t))^T$, then we have

$$\begin{aligned} & N_i - \epsilon \le x_i(t) \le M_i + \epsilon, \\ & N_{n+1} - \epsilon \le y(t) \le M_{n+1} + \epsilon, \quad t \in \mathbb{R}, i = 1,2,\ldots,n. \end{aligned}$$

From differential equations (5.3) and the arbitrariness of ϵ, we can easily see that $x^*(t)$ is the solution of the hull equations (5.1) and $N_i \le x_i^*(t) \le M_i$ for all $t \in \mathbb{R}$, $i = 1, 2, \dots, n$. Hence each hull equation of the almost periodic system (1.2) has at least a strictly positive solution.

Now we prove the uniqueness of the strictly positive solution of each hull equations (5.1). Suppose that the hull equations (5.1) have two arbitrary strictly positive solutions $x(t) = (x_1(t), x_2(t), \dots, x_n(t), y(t))^T$ and $x^*(t) = (x_1^*(t), x_2^*(t), \dots, x_n^*(t), y^*(t))^T$, which satisfy

$$\begin{aligned} &N_i - \epsilon \le x_i(t), x_i^*(t) \le M_i + \epsilon, \\ &N_{n+1} - \epsilon \le y(t), y^*(t) \le M_{n+1} + \epsilon, \quad t \in \mathbb{R}, i = 1, 2, \dots, n. \end{aligned}$$

Similar to Theorem 4.1, we define a Lyapunov functional

$$V^*(t) = V_1^*(t) + V_2^*(t) + V_3^*(t), \quad \forall t \in \mathbb{R},$$

where

$$\begin{aligned} V_1^*(t) &= \sum_{i=1}^n \lambda_i \left|\ln x_i(t) - \ln x_i^*(t)\right| + \lambda_{n+1}\left|\ln y(t) - \ln y^*(t)\right|, \\ V_2^*(t) &= \sum_{i=1}^n \int_{t-\tau_i(t)}^t \frac{\lambda_i b_i^*(\delta_i^{*-1}(s))}{1 - \dot{\tau^*}_i(\delta_i^{*-1}(s))} \left|x_i(s) - x_i^*(s)\right| \,\mathrm{d}s \\ &\quad + \int_{t-\tau_{n+1}^*(t)}^t \frac{\lambda_{n+1} b_{n+1}^*(\delta_{n+1}^{*-1}(s))}{1 - \dot{\tau^*}_{n+1}(\delta_{n+1}^{*-1}(s))} \left|y(s) - y^*(s)\right| \,\mathrm{d}s, \\ V_3^*(t) &= \sum_{i=1}^n \lambda_i \int_{-\sigma_i}^0 \int_{t+s}^t k_i^*(l-s, s)\left|x_i(l) - x_i^*(l)\right| \,\mathrm{d}l\,\mathrm{d}s \\ &\quad + \lambda_{n+1} \int_{-\sigma_{n+1}}^0 \int_{t+s}^t k_{n+1}^*(l-s, s)\left|y(l) - y^*(l)\right| \,\mathrm{d}l\,\mathrm{d}s, \end{aligned}$$

where δ_j^{*-1} is an inverse function of τ_j^*, $j = 1, 2, \dots, n+1$. Similar to the argument in (4.4), one has

$$D^+ V^*(t) \le -\Theta \left[\sum_{i=1}^n \left|x_i(t) - x_i^*(t)\right| + \left|y(t) - y^*(t)\right|\right], \quad \forall t \in \mathbb{R}.$$

Summing both sides of the above inequality from t to 0, we have

$$\Theta \int_t^0 \left[\sum_{i=1}^n \left|x_i(s) - x_i^*(s)\right| + \left|y(s) - y^*(s)\right|\right] \mathrm{d}s \le V^*(t) - V(0), \quad \forall t \le 0.$$

Note that V^* is bounded. Hence we have

$$\int_{-\infty}^0 \left[\sum_{i=1}^n \left|x_i(s) - x_i^*(s)\right| + \left|y(s) - y^*(s)\right|\right] \mathrm{d}s < \infty,$$

which implies that

$$\lim_{s\to-\infty}\sum_{i=1}^{n}\left|x_i(s)-x_i^*(s)\right| = \lim_{s\to-\infty}\left|y(s)-y^*(s)\right| = 0, \quad i=1,2,\ldots,n.$$

For arbitrary $\epsilon_0 > 0$, there exists a positive constant L such that

$$\max\left\{\left|x_i(t)-x_i^*(t)\right|,\left|y(t)-y^*(t)\right|\right\} < \epsilon_0, \quad \forall t < -L, i=1,2,\ldots,n.$$

Hence, one has

$$V_1^*(t) \le \sum_{i=1}^{n+1}\frac{\lambda_i\epsilon_0}{N_i}, \quad \forall t < -L,$$

$$V_2^*(t) \le \sum_{i=1}^{n+1}\tau_i^u\frac{\lambda_i b_i^u}{1-\sup_{t\in\mathbb{R}}\dot{\tau}_i(t)}\epsilon_0, \quad \forall t < -L,$$

$$V_3^*(t) \le \sum_{i=1}^{n+1}\lambda_i\int_{-\sigma_i}^{0}(-s)k_i^u(s)\,\mathrm{d}s\epsilon_0, \quad \forall t < -L,$$

which imply that there exists a positive constant ρ such that

$$V^*(t) < \rho\epsilon_0, \quad \forall t < -L.$$

So

$$\lim_{t\to-\infty}V^*(t) = 0.$$

Note that $V^*(t)$ is a nonincreasing function on $\mathbb{R}$, and then $V^*(t)\equiv 0$. That is,

$$x_i(t) = x_i^*(t), \qquad y(t) = y^*(t), \quad \forall t\in\mathbb{R}, i=1,2,\ldots,n.$$

Therefore, each hull equation of system (1.2) has a unique strictly positive solution.

In view of the above discussion, any hull equation of system (1.2) has a unique strictly positive solution. By Lemma 5.1, system (1.2) has a unique strictly positive almost periodic solution. The proof is completed. □

By Theorem 4.1 and Lemma 5.2, we obtain the following.

Theorem 5.1 *Suppose that* (H_1)-(H_8) *hold, then system* (1.2) *admits a unique strictly positive almost periodic solution, which is globally asymptotically stable.*

Remark 5.1 Theorem 5.1 gives sufficient condition for the global asymptotical stability of a unique positive almost periodic solution of system (1.2). Therefore, Theorem 5.1 extends the corresponding result in [16] and provides a possible method to study the existence, uniqueness, and stability of positive almost periodic solution of the models with impulsive perturbations in biological populations.

6 An example and numerical simulations

Example 6.1 Consider the following Lotka-Volterra type predator-prey dispersal system with impulsive effects:

$$\begin{cases} \dot{x}_1(t) = x_1(t)[10 - (5 + \sin(\sqrt{2}t))x_1(t) - 0.1x_1(t-1) - \frac{0.02y(t)}{1+x_1(t)}] \\ \qquad\quad + 0.3[x_2(t) - x_1(t)], \\ \dot{x}_2(t) = x_2(t)[8 + \cos(\sqrt{3}t) - 4x_2(t) - \int_{-0.1}^{0} x_2(t+s)\,ds] \\ \qquad\quad + 0.1\cos(\sqrt{5}t)[x_1(t) - x_2(t)], \\ \dot{y}(t) = y(t)[-0.01|\cos(\sqrt{5}t)| + \frac{2x_1(t)}{1+x_1(t)} - 2y(t)], \quad t \neq t_k, \\ \Delta x_i(t_k) = -0.4x_i(t_k), \quad i = 1, 2, \\ \Delta y(t_k) = -0.5y(t_k), \quad \{t_k : k \in \mathbb{Z}\} \subset \{10k : k \in \mathbb{Z}\}. \end{cases} \tag{6.1}$$

Then system (6.1) is uniformly persistent and has a unique globally asymptotically stable almost periodic solution.

Proof Corresponding to system (1.2), we have $r^u\theta - \xi^l = 90 - 0.5 > 0$ and $r_y^u\theta - \xi_3^l = 1.6 \times 10 - 0.7 > 0$. Then (H_6) in Proposition 3.1 holds. By calculation, we obtain $M_1 = M_2 \approx 3.7$, $M_3 \approx 1.53$. Further, $p_1 = 10 - 0.3 - 0.02 \times 1.53 \geq 0.5 = \xi_1^u A$, $N_1 \approx 0.89$, $p_2 = 7 - 0.1 - 0.1 \times 0.4 \geq 0.5 = \xi_2^u A$, $p_3 = -0.01 + \frac{2\times 0.89}{1+0.89} \geq 0.7 = \xi_3^u A$, which imply that (H_7) in Proposition 3.2 holds. Obviously, (H_1)-(H_5) in Theorem 3.1 hold and system (6.1) is uniformly persistent (see Figure 1).

Taking $\lambda_1 = \lambda_2 = 1$, $\lambda_3 = 0.1$, corresponding to system (1.2), we get

$$\begin{aligned} &\inf_{t\in\mathbb{R}}\biggl[\lambda_1 a_1(t) - \frac{\lambda_1 b_1(\delta_1^{-1}(t))}{1 - \dot{\tau}_1(\delta_1^{-1}(t))} - \lambda_1 \int_{-\sigma_1}^{0} k_1(t-s,s)\,ds \\ &\quad - \frac{\alpha(t)c(t)M_{n+1}}{[1+\alpha(t)N_1]^2} - \sum_{j=1}^{n} \frac{\lambda_1 D_{j1}(t)}{N_1} - \frac{\lambda_{n+1} f(t)}{1+\alpha(t)N_1}\biggr] \\ &\geq 10 - 0.1 - 0 - \frac{0.02 \times 1.53}{(1+0.89)^2} - \frac{0.3}{0.89} - \frac{0.2}{1+0.89} > 0, \end{aligned}$$

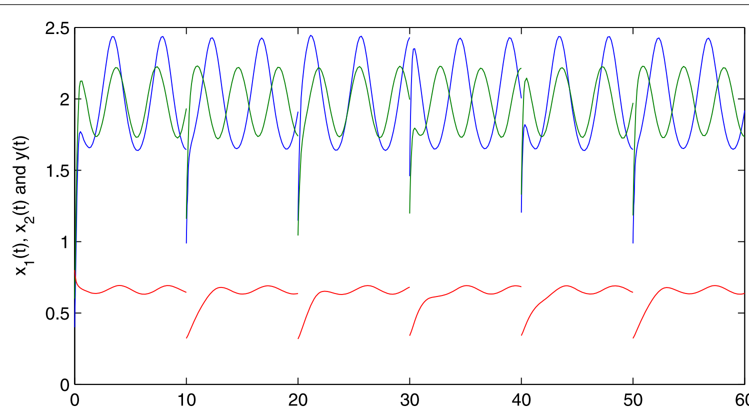

Figure 1 Uniform persistence and almost periodic oscillation of system (6.1).

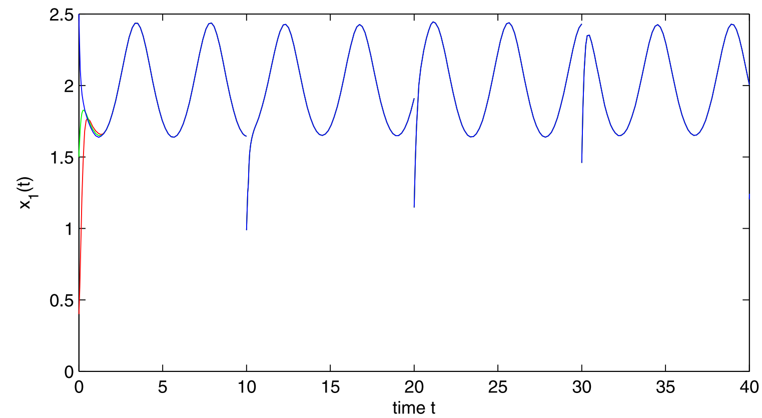

Figure 2 Global asymptotical stability of state variable x_1 of system (6.1).

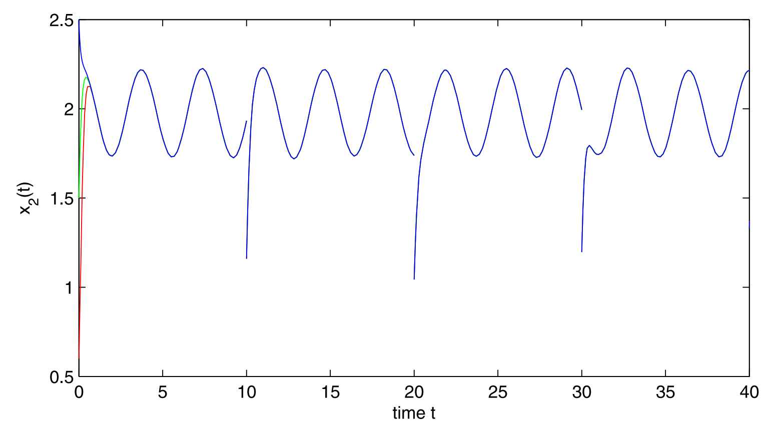

Figure 3 Global asymptotical stability of state variable x_2 of system (6.1).

$$\begin{aligned}
&\inf_{t\in\mathbb{R}}\left[\lambda_i a_i(t)-\frac{\lambda_i b_i(\delta_i^{-1}(t))}{1-\dot{\tau}_i(\delta_i^{-1}(t))}-\lambda_i\int_{-\sigma_i}^{0}k_i(t-s,s)\,\mathrm{d}s-\sum_{j=1}^{n}\frac{\lambda_j D_{ij}(t)}{N_j}-\frac{\lambda_{n+1}f(t)}{1+\alpha(t)N_1}\right]\\
&\quad\geq 7-0-0.1-\frac{0.1}{0.89}-\frac{0.2}{1+0.89}>0,\\
&\inf_{t\in\mathbb{R}}\left[\lambda_{n+1}a_{n+1}(t)-\frac{\lambda_{n+1}b_{n+1}(\delta_{n+1}^{-1}(t))}{1-\dot{\tau}_{n+1}(\delta_{n+1}^{-1}(t))}-\lambda_{n+1}\int_{-\sigma_{n+1}}^{0}k_{n+1}(t-s,s)\,\mathrm{d}s-\frac{c(t)}{1+\alpha(t)N_1}\right]\\
&\quad\geq 0.2-0-0-\frac{0.02}{1+0.89}>0.
\end{aligned}$$

Hence (H_8) in Theorem 5.1 is satisfied. By Theorem 5.1, system (6.1) has a unique globally asymptotically stable almost periodic solution (see Figures 2-4). This completes the proof. □

7 Conclusion

By applying the comparison theorem, the Lyapunov functional, and almost periodic functional hull theorem of the impulsive differential equations, this paper gives some new sufficient conditions for the uniform persistence, global asymptotical stability, and almost

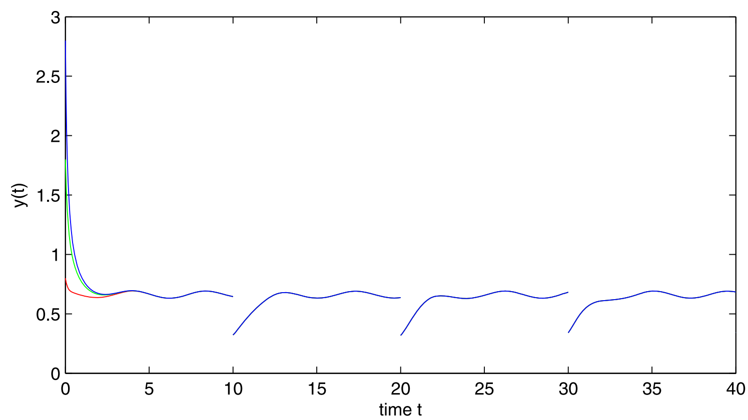

Figure 4 Global asymptotical stability of state variable *y* of system (6.1).

periodic solution to a nonautonomous dispersal competition system with impulsive effects. Theorem 3.1 and Theorem 4.1 indicate that the distance θ between impulse points, the values of the impulse coefficients h_{ik} ($i = 1, 2, \ldots, n$, $k \in \mathbb{Z}$), and the number A of the impulse points in each interval of length 1 are harmful for the uniform persistence and existence of a unique globally asymptotically stable positive almost periodic solution for the model. The main results obtained in this paper are completely new and the method used in this paper provides a possible method to study the uniform persistence and existence of a unique globally asymptotically stable positive almost periodic solution of the models with impulsive perturbations in biological populations.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Each of the authors contributed to each part of this study equally and read and approved the final version of the manuscript.

Author details
[1]School of Mathematics and Computer Science, Panzhihua University, Panzhihua, Sichuan 617000, China. [2]Department of Mathematics, Aba Teachers College, Wenchuan, Sichuan 623002, China.

Acknowledgements
The authors thank the reviewer for his constructive remarks that led to the improvement of the original manuscript.

Received: 14 February 2014 Accepted: 26 September 2014 Published: 14 Oct 2014

References

1. Chen, S, Wang, T, Zhang, J: Positive periodic solution for non-autonomous competition Lotka-Volterra patch system with time delay. Nonlinear Anal., Real World Appl. **5**, 409-419 (2004)
2. Xiao, Y, Chen, L, Tang, S: Permanence and periodic solution in an integrodifferential system with discrete diffusion. J. Syst. Sci. Complex. **16**, 114-121 (2003)
3. Cui, J, Chen, L: The effect of diffusion on the time varying logistic population growth. Comput. Math. Appl. **36**, 1-9 (1998)
4. Levin, SA: Dispersion and population interactions. Am. Nat. **108**, 207-228 (1974)
5. Meng, XZ, Chen, LS: Periodic solution and almost periodic solution for a nonautonomous Lotka-Volterra dispersal system with infinite delay. J. Math. Anal. Appl. **339**, 125-145 (2008)
6. Kouche, M, Tatar, N: Existence and global attractivity of a periodic solution to a nonautonomous dispersal system with delays. Appl. Math. Model. **31**, 780-793 (2007)
7. Dong, LZ, Chen, LS, Shi, PL: Periodic solutions for a two-species nonautonomous competition system with diffusion and impulses. Chaos Solitons Fractals **32**, 1916-1926 (2007)
8. Berryman, AA: The origins and evolution of predator-prey theory. Ecology **75**, 1530-1535 (1992)
9. Sun, YG, Saker, SH: Positive periodic solutions of discrete three-level food-chain model of Holling type II. Appl. Math. Comput. **180**, 353-365 (2006)

10. Li, YK, Lu, LH, Zhu, XY: Existence of periodic solutions in n-species food-chain system with impulsive. Nonlinear Anal., Real World Appl. **7**, 414-431 (2006)
11. Xu, R, Chen, LS, Hao, FL: Periodic solutions of a discrete time Lotka-Volterra type food-chain model with delays. Appl. Math. Comput. **171**, 91-103 (2005)
12. Shen, CX: Permanence and global attractivity of the food-chain system with Holling IV type functional response. Appl. Math. Comput. **194**, 179-185 (2007)
13. Baek, H: A food chain system with Holling type IV functional response and impulsive perturbations. Comput. Math. Appl. **60**, 1152-1163 (2010)
14. Cui, GH, Yan, XP: Stability and bifurcation analysis on a three-species food chain system with two delays. Commun. Nonlinear Sci. Numer. Simul. **16**, 3704-3720 (2011)
15. Liu, ZJ, Zhong, SM, Liu, XY: Permanence and periodic solutions for an impulsive reaction-diffusion food-chain system with ratio-dependent functional response. Commun. Nonlinear Sci. Numer. Simul. **19**, 173-188 (2014)
16. Meng, XZ, Chen, LS: Almost periodic solution of non-autonomous Lotka-Volterra predator-prey dispersal system with delays. J. Theor. Biol. **243**, 562-574 (2006)
17. Lakshmikantham, V, Bainov, DD, Simeonov, PS: Theory of Impulsive Differential Equations. World Scientific, Singapore (1989)
18. Stamov, GT: Almost Periodic Solutions of Impulsive Differential Equations. Lecture Notes in Mathematics (2012)
19. Samoilenko, AM, Perestyuk, NA: Impulsive Differential Equations. World Scientific, Singapore (1995)
20. Bainov, DD, Simeonov, PS: Impulsive Differential Equations: Periodic Solutions and Applications. Longman, Harlow (1993)
21. Jin, Z, Han, MA, Li, GH: The persistence in a Lotka-Volterra competition systems with impulsive. Chaos Solitons Fractals **24**, 1105-1117 (2005)
22. Stamov, GT: On the existence of almost periodic solutions for the impulsive Lasota-Wazewska model. Appl. Math. Lett. **22**, 516-520 (2009)
23. Ahmad, S, Stamov, GT: Almost periodic solutions of N-dimensional impulsive competitive systems. Nonlinear Anal., Real World Appl. **10**, 1846-1853 (2009)
24. Liu, ZJ, Chen, LS: Periodic solution of a two-species competitive system with toxicant and birth pulse. Chaos Solitons Fractals **32**, 1703-1712 (2007)
25. Zhang, TW, Li, YK: Positive periodic solutions for a generalized impulsive n-species Gilpin-Ayala competition system with continuously distributed delays on time scales. Int. J. Biomath. **4**, 23-34 (2011)
26. He, MX, Chen, FD, Li, Z: Almost periodic solution of an impulsive differential equation model of plankton allelopathy. Nonlinear Anal., Real World Appl. **11**, 2296-2301 (2010)
27. Zhang, TW, Li, YK, Ye, Y: On the existence and stability of a unique almost periodic solution of Schoener's competition model with pure-delays and impulsive effects. Commun. Nonlinear Sci. Numer. Simul. **17**, 1408-1422 (2012)
28. Ahmad, S, Stamov, GT: Almost periodic solutions of N-dimensional impulsive competitive systems. Nonlinear Anal., Real World Appl. **10**, 1846-1853 (2009)
29. Ahmad, S, Stamov, GT: On almost periodic processes in impulsive competitive systems with delay and impulsive perturbations. Nonlinear Anal., Real World Appl. **10**, 2857-2863 (2009)

10.1186/1687-1847-2014-264
Cite this article as: Xu and Wu: Dynamics of a nonautonomous Lotka-Volterra predator-prey dispersal system with impulsive effects. *Advances in Difference Equations* 2014, 2014:264