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Alternate control systems

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Abstract

The exponential stability of a class of nonlinear systems by means of alternate control is studied. An exponential stability criterion is given in terms of a set of linear matrix inequalities. Numerical simulations are presented to verify the correction of the obtained results.

Keywords: alternate control system; exponential stabilization; Chua's oscillator

1 Introduction

There are many methods to stabilize a nonlinear system. For example, impulsive control methods [1–4], switching control methods [5–9], *etc.* Intermittent control methods are special cases of switching control methods and have been studied by many researchers, *e.g.*, [10–15]. Within the intermittent control, one adds a continuous control during the first part of the period while in the other part of the period there is no control. This method is available for some cases, but it costs time. For other cases in which the time is very important, this method is not of use. So we advise to add two different controls alternately to the system. We call this system alternate control system. Figure 1 and Figure 2 show the working principles of intermittent control system and alternate control system, from which we conclude that alternate control system is a generalization of intermittent one.

In this paper, we first investigate the stability of the alternate control system, then by using the stability criterion obtained we study the stability of Chua's oscillator. Also, numerical simulations are illustrated to show the effectiveness of the results.

The rest of the paper is organized as follows. In Section 2, we formulate the problem of alternate control system and introduce some notations and lemmas. We then establish, in Section 3, an exponential stability criterion. In Section 4, we discuss the alternate control of Chua's oscillator. Lastly, we conclude the paper.

2 Problem formulation and preliminaries

Consider a class of nonlinear systems described by

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x(t)) + u(t), \\ x(t_0) = x_0, \end{cases} \quad (1)$$

where $x \in R^n$ presents state vector, $f : R^n \rightarrow R^n$ is a continuous nonlinear function satisfying $f(0) = 0$ and there exists a diagonal matrix $L = \text{diag}(l_1, l_2, \dots, l_n) \geq 0$ such that $\|f(x)\|^2 \leq x^T Lx$ for any $x \in R^n$, $A \in R^{n \times n}$ is constant matrix, $u(t)$ denotes the external input of system (1).

Figure 1 Intermittent control: in the first part of the period there is a control $Kx(t)$ and in the other part there is not (O means that the input control is 0).

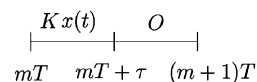
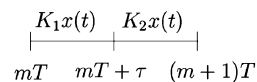


Figure 2 Alternate control: in the first part of the period there is a control $K_1x(t)$ and in the other part there is a control $K_2x(t)$.



For stabilizing the origin of the system (1) by means of a periodically alternate control, we assume that the control imposed on the system is of the following form:

$$u(t) = \begin{cases} K_1x(t), & mT \leq t < mT + \tau, \\ K_2x(t), & mT + \tau \leq t < (m + 1)T, \end{cases} \quad (2)$$

where $K_1, K_2 \in R^{n \times n}$ are constant matrices, $T > 0$ denotes the control period, $\tau \in (0, T)$ is a constant.

Our target is to design suitable T, τ, K_1 , and K_2 such that the system (1) can be stabilized at the origin.

By the control law (2), the system (1) can be rewritten as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x(t)) + K_1x(t), & mT \leq t < mT + \tau, \\ \dot{x}(t) = Ax(t) + f(x(t)) + K_2x(t), & mT + \tau \leq t < (m + 1)T. \end{cases} \quad (3)$$

It is obvious that the system (3) is a classical switched system where the switching rule only depends on the time.

Remark 1 When $K_2(t) = 0$, the alternate control system (3) becomes the classical intermittent control system [10].

In the sequel, we will use the following two lemmas.

Lemma 1 (Sanchez and Perez [16]) *Given any real matrices $\Sigma_1, \Sigma_2, \Sigma_3$ of appropriate dimensions and a scalar $\epsilon \geq 0$ such that $0 < \Sigma_3 = \Sigma_3^T$, the following inequality holds:*

$$\Sigma_1^T \Sigma_2 + \Sigma_2^T \Sigma_1 \leq \epsilon \Sigma_1^T \Sigma_3 \Sigma_1 + \epsilon^{-1} \Sigma_2^T \Sigma_3^{-1} \Sigma_2. \quad (4)$$

Lemma 2 (Boyd et al. [17]) *The LMI*

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0,$$

where $Q(x) = Q^T(x)$, $R(x) = R^T(x)$, and $S(x)$ depend affinely on x , is equivalent to

$$R(x) > 0, \quad Q(x) - S(x)R^{-1}(x)S^T(x) > 0.$$

Throughout this paper, we use $P^T, \lambda_M(P)$, and $\lambda_m(P)$ to denote the transpose, the maximum eigenvalue and the minimum eigenvalue of a square matrix P , respectively. $\|x\|$ is

used to denote the Euclidean norm of the vector x . The matrix norm $\| \cdot \|$ is also referred to the Euclidean norm. We use $P > 0$ ($< 0, \leq 0, \geq 0$) to denote a symmetrical positive (negative, semi-negative, semi-positive) definite matrix P . $f(x(t_1^-))$ is defined by $f(x(t_1^-)) = \lim_{t \rightarrow t_1^-} f(x(t))$.

3 Main results

Theorem 1 *If there exist a symmetric and positive definite matrix $P \in R^{n \times n}$, positive scalar constants $g_1 > 0, \epsilon_1 > 0, \epsilon_2 > 0$, and scalar constant $g_2 \in R$ such that the following hold:*

- (1) $PA + A^T P + PK_1 + K_1^T P + \epsilon_1 P^2 + \epsilon_1^{-1} L + g_1 P \leq 0$,
- (2) $PA + A^T P + PK_2 + K_2^T P + \epsilon_2 P^2 + \epsilon_2^{-1} L - g_2 P \leq 0$,
- (3) $g_1 \tau - g_2(T - \tau) > 0$,

then the origin of the system (3) is exponentially stable, and

$$\|x(t)\| < \sqrt{\frac{\lambda_M(P)}{\lambda_m(P)}} \|x_0\| \exp[-\gamma(t - T)],$$

where $\gamma = \frac{g_1 \tau - g_2(T - \tau)}{2T}$, for any $t > 0$.

Proof Let us construct the following Lyapunov function:

$$V(x(t)) = x^T(x)Px(t), \tag{5}$$

from which we obtain

$$\lambda_m(P)\|x(t)\|^2 \leq V(x(t)) \leq \lambda_M(P)\|x(t)\|. \tag{6}$$

If $mT \leq t < mT + \tau$, then by (3), (4), and (5) we have

$$\begin{aligned} \dot{V}(x) &= 2x^T P \dot{x} \\ &= 2x^T P [Ax + f(x) + K_1 x] \\ &= 2x^T PAx + 2x^T Pf(x) + 2x^T PK_1 x \\ &= x^T [PA + A^T P + PK_1 + K_1^T P]x + 2x^T Pf(x) \\ &\leq x^T [PA + A^T P + PK_1 + K_1^T P]x \\ &\quad + \epsilon_1 x^T P^2 x + \epsilon_1^{-1} x^T L x \\ &= -g_1 V(x) + x^T [PA + A^T P + PK_1 + K_1^T P \\ &\quad + \epsilon_1 P^2 + \epsilon_1^{-1} L + g_1 P]x \\ &\leq -g_1 V(x), \end{aligned}$$

which implies that

$$V(x(t)) \leq V(x((mT)^-)) \exp(-g_1(t - mT)). \tag{7}$$

Similarly, if $mT + \tau \leq t < (m + 1)T$, then we have

$$\begin{aligned} \dot{V}(x) &= 2x^T P \dot{x} \\ &\leq g_2 V(x) + x^T [PA + A^T P + PK_2 + K_2^T P + \epsilon_2 P^2 + \epsilon_2^{-1} L - g_2 P] x \\ &\leq g_2 V(x), \end{aligned}$$

which implies that

$$V(x(t)) \leq V(x((mT + \tau)^-)) \exp(g_2(t - mT - \tau)). \quad (8)$$

It follows from (7) and (8) that:

(1) If $0 \leq t < \tau$, then we have

$$V(x(t)) \leq V(x_0) \exp(-g_1 t).$$

So

$$V(x(\tau^-)) \leq V(x_0) \exp(-g_1 \tau).$$

(2) If $\tau \leq t < T$, then we have

$$\begin{aligned} V(x(t)) &\leq V(x(\tau^-)) \exp(g_2(t - \tau)) \\ &\leq V(x_0) \exp(-g_1 \tau + g_2(t - \tau)). \end{aligned}$$

So

$$V(x(T^-)) \leq V(x_0) \exp(-g_1 \tau + g_2(T - \tau)).$$

(3) If $T \leq t < T + \tau$, then we have

$$\begin{aligned} V(x(t)) &\leq V(x(T^-)) \exp(-g_1(t - T)) \\ &\leq V(x_0) \exp(-g_1 \tau - g_1(t - T) + g_2(T - \tau)). \end{aligned}$$

So

$$V(x((T + \tau)^-)) \leq V(x_0) \exp(-2g_1 \tau + g_2(T - \tau)).$$

(4) If $T + \tau \leq t < 2T$, then we have

$$\begin{aligned} V(x(t)) &\leq V(x((T + \tau)^-)) \exp(g_2(t - T - \tau)) \\ &\leq V(x_0) \exp(-2g_1 \tau + g_2(T - \tau) + g_2(t - T - \tau)). \end{aligned}$$

So

$$V(x((2T)^-)) \leq V(x_0) \exp(-2g_1 \tau + 2g_2(T - \tau)).$$

(5) If $2T \leq t < 2T + \tau$, then we have

$$\begin{aligned} V(x(t)) &\leq V(x((2T)^-)) \exp(-g_1(t - 2T)) \\ &\leq V(x_0) \exp(-2g_1\tau - g_1(t - 2T) + 2g_2(T - \tau)). \end{aligned}$$

So

$$V(x((2T + \tau)^-)) \leq V(x_0) \exp(-3g_1\tau + 2g_2(T - \tau)).$$

(6) If $2T + \tau \leq t < 3T$, then we have

$$\begin{aligned} V(x(t)) &\leq V(x((2T + \tau)^-)) \exp(g_2(t - 2T - \tau)) \\ &\leq V(x_0) \exp(-3g_1\tau + 2g_2(T - \tau) + g_2(t - 2T - \tau)). \end{aligned}$$

So

$$V(x((3T)^-)) \leq V(x_0) \exp(-3g_1\tau + 3g_2(T - \tau)).$$

By induction, we have:

(7) If $mT \leq t < mT + \tau$, i.e., $\frac{t-\tau}{T} < m \leq \frac{t}{T}$, then we have

$$V(x(t)) \leq V(x_0) \exp(-mg_1\tau - g_1(t - mT) + mg_2(T - \tau)). \tag{9}$$

So

$$V(x((mT + \tau)^-)) \leq V(x_0) \exp(-(m + 1)g_1\tau + mg_2(T - \tau)).$$

(8) If $mT + \tau \leq t < (m + 1)T$, i.e., $\frac{t}{T} < m + 1 \leq \frac{t+T-\tau}{T}$, then we have that

$$\begin{aligned} V(x(t)) &\leq V(x((mT + \tau)^-)) \exp(g_2(t - mT - \tau)) \\ &\leq V(x_0) \exp(-(m + 1)g_1\tau + mg_2(T - \tau) \\ &\quad + g_2(t - mT - \tau)). \end{aligned} \tag{10}$$

From (9) we know that

$$\begin{aligned} V(x(t)) &\leq V(x_0) \exp(-mg_1\tau + mg_2(T - \tau)) \\ &= V(x_0) \exp(-(g_1\tau - g_2(T - \tau))m) \\ &< V(x_0) \exp\left(-\left(g_1\tau - g_2(T - \tau)\right)\frac{t - \tau}{T}\right) \\ &< V(x_0) \exp\left(-\left(g_1\tau - g_2(T - \tau)\right)\frac{t - T}{T}\right), \end{aligned} \tag{11}$$

where $mT \leq t < mT + \tau$.

From (10) we know that

Case 1. If $g_2 > 0$, then

$$\begin{aligned} V(x(t)) &\leq V(x_0) \exp(-(m+1)g_1\tau + (m+1)g_2(T-\tau)) \\ &< V(x_0) \exp\left(-\left(g_1\tau - g_2(T-\tau)\right)\frac{t}{T}\right) \\ &\leq V(x_0) \exp\left(-\left(g_1\tau - g_2(T-\tau)\right)\frac{t-\tau}{T}\right) \\ &< V(x_0) \exp\left(-\left(g_1\tau - g_2(T-\tau)\right)\frac{t-T}{T}\right). \end{aligned}$$

Case 2. If $g_2 \leq 0$, then

$$\begin{aligned} V(x(t)) &\leq V(x_0) \exp(-(m+1)g_1\tau + mg_2(T-\tau)) \\ &\leq V(x_0) \exp(-mg_1\tau + mg_2(T-\tau)) \\ &= V(x_0) \exp(-(g_1\tau - g_2(T-\tau))m) \\ &< V(x_0) \exp\left(-\left(g_1\tau - g_2(T-\tau)\right)\frac{t-T}{T}\right). \end{aligned}$$

So, for any $g_2 \in R$, we have

$$V(x(t)) < V(x_0) \exp\left(-\left(g_1\tau - g_2(T-\tau)\right)\frac{t-T}{T}\right), \tag{12}$$

where $mT + \tau \leq t < (m+1)T$.

It follows from (11) and (12) that, for any $t > 0$,

$$V(x(t)) < V(x_0) \exp\left(-\left(g_1\tau - g_2(T-\tau)\right)\frac{t-T}{T}\right). \tag{13}$$

By (5), (6), and (13), we conclude that

$$\|x(t)\| < \sqrt{\frac{\lambda_M(P)}{\lambda_m(P)}} \|x_0\| \exp[-\gamma(t-T)],$$

where $\gamma = \frac{g_1\tau - g_2(T-\tau)}{2T}$, for any $t > 0$.

So we finish the proof. □

From Lemma 2, we know that the two conditions of Theorem 1 are equivalent to the following two LMIs, respectively:

$$\begin{bmatrix} PA + A^T P + PK_1 + K_1^T P + \epsilon_1^{-1} L + g_1 P & -P \\ -P & -\epsilon_1^{-1} I \end{bmatrix} \leq 0, \tag{14}$$

$$\begin{bmatrix} PA + A^T P + PK_2 + K_2^T P + \epsilon_2^{-1} L - g_2 P & -P \\ -P & -\epsilon_2^{-1} I \end{bmatrix} \leq 0. \tag{15}$$

4 Numerical example

The original and dimensionless form of a Chua's oscillator [18] is given by

$$\begin{cases} \dot{x}_1 = \alpha(x_2 - x_1 - g(x_1)), \\ \dot{x}_2 = x_1 - x_2 + x_3, \\ \dot{x}_3 = -\beta x_2, \end{cases} \quad (16)$$

where α and β are parameters and $g(x)$ is the piecewise linear characteristics of Chua's diode, which is defined by

$$g(x_1) = bx_1 + 0.5(a - b)(|x_1 + 1| - |x_1 - 1|), \quad (17)$$

where $a < b < 0$ are two constants.

In this section, we set the system parameters as $\alpha = 9.2156$, $\beta = 15.9946$, $a = -1.24905$, and $b = -0.75735$, which make Chua's circuit (16) chaotic [18]. Figure 3 shows the chaotic phenomenon of Chua's oscillator with the initial condition $x(0) = (5, 1, -3)'$.

We rewrite the system (16) as follows:

$$\dot{x} = Ax + f(x), \quad (18)$$

where

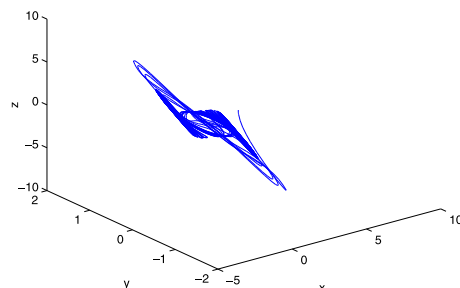
$$A = \begin{bmatrix} -\alpha - \alpha b & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix},$$

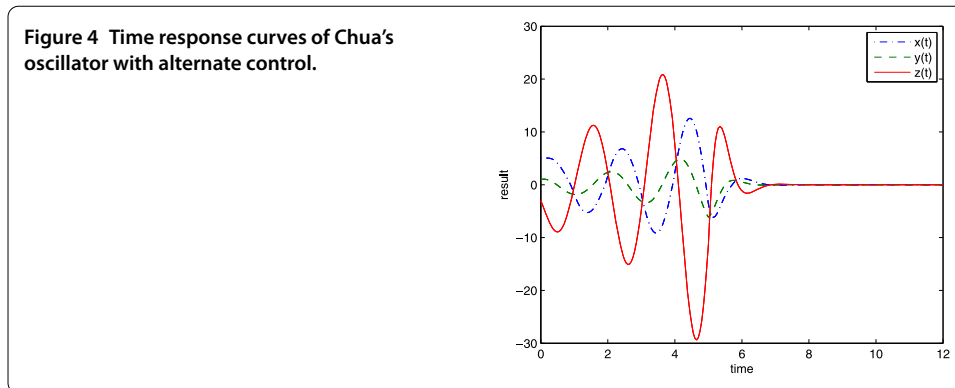
$$f(x) = \begin{bmatrix} -0.5\alpha(a - b)(|x_1 + 1| - |x_1 - 1|) \\ 0 \\ 0 \end{bmatrix}.$$

So

$$\begin{aligned} \|f(x)\|^2 &= 0.25\alpha^2(a - b)^2[(x_1 + 1)^2 + (x_1 - 1)^2 - 2|(x_1 + 1)(x_1 - 1)|] \\ &= 0.5\alpha^2(a - b)^2(x_1^2 + 1 - |x_1^2 - 1|) \\ &= \begin{cases} \alpha^2(a - b)^2, & x_1^2 > 1, \\ \alpha^2(a - b)^2x_1^2, & x_1^2 \leq 1 \end{cases} \\ &\leq \alpha^2(a - b)^2x_1^2. \end{aligned}$$

Figure 3 The chaotic phenomenon of Chua's oscillator with the initial condition $x(0) = (5, 1, -3)'$.





Thus we can choose $L = \text{diag}(\alpha^2(a - b)^2, 0, 0)$.

Choosing

$$K_1 = \text{diag}(-50, -40, -30),$$

$$K_2 = \text{diag}(-10, -20, -10).$$

With $T = 10$ and $\tau = 5$, solving LMIs (14), (15) and inequality $g_1 \tau - g_2(T - \tau) > 0$, we obtain a feasible solution:

$$\epsilon_1 = \epsilon_2 = 0.5, \quad g_1 = 21, \quad g_2 = 20,$$

and

$$P = \begin{bmatrix} 0.9735 & 0.1652 & 0.0040 \\ 0.1652 & 0.1648 & -0.0412 \\ 0.0040 & -0.0412 & 0.1423 \end{bmatrix}.$$

Thus by the previous theorem we see that the origin of the system (3) is exponentially stable. The time response curves of Chua's oscillator with alternate control is shown in Figure 4.

5 Conclusions

This paper gives a new model of control system, namely alternate control system. A stability criterion is given in terms of linear matrix inequalities. By the new method, the chaotic Chua circuit is controlled.

Obviously, there is no *rest time* in an alternate control system. By comparing our model with the traditional intermittent control system, we know that our model is a generalization of intermittent control system. The proposed method can be applied to linear and nonlinear systems.

This paper considers systems without delay. For delayed systems [19–21], we know that the methods used to deal with them are different from ones of the systems without delay. We are ready to focus on this aspect in future papers.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

CL has proposed the ideal of *alternate control*. YF has proved the main theory and prepared the paper with latex. TH has provided all the figures of the paper. WZ has given some advice to improve the paper. All authors have read and approved the final manuscript.

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Acknowledgements

This research is supported by the Natural Science Foundation of China (grant No. 61374078), NPRP grant # NPRP 4-1162-1-181 from the Qatar National Research Fund (a member of Qatar Foundation), Scientific & Technological Research Foundation of Chongqing Municipal Education Commission (grant Nos. KJ1401006, KJ1401019), the Fundamental Research Funds for the Central Universities (grant No. XDJK2015D004) and Key Program of Chongqing Three Gorges University (grant No. 14ZD18).

Received: 9 August 2014 Accepted: 19 November 2014 Published: 03 Dec 2014

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10.1186/1687-1847-2014-305

Cite this article as: Feng et al.: Alternate control systems. *Advances in Difference Equations* 2014, **2014**:305