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Stability analysis of Markovian jumping impulsive stochastic delayed RDCGNNs with partially known transition probabilities

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Abstract

This paper considers the robust stability for a class of Markovian jump impulsive stochastic delayed reaction-diffusion Cohen-Grossberg neural networks with partially known transition probabilities. Based on the Lyapunov stability theory and linear matrix inequality (LMI) techniques, some robust stability conditions guaranteeing the global robust stability of the equilibrium point in the mean square sense are derived. To reduce the conservatism of the stability conditions, improved Lyapunov-Krasovskii functional and free-connection weighting matrices are introduced. An example shows the proposed theoretical result is feasible and effective.

Keywords: impulsive; stochastic reaction-diffusion neural networks; asymptotical stability; Markovian jump; mixed time delays

1 Introduction

During the last decades, neural networks (NNs) with time delays have received considerable attention, due to time delays are existed in many fields, for instance, finite switching speeds of amplifiers and transmission of signals in a network, which effect the system performance. So, the stability analysis of NNs with time delays has attracted more and more attention of the researchers [1–3]. As is well known, uncertainties are inevitable in NNs because of the existence of modeling errors, external disturbance and parameter fluctuation in practice. Therefore, it is important to ensure the stability of the designed networks in the presence of such uncertainties. Accordingly, many sufficient conditions guaranteeing the robust stability of delayed NNs have been derived in [2, 3]. On the other hand, impulsive phenomena can be found in a wide variety of evolutionary process, particularly the state of the networks is subject to instantaneous perturbations, in implementation of electronic networks, which may be caused by the switching phenomenon, frequency change or other sudden noise, that is, it exhibits impulsive effects [3–6]. NNs are often subject to impulsive perturbations that in turn affect dynamical behaviors of the systems. Thus, it is necessary to take both time delays and impulsive effects into account on dynamical behaviors of NNs [3, 7, 8]. However, the diffusion phenomena could not be ignored in NNs and electric circuits when electrons are moving in a non-uniform electromagnetic field. Therefore, it is essential to consider the state variables varying with the time and space variables. The NNs with diffusion terms can commonly be expressed by partial differential equations [9–25]. In particular, by using delay differential inequality with impulses, the authors in [9–11] have derived sufficient stability conditions of the equilibrium point for impulsive reaction-diffusion NNs (RDNNs) with delays and Neumann boundary conditions. In [12], under some suitable assumptions, utilizing a matrix decomposition and linear matrix inequality (LMI) method, the authors have proposed some the new global asymptotic stability sufficient conditions for RDNNs with continuously distributed delays.

In the real world, a system is usually affected by unknown disturbances, which may be regarded as stochastic effects. Consequently, it is of significant importance to study stochastic effects for the NNs. In recent years, the dynamic behaviors of stochastic RDNNs, especially the stability of stochastic RDNNs, have become a hot study topic. For example, the authors in [15–17] have obtained some criteria to guarantee the almost sure exponential stability, and mean square exponential stability for RDNNs with continuously distributed delays and stochastic influence.

Markovian jump systems (MJSs) involve both time-evolving and event-driven mechanisms, which can be employed to model abrupt phenomena such as random failures and repairs of the components, changes in the interconnections of subsystems, sudden environment changes, *etc*. The issues of stability for RDNNs have been well investigated [18–22]. Particularly, the stability of stochastic RDNNs with Markovian jumping has been studied in [21, 22], delay-dependent stability criteria were derived. It is noted that the authors in [21, 22] did not take impulsive phenomena and diffusion effects into account on the dynamic behaviors of RDNNs. Moreover, the obtained results are independent on the measure of the space. However, the stability analysis problem for Markovian jump impulsive stochastic reaction-diffusion Cohen-Grossberg NNs (RDCGNNs) with partially known transition probabilities and mixed time delays, has received little attention, in despite its practical importance.

Motivated by the above discussions, the objective of this paper is to study the asymptotical stability of the equilibrium point in the mean square sense for Markovian jump impulsive stochastic RDCGNNs with partially known transition probabilities and mixed time delays. In this paper, some novel criteria for the asymptotical stability in the mean square sense are derived by employing a new Lyapunov-Krasovskii functional and LMI approach. To reduce the conservatism of the stability conditions, improved Lyapunov-Krasovskii functional and free-connection weighting matrices are introduced. The obtained criteria are dependent on delays and the reaction-diffusion terms. The results of this paper are new and they complement previously known results.

To begin with, we introduce some notation and recall some basic definitions.

The superscript 'T' stands for matrix transposition; R^n denotes the n-dimensional Euclidean space. Vector $X \in R^n$, its norm is defined as $|X| = \sqrt{X^T X}$. For symmetric matrices A and B, the notation $A > B$ ($A \geq B$) means that $A - B$ is positive definite (positive-semidefinite). The symmetric terms in a symmetric matrix are denoted by $*$. Mathematical expectation will be denoted by $E[\cdot]$. trace($\cdot$) denotes the trace of the corresponding matrix. I denotes the identity matrix with compatible dimensions. N^* is the set of positive integers, $Z^+ = \{1, 2, \dots, n\}$.

$PC[\Lambda \times \Omega, R^n] = \{u(t,x) : \Lambda \times \Omega \to R^n | u(t,x)$ is continuous at $t \neq t_k$, $u(t_k^+, x) = u(t_k, x)$ and $u(t_k^-, x)$ exists for $t, t_k \in \Lambda$, $k \in N^*\}$, where $\Lambda \subset R$ is an interval. $PC[\Omega] = \{\varphi : (-\infty, 0] \times \Omega \to R^n | \varphi(s^+, x) = \varphi(s, x)$ for $s \in (-\infty, 0]$, $\varphi(s^-, x)$ exists for $s \in (-\infty, 0]$, $\varphi(s^-, x) = \varphi(s, x)$ for all but at most countable points $s \in (-\infty, 0]\}$.

For $\varphi(s,x) = (\varphi_1(s,x),\dots,\varphi_n(s,x))^T \in PC[\Omega]$, the norm on $PC[\Omega]$ is defined by

$$\|\varphi\|_2 = \left(\int_\Omega \sum_{i=1}^n \sup_{-\infty\le s\le 0} \left|\varphi_i(s,x)\right|^2 dx\right)^{\frac{1}{2}}.$$

For $u(t,x) = (u_1(t,x),u_2(t,x),\dots,u_n(t,x))^T \in R^n$, we define

$$\left\|u(t,x)\right\|_2 = \left[\int_\Omega \sum_{i=1}^n \left|u_i(t,x)\right|^2 dx\right]^{\frac{1}{2}}.$$

$L^2_{F_t}[\Omega]$ denotes the family of all bounded F_t-measurable, $PC[\Omega]$-valued stochastic variables $\varphi(s,x)$ such that $\int_\Omega \int_{-\infty}^0 E[|\varphi(s,x)|^2]\,ds\,dx < \infty$. Let $(\bar{\Omega}, F, \{F_t\}_{t\ge 0}, P)$ be a complete probability space $(\bar{\Omega}, F, \{F_t\}_{t\ge 0}, P)$ with a filtration $\{F_t\}_{t\ge 0}$ satisfying the usual conditions.

2 Problem formulation and preliminaries

In this paper, we consider the following Markovian jump impulsive stochastic delayed RD-CGNNs with partially known transition probabilities

$$\begin{aligned} du(t,x) = {}& \sum_{l=1}^m \frac{\partial}{\partial x_l}\left(D_l \frac{\partial u(t,x)}{\partial x_l}\right) dt - \tilde{\alpha}\bigl(u(t,x),r(t)\bigr)\Bigl[a\bigl(u(t,x),r(t)\bigr) \\ & - B\bigl(r(t)\bigr)f\bigl(u(t,x)\bigr) - C\bigl(r(t)\bigr)g\bigl(u\bigl(t-d(t),x\bigr)\bigr) - E\bigl(r(t)\bigr) \\ & \times \int_{-\infty}^t K(t-s)h\bigl(u(s,x)\bigr)\,ds + J\Bigr] dt \\ & + \sigma\left(t,x,u(t,x),u\bigl(t-d(t),x\bigr),\int_{-\infty}^t K(t-s)h\bigl(u(s,x)\bigr)\,ds, r(t)\right) dw(t), \\ & t \ge t_0 \ge 0, t \ne t_k, x \in \Omega, \end{aligned} \tag{1}$$

$$u(t_k,x) = I_k u\bigl(t_k^-,x\bigr), \quad t = t_k, \tag{2}$$

$$\frac{\partial u(t,x)}{\partial \bar{n}} = 0, \quad (t,x) \in [0,+\infty) \times \partial\Omega, \tag{3}$$

$$u(t_0+s,x) = \varphi(s,x), \quad (s,x) \in (-\infty,0] \times \Omega, \tag{4}$$

where $x = (x_1,x_2,\dots,x_m)^T \in \Omega \subset R^m$, $\Omega = \{x||x_i| < d_l, l = 1,2,\dots,m\}$ is a compact set with smooth boundary $\partial\Omega$ and measure $\operatorname{mes}\Omega > 0$; $u(t,x)$ denotes the state vector associated with the n neurons; $\tilde{\alpha}(u(t,x),r(t))$ represents an amplification function, and $a(u(t,x),r(t))$ is the behaved function. $f(u(t,x))$, $g(u(t,x))$, and $h(u(s,x))$ are the neuron activation functions, and $J = (J_1,J_2,\dots,J_n)^T$ denotes a constant external input vector. $B(r(t)) = (b_{ij}(r(t)))_{n\times n}$, $C(r(t)) = (c_{ij}(r(t)))_{n\times n}$, and $E(r(t)) = (e_{ij}(r(t)))_{n\times n}$ are the connection weight matrix, the time-varying delay connection weight matrix, and the distributed delay connection weight matrix, respectively. $d(t)$ denotes the time-varying delay, $d(t)$ is assumed to satisfy $0 \le d(t) \le d$, $\dot{d}(t) \le \mu$, where d and μ are constants; $K(t-s) = \operatorname{diag}[k_1(t-s),\dots,k_n(t-s)]$ and the delay kernel $k_j(\cdot)$ is a real value non-negative continuous function defined on $[0,+\infty)$ and such that $\int_0^{+\infty} k_j(\theta)\,d\theta = 1$. $D_l = \operatorname{diag}(D_{1l},D_{2l},\dots,D_{nl})$, $D_{il} = D_{il}(t,x,u) \ge 0$, stand for transmission diffusion operator along the ith neurons,

$i,j \in Z^+$. $\sigma(\cdot) = \sigma(t,x,u(t,x),u(t-d(t),x),\int_{-\infty}^{t} K(t-s)h(u(s,x))\,ds, r(t))$ is the noise intensity matrix; $w(t) = (w_1(t),\ldots,w_n(t))^T$ is a standard Brownian motion; $I_k = (I_{k_1},\ldots,I_{k_n})^T$ is the impulse gain matrix at the moments of time t_k, $k \in N^*$; $u(t_k,x)$ is the impulse at moment t_k and in space x. The discrete set $\{t_k\}$ satisfies $0 \le t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots$, $\lim_{k\to\infty} t_{k+1} = \infty$; $u(t^-,x)$ and $u(t^+,x)$ denote the left-hand limit and the right-hand limit of $u(t,x)$ at time t, respectively; $\bar{n}$ is the outer normal vector of $\partial\Omega$, $\varphi(s,x) = (\varphi_1(s,x),\ldots,\varphi_n(s,x))^T$ on $[(-\infty,0] \times \Omega, R^n]$ in $L^2_{F_t}[\Omega]$.

Let $r(t)$, $t \ge 0$, be a right-continuous Markovian chain on the probability space which takes values in the finite space $S = \{1,2,\ldots,N\}$ with generator $\Gamma = (\gamma_{ij})_{N\times N}$ given by

$$P\{r(t+\delta) = j | r(t) = i\} = \begin{cases} \gamma_{ij}\delta + o(\delta), & \text{if } i \neq j, \\ 1 + \gamma_{ij}\delta + o(\delta), & \text{if } i = j. \end{cases}$$

Here $\delta > 0$ and $\lim_{\delta\to 0} o(\delta)/\delta = 0$, $\gamma_{ij} \ge 0$ is the transition rate from i to j if $i \neq j$ and $\gamma_{ii} = -\sum_{i\neq j} \gamma_{ij}$.

Since the transition probability is relation to the transition rates for the continuous-time MJSs, the concept of partly unknown transition probabilities proposed in [26–29] means that no knowledge of unknown elements in matrix Θ is required. For example, the transition rate matrix Θ for system (1) with N operation modes may be described as

$$\Theta = \begin{bmatrix} \gamma_{11} & ? & \gamma_{13} & \cdots & \gamma_{1N} \\ ? & ? & \gamma_{23} & \cdots & \gamma_{2N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ ? & \gamma_{N2} & ? & \cdots & \gamma_{NN} \end{bmatrix}, \tag{5}$$

where '?' represents the unknown transition rate. For notation clarity, $\forall i \in S$, the set Ξ^i denotes $\Xi^i = \Xi^i_k \cup \Xi^i_{uk}$ with $\Xi^i_k = \{j : \gamma_{ij} \text{ is known for } j \in S\}$, $\Xi^i_{uk} = \{j : \gamma_{ij} \text{ is unknown for } j \in S\}$. Moreover, when $\Xi^i_k \neq \phi$, it is further expressed as

$$\Xi^i_k = \{k^i_1, k^i_2, \ldots, k^i_n\},$$

where $1 \le n \le N$, $n \in N^*$ and $k^i_j \in N^*$, $1 \le k^i_j \le N$, $j = 1,2,\ldots,n$ represent the jth known element of the set Ξ^i_k in the ith row of the transition rate matrix Θ.

Remark 1 In [19, 25], the authors considered stability RDNNs with Markovian jumping parameters. It is noted that the jumping process was commonly assumed to be completely available ($\Xi^i_{uk} = \phi$, $\Xi^i_k = \Xi^i$) in [19, 25]. However, in most cases the transition probabilities of MJSs, are not exactly known. Recently, a considerable amount of attention has been paid to studying the stability and stabilization of general MJSs governed by ordinary differential equations with partly unknown transition probabilities [27–29]. As is well known, the stability analysis of partial differential equations with partial information on transition probabilities is more complicated, very few results on such systems have appeared. In this paper, the new stability criteria for a class of novel Markovian jump impulsive stochastic delayed CGRDNNs with partial information on transition probability are proposed.

For the sake of simplicity, we write $r(t) = i \in S$, the matrices $B(r(t))$, $C(r(t))$, $E(r(t))$ and $\sigma(\cdot)$ will be written as B_i, C_i, E_i and $\sigma_i(\cdot)$, respectively. Hence, (1) and (2) can be rewritten as the follows:

$$du(t,x) = \sum_{l=1}^{n} \frac{\partial}{\partial x_l}\left(D_l \frac{\partial u(t,x)}{\partial x_l}\right) dt - \tilde{\alpha}\big(u(t,x),i\big)\Big[A\big(u(t,x),i\big) - B_i f\big(u(t,x)\big)$$
$$- C_i g\big(u\big(t-d(t),x\big)\big) - E_i \int_{-\infty}^{t} K(t-s)h\big(u(s,x)\big)\,ds + J\Big]\,dt$$
$$+ \sigma_i\left(t,x,u(t,x),u\big(t-d(t),x\big),\int_{-\infty}^{t} K(t-s)h\big(u(s,x)\big)\,ds\right) dw(t),$$
$$t \geq t_0 \geq 0, t \neq t_k, x \in \Omega, \tag{6}$$
$$u(t_k,x) = I_k u\big(t_k^-,x\big), \quad t = t_k, x \in \Omega. \tag{7}$$

The main aim of this paper is to investigate the stability of system (1)-(4). Let $u^* = (u_1^*, u_2^*, \dots, u_n^*)^T$ be the equilibrium point of system (1)-(4). Now, we set $z(t) = u(t,x) - u^*$, which yields the following system:

$$dz(t) = \sum_{l=1}^{n} \frac{\partial}{\partial x_l}\left(D_l \frac{\partial z(t)}{\partial x_l}\right) dt - \alpha\big(z(t),i\big)\Big[A\big(z(t),i\big) - B_i \bar{f}\big(z(t)\big)$$
$$- C_i \bar{g}\big(z\big(t-d(t)\big)\big) - E_i \int_{-\infty}^{t} K(t-s)\bar{h}\big(z(t)\big)\,ds\Big]\,dt$$
$$+ \sigma_i\left(t,x,z(t)+u^*,z\big(t-d(t)\big)+u^*,\int_{-\infty}^{t} K(t-s)h\big(z(s)+u^*\big)\,ds\right) dw(t), \tag{8}$$
$$z(t_k) = I_k z\big(t_k^-\big), \quad t = t_k, \tag{9}$$
$$\frac{\partial z(t)}{\partial \bar{n}}\Big|_{\partial\Omega} = 0, \quad t \geq t_0, \tag{10}$$
$$z(t_0+s) = \psi(s), \qquad (s,x) \in (-\infty,0] \times \Omega, \tag{11}$$

where

$$\alpha\big(z(t),i\big) = \operatorname{diag}\big(\alpha_1\big(z_1(t),i\big),\dots,\alpha_n\big(z_n(t),i\big)\big),$$
$$\alpha_j\big(z_j(t),i\big) = \alpha_j\big(z_j(t)+u_j^*,i\big), \qquad A\big(z(t),i\big) = \big[A_1\big(z_1(t),i\big),\dots,A_n\big(z_n(t),i\big)\big]^T,$$
$$A_j\big(z_j(t),i\big) = a_j\big(z_j(t)+u_j^*,i\big) - a_j\big(u_j^*,i\big), \quad j = 1,2,\dots,n,$$
$$\bar{f}_j\big(z(t)\big) = f_j\big(z(t)+u^*\big) - f_j\big(u^*\big), \qquad \bar{g}_j\big(z(t)\big) = g_j\big(z(t)+u^*\big) - g_j\big(u^*\big),$$
$$\bar{h}_j\big(z(s)\big) = h_j\big(z(s)+u^*\big) - h_j\big(u^*\big), \qquad \bar{f}\big(z(t)\big) = \big(\bar{f}_1\big(z(t)\big),\dots,\bar{f}_n\big(z(t)\big)\big)^T,$$
$$\bar{g}\big(z(t)\big) = \big(\bar{g}_1\big(z(t)\big),\dots,\bar{g}_n\big(z(t)\big)\big)^T, \qquad \bar{h}\big(z(s)\big) = \big(\bar{h}_1\big(z(s)\big),\dots,\bar{h}_n\big(z(s)\big)\big)^T,$$
$$\psi_j(s) = u_j(t_0+s,x) - u_j^*, \qquad \psi(s) = u(t_0+s,x) - u^*.$$

Throughout this paper, the following assumptions are made.

(A1) There exist positive constants $\delta_j(i)$, such that

$$z_j(t)A_j\big(z_j(t),i\big) \geq \delta_j(i) z_j^2(t),$$

for all $i \in S$ and $j \in Z^+$.

(A2) The amplification function $\alpha_j(z_j(t), i)$ is positive and satisfies the following condition:

$$0 < \bar{\alpha}_j(i) \le \alpha_j\bigl(z_j(t), i\bigr) \le \hat{\alpha}_j(i), \qquad \bar{\alpha}(i) = \min_{1\le j\le n} \bar{\alpha}_j(i),$$
$$\hat{\alpha}(i) = \max_{1\le j\le n} \hat{\alpha}_j(i), \quad j \in Z^+, i \in S.$$

(A3) There exist positive diagonal matrices $L^f = \operatorname{diag}(L_1^f, \ldots, L_n^f)$, $L^g = \operatorname{diag}(L_1^g, \ldots, L_n^g)$, $L^h = \operatorname{diag}(L_1^h, \ldots, L_n^h)$, such that

$$0 \le \frac{f_j(\xi_1) - f_j(\xi_2)}{\xi_1 - \xi_2} \le L_j^f, \qquad 0 \le \frac{g_j(\xi_1) - g_j(\xi_2)}{\xi_1 - \xi_2} \le L_j^g, \qquad 0 \le \frac{h_j(\xi_1) - h_j(\xi_2)}{\xi_1 - \xi_2} \le L_j^h,$$

for all $\xi_1, \xi_2 \in R$, $\xi_1 \ne \xi_2$, $j \in Z^+$.

(A4) There exist positive definite matrices Σ_{i1}, Σ_{i2}, and Σ_{i3} $(i \in S)$ such that

$$\operatorname{tr}\bigl[\bigl(\sigma_i(t, x, \zeta_1, \zeta_2, \zeta_3)\bigr)^T \bigl(\sigma_i(t, x, \zeta_1, \zeta_2, \zeta_3)\bigr)\bigr] \le \rho_i \zeta_1^T \Sigma_{i1} \zeta_1 + \rho_i \zeta_2^T \Sigma_{i2} \zeta_2 + \rho_i \zeta_3^T \Sigma_{i3} \zeta_3$$

for all $\rho_i \in R^+$, $\zeta_1, \zeta_2, \zeta_3 \in R^n$.

(A5) $\sigma_i(t, x, u^*, u^*, \int_{-\infty}^{t} K(t-s) h(u^*(s, x))\, ds) = 0$, where u^* is the equilibrium point of system (1)-(4).

By assumptions (A1)-(A4), it is not difficult to prove that there exists a unique equilibrium point $u^* = (u_1^*, u_2^*, \ldots, u_n^*)^T$ for system (1)-(4) based on Mawhin's continuation theorem [30].

Clearly, $\bar{f}_j(z_j(t))$, $\bar{g}_j(z_j(t))$, and $\bar{h}_j(z_j(s))$ satisfy (A3). Thus, the stability problem of system (1)-(4) is equivalent to the stability problem of system (8)-(11).

Lemma 1 [14] *Let Ω be a cube $|x_i| < d_l$ $(l = 1, \ldots, m)$ and let $h(x)$ be a real-valued function belonging to $C^1(\Omega)$ which vanish on the boundary $\partial\Omega$ of Ω, i.e., $h(x)|_{\partial\Omega} = 0$. Then*

$$\int_\Omega h^2(x)\, dx \le d_l^2 \int_\Omega \left| \frac{\partial h}{\partial x_i} \right| dx.$$

Lemma 2 [31] *For any real matrices X and Y, the following matrix inequality holds*:

$$X^T Y + Y^T X \le X^T X + Y^T Y.$$

Lemma 3 [31] *Given one positive definite matrix $X_2 > 0$ and constant matrices X_1, X_3, where $X_1 = X_1^T$, then $X_1 + X_3^T X_2^{-1} X_3 < 0$ if and only if*

$$\begin{pmatrix} X_1 & X_3^T \\ X_3 & -X_2 \end{pmatrix} < 0 \quad \text{or} \quad \begin{pmatrix} -X_2 & X_3 \\ X_3^T & X_1 \end{pmatrix} < 0.$$

3 Main results

Theorem 1 *Under assumptions* (A1)-(A4), *if there exist positive definite diagonal matrices P_i, $\tilde{Q}$, positive definite symmetry matrices Q, G, positive definite diagonal matrices M_1, M_2 with appropriate dimensions, $\Lambda_i = \Lambda_i^T$ and scalar $\rho_i > 0$, such that the following LMIs hold*:

$$I_k^T P_j I_k - P_i < 0, \tag{12}$$

and

$$P_i \le \rho_i I, \tag{13}$$

$$\begin{bmatrix} \Xi_i & Y_i \\ * & -[1/(\sqrt{3}\hat{\alpha}(i))^2]I \end{bmatrix} < 0, \tag{14}$$

where

$$Y_i = [P_i \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T,$$

$$\Xi_i = \begin{bmatrix} \alpha_{11} & 0 & \alpha_{13} & 0 & 0 \\ * & \alpha_{22} & 0 & 0 & 0 \\ * & * & \alpha_{33} & 0 & 0 \\ * & * & * & \alpha_{44} & 0 \\ * & * & * & * & \alpha_{55} \end{bmatrix} < 0,$$

$$P_j - \Lambda_i \le 0, \quad j \in \Xi_{uk}^i, j \ne i, \tag{15}$$

$$P_i - \Lambda_i \ge 0, \quad j \in \Xi_{uk}^i, j = i, \tag{16}$$

where

$$\begin{aligned} \alpha_{11} &= -P_iA_i - A_iP_i - 2P_iD^* + Q + L^gGL^g \\ &\quad + \sum_{j\in\Xi_k^i} \gamma_{ij}(P_j - \Lambda_i) - 2\bar{\alpha}_j(i)P_iU_i + L^h\tilde{Q}L^h + \rho_i\Sigma_{i1}^T\Sigma_{i1}, \end{aligned}$$

$$\alpha_{13} = L^fM_1^T, \qquad \alpha_{22} = -(1-\mu)Q + L^gM_2^TL^g + \rho_i\Sigma_{i2}^T\Sigma_{i2},$$

$$\alpha_{33} = B_i^TB_i - 2M_1, \qquad \alpha_{44} = -(1-\mu)G - M_2 + C_i^TC_i,$$

$$\alpha_{55} = E_i^TE_i - \tilde{Q} + \rho_i\Sigma_{i3}^T\Sigma_{i3}, \qquad D^* = \operatorname{diag}\left(\sum_{l=1}^m \frac{D_{1l}}{d_l^2}, \ldots, \sum_{l=1}^m \frac{D_{nl}}{d_l^2}\right),$$

then equilibrium point u^* *of system* (1)-(4) *is asymptotical stability in the mean square sense.*

Proof Consider the Lyapunov-Krasovskii functional

$$V\big(t,z(t),i\big) = V_1\big(t,z(t),i\big) + V_2\big(t,z(t),i\big) + V_3\big(t,z(t),i\big),$$

where $V_1(t,z(t),i) = \int_\Omega z(t)^TP_iz(t)\,dx$,

$$\begin{aligned} V_2\big(t,z(t),i\big) &= \int_\Omega\int_{t-d(t)}^t z(s)^TQz(s)\,ds\,dx + \int_\Omega\int_{t-d(t)}^t \bar{g}\big(z(s)\big)^TG\bar{g}\big(z(s)\big)\,ds\,dx, \\ V_3\big(t,z(t),i\big) &= \int_\Omega\sum_{j=1}^n q_j\int_0^\infty K_j(\theta)\int_{t-\theta}^t \bar{h}_j^2\big(z_j(s)\big)\,ds\,d\theta\,dx, \quad t \ne t_k, x \in \Omega, \end{aligned} \tag{17}$$

and $\tilde{Q} = \operatorname{diag}(q_1, q_2, \ldots, q_n)$ is positive diagonal matrix.

By the Itô formula, we can calculate $LV(t,z(t),i)$ along trajectories of the system (8)-(11), then we have

$$\begin{aligned} &LV\bigl(t,z(t),i\bigr) \\ &\quad= V_t\bigl(t,z(t),i\bigr) + V_z\bigl(t,z(t),i\bigr)\Biggl\{\sum_{l=1}^{n}\frac{\partial}{\partial x_l}\biggl(D_l\frac{\partial z(t)}{\partial x_l}\biggr)\,dt - \alpha\bigl(z(t),i\bigr) \\ &\qquad\times\biggl[A\bigl(z(t),i\bigr) - B_i\bar{f}\bigl(z(t)\bigr) - C_i\bar{g}\bigl(z\bigl(t-d(t)\bigr)\bigr) - E_i\int_{-\infty}^{t}K(t-s)\bar{h}\bigl(z(s)\bigr)\,ds\biggr]\Biggr\} \\ &\qquad+ \frac{1}{2}\operatorname{trace}\bigl[\sigma_i^T(\cdot)V_{zz}(t,z,i)\sigma_i(\cdot)\bigr] + \sum_{j=1}^{N}\gamma_{ij}V\bigl(t,z(t),j\bigr), \end{aligned} \tag{18}$$

where

$$V_t\bigl(t,z(t),i\bigr) = \frac{\partial V(t,z(t),i)}{\partial t}, \qquad V_z\bigl(t,z(t),i\bigr) = \biggl(\frac{\partial V(t,z(t),i)}{\partial z_1},\dots,\frac{\partial V(t,z(t),i)}{\partial z_n}\biggr), \quad \text{and}$$

$$V_{zz}\bigl(t,z(t),i\bigr) = \biggl(\frac{\partial^2 V(t,z(t),i)}{\partial z_i\,\partial z_j}\biggr)_{n\times n}.$$

For $t = t_k$, we obtain

$$\begin{aligned} &V\bigl(t_k,z(t_k),j\bigr) - V\bigl(t_k^-,z\bigl(t_k^-\bigr),i\bigr) \\ &\quad= \int_\Omega z(t_k)^T P_j z(t_k)\,dx - \int_\Omega z\bigl(t_k^-\bigr)^T P_i z\bigl(t_k^-\bigr)\,dx \\ &\qquad+ \int_\Omega\int_{t_k-d(t_k)}^{t_k} z(s)^T Q z(s)\,ds\,dx - \int_\Omega\int_{t_k^- - d(t_k^-)}^{t_k^-} z(s)^T Q z(s)\,ds\,dx \\ &\qquad+ \int_\Omega\int_{t_k-d(t_k)}^{t_k} \bar{g}\bigl(z(s)\bigr)^T G\bar{g}\bigl(z(s)\bigr)\,ds\,dx - \int_\Omega\int_{t_k^- - d(t_k^-)}^{t_k^-} \bar{g}\bigl(z(s)\bigr)^T G\bar{g}\bigl(z(s)\bigr)\,ds\,dx \\ &\qquad+ \int_\Omega\sum_{j=1}^{n} q_j\int_0^\infty K_j(\theta)\int_{t_k-\theta}^{t_k}\bar{h}_j^2\bigl(z_j(s)\bigr)\,ds\,d\theta\,dx \\ &\qquad- \int_\Omega\sum_{j=1}^{n} q_j\int_0^\infty K_j(\theta)\int_{t_k^- -\theta}^{t_k^-}\bar{h}_j^2\bigl(z_j(s)\bigr)\,ds\,d\theta\,dx \\ &\quad= \int_\Omega\bigl[z(t_k)^T P_j z(t_k) - z\bigl(t_k^-\bigr)^T P_i z\bigl(t_k^-\bigr)\bigr]\,dx \\ &\quad= \int_\Omega\bigl[z\bigl(t_k^-\bigr)^T\bigl(I_k^T P_j I_k - P_i\bigr)z\bigl(t_k^-\bigr)\bigr]\,dx < 0. \end{aligned}$$

Then

$$V\bigl(t_k,z(t_k),j\bigr) < V\bigl(t_k^-,z\bigl(t_k^-\bigr),i\bigr). \tag{19}$$

For $t \neq t_k$, by the infinitesimal operator of $LV(t,z,i)$ (see [32]) along the trajectory of system (8)-(11) and (18), we can obtain

$$
\begin{aligned}
&LV\big(t,z(t),i\big) \\
&\quad = 2z(t)^T P_i \left\{ \sum_{l=1}^{n} \frac{\partial}{\partial x_l}\left(D_l \frac{\partial z(t)}{\partial x_l}\right) dt - \alpha\big(z(t),i\big) \right. \\
&\qquad \left. \times \left[A\big(z(t),i\big) - B_i\bar{f}\big(z(t)\big) - C_i\bar{g}\big(z\big(t-d(t)\big)\big) - E_i \int_{-\infty}^{t} K(t-s)\bar{h}\big(z(s)\big)\,ds \right] \right\} \\
&\qquad + \operatorname{trace}\big[\sigma_i^T(\cdot)P_i\sigma_i(\cdot)\big] + \sum_{j=1}^{N}\gamma_{ij}z(t)^T P_j z(t) \\
&\qquad + \int_\Omega \big[z(t)^T Q z(t) - \big(1-\dot{d}(t)\big)z\big(t-d(t)\big)^T Q z\big(t-d(t)\big)\big]\,dx \\
&\qquad + \int_\Omega \big[\bar{g}\big(z(t)\big)^T G\bar{g}\big(z(t)\big) - \big(1-\dot{d}(t)\big)\bar{g}\big(z\big(t-d(t)\big)\big)^T G\bar{g}\big(z\big(t-d(t)\big)\big)\big]\,dx \\
&\qquad + \int_\Omega \sum_{j=1}^{n} q_j \int_0^\infty K_j(\theta)\bar{h}_j^2\big(z_j(t)\big)\,d\theta\,dx - \int_\Omega \sum_{j=1}^{n} q_j \int_0^\infty K_j(\theta)\bar{h}_j^2\big(z_j(t-\theta)\big)\,d\theta\,dx \\
&\quad \le 2\int_\Omega \left\{ z(t)^T P_i \left\{ \sum_{l=1}^{n} \frac{\partial}{\partial x_l}\left(D_l \frac{\partial z(t)}{\partial x_l}\right) - \alpha\big(z(t),i\big)\left[A\big(z(t),i\big) - B_i\bar{f}\big(z(t)\big) \right.\right.\right. \\
&\qquad \left.\left. - C_i\bar{g}\big(z\big(t-d(t)\big)\big) - E_i \int_{-\infty}^{t} K(t-s)\bar{h}\big(z(s)\big)\,ds \right]\right\} + \sum_{j=1}^{N}\gamma_{ij}z(t)^T P_j z(t) \\
&\qquad + \rho_i z(t)^T \Sigma_{i1}^T\Sigma_{i1}z(t) + \rho_i z\big(t-d(t)\big)^T \Sigma_{i2}^T\Sigma_{i2} z\big(t-d(t)\big) \\
&\qquad \left. + \rho_i\left(\int_{-\infty}^{t} K(t-s)\bar{h}\big(z(s)\big)\,ds\right)^T \Sigma_{i3}^T\Sigma_{i3}\int_{-\infty}^{t} K(t-s)\bar{h}\big(z(s)\big)\,ds \right\} dx \\
&\qquad + \int_\Omega \big[z(t)^T Q z(t) - \big(1-\dot{d}(t)\big)z\big(t-d(t)\big)^T Q z\big(t-d(t)\big)\big]\,dx \\
&\qquad + \int_\Omega \big[\bar{g}\big(z(t)\big)^T G\bar{g}\big(z(t)\big) - \big(1-\dot{d}(t)\big)\bar{g}\big(z\big(t-d(t)\big)\big)^T G\bar{g}\big(z\big(t-d(t)\big)\big)\big]\,dx \\
&\qquad + \int_\Omega \sum_{j=1}^{n} q_j \int_0^\infty K_j(\theta)\bar{h}_j^2\big(z_j(t)\big)\,d\theta\,dx \\
&\qquad - \int_\Omega \sum_{j=1}^{n} q_j \int_0^\infty K_j(\theta)\bar{h}_j^2\big(z_j(t-\theta)\big)\,d\theta\,dx.
\end{aligned}
\tag{20}
$$

From (A3), we have

$$
\begin{aligned}
&\bar{f}\big(z(t)\big)^T M_1 L^f z(t) - \bar{f}\big(z(t)\big)^T M_1 \bar{f}\big(z(t)\big) \ge 0, \\
&z\big(t-d(t)\big)^T L^g M_2 L^g z\big(t-d(t)\big) - \bar{g}\big(z\big(t-d(t)\big)\big)^T M_2 \bar{g}\big(z\big(t-d(t)\big)\big) \ge 0,
\end{aligned}
\tag{21}
$$

where M_1 and M_2 are positive definite diagonal matrices.

Considering the situation that the information of transition probabilities is not accessible completely, the following equalities are satisfied for arbitrary matrices $\Lambda_i = \Lambda_i^T$: due to

$\sum_{j=1}^{N}\gamma_{ij}=0$

$$-z(t)^T\sum_{j=1}^{N}\gamma_{ij}\Lambda_i z(t)=0. \tag{22}$$

By (18), (20), and (21), we can conclude

$$\begin{aligned}
&LV\bigl(t,z(t),i\bigr)\\
&\quad\le 2\int_\Omega\Biggl\{z(t)^TP_i\Biggl\{\sum_{l=1}^{n}\frac{\partial}{\partial x_l}\biggl(D_l\frac{\partial z(t)}{\partial x_l}\biggr)-\alpha\bigl(z(t),i\bigr)\Bigl[A\bigl(z(t),i\bigr)-B_i\bar{f}\bigl(z(t)\bigr)\\
&\qquad-C_i\bar{g}\bigl(z\bigl(t-d(t)\bigr)\bigr)-E_i\biggl(\int_{-\infty}^{t}K(t-s)\bar{h}\bigl(z(s)\bigr)\,ds\biggr)\Bigr]\Biggr\}+z(t)^T\sum_{j\in\Xi_k^i}\gamma_{ij}(P_j-\Lambda_i)z(t)\\
&\qquad+z(t)^T\sum_{j\in\Xi_{uk}^i}\gamma_{ij}(P_j-\Lambda_i)z(t)+\rho_i z(t)^T\Sigma_{i1}^T\Sigma_{i1}z(t)\\
&\qquad+\rho_i z\bigl(t-d(t)\bigr)^T\Sigma_{i2}^T\Sigma_{i2}z\bigl(t-d(t)\bigr)\\
&\qquad+\rho_i\biggl(\int_{-\infty}^{t}K(t-s)\bar{h}\bigl(z(s)\bigr)\,ds\biggr)^T\Sigma_{i3}^T\Sigma_{i3}\biggl(\int_{-\infty}^{t}K(t-s)\bar{h}\bigl(z(s)\bigr)\,ds\biggr)\Biggr\}\,dx\\
&\qquad+\int_\Omega\bigl[z(t)^TQz(t)-(1-\mu)z\bigl(t-d(t)\bigr)^TQz\bigl(t-d(t)\bigr)\bigr]\,dx\\
&\qquad+\int_\Omega\bigl[z(t)^TL^gGL^gz(t)-(1-\mu)\bar{g}\bigl(t-d(t)\bigr)^TG\bar{g}\bigl(t-d(t)\bigr)\bigr]\,dx\\
&\qquad+\int_\Omega\biggl[z(t)^TL^h\tilde{Q}L^hz(t)-\biggl(\int_{-\infty}^{t}K(t-s)\bar{h}\bigl(z(s)\bigr)\,ds\biggr)^T\\
&\qquad\times\tilde{Q}\biggl(\int_{-\infty}^{t}K(t-s)\bar{h}\bigl(z(s)\bigr)\,ds\biggr)\biggr]\,dx\\
&\qquad+2\int_\Omega\bigl[\bar{f}\bigl(z(t)\bigr)^TM_1L^fz(t)-\bar{f}\bigl(z(t)\bigr)^TM_1\bar{f}\bigl(z(t)\bigr)\bigr]\,dx\\
&\qquad+\int_\Omega\bigl[z\bigl(t-d(t)\bigr)^TL^gM_2L^gz\bigl(t-d(t)\bigr)-\bar{g}\bigl(z\bigl(t-d(t)\bigr)\bigr)^TM_2\bar{g}\bigl(z\bigl(t-d(t)\bigr)\bigr)\bigr]\,dx.
\end{aligned} \tag{23}$$

According to Green's formula and the Dirichlet boundary condition, we get

$$\int_\Omega\sum_{l=1}^{m}z_i(t)\frac{\partial}{\partial x_l}\biggl(D_{il}\frac{\partial z_i(t)}{\partial x_l}\biggr)\,dx=-\sum_{l=1}^{m}\int_\Omega D_{il}\biggl(\frac{\partial z_i(t)}{\partial x_l}\biggr)^2\,dx. \tag{24}$$

Furthermore, from Lemma 1, we have

$$\begin{aligned}
-\sum_{l=1}^{m}\int_\Omega D_{il}\biggl(\frac{\partial z_i(t)}{\partial x_l}\biggr)^2\,dx&\le-\int_\Omega\sum_{l=1}^{m}\frac{D_{il}}{d_l^2}\bigl(z_i(t)\bigr)^2\,dx\\
&\le-\int_\Omega\sum_{l=1}^{m}\frac{\min_{i\in N}(D_i)}{d_l^2}\bigl(z_i(t)\bigr)^2\,dx,
\end{aligned} \tag{25}$$

where $D_i=\min_{1\le l\le m}\{D_{il}\}$.

From (A1), (A2), and Lemma 2, we have

$$\begin{aligned}
&-2z(t)^T P_i\alpha\bigl(z(t),i\bigr)A\bigl(z(t),i\bigr)\\
&\quad= -2\sum_{j=1}^{n} z_j(t)P_{ij}\alpha_j\bigl(z_j(t),i\bigr)A_j\bigl(z_j(t),i\bigr)\\
&\quad\le -2\bar{\alpha}_j(i)\sum_{j=1}^{n} P_{ij}\delta_j(i)z_j^2(t) = -2\bar{\alpha}_j(i)z(t)^T P_i U_i z(t), \qquad (26)\\
&2z(t)^T P_i\alpha\bigl(z(t),i\bigr)B_i\bar{f}\bigl(z(t)\bigr)\\
&\quad\le z(t)^T P_i\alpha\bigl(z(t),i\bigr)\alpha\bigl(z(t),i\bigr)P_i z(t) + \bar{f}\bigl(z(t)\bigr)^T B_i^T B_i\bar{f}\bigl(z(t)\bigr)\\
&\quad\le \hat{\alpha}(i)^2 z(t)^T P_i^2 z(t) + \bar{f}\bigl(z(t)\bigr)^T B_i^T B_i\bar{f}\bigl(z(t)\bigr), \qquad (27)
\end{aligned}$$

where $U_i = \operatorname{diag}\{\delta_1(i),\delta_2(i),\dots,\delta_n(i)\}$.

As in the proof of the above inequality, we obtain

$$\begin{aligned}
&2z(t)^T P_i\alpha\bigl(z(t),i\bigr)C_i\bar{g}\bigl(z\bigl(t-d(t)\bigr)\bigr)\\
&\quad\le \hat{\alpha}(i)^2 z(t)^T P_i^2 z(t) + \bar{g}\bigl(z\bigl(t-d(t)\bigr)\bigr)^T C_i^T C_i\bar{g}\bigl(z\bigl(t-d(t)\bigr)\bigr), \qquad (28)\\
&2z(t)^T P_i\alpha\bigl(z(t),i\bigr)E_i\int_{-\infty}^{t} K(t-s)\bar{h}\bigl(z(s)\bigr)\,ds\\
&\quad\le \hat{\alpha}(i)^2 z(t)^T P_i^2 z(t) + \biggl(\int_{-\infty}^{t} K(t-s)\bar{h}\bigl(z(s)\bigr)\,ds\biggr)^T\\
&\qquad\times E_i^T E_i\biggl(\int_{-\infty}^{t} K(t-s)\bar{h}\bigl(z(s)\bigr)\,ds\biggr). \qquad (29)
\end{aligned}$$

From (21)-(29), we have

$$\begin{aligned}
&LV\bigl(t,z(t),i\bigr)\\
&\quad\le \int_\Omega z(t)^T\biggl(-P_iA_i - A_iP_i - 2P_iD^* + Q + L^gGL^g + \sum_{j\in\Xi_k^i}\gamma_{ij}(P_j-\Lambda_i)\\
&\qquad- 2\bar{\alpha}_j(i)P_iU_i + L^h\tilde{Q}L^h + \rho_i\Sigma_{i1}^T\Sigma_{i1}\biggr)z(t)\,dx + \int_\Omega z(t)^T\sum_{j\in\Xi_{uk}^i}\gamma_{ij}(P_j-\Lambda_i)z(t)\,dx\\
&\qquad+ \int_\Omega z(t)^T L^f M_1^T\bar{f}\bigl(z(t)\bigr)\,dx + \int_\Omega \bar{f}\bigl(z(t)\bigr)^T M_1L^f z(t)\,dx\\
&\qquad+ \int_\Omega z\bigl(t-d(t)\bigr)^T\bigl[-(1-\mu)Q + L^gM_2^TL^g + \rho_i\Sigma_{i2}^T\Sigma_{i2}\bigr]z\bigl(t-d(t)\bigr)\,dx\\
&\qquad- \int_\Omega \bar{g}\bigl(z\bigl(t-d(t)\bigr)\bigr)^T\bigl[(1-\mu)G + M_2 - C_i^TC_i\bigr]\bar{g}\bigl(z\bigl(t-d(t)\bigr)\bigr)\,dx\\
&\qquad+ \int_\Omega \bar{f}\bigl(z(t)\bigr)^T\bigl(B_i^TB_i - 2M_1\bigr)\bar{f}\bigl(z(t)\bigr)\,dx\\
&\qquad+ \int_\Omega\biggl(\int_{-\infty}^{t} K(t-s)\bar{h}\bigl(z(s)\bigr)\,ds\biggr)^T\bigl(E_i^TE_i - \tilde{Q} + \rho_i\Sigma_{i3}^T\Sigma_{i3}\bigr)
\end{aligned}$$

$$\times \left(\int_{-\infty}^{t} K(t-s)\bar{h}\big(z(s)\big)\,ds\right)dx$$
$$= \int_{\Omega} \big(\eta^T \Xi_1 \eta + 3\hat{\alpha}(i)^2 z(t)^T P_i^2 z(t)\big)\,dx + \int_{\Omega} z(t)^T \sum_{j\in\Xi_{uk}^i} \gamma_{ij}(P_j - \Lambda_i) z(t)\,dx, \tag{30}$$

where

$$\eta = \begin{pmatrix} z(t)^T & z(t-d(t))^T & \bar{f}(z(t))^T & \bar{g}(z(t-d(t)))^T & (\int_{-\infty}^{t} K(t-s)\bar{h}(z(s))\,ds)^T \end{pmatrix}^T,$$
$$D^* = \operatorname{diag}\left(\sum_{l=1}^{m} \frac{D_{1l}}{d_l^2}, \ldots, \sum_{l=1}^{m} \frac{D_{nl}}{d_l^2}\right).$$

By the conditions of Theorem 1, note that $\gamma_{ii} = -\sum_{j=1,j\neq i}^{N} \gamma_{ij}$ and $\gamma_{ij} \geq 0$ for all $j \neq i$, that is, $\gamma_{ii} < 0$ for all $i \in S$. Hence, when $i \in \Xi_k^i$, from (12)-(15) and $\eta \neq 0$, we can derive

$$LV\big(t, z(t), i\big) < 0. \tag{31}$$

On the other hand, if $i \in \Xi_{uk}^i$, according to (12)-(16) and $\eta \neq 0$, we have (25) holds.

For $t \neq t_k$, in view of (19) and (31), by using the mathematical induction, we know that (32) is true for all i, j, $r(0) = i_0 \in S$ and $k \in N^*$, $k \geq 1$.

$$\begin{aligned} EV\big(t_k, z(t_k), j\big) &< EV\big(t_k^-, z\big(t_k^-\big), i\big) < EV\big(t_{k-1}, z(t_{k-1}), r(t_{k-1})\big) \\ &< EV\big(t_{k-1}^-, z\big(t_{k-1}^-\big), r\big(t_{k-1}^-\big)\big) < \cdots < EV\big(t_0, z(t_0), r(0)\big). \end{aligned} \tag{32}$$

Then system (1)-(4) is asymptotic stability in the mean square sense. This completes the proof. □

Remark 2 In proof of above Theorem 1, the new Lyapunov functional to construct is more general. The criteria in [21, 22] are independent on the measure of the space and diffusion effects. However, in this paper, the obtained results are dependent on the measure of the space and diffusion effects. The idea of free-connection weighting matrix is introduced, these methods mentioned above are not considered in other literature and may lead to derive an improved feasible region for delay-dependent and space-dependent stability criteria. Therefore, it is shown that the newly obtained results are less conservative and more applicable than the existing corresponding ones.

Remark 3 It is noted that the authors in [21, 22, 25] do not take impulsive phenomena and the distributed time-varying delay into account on studying the stability of RDNNs with Markovian jumping. When Markovian jumping occurs at the impulsive time instants, the stability analysis approach for Markovian jump impulsive stochastic delayed CGRDNNs with partially known transition probabilities is derived in Theorem 1. In fact, Markovian jumping could occur at any moment. If Markovian jumping does not occur at the impulsive time instants, the Lyapunov parameter P_j should be rewritten as P_i in (17).

Remark 4 It is well known that time delays inevitably exist in electronic NNs due to the finite switching speed of amplifiers. Since a NN usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths, it is desired to model them by introducing continuously distributed delays over a certain duration of time, such that the distant past has less influence compared to the recent behavior of the state [1, 21, 25]. In [25], the authors investigated the robust stochastic exponential stability for RDCGNNs with Markovian jumping parameters and mixed delays. It is noted that [25] investigated RDCGNNs with bounded distributed delays. Though delays arise frequently in practical applications, it is difficult to measure them precisely. In most situations, delays are variable, and in fact unbounded. That is, the entire history affects the present. Such delay terms, more suitable to practical neural nets, are called unbounded delays [33, 34]. So delay terms with time-varying and distributed delays are more suitable for practical NNs. [34] studied the problem of global exponential stability for a class of impulsive NNs with bounded and unbounded delays. To the best of our knowledge, few authors have considered stability of Markovian jump impulsive stochastic RDCGNNs with partially known transition probabilities and unbounded distributed delays described by the stochastic non-linear integro-differential equations.

Consider globally robustly asymptotic stability in the mean square sense of the following Markovian jump impulsive stochastic delayed CGRDNNs with unknown parameters and partially known transition probabilities:

$$\begin{aligned} dz(t) = {} & \sum_{l=1}^{n} \frac{\partial}{\partial x_l}\left(D_l \frac{\partial z(t)}{\partial x_l}\right) dt - \alpha\big(z(t), i\big)\Big[A\big(z(t), i\big) - \big(B_i + \Delta B_i(t)\big)\bar{f}\big(z(t)\big) \\ & - \big(C_i + \Delta C_i(t)\big)\bar{g}\big(z\big(t - d(t)\big)\big) - \big(E_i + \Delta E_i(t)\big) \int_{-\infty}^{t} K(t-s)\bar{h}\big(z(t)\big)\, ds\Big]\, dt \\ & + \sigma_i\left(t, x, z(t) + u^*, z\big(t - d(t)\big) + u^*, \int_{-\infty}^{t} K(t-s) h\big(z(s) + u^*\big)\, ds\right) dw(t), \end{aligned} \tag{33}$$

where some parameters and variables are introduced in system (8)-(11), the perturbed matrices $\Delta B_i(t)$, $\Delta C_i(t)$, and $\Delta E_i(t)$ are unknown matrices denoting time-varying parameter uncertainties and such that

$$\begin{bmatrix} \Delta B_i(t) & \Delta C_i(t) & \Delta E_i(t) \end{bmatrix} = MF(t)[N_{1i} \quad N_{2i} \quad N_{3i}], \tag{34}$$

where M, N_{1i}, N_{2i}, and N_{3i} are known real constant matrices and $F(t)$ is the unknown real time-varying matrix-valued function satisfying

$$F(t)^T F(t) \le I, \qquad \text{for all } t \ge 0.$$

Definition 1 For the uncertain Markovian jump impulsive stochastic delayed CGRDNNs (33) with (9)-(11) and initial functions $\varphi \in L^2_{F_0}((-\infty, 0] \times \Omega; R^n)$, the equilibrium point is said to be globally robustly asymptotically stable in the mean square sense, if for all admissible uncertainties holds (34) and every network mode the following relation is satisfied:

$$\lim_{t\to\infty} E\big\|u(t,x) - u^*\big\|_2^2 = 0.$$

Theorem 2 *Under assumptions* (A1)-(A4), *if there exist positive definite diagonal matrices* P_i, $\tilde{Q}$, *positive definite symmetry matrices* Q, G, *positive definite diagonal matrices* M_1, M_2 *with appropriate dimensions and scalar* $\rho_i > 0$, *such that the following LMIs hold*:

$$I_k^T P_j I_k - P_i < 0, \tag{35}$$

and

$$P_i \leq \rho_i I, \tag{36}$$

$$\begin{bmatrix} \tilde{\Xi}_i & \tilde{Y}_i \\ * & -[1/(\sqrt{3}\hat{\alpha}(i))^2]I \end{bmatrix} < 0, \tag{37}$$

where

$$\tilde{Y}_i = \begin{bmatrix} P_i(I+M) & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$\tilde{\Xi}_i = \begin{bmatrix} \alpha_{11} & 0 & \alpha_{13} & 0 & 0 \\ * & \alpha_{22} & 0 & 0 & 0 \\ * & * & \tilde{\alpha}_{33} & 0 & 0 \\ * & * & * & \tilde{\alpha}_{44} & 0 \\ * & * & * & * & \tilde{\alpha}_{55} \end{bmatrix} < 0,$$

$$P_j - \Lambda_i \leq 0, \quad j \in \Xi_{uk}^i, j \neq i, \tag{38}$$

$$P_i - \Lambda_i \geq 0, \quad j \in \Xi_{uk}^i, j = i, \tag{39}$$

where

$$\tilde{\alpha}_{33} = B_i^T B_i - 2M_1 + N_{1i}^T N_{1i},$$

$$\tilde{\alpha}_{44} = -(1-\mu)G - M_2 + C_i^T C_i + N_{2i}^T N_{2i},$$

$$\tilde{\alpha}_{55} = E_i^T E_i - \tilde{Q} + \rho_i \Sigma_{i3}^T \Sigma_{i3} + N_{3i}^T N_{3i},$$

the other notations are the same as those in Theorem 1, *then system* (33) *with* (9)-(11) *is globally robustly asymptotical stability in the mean square sense*.

Proof We use the same Lyapunov-Krasovskii functional as defined in (17) to derive the stability result. From the Itô formula, we can calculate $LV(t, z(t), i)$ along the trajectories of the system (33) with (2)-(4). To end this, by Theorem 1, we only need to estimate the following inequalities:

$$\begin{aligned} &2z(t)^T P_i \alpha\big(z(t), i\big) \Delta B_i(t) \bar{f}\big(z(t)\big) \\ &\quad = 2z(t)^T P_i \alpha\big(z(t), i\big) M F(t) N_{1i} \bar{f}\big(z(t)\big) \\ &\quad \leq z(t)^T P_i \alpha\big(z(t), i\big) M F(t) F(t)^T M^T \alpha\big(z(t), i\big) P_i z(t) + \bar{f}\big(z(t)\big)^T N_{1i}^T N_{1i} \bar{f}\big(z(t)\big) \\ &\quad \leq \hat{\alpha}(i)^2 z(t)^T P_i M M^T P_i z(t) + \bar{f}\big(z(t)\big)^T N_{1i}^T N_{1i} \bar{f}\big(z(t)\big), \end{aligned}$$

and as in the proof of the above inequality, we obtain

$$\begin{aligned}
&2z(t)^T P_i\alpha\bigl(z(t),i\bigr)\Delta C_i(t)\bar{g}\bigl(z\bigl(t-d(t)\bigr)\bigr)\\
&\quad\le \hat{\alpha}(i)^2 z(t)^T P_i MM^T P_i z(t) + \bar{g}\bigl(z\bigl(t-d(t)\bigr)\bigr)^T N_{2i}^T N_{2i}\bar{g}\bigl(z\bigl(t-d(t)\bigr)\bigr),\\
&2z(t)^T P_i\alpha\bigl(z(t),i\bigr)\Delta E_i(t)\int_{-\infty}^{t} K(t-s)\bar{h}\bigl(z(s)\bigr)\,ds\\
&\quad\le \hat{\alpha}(i)^2 z(t)^T P_i MM^T P_i z(t)\\
&\qquad+\left(\int_{-\infty}^{t} K(t-s)\bar{h}\bigl(z(s)\bigr)\,ds\right)^T N_{3i}^T N_{3i}\left(\int_{-\infty}^{t} K(t-s)\bar{h}\bigl(z(s)\bigr)\,ds\right).
\end{aligned}$$

Therefore, under condition (35)-(39), system (33) with (9)-(11) is globally robustly asymptotical stability in the mean square sense with respect to the uncertain parameters $\Delta B_i(t)$, $\Delta C_i(t)$ and $\Delta E_i(t)$. □

Remark 5 In [10, 12, 13, 17, 21], the authors discussed the stability problem for delayed RDNNs. It should be noted that delayed RDNNs studied in these papers noise disturbances and Markovian jump parameters were also not considered. It is known that the noise disturbance is a major source of instability and poor performances in RDNNs. Furthermore, noise disturbances are ubiquitous in real neural networks. On the other hand, it is recognized that Markovian jump systems with partly unknown transition probabilities are more reasonable to model many practical systems where they may experience abrupt changes in their structure and accepted manuscript parameters than systems without Markovian jump parameters. In addition, systems with Markovian jump parameters can be regarded as an extension of systems without Markovian jump parameters. Thus, the model discussed in this paper is more general and meaningful than the ones in [10, 13]. The proposed systems are more general and include the cases of systems with completely known or unknown transition probabilities.

4 An illustrative example

In this section, we provide the effectiveness of the proposed stability criterion through solving a numerical example. Here, we consider the system (1)-(4) with three modes on $\Omega = \{(x_1,x_2)^T | 0 < x_l < 1/2, l = 1,2\}$. These parameters are described as

$$\begin{aligned}
&a(u,1)=\begin{bmatrix}6 & 0.1\\ 0.6 & 13\end{bmatrix}u,\qquad a(u,2)=\begin{bmatrix}15 & 0.6\\ 0.7 & 10\end{bmatrix}u,\qquad a(u,3)=\begin{bmatrix}10 & 0.5\\ 0.6 & 11\end{bmatrix}u,\\
&B_1=\begin{bmatrix}0.1 & 0.2\\ 0.2 & 0.1\end{bmatrix},\qquad B_2=\begin{bmatrix}0.5 & 0.3\\ 0.5 & 0.1\end{bmatrix},\\
&B_3=\begin{bmatrix}0.3 & 0.4\\ 0.2 & 0.1\end{bmatrix},\qquad C_i=\begin{bmatrix}0.5 & 0\\ 0 & 0.5\end{bmatrix},\\
&\alpha(u,i)=I,\qquad E_1=E_2=E_3=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\qquad \Sigma_{11}=\Sigma_{12}=\Sigma_{13}=0.12I,\\
&\Sigma_{21}=\Sigma_{22}=\Sigma_{23}=0.8I,\qquad \Sigma_{31}=\Sigma_{32}=\Sigma_{33}=1.1I,\qquad \tau_0=d_0=0.6,\qquad \rho_i=3,
\end{aligned}$$

$$\mu = d = 0.2, \qquad D_i = D_i^* = 1, \qquad R = -I,$$
$$I_1 = I_2 = 0.1I, \qquad \rho_i = 3, \quad i = 1, 2, 3.$$

Let $f(\eta) = g(\eta) = h(\eta) = (\hat{g}_1(\eta), \ldots, \hat{g}_n(\eta))^T$, $\hat{g}_j(\eta) = \frac{1}{2}(|\eta + 1| + |\eta - 1|)$, $j = 1, 2, 3$.

Clearly, we get $L^f = L^g = L^h = I$.

The transition probability matrices of the form (4) are given by

$$\Theta = \begin{bmatrix} -0.3 & ? & ? \\ ? & 0.1 & ? \\ ? & ? & 0.15 \end{bmatrix}.$$

In (12)-(16), by applying the MATLAB LMI Control Toolbox, we obtain the feasible solution as follows:

$$P_1 = \begin{bmatrix} 1.8934 & 0 \\ 0 & 1.8934 \end{bmatrix}, \qquad P_2 = \begin{bmatrix} 2.3023 & 0 \\ 0 & 2.3023 \end{bmatrix}, \qquad P_3 = \begin{bmatrix} 1.8206 & 0 \\ 0 & 1.8206 \end{bmatrix},$$
$$\tilde{Q} = \begin{bmatrix} 5.2686 & 0 \\ 0 & 5.2686 \end{bmatrix}, \qquad Q = \begin{bmatrix} 6.6737 & 0 \\ 0 & 6.6737 \end{bmatrix}, \qquad G = \begin{bmatrix} 1.0944 & 0.4274 \\ 0.4274 & 5.3632 \end{bmatrix},$$
$$\Lambda_1 = \begin{bmatrix} 4.7422 & 0.3773 \\ 0.3773 & 14.4768 \end{bmatrix}, \qquad \Lambda_2 = \begin{bmatrix} 1.1483 & -0.0015 \\ -0.0015 & 1.1651 \end{bmatrix},$$
$$\Lambda_3 = \begin{bmatrix} 9.1701 & 1.0125 \\ 1.0125 & 8.5974 \end{bmatrix},$$
$$M_1 = \begin{bmatrix} 2.0757 & 0 \\ 0 & 2.0757 \end{bmatrix}, \qquad M_2 = \begin{bmatrix} 1.9995 & 0 \\ 0 & 1.9995 \end{bmatrix}.$$

Therefore, it follows from Theorem 1 that system (1)-(4) has asymptotic stability in the mean square sense.

5 Conclusions

In this paper, we have dealt with an interesting and important problem of globally robustly asymptotical stability in the mean square sense for a class of Markovian jump impulsive stochastic delayed CGRDNNs with unknown parameters and partially known transition probabilities. By applying the Lyapunov stability analysis approach, some novel stability criteria for the system were proposed. The obtained criteria are dependent on delays and reaction-diffusion terms. The results of this paper are new and they complement previously known results. Therefore, it is shown that the newly obtained results are less conservative and more applicable than the corresponding existing ones.

Competing interests
The author declares that he has no competing interests.

Author's contributions
The studies and manuscript of this paper were written by Weiyuan Zhang independently.

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