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Asymptotic behavior of a multigroup SIS epidemic model with stochastic perturbation

Jing Fu¹, Qixing Han^{1,2*}, Yuguo Lin^{2,3} and Daqing Jiang^{2,4}

*Correspondence:

hanqixing123@163.com

¹School of Mathematics,
Changchun Normal University,
Changchun, 130032, China

²School of Mathematics and
Statistics, Northeast Normal
University, Changchun, 130024,
China

Full list of author information is
available at the end of the article

Abstract

In this paper, we introduce stochasticity into a multigroup SIS model. We present the sufficient condition for the exponential extinction of the disease and prove that the noises significantly raise the threshold of a deterministic system. In the case of persistence, we prove that there exists an invariant distribution which is ergodic.

MSC: 60H10; 93E15; 34E10

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1 Introduction

Mathematical models have become important tools in analyzing the spread and control of infectious diseases. In many epidemiological models it is assumed that the population being considered is uniform and homogeneously mixing. However, when dealing with a heterogeneous population, it is appropriate to divide the whole population into subpopulations, each of which is homogeneous. So far a lot of research has been done on various forms of multigroup models, see, *e.g.*, [1–11].

Lajmanovich and Yorke [12] proposed a deterministic model for the spread of gonorrhoea. Since the spread of gonorrhoea in a population is highly nonuniform, they developed a deterministic SIS (susceptible-infective-susceptible) model with n groups. Because of there being no immunes and negligibly few incubating the disease, they assume the population of every subpopulation is constant in size, *i.e.* $S_k + I_k = N_k$, where S_k, I_k denote the susceptible and infective population at time t in the k th group, respectively, and N_k the total size of the population of k th group. Then the multigroup SIS model takes the form of

$$\frac{dI_k}{dt} = \sum_{j=1}^n \beta_{kj} I_j (N_k - I_k) - \alpha_k I_k, \quad k = 1, 2, \dots, n, \quad (1.1)$$

where α_k is the recovery rate, β_{kj} the contact rate of the k th group's susceptibles with the j th group's infectives. Here α_k, β_{kj} , and N_k are positive constants.

Since gonorrhoea is in essence a nonseasonal disease with less than a 10 percent seasonal component in the variation [13], the authors [12] give the model (1.1) time-independent coefficients. However, also according to [13], the probability of one susceptible contacting one infective does vary with the time of year, although this does not significantly fluctuate.

For more reality, we introduce randomness into the model (1.1) by replacing the parameters β_{kk} by $\beta_{kk} \rightarrow \beta_{kk} + \sigma_k dB_k(t)$, where $B_k(t), k = 1, 2, \dots, n$ are standard Brownian motions with $B_k(0) = 0$, and the intensities of white noises $\sigma_k^2 > 0$. Then the stochastic version of the model (1.1) takes the following form:

$$dI_k = \left[\sum_{j=1}^n \beta_{kj} I_j (N_k - I_k) - \alpha_k I_k \right] dt + \sigma_k I_k (N_k - I_k) dB_k(t), \quad k = 1, 2, \dots, n. \tag{1.2}$$

Ideally, all of the transmission coefficients β_{kj} should suffer from white noise. If we really do that, the solution will be very sensitive to white noise. The solution may be negative or explosive. But for a population system, we always expect there is a positive solution. This is the main reason why we only introduce white noise into $\beta_{kk}, k = 1, 2, \dots, n$.

Recently, Gray *et al.* [14] discussed a stochastic SIS model (1.2) with one group. They proved that this SIS model has a unique global positive solution and establish conditions for the extinction and persistence of the disease. In the case of persistence they showed the existence of a stationary distribution and derived expressions for its mean and variance.

The aim of this paper is to study the asymptotical behavior of the solutions of system (1.2). The rest of this paper is organized as follows. In Section 2, we show that system (1.2) has a unique nonnegative solution. In Section 3, we present the sufficient condition for the exponential extinction of the disease. Section 4 focuses on the persistence of the disease. We show there is a stationary distribution for system (1.2) and it is ergodic.

Throughout this paper, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions (*i.e.* it is right continuous and \mathcal{F}_0 contains all P -null sets).

In general, consider the d -dimensional stochastic differential equation,

$$dx(t) = f(x(t), t) dt + g(x(t), t) dB(t) \quad \text{on } t \geq t_0 \tag{1.3}$$

with initial value $x(t_0) = x_0 \in \mathbb{R}^d$. Define the differential operator L associated with (1.3) by

$$L = \frac{\partial}{\partial t} + \sum_{k=1}^d f_k(x, t) \frac{\partial}{\partial x_k} + \frac{1}{2} \sum_{k,j=1}^d [g^T(x, t)g(x, t)]_{kj} \frac{\partial^2}{\partial x_k \partial x_j}.$$

By Itô's formula, if $x(t) \in \mathbb{R}^d$, then $dV(x(t), t) = LV(x(t), t) dt + V_x(x(t), t)g(x(t), t) dB(t)$.

2 Existence and uniqueness of positive solution

To investigate the dynamical behavior, the first concern is whether the solution has a global existence. Moreover, for an epidemics model, whether the solution is nonnegative is also considered, that is, we need to show the solution is global and nonnegative. In this section, we show global existence and uniqueness of a positive solution of system (1.2). From now on, we denote the solution $(I_1(t), I_2(t), \dots, I_n(t))$ of system (1.2) as $Y(t)$. Denote

$$\Gamma = \{x \in \mathbb{R}^n : 0 < x_k < N_k \text{ for all } 1 \leq k \leq n\}.$$

Theorem 2.1 *If $(\beta_{kj})_{n \times n}$ is irreducible, then for any initial value $Y(0) \in \Gamma$, there is a unique solution $Y(t)$ of system (1.2) on $t \geq 0$, and the solution will remain in Γ with probability 1.*

Proof Since the coefficients of system (1.2) satisfy the local Lipschitz condition, there is a unique local solution $Y(t) \in \Gamma$ on $t \in [0, \tau_e)$, where τ_e is the explosion time. To show this solution is global, we need to show that $\tau_e = +\infty$ a.s. Define C^2 -function $V(Y) : \Gamma \rightarrow R_+$ by

$$\begin{aligned} V(Y) &= \sum_{k=1}^n [(N_k - I_k) - 1 - \log(N_k - I_k) + I_k - 1 - \log I_k] \\ &= \sum_{k=1}^n [N_k - 2 - \log((N_k - I_k)I_k)]. \end{aligned}$$

By direct calculation, we get

$$\begin{aligned} LV &= \sum_{k=1}^n \left[-\frac{N_k - 2I_k}{I_k(N_k - I_k)} \left(-\alpha_k I_k + \sum_{j=1}^n \beta_{kj} I_j (N_k - I_k) \right) + \frac{\sigma_k^2}{2} [2I_k(N_k - I_k) + (N_k - 2I_k)^2] \right] \\ &= \sum_{k=1}^n \left(\frac{\alpha_k(N_k - 2I_k)}{N_k - I_k} - \frac{N_k - 2I_k}{I_k} \sum_{j=1}^n \beta_{kj} I_j + \frac{\sigma_k^2}{2} (-2I_k N_k + 2I_k^2 + N_k^2) \right) \\ &\leq \sum_{k=1}^n \left(\alpha_k + \sum_{j=1}^n \beta_{kj} I_j + \frac{\sigma_k^2 N_k^2}{2} \right) \leq \sum_{k=1}^n \left(\alpha_k + \sum_{j=1}^n \beta_{kj} N_j + \frac{\sigma_k^2 N_k^2}{2} \right) := M. \end{aligned} \tag{2.1}$$

By using similar arguments to Theorem 3.1 of Gray *et al.* [14], we get $\tau_e = +\infty$ a.s. This completes the proof. \square

Remark 2.1 An $n \times n$ matrix (a_{ij}) is irreducible if for any nonempty subset S of $\{1, \dots, n\}$ with a nonempty complement S' , there exist i in S and j in S' such that $a_{ij} \neq 0$.

Remark 2.2 From the proof of Theorem 2.1, we obtain $LV \leq M$. Let $\tilde{V} = V + M$. Then $L\tilde{V} \leq \tilde{V}$ and it is clear that $\inf_{Y \in \Gamma \setminus D_k} \tilde{V}(Y) \rightarrow \infty$ as $k \rightarrow \infty$, where $D_k = (\frac{1}{k}, N_1 - \frac{1}{k}) \times (\frac{1}{k}, N_2 - \frac{1}{k}) \times \dots \times (\frac{1}{k}, N_n - \frac{1}{k})$. Hence, by Remark 2 of Theorem 4.1 of Hasminskii [15], p.86, we find that the solution $Y(t)$ is a homogeneous Markov process in Γ .

3 Exponential extinction of infectious disease

It is clear that $P_0 = (0, 0, \dots, 0)$ is the disease-free equilibrium of system (1.2). For system (1.1), P_0 is globally stable if $\mathcal{R}_0 \leq 1$. Hence, it is interesting to study the disease-free equilibrium for controlling the infectious disease. In this section, we present sufficient conditions for the disease to extinct exponentially for system (1.2).

Theorem 3.1 Assume $(\beta_{kj})_{n \times n}$ is irreducible. If $\beta_{kk}/N_k \geq \sigma_k^2$, $k = 1, 2, \dots, n$ and

$$\mathcal{R}_0 < 1 + \left(2 \max_{1 \leq k \leq n} \{ \alpha_k \} \sum_{k=1}^n (\sigma_k N_k)^{-2} \right)^{-1}, \tag{3.1}$$

then the solution $Y(t)$ of system (1.2) with initial value $Y(0) \in \Gamma$ will exponentially tend to P_0 almost surely. Here $\mathcal{R}_0 = \rho(M_0)$ (the spectral radius of M_0), $M_0 = (\beta_{kj} N_k / \alpha_k)_{n \times n}$.

Proof Since $(\beta_{kj})_{n \times n}$ is irreducible, $\beta_{kj} \geq 0$, and $\sigma_k > 0$, $k, j = 1, 2, \dots, n$, M_0 is also non-negative and irreducible. By Theorem 1.4 of [16], p.27, there is a left eigenvector of M_0

corresponding to $\rho(M_0)$, which is denoted as $(\omega_1, \omega_2, \dots, \omega_n)$ and $\omega_k > 0, k = 1, 2, \dots, n$, i.e.

$$(\omega_1, \omega_2, \dots, \omega_n)\rho(M_0) = (\omega_1, \omega_2, \dots, \omega_n)M_0. \tag{3.2}$$

Define the C^2 -function $V : \Gamma \rightarrow R_+$ by $V(Y) = \sum_{k=1}^n a_k I_k$, $a_k = \omega_k / \alpha_k$. By Itô's formula, we compute

$$d(\log V) = L(\log V) dt + \frac{1}{V} \sum_{k=1}^n a_k \sigma_k (N_k - I_k) I_k dB_k, \tag{3.3}$$

where

$$\begin{aligned} L(\log V) &= \frac{1}{V} \sum_{k=1}^n a_k \left[\sum_{j=1}^n \beta_{kj} (N_k - I_k) I_j - \alpha_k I_k \right] - \frac{\sum_{k=1}^n a_k^2 \sigma_k^2 (N_k - I_k)^2 I_k^2}{2V^2} \\ &\leq \frac{1}{V} \sum_{k=1}^n a_k \left[\sum_{j=1}^n \beta_{kj} N_k I_j - \alpha_k I_k \right] - \frac{1}{V} \sum_{k=1}^n a_k \beta_{kk} I_k^2 - \frac{\sum_{k=1}^n a_k^2 \sigma_k^2 (N_k - I_k)^2 I_k^2}{2V^2} \\ &= H_1 + H_2. \end{aligned}$$

Here $H_1 := \frac{1}{V} \sum_{k=1}^n a_k [\sum_{j=1}^n \beta_{kj} N_k I_j - \alpha_k I_k]$, $H_2 := -\frac{1}{V} \sum_{k=1}^n a_k \beta_{kk} I_k^2 - \frac{\sum_{k=1}^n a_k^2 \sigma_k^2 (N_k - I_k)^2 I_k^2}{2V^2}$.

In view of the definition of a_k and (3.2), we have

$$\begin{aligned} H_1 &= \frac{1}{V} \left[\sum_{k=1}^n \sum_{j=1}^n a_k \beta_{kj} N_k I_j - \sum_{k=1}^n a_k \alpha_k I_k \right] \\ &= \frac{1}{V} \left[\sum_{k=1}^n \sum_{j=1}^n \omega_k \frac{\beta_{kj} N_k}{\alpha_k} I_j - \sum_{k=1}^n \omega_k I_k \right] \\ &= \frac{1}{V} (\omega_1, \omega_2, \dots, \omega_n) (M_0(I_1, I_2, \dots, I_n)^T - (I_1, I_2, \dots, I_n)^T) \\ &= \frac{1}{V} (\mathcal{R}_0 - 1) \sum_{k=1}^n \omega_k I_k = \frac{1}{V} (\mathcal{R}_0 - 1) \sum_{k=1}^n \alpha_k a_k I_k \\ &\leq \max_{1 \leq k \leq n} \{\alpha_k\} (\mathcal{R}_0 - 1) \mathbf{1}_{\mathcal{R}_0 \geq 1} := m_1. \end{aligned}$$

By the condition $\beta_{kk} \geq \sigma_k^2 N_k, k = 1, 2, \dots, n$, we have

$$\begin{aligned} H_2 &\leq -\frac{1}{2V^2} \sum_{k=1}^n 2a_k^2 \beta_{kk} I_k^3 - \frac{1}{2V^2} \sum_{k=1}^n a_k^2 \sigma_k^2 (N_k^2 - 2N_k I_k + I_k^2) I_k^2 \\ &= -\frac{1}{2V^2} \left[\sum_{k=1}^n 2a_k^2 (\beta_{kk} - \sigma_k^2 N_k) I_k^3 + \sum_{k=1}^n a_k^2 \sigma_k^2 N_k^2 I_k^2 + \sum_{k=1}^n a_k^2 \sigma_k^2 I_k^2 I_k^2 \right] \\ &\leq -\frac{1}{2V^2} \sum_{k=1}^n a_k^2 \sigma_k^2 N_k^2 I_k^2 = -\frac{\sum_{k=1}^n a_k^2 \sigma_k^2 N_k^2 I_k^2}{2(\sum_{k=1}^n \frac{a_k}{\sigma_k N_k} \sigma_k N_k I_k)^2} \\ &\leq -\frac{\sum_{k=1}^n a_k^2 \sigma_k^2 N_k^2 I_k^2}{2(\sum_{k=1}^n a_k^2 \sigma_k^2 N_k^2 I_k^2)(\sum_{k=1}^n \frac{1}{\sigma_k^2 N_k^2})} = -\left(2 \sum_{k=1}^n (\sigma_k N_k)^{-2} \right)^{-1} := m_2. \end{aligned}$$

Then $L(\log V) \leq m_1 + m_2$, which combined with (3.3) yields

$$\frac{\log V(t)}{t} \leq \frac{\log V(0)}{t} + m_1 + m_2 - \sum_{k=1}^n a_k \sigma_k \frac{\int_0^t (N_k - I_k) I_k / V dB_k}{t}.$$

By the strong law of large numbers (Lemma 2.6 in [17]), we get

$$\lim_{t \rightarrow +\infty} \frac{\int_0^t (N_k - I_k) I_k / V dB_k}{t} = 0 \quad \text{a.s.}$$

Hence, $\lim_{t \rightarrow \infty} \frac{\log V}{t} \leq m_1 + m_2$ a.s. Besides, (3.1) implies $m_1 + m_2 < 0$. Thus, the proof of Theorem 3.1 is complete. □

Remark 3.1 Theorem 3.1 tells us the disease will die out if

$$\tilde{\mathcal{R}} = \mathcal{R}_0 - \left(2 \max_{1 \leq k \leq n} \{\alpha_k\} \sum_{k=1}^n (\sigma_k N_k)^{-2} \right)^{-1} \leq 1$$

and the noise intensity is not large. In [12], the disease will extinct when $\mathcal{R}_0 \leq 1$. Therefore, the conditions for the disease to become extinct in system (1.2) are weaker than the corresponding deterministic model.

4 Ergodicity of system (1.2)

For a deterministic system, we always discuss the global attractivity of the positive equilibrium of the system. However, there is no positive equilibrium for system (1.2). In this section, we show there is a stationary distribution for system (1.2) when the white noise is small, which in turn implies the stability in stochastic sense. To begin with, we present a well-known result, due to Hasminskii [15].

Let $X(t)$ be a homogeneous Markov process in E_l (E_l denotes the Euclidean l -space) described by the stochastic equation

$$dX(t) = b(X) dt + \sum_{r=1}^k \sigma_r(X) dB_r(t).$$

The diffusion matrix is

$$A(x) = (a_{ij}(x)), \quad a_{ij}(x) = \sum_{r=1}^k \sigma_r^i(x) \sigma_r^j(x).$$

Assumption B There exists a bounded domain $U \subset E_l$ with regular boundary, having the following properties:

- (B.1) In the domain U and some neighborhood thereof, the smallest eigenvalue of the diffusion matrix $A(x)$ is bounded away from zero.
- (B.2) If $x \in E_l \setminus U$, the mean time τ at which a path issuing from x reaches the set U is finite, and $\sup_{x \in K} E_x \tau < \infty$ for every compact subset $K \subset E_l$.

Lemma 4.1 *If (B) holds, then the Markov process $X(t)$ has a stationary distribution $\mu(\cdot)$. Let $f(\cdot)$ be a function integrable with respect to the measure μ . Then*

$$P_x \left\{ \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T f(X(t)) dt = \int_{E_I} f(x) \mu(dx) \right\} = 1 \quad \text{for all } x \in E_I.$$

Theorem 4.1 *Assume $(\beta_{kj})_{n \times n}$ is irreducible and $\mathcal{R}_0 = \rho(M_0) > 1$. If*

$$\sum_{k=1}^n \bar{c}_k \sigma_k^2 I_k^* N_k^2 / 2 < \min \{ \bar{c}_k \beta_{kk} I_k^{*2}, \bar{c}_k \beta_{kk} (N_k - I_k^*)^2, k = 1, 2, \dots, n \}, \tag{4.1}$$

then, for any initial value $Y(0) \in \Gamma$, there is a stationary distribution $\mu(\cdot)$ for system (1.2) and it has the ergodic property, where $M_0 = (\beta_{kj} N_k / \alpha_k)_{n \times n}$, $P^ = (I_1^*, I_2^* \dots, I_n^*) \in \text{int } \Gamma$ is the endemic equilibrium of system (1.1), and the $\bar{c}_k, k = 1, 2, \dots, n$ denote the cofactor of the k th diagonal element of L_B , $B = (\bar{\beta}_{kj})_{n \times n} = (I_j^* (N_k - I_k^*) \beta_{kj})_{n \times n}$; L_B is defined as*

$$L_B = \begin{bmatrix} \sum_{k \neq 1} \bar{\beta}_{1k} & -\bar{\beta}_{12} & \cdots & -\bar{\beta}_{1n} \\ -\bar{\beta}_{21} & \sum_{k \neq 2} \bar{\beta}_{2k} & \cdots & -\bar{\beta}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\bar{\beta}_{n1} & -\bar{\beta}_{n2} & \cdots & \sum_{k \neq n} \bar{\beta}_{nk} \end{bmatrix}.$$

Proof Since $\mathcal{R}_0 > 1$, from Theorem 3.1 of [12], system (1.1) has a unique endemic equilibrium $P^* \in \text{int } \Gamma$. Then

$$\sum_{j=1}^n \beta_{kj} I_j^* (N_k - I_k^*) - \alpha_k I_k^* = 0, \quad k = 1, 2, \dots, n.$$

Let

$$V = \sum_{k=1}^n \bar{c}_k V_k,$$

where

$$V_k = I_k - I_k^* - I_k^* \log(I_k / I_k^*).$$

By direct calculation, we get

$$\begin{aligned} LV_k &= \left(1 - \frac{I_k^*}{I_k} \right) \left(\sum_{j=1}^n \beta_{kj} (N_k - I_k) I_j - \alpha_k I_k \right) + \sigma_k^2 I_k^* (N_k - I_k)^2 / 2 \\ &= \left(1 - \frac{I_k^*}{I_k} \right) \left(\sum_{j=1}^n \beta_{kj} (N_k - I_k) I_j - \frac{I_k}{I_k^*} \sum_{j=1}^n \beta_{kj} (N_k - I_k^*) I_j^* \right) + \sigma_k^2 I_k^* (N_k - I_k)^2 / 2 \\ &= \left(1 - \frac{I_k^*}{I_k} \right) \sum_{j=1}^n \beta_{kj} \left[(N_k - I_k) I_j - \frac{I_k}{I_k^*} (N_k - I_k^*) I_j^* \right] + \sigma_k^2 I_k^* (N_k - I_k)^2 / 2. \end{aligned}$$

Obviously,

$$\begin{aligned} (N_k - I_k)I_j - \frac{I_k}{I_k^*}(N_k - I_k^*)I_j^* &= (N_k - I_k)I_j - (N_k - I_k^*)I_j + (N_k - I_k^*)I_j - \frac{I_k}{I_k^*}(N_k - I_k^*)I_j^* \\ &= (I_k^* - I_k)I_j + (N_k - I_k^*)\left(I_j - \frac{I_k}{I_k^*}I_j^*\right). \end{aligned}$$

Hence,

$$\begin{aligned} LV_k &= \left(1 - \frac{I_k}{I_k^*}\right) \sum_{j=1}^n \beta_{kj} \left[(I_k^* - I_k)I_j + (N_k - I_k^*)\left(I_j - \frac{I_k}{I_k^*}I_j^*\right) \right] + \sigma_k^2 I_k^* (N_k - I_k)^2 / 2 \\ &= -\frac{1}{I_k} \sum_{j=1}^n \beta_{kj} (I_k - I_k^*)^2 I_j + I_j^* (N_k - I_k^*) \sum_{j=1}^n \beta_{kj} \left(1 - \frac{I_k}{I_k^*}\right) \left(\frac{I_j}{I_k^*} - \frac{I_k}{I_k^*}\right) \\ &\quad + \sigma_k^2 I_k^* (N_k - I_k)^2 / 2 \\ &\leq -\beta_{kk} (I_k - I_k^*)^2 + \sum_{j=1}^n \bar{\beta}_{kj} \left(1 - \frac{I_k}{I_k^*} + \frac{I_j}{I_j^*} - \frac{I_k^* I_j}{I_k I_j^*}\right) + \sigma_k^2 I_k^* N_k^2 / 2, \end{aligned}$$

where $\bar{\beta}_{kj} := I_j^* (N_k - I_k^*) \beta_{kj}$. Consequently,

$$\begin{aligned} LV &\leq -\sum_{k=1}^n \bar{c}_k \beta_{kk} (I_k - I_k^*)^2 + \sum_{k=1}^n \sum_{j=1}^n \bar{c}_k \bar{\beta}_{kj} \left(1 - \frac{I_k}{I_k^*} + \frac{I_j}{I_j^*} - \frac{I_k^* I_j}{I_k I_j^*}\right) \\ &\quad + \sum_{k=1}^n \bar{c}_k \sigma_k^2 I_k^* N_k^2 / 2. \end{aligned} \tag{4.2}$$

By Theorem 2.3 of Li and Shuai [8], we know

$$\begin{aligned} \sum_{k=1}^n \bar{c}_k \left(\sum_{j=1}^n \bar{\beta}_{kj} \frac{I_j}{I_j^*} - \sum_{j=1}^n \bar{\beta}_{kj} \frac{I_k}{I_k^*} \right) &= 0 \quad \text{and} \\ \sum_{k=1}^n \bar{c}_k \left(\sum_{j=1}^n \bar{\beta}_{kj} \log \frac{I_j}{I_j^*} - \sum_{j=1}^n \bar{\beta}_{kj} \log \frac{I_k}{I_k^*} \right) &= 0. \end{aligned} \tag{4.3}$$

Besides, note that $a - 1 - \log a \geq 0$ for $a > 0$, then

$$1 - \frac{I_k^* I_j}{I_k I_j^*} \leq -\log \frac{I_k^* I_j}{I_k I_j^*} = -\log \frac{I_k^*}{I_k} - \log \frac{I_j}{I_j^*}. \tag{4.4}$$

Successively substituting (4.4) and (4.3) into (4.2) yields

$$LV \leq -\sum_{k=1}^n \bar{c}_k \beta_{kk} (I_k - I_k^*)^2 + \sum_{k=1}^n \bar{c}_k \sigma_k^2 I_k^* N_k^2 / 2.$$

The condition (4.1) implies that the ellipsoid

$$-\sum_{k=1}^n \bar{c}_k \beta_{kk} (I_k - I_k^*)^2 + \sum_{k=1}^n \bar{c}_k \sigma_k^2 I_k^* N_k^2 / 2 = 0$$

lies entirely in Γ . We can take U to be a neighborhood of the ellipsoid with $\bar{U} \subseteq \Gamma$, so for $Y \in \Gamma \setminus U$, $LV \leq -K$ (K is a positive constant), which implies that the condition (B.2) in Lemma 4.1 is satisfied (see Chapter 3, p.103 of [18]).

The diffusion matrix of system (1.2) is written

$$(a_{ij})_{n \times n} = \text{diag}(\sigma_1^2(N_1 - I_1)^2 I_1^2, \sigma_2^2(N_2 - I_2)^2 I_2^2, \dots, \sigma_n^2(N_n - I_n)^2 I_n^2).$$

For any bounded domain D with $\bar{D} \subset \Gamma$ there is $M = \min\{\sigma_i^2(N_i - I_i)^2 I_i^2, i = 1, 2, \dots, n, Y \in \bar{D}\} > 0$ such that $\sum_{i,j=1}^n a_{ij} \xi_i \xi_j = \sum_{i=1}^n \sigma_i^2(N_i - I_i)^2 I_i^2 \xi_i^2 \geq M \|\xi\|^2$ for all $Y \in \bar{D}$, $\xi \in \mathbb{R}^n$, which implies that the condition (B.1) is also satisfied (refer to [19], p.1163 for details). Therefore, by Lemma 4.1, system (1.2) has a stationary distribution $\mu(\cdot)$ and it is ergodic. \square

Remark 4.1 Theorem 4.1 shows that, under some conditions imposed on the parameters, if the noise intensity is small, there exists a stationary distribution for system (1.2), which reveals that the disease will prevail.

5 Conclusions

In this paper, we investigate a multigroup SIS model with the effect of environmental white noise. We obtain the sufficient condition for the extinction of the disease, and we obtain the criteria for the existence of the invariant distribution of system (1.2). Some interesting topics deserve further investigation. For instance, the coefficients of stochastic differential equations can be modeled by fuzzy sets, and this leads to stochastic differential equations with fuzziness [20, 21]. We leave it for future investigation.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

The authors have contributed to the manuscript on an equal basis. All authors read and approved the final manuscript.

Author details

¹School of Mathematics, Changchun Normal University, Changchun, 130032, China. ²School of Mathematics and Statistics, Northeast Normal University, Changchun, 130024, China. ³School of Mathematics and Statistics, Beihua University, Jilin, 132013, China. ⁴College of Science, China University of Petroleum (East China), Qingdao, 266580, China.

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