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Dynamic analysis of stochastic fuzzy Cohen-Grossberg neural networks with time-varying delays

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Abstract

This paper is concerned with the problem of stochastic stability for a class of fuzzy Cohen-Grossberg neural networks, in which the interconnections and delays are time-varying. Based on a Lyapunov function and the Ito differential formula, a set of novel sufficient conditions on the pth moment exponential stability of the equilibrium of the system is derived. Moreover, an illustrative example is given to demonstrate the effectiveness of the results obtained.

Keywords: fuzzy Cohen-Grossberg neural networks; global pth moment exponential stability; time-varying delays; Ito differential formula

1 Introduction

Recently Cohen and Grossberg neural networks [1] have been extensively studied and applied in many different fields such as associative memory, signal processing, and some optimization problems. In such applications, it is of prime importance to ensure that the designed neural networks are stable [2]. In practice, due to the finite speeds of the switching and transmission of signals, time delays do exist in a working network and thus should be incorporated into the model equation [3–12]. In addition to the delay effects, studies have been intensively focused on stochastic models [13–18]. It has been realized that the synaptic transmission is a noisy process brought about by random fluctuations from the release of neurotransmitters and other probabilistic causes, and it is of great significance to consider stochastic effects on the stability of neural networks described by stochastic functional differential equations.

Stochastic effects constitute another source of disturbances or uncertainties in real systems. A lot of dynamical systems have variable structures subject to stochastic abrupt changes, which may result from abrupt phenomena such as stochastic failures and repairs of the components, changes in the interconnections of subsystems or sudden environment switching. Therefore, stochastic perturbations should be taken into account when modeling neural networks. In recent years, the dynamic analysis of stochastic systems (including neural networks) with delays has been an attractive topic for many researchers, and a large number of stability criteria of these systems have been reported; see *e.g.* [19–26] and the references therein.

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In this paper, I would like to integrate fuzzy operations into Cohen-Grossberg neural networks. Speaking of fuzzy operations, Yang and Yang [27] first introduced fuzzy cellular neural networks (FCNNs) combining those operations with cellular neural networks. So far researchers have found that FCNNs are useful in image processing, and some results have been reported on the stability and periodicity of FCNNs [26–34].

However, to the best of my knowledge, few authors have considered the problem of the pth moment exponential stability and almost sure exponential stability of stochastic nonautonomous fuzzy Cohen-Grossberg neural networks. In fact, in the process of the electronic circuits' applications, assuring a constant connection matrix and delays is unrealistic. Therefore, in this sense, time-varying connection matrix and delays will be better candidates for modeling neural information processing.

Motivated by the above discussions, this paper is concerned with the following stochastic fuzzy Cohen-Grossberg neural networks with time-varying delays:

$$
dx_i(t) = -a_i\big(x_i(t)\big)\Bigg[b_i\big(x_i(t)\big) - \sum_{j=1}^{n} c_{ij}(t)f_j\big(x_j(t)\big) - \bigwedge_{j=1}^{n} \alpha_{ij}(t)g_j\big(x_j\big(t-\tau_j(t)\big)\big)
$$

$$
- \bigvee_{j=1}^{n} \beta_{ji}(t)g_j\big(x_j\big(t-\tau_j(t)\big)\big) + I_i(t) \Bigg] dt + \sum_{j=1}^{n} \sigma_{ij}\big(x_j(t)\big)\,d\omega_j(t). \tag{1}
$$

For $i = 1, 2, \ldots, n$, where n corresponds to the number of units in the neural networks, $x_i(t)$ corresponds to the state of the ith neuron. $f_j(\cdot)$, $g_j(\cdot)$ are signal transmission functions. $\tau_j(t)$ corresponds to the time delay required in processing and satisfies $0 \le \tau_j(t) \le \tau$ (τ is a constant). $a_i(x_i(t))$ represents an amplification function at time t. $b_i(x_i(t))$ is an appropriately behaved function at time t such that the solutions of model (1) remain bounded; $c_{ij}(t)$ represents the elements of the feedback template. $I_i(t) = \tilde{I}_i(t) + \bigwedge_{j=1}^{n} T_{ij}(t)u_j(t) + \bigvee_{j=1}^{n} H_{ij}(t)u_j(t)$. $\alpha_{ij}(t)$, $\beta_{ij}(t)$, $T_{ij}(t)$, and $H_{ij}(t)$ are elements of the fuzzy feedback MIN template and the fuzzy feedback MAX template, fuzzy feed-forward MIN template, and fuzzy feed-forward MAX template, respectively; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively; $u_j(t)$ denotes the external input of the ith neurons. $\tilde{I}_i(t)$ is the external bias of the ith unit. $\sigma_{ij}(\cdot)$ is the diffusion coefficient, $\sigma_i = (\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{in})$: $\omega(t) = (\omega_1(t), \omega_2(t), \ldots, \omega_n(t))^T$ is an n-dimensional Brownian motion defined on a complete probability space $(\Omega, F, \{F_t\}_{t\ge0}, P)$ with a filtration $\{F_t\}_{t\ge0}$ satisfying the usual conditions (*i.e.*, it is right continuous and F_0 contains all P-null sets).

Obviously, model (1) is quite general, and it includes several well-known neural networks models as its special cases such as Hopfield neural networks, cellular neural networks, and bidirectional association memory neural networks [12, 24]. There are at least three different types of stochastic stability to describe the limiting behaviors of stochastic differential equations: stability in probability, moment stability, and almost sure stability (see [23, 35]). When designing an associative memory neural network, we should make the convergence speed as high as possible to ensure the quick convergence of the network operation. Therefore, pth moment ($p \ge 2$) exponential stability and almost sure exponential stability are most useful concepts as they imply that the solutions will tend to the trivial solution exponentially fast. This motivates us to study the pth moment exponential stability for system (1).

The rest of this paper is organized as follows. In Section 2, the basic assumptions and preliminaries are introduced. In Section 3, the criterion for the pth moment ($p \ge 2$) ex-

ponential stability for system (1) is derived by using the Lyapunov function method and Ito differential inequality. An illustrative example is given in Section 4. Conclusions are drawn in Section 5.

2 Preliminaries and some assumptions

For convenience, we introduce several notations. Let $C = C((-\infty, 0], R^n)$ be the Banach space of continuous function which map into R^n with the topology of uniform convergence. For any $x(t) = (x_1(t), x_2(t), \ldots, c_n(t))^T \in R^n$, we define $\|x(t)\| = \|x(t)\|_p = (\sum_{i=1}^n |x_i(t)|^p)^{\frac{1}{p}}$ $(1 < p < \infty)$.

The initial conditions for system (1) are $x(s) = \varphi(s)$, $-\tau \leq s \leq 0$, $\varphi \in L_{F_0}^P((-\tau, 0], R^n)$, where $L_{F_0}^P((-\tau, 0], R^n)$ is R^n-valued stochastic process $\varphi(s)$, $-\tau \leq s \leq 0$, $\varphi(s)$ is F_0 measurable, $\int_{-\tau}^0 E[|\varphi(s)|^p]\,ds < \infty$.

Throughout the paper, we make the following assumptions.

(A1) There exist positive constants $\underline{a}_i, \overline{a}_i$ such that

$$0 < \underline{a}_i \leq a_i(x) < \overline{a}_i, \quad \forall x \in R, i = 1, 2, \ldots, n. \tag{2}$$

(A2) The signal transmission functions $f_j(\cdot)$, $g_j(\cdot)$ $(j = 1, 2, \ldots, n)$ are Lipschitz continuous on R with Lipschitz constants μ_j and ν_j, namely, for any $u, v \in R$,

$$\left|f_j(u) - f_j(v)\right| \leq \mu_j |u - v|, \qquad \left|g_i(u) - g_i(v)\right| \leq \nu_i |u - v|, \qquad f_j(0) = g_j(0) = 0.$$

(A3) $b_i(\cdot) \in C(R, R)$ and there exist positive constants h_i such that

$$\frac{b_i(u) - b_i(v)}{u - v} \geq h_i, \quad \forall u \neq v, i = 1, 2, \ldots, n.$$

(A4) $\sigma(x(t)) = (\sigma_{ij}(x_j(t)))_{n \times n}$ $(i, j = 1, 2, \ldots, n)$, there exist nonnegative numbers s_i, $i = 1, 2, \ldots, n$, such that

$$\text{trace}\left[\sigma^T(x)\sigma(x)\right] \leq \sum_{i=1}^n s_i x_i^2. \tag{3}$$

Remark 2.1 The activation functions are generally assumed to be continuous, differentiable, and monotonically increasing, such as the functions of sigmoid type. These restrictive conditions are no longer needed in this paper. Instead, only the Lipschitz condition is imposed in assumption (A2). Note that the type of activation functions in (A2) has already been used in numerous papers.

If $V(t, x) \in C^{2,1}([-\tau, \infty) \times R^n; R^+)$, according to the Ito formula, we define an operator LV associated with (1) as

$$
\begin{aligned}
LV(t, x) = {} & V_t(t, x) + \sum_{i=1}^n V_{x_i}(t, x) \Bigg\{ -a_i x_i(t) \Bigg[b_i\big(x_i(t)\big) - \sum_{j=1}^n c_{ij}(t) f_j\big(x_j(t)\big) \\
& - \bigwedge_{j=1}^n \alpha_{ij}(t) g_j\big(x_j\big(t - \tau_j(t)\big)\big) - \bigvee_{j=1}^n \beta_{ij}(t) g_j\big(x_j\big(t - \tau_j(t)\big)\big) + I_i(t) \Bigg] dt \Bigg\} \\
& + \frac{1}{2} \text{trace}\big[\sigma^T V_{xx}(t, x)\sigma\big],
\end{aligned}
$$

where

$$V_t(t,x) = \frac{\partial V(t,x)}{\partial t}, \qquad V_{x_i}(t,x) = \frac{\partial V(t,x)}{\partial x_i}, \qquad V_{xx}(t,x) = \left(\frac{\partial V(t,x)}{\partial x_i \, \partial x_j} \right)_{n \times n}.$$

Definition 2.1 The equilibrium x^* of system (1) is said to be global pth moment exponentially stable, if there exist positive constants $M \geq 1$, $\lambda > 0$ such that

$$E\big(\|x(t) - x^*\|^p \big) \leq M \|\varphi - x^*\|_{L^p}^p e^{-\lambda(t-t_0)}, \quad t > t_0, \forall x_0 \in R^n, \tag{4}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is any solution of model (1), $p \geq 2$ is a constant; when $p = 2$, it is usually said to be exponential stability in mean square.

Lemma 2.1 [34] *Suppose x and y are two states of system* (1), *then we have*

$$\left| \bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j(x) - \bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j(y) \right| \leq \sum_{j=1}^{n} |\alpha_{ij}(t)| |g_j(x) - g_j(y)|$$

and

$$\left| \bigvee_{j=1}^{n} \beta_{ij}(t) g_j(x) - \bigvee_{j=1}^{n} \beta_{ij}(t) g_j(y) \right| \leq \sum_{j=1}^{n} |\beta_{ij}(t)| |g_j(x) - g_j(y)|.$$

Lemma 2.2 *If $a_i > 0$ $(i = 1, 2, \ldots, m)$, denote p nonnegative real numbers, then*

$$a_1 a_2 \cdots a_m \leq \frac{a_1^p + a_2^p + \cdots + a_m^p}{p}, \tag{5}$$

where $p \geq 1$ denotes an integer. A particular form of (5) *is*

$$a_1^{p-1} a_2 \leq \frac{(p-1)a_1^p}{p} + \frac{a_2^p}{p}.$$

3 Main results

In this section, we will consider the existence and the global pth moment exponential stability of system (1).

Theorem 3.1 *Under condition* (A1)-(A4), *if there exist a positive diagonal matrix $D = \mathrm{diag}(d_1, d_2, \ldots, d_n)$ and two constants $0 < N_2$, $0 < u < 1$, such that*

$$0 < N_2 \leq N_2(t) \leq u N_1(t), \quad t \geq t_0,$$

where

$$N_1(t) = \min_{1 \leq i \leq n} \left\{ p \underline{a}_i h_i - \sum_{j=1}^{n} \overline{a}_i (p-1) |c_{ij}(t)| \mu_j - \sum_{j=1}^{n} \overline{a}_j |c_{ji}(t)| \mu_j \right.$$

$$- \sum_{j=1}^{n} \overline{a}_i (p-1) \big(|\alpha_{ij}(t)| + |\beta_{ij}(t)| \big) v_j$$

$$- \sum_{j=1}^{n} \frac{(p-1)(p-2)}{2} s_j - \sum_{j=1}^{n} \frac{d_j}{d_i}(p-1)s_i \bigg\},$$

$$N_2(t) = \max_{1 \leq i \leq n} \sum_{j=1}^{n} \frac{d_j}{d_i} \bar{a}_i \big(\big| \alpha_{ij}(t) \big| + \big| \beta_{ij}(t) \big| \big) v_j,$$

then $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ is a unique equilibrium which is globally pth moment exponentially stable, where $p \geq 2$ denotes a positive constant. When $p = 2$, the equilibrium x^* of system (1) has exponential stability in mean square.

Proof The proof of the existence and uniqueness of the equilibrium for system is similar to that of [33]. So we omit it.

Suppose that $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ is the unique equilibrium of system (1). Set $y_i(t) = x_i(t) - x_i^*$, $\tilde{\sigma}_{ij}(y_j(t)) = \sigma_{ij}(y_j(t) + x_j^*) - \sigma_{ij}(x_j^*)$, then system (1) can be transformed into the following equation:

$$dy_i(t) = -a_i\big(y_i(t) + x_i^*\big)\bigg[b_i\big(y_i(t) + x_i^*\big) - b_i\big(x_i^*\big)$$

$$- \sum_{j=1}^{n} c_{ij}(t)\big(f_j\big(y_j(t) + x_j^*\big) - f_j\big(x_j^*\big)\big)$$

$$- \bigg(\bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j\big(y_j\big(t - \tau_j(t)\big) + x_j^*\big) - \bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j\big(x_j^*\big) \bigg)$$

$$- \bigg(\bigvee_{j=1}^{n} \beta_{ji}(t) g_j\big(y_j\big(t - \tau_j(t)\big) + x_j^*\big) - \bigvee_{j=1}^{n} \beta_{ji}(t) g_j\big(x_j^*\big) \bigg) \bigg] dt$$

$$+ \sum_{j=1}^{n} \tilde{\sigma}_{ij}\big(y_j(t)\big) d\omega_j(t), \quad t \geq t_0, i = 1, 2, \ldots, n. \tag{6}$$

Consider the following Lyapunov function:

$$V(t, y) = \sum_{i=1}^{n} d_i \big| y_i(t) \big|^p = \sum_{i=1}^{n} d_i \big| x_i(t) - x_i^* \big|^p, \quad p \geq 2. \tag{7}$$

Calculating the operator $LV(t, y(t))$, and using Lemma 2.2, associated with system (6), it has the form

$$LV\big(t, y(t)\big) = p \sum_{i=1}^{n} d_i \big| y_i(t) \big|^{p-1} \operatorname{sgn}\{y_i(t)\} \bigg\{ -a_i\big(y_i(t) + x_i^*\big)\bigg[b_i\big(y_i(t) + x_i^*\big) - b_i\big(x_i^*\big)$$

$$- \sum_{j=1}^{n} c_{ij}(t)\big(f_j\big(y_j(t) + x_j^*\big) - f_j\big(x_j^*\big)\big)$$

$$- \bigg(\bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j\big(y_j\big(t - \tau_j(t)\big) + x_j^*\big) - \bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j\big(x_j^*\big) \bigg)$$

$$- \bigg(\bigvee_{j=1}^{n} \beta_{ji}(t) g_j\big(y_j\big(t - \tau_j(t)\big) + x_j^*\big) - \bigvee_{j=1}^{n} \beta_{ji}(t) g_j\big(x_j^*\big) \bigg) \bigg] \bigg\}$$

$$+ \frac{p(p-1)}{2} \sum_{i=1}^{n} \left|y_i(t)\right|^{p-2} \sum_{j=1}^{n} \tilde{\sigma}_{ij}\left(y_i(t)\right)$$

$$\leq -p \sum_{i=1}^{n} d_i \left|y_i(t)\right|^{p-1} a_i\left(y_i(t) + x_i^*\right) h_i y_i(t) \operatorname{sgn}\{y_i(t)\}$$

$$+ p \sum_{i=1}^{n} d_i \left|y_i(t)\right|^{p-1} a_i\left(y_i(t) + x_i^*\right) \left\{ \sum_{j=1}^{n} c_{ij}(t) f_j\left(y_j(t)\right) \operatorname{sgn}\{y_i(t)\} \right.$$

$$+ \sum_{j=1}^{n} \left|\alpha_{ij}(t)\right| \left|g_j\left(y_j\left(t - \tau_j(t)\right) + x_j^*\right) - g_j\left(x_j^*\right)\right| \operatorname{sgn}\{y_i(t)\}$$

$$\left. + \sum_{j=1}^{n} \left|\beta_{ij}(t)\right| \left|g_j\left(y_j\left(t - \tau_j(t)\right) + x_j^*\right) - g_j\left(x_j^*\right)\right| \operatorname{sgn}\{y_i(t)\} \right\}$$

$$+ \frac{p(p-1)}{2} \sum_{i=1}^{n} d_i y_i^{p-2}(t) \sum_{j=1}^{n} \sigma_{ij}^2 \operatorname{sgn}\{y_i(t)\}$$

$$\leq -p \sum_{i=1}^{n} d_i \left|y_i(t)\right|^{p-1} \underline{a}_i h_i \left|y_i(t)\right| + p \sum_{i=1}^{n} d_i \left|y_i(t)\right|^{p-1} \overline{a}_i \sum_{j=1}^{n} \left|c_{ij}(t)\right| \mu_j \left|y_j(t)\right|$$

$$+ p \sum_{i=1}^{n} d_i \left|y_i(t)\right|^{p-1} \overline{a}_i \sum_{j=1}^{n} \left(\left|\alpha_{ij}(t)\right| + \left|\beta_{ij}(t)\right|\right) v_j \left|y_j(t)\right|$$

$$+ \frac{p(p-1)}{2} \sum_{i=1}^{n} d_i \left|y_i(t)\right|^{p-2} \sum_{j=1}^{n} s_j y_j^2(t)$$

$$= - \sum_{i=1}^{n} d_i \left\{ p \underline{a}_i h_i - \sum_{j=1}^{n} \overline{a}_i (p-1) \left|c_{ij}(t)\right| \mu_j - \sum_{j=1}^{n} \overline{a}_j \left|c_{ji}(t)\right| \mu_j \right.$$

$$- \sum_{j=1}^{n} \overline{a}_i (p-1) \left(\left|\alpha_{ij}(t)\right| + \left|\beta_{ij}(t)\right|\right) v_j - \sum_{j=1}^{n} \frac{(p-1)(p-2)}{2} s_j$$

$$\left. - \sum_{j=1}^{n} \frac{d_j}{d_i} (p-1) s_i \right\} \left|y_i(t)\right|^p$$

$$+ \sum_{i=1}^{n} d_i \sum_{j=1}^{n} \frac{d_j}{d_i} \overline{a}_i \left(\left|\alpha_{ij}(t)\right| + \left|\beta_{ij}(t)\right|\right) v_j \left|y_i\left(t - t_i(t)\right)\right|^p$$

$$\leq -N_1(t) V\left(t, y(t)\right) + N_2(t) \sup_{t-\tau \leq t \leq t} V\left(s, y(s)\right), \tag{8}$$

where

$$N_1(t) = \min_{1 \leq i \leq n} \left\{ p \underline{a}_i h_i - \sum_{j=1}^{n} \overline{a}_i (p-1) \left|c_{ij}(t)\right| \mu_j - \sum_{j=1}^{n} \overline{a}_j \left|c_{ji}(t)\right| \mu_j \right.$$

$$\left. - \sum_{j=1}^{n} \overline{a}_i (p-1) \left(\left|\alpha_{ij}(t)\right| + \left|\beta_{ij}(t)\right|\right) v_j - \sum_{j=1}^{n} \frac{(p-1)(p-2)}{2} s_j - \sum_{j=1}^{n} \frac{d_j}{d_i} (p-1) s_i \right\},$$

$$N_2(t) = \max_{1 \leq i \leq n} \sum_{j=1}^{n} \frac{d_j}{d_i} \overline{a}_i \left(\left|\alpha_{ij}(t)\right| + \left|\beta_{ij}(t)\right|\right) v_j.$$

Applying the Ito formula, for $t \geq t_0$, we obtain

$$V\big(t + \delta, y(t + \delta)\big) - V\big(t, y(t)\big)$$
$$= \int_t^{t+\delta} LV\big(s, y(s)\big)\, ds + \int_t^{t+\delta} V_y\big(s, y(s)\big)\sigma\big(s, y(s)\big)\, d\omega(s). \tag{9}$$

Since $E[V_x(s, y(s))\sigma(s, y(s))\, d\omega(s)] = 0$, taking expectations on both sides of the equality (9) and applying the inequality (8) yields

$$E\big(V\big(t + \delta, y(t + \delta)\big)\big) - E\big(V\big(t, y(t)\big)\big)$$
$$\leq \int_t^{t+\delta} \left[-N_1(t)E\big(V\big(s, y(s)\big)\big) + N_2(t)E\Big(\sup_{s-\tau \leq \theta \leq s} V\big(\theta, y(\theta)\big)\Big) \right] ds. \tag{10}$$

The Dini derivative D^+ is

$$D^+ E\big(V\big(t, y(t)\big)\big) = \lim_{\delta \to 0^+} \sup \frac{E(V(t + \delta, y(t + \delta))) - E(V(t, y(t)))}{\delta}. \tag{11}$$

Denote $z(t) = E(V(t, y(t)))$, and (10) leads directly to

$$D^+ z(t) \leq -N_1(t)z(t) + N_2(t)\|z_t\|^p. \tag{12}$$

Hence, from Lemma 3.2 of [22], we have

$$z(t) \leq \big\|z(t_0)\big\|^p e^{-\lambda(t-t_0)}.$$

Namely,

$$E\big[\big\|x(t) - x^*\big\|^p\big] \leq M\big\|\varphi - x^*\big\|^p e^{-\lambda(t-t_0)}, \quad t \geq t_0,$$

where $M = \frac{\max_{1 \leq i \leq n}\{d_i\}}{\min_{1 \leq i \leq n}\{d_i\}} > 1$. λ is the unique positive solution of the following equation:

$$\lambda = N_1(t) - N_2(t)e^{\lambda \tau}.$$

Therefore the equilibrium x^* of system (1) is pth moment exponentially stable. The proof is completed. $\qquad\square$

Specially, suppose that $c_{ij}(t) = c_{ij}$, $\alpha_{ij}(t) = \alpha_{ij}$, $\beta_{ij}(t) = \beta_{ij}$ $(i, j = 1, 2, \dots, n)$; system (1) becomes the stochastic fuzzy Cohen-Grossberg neural networks with time-varying delays,

$$dx_i(t) = -a_i\big(x_i(t)\big)\left[b_i\big(x_i(t)\big) - \sum_{j=1}^n c_{ij}f_j\big(x_j(t)\big) - \bigwedge_{j=1}^n \alpha_{ij}g_j\big(x_j(t - \tau_j(t))\big) \right.$$
$$\left. - \bigvee_{j=1}^n \beta_{ji}g_j\big(x_j(t - \tau_j(t))\big) + I_i(t) \right] dt + \sum_{j=1}^n \sigma_{ij}\big(x_j(t)\big)\, d\omega_j(t). \tag{13}$$

For (13), we have the following corollary by Theorem 3.1.

Corollary 3.1 *If assumptions* (A1)-(A4) *hold, and there are constants* $N_i > 0$ $(i = 1, 2)$, $0 < u < 1$ *such that*

$$0 < N_2 < uN_1,$$

where

$$N_1 = \min_{1 \le i \le n} \left\{ p\underline{a}_i h_i - \sum_{j=1}^n \overline{a}_i(p-1)|c_{ij}|\mu_j - \sum_{j=1}^n \overline{a}_j|c_{ji}|\mu_j \right.$$

$$\left. - \sum_{j=1}^n \overline{a}_i(p-1)\big(|\alpha_{ij}| + |\beta_{ij}|\big)v_j - \sum_{j=1}^n \frac{(p-1)(p-2)}{2}s_j - \sum_{j=1}^n \frac{d_j}{d_i}(p-1)s_i \right\},$$

$$N_2 = \max_{1 \le i \le n} \sum_{j=1}^n \frac{d_j}{d_i}\overline{a}_i\big(|\alpha_{ij}| + |\beta_{ij}|\big)v_j,$$

then the unique equilibrium $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ *of system* (13) *is globally pth moment exponentially stable.*

Remark 3.1 The delay functions $\tau_j(t)$ considered in this paper only need to be bounded and can be nondifferential. This generalized some published results in [20]. It should be noted that the stability of system (1) is dependent on the magnitude of noise, therefore, stochastic noise fluctuation is one of the very important aspects in designing a stable network and should be considered adequately.

Remark 3.2 Compared with [20, 21], the method in this paper does not resort to the semimartingale convergence theorem. Since system (1) does not require the delays to be constants, furthermore, the model is nonautonomous and includes fuzzy operation, it is clear that the results obtained in [12, 14, 20–23] cannot be applicable to system (1). This implies that the results of this paper are essentially new and complement some corresponding ones already known.

4 An example

Example 4.1 Consider the following impulsive stochastic fuzzy neural networks with time-varying delays and distributed delays:

$$\begin{cases} dx_1(t) = -(3 + \cos x_1(t))[11x_1(t) - c_{11}(t)f_1(x_1(t)) - c_{12}(t)f_2(x_2(t)) \\ \qquad - \bigwedge_{j=1}^2 \alpha_{1j}(t)g_j(x_j(t - \tau_j(t))) + I_1(t) + \bigwedge_{j=1}^2 T_{1j}(t)u_j(t) \\ \qquad - \bigvee_{j=1}^2 \beta_{1j}(t)g_j(x_j(t - \tau_j(t)))) + \bigvee_{j=1}^2 H_{1j}(t)u_j(t)] \, dt \\ \qquad + \sigma_{11}(x_1(t)) \, d\omega_1 + \sigma_{12}(x_2(t)) \, d\omega_2, \\ dx_2(t) = -(2 + \sin x_2(t))[17x_2(t) - c_{21}(t)f_1(x_1(t)) - c_{22}(t)f_2(x_2(t)) \\ \qquad - \bigwedge_{j=1}^2 \alpha_{2j}(t)g_j(x_j(t - \tau_j(t))) + I_2(t) + \bigwedge_{j=1}^2 T_{2j}(t)u_j(t) \\ \qquad - \bigvee_{j=1}^2 \beta_{2j}(t)g_j(x_j(t - \tau_j(t))) + \bigvee_{j=1}^2 H_{2j}(t)u_j(t)] \, dt \\ \qquad + \sigma_{21}(x_1(t)) \, d\omega_1 + \sigma_{22}(x_2(t)) \, d\omega_2, \end{cases} \tag{14}$$

where

$$\begin{pmatrix} c_{11}(t) & c_{12}(t) \\ c_{21}(t) & c_{22}(t) \end{pmatrix} = \begin{pmatrix} 0.1 & 0.7 \\ 0.6 & 0.3 \end{pmatrix}, \qquad \begin{pmatrix} \alpha_{11}(t) & \alpha_{12}(t) \\ \alpha_{21}(t) & \alpha_{22}(t) \end{pmatrix} = \begin{pmatrix} \frac{5}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{3}{4} \end{pmatrix},$$

$$\begin{pmatrix} \beta_{11}(t) & \beta_{12}(t) \\ \beta_{21}(t) & \beta_{22}(t) \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{2}{3} & \frac{3}{4} \end{pmatrix},$$

$$f_i(r) = g_i(r) = \frac{1}{2}\big(|r+1| - |r-1|\big), \qquad \tau_j(t) = 0.3|\sin t| + 0.1, \quad i, j = 1, 2,$$

$$\sigma_{11}(x) = 0.2x, \qquad \sigma_{12}(x) = 0.3x, \qquad \sigma_{21}(x) = 0.1x, \qquad \sigma_{22}(x) = 0.2x,$$

$$T_{ij}(t) = H_{ij}(t) = u_j(t) = 0.8 + 2t, \qquad I_i(t) = 2 + 3t \quad (i, j = 1, 2).$$

Obviously, system (14) satisfies assumptions (A1)-(A3) with

$$\underline{a}_1 = 2, \qquad \overline{a}_1 = 4, \qquad \underline{a}_2 = 1, \qquad \overline{a}_2 = 3,$$

$$h_1 = 11, \qquad h_2 = 17, \qquad \mu_i = \nu_i = 1 \quad (i = 1, 2).$$

It can easily be checked that the assumption (A4) is satisfied with $s_1 = 0.05$, $s_2 = 0.13$. Let $p = 2$. It is easy to compute $N_1 = 19.97$, $N_2 = 10$. There exists a positive number $0 < u = 0.6 < 1$ such that $0 < N_2 = 10 < uN_1 = 0.6 \times 19.97 = 11.98$. Clearly, all conditions of Corollary 3.1 are satisfied. Thus system (14) has a unique equilibrium point x^* which is globally mean square exponential stable.

5 Conclusions

In this paper, we have studied the existence, uniqueness, and pth moment exponential stability of the equilibrium point for stochastic fuzzy Cohen-Grossberg neural networks with time-varying delays. Some sufficient conditions set up here are easily verified and these conditions are correlated with parameters and the magnitude of noise the system (1). The obtained criteria can be applied to design globally mean square exponentially stable fuzzy Cohen-Grossberg neural networks.

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