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On local aspects of sensitivity in topological dynamics

Lei Liu[1*], Haitao Li[2] and Jinli Pang[3]

*Correspondence:
mathliulei@163.com
[1]School of Mathematics and Information Science, Shangqiu Normal University, Shangqiu, Henan 476000, P.R. China
Full list of author information is available at the end of the article

Abstract
In this paper along with the research on weakly mixing sets and transitive sets, we introduce a local aspect of sensitivity in topological dynamics and give the concept of an s-set. It is shown that a weakly mixing set is an s-set. A transitive set with the set of periodic points being dense is an s-set. In particular, a transitive set is an s-set for interval maps. Moreover, we discuss s-sets for set-valued discrete dynamical systems.

MSC: 54H20; 37B20

Keywords: transitive set; weakly mixing set; s-set; set-valued discrete dynamical system

1 Introduction

A topological dynamical system (abbreviated by TDS) is a pair (X,f), where X is a compact metric space with metric d and $f : X \to X$ is a continuous map. When X is finite, it is a discrete space and there is no non-trivial convergence at all. Hence, we assume that X contains infinitely many points. Let $\mathbb{N}^+$ denote the set of all positive integers and let $\mathbb{N} = \mathbb{N}^+ \cup \{0\}$.

Transitivity, weak mixing, and sensitive dependence on initial conditions (see [1–4]) are global characteristics of topological dynamical systems. Let (X,f) be a TDS, (X,f) is *(topologically) transitive* if for any nonempty open subsets U and V of X there exists an $n \in \mathbb{N}$ such that $f^n(U) \cap V \neq \emptyset$. (X,f) is *(topologically) mixing* if for any nonempty open subsets U and V of X, there exists an $N \in \mathbb{N}$ such that $f^n(U) \cap V \neq \emptyset$ for all $n \in \mathbb{N}$ with $n \geq N$. (X,f) is *(topologically) weakly mixing* if for any nonempty open subsets U_1, U_2, V_1, and V_2 of X, there exists an $n \in \mathbb{N}$ such that $f^n(U_1) \cap V_1 \neq \emptyset$ and $f^n(U_2) \cap V_2 \neq \emptyset$. It follows from these definitions that mixing implies weak mixing, which in turn implies transitivity. A map f is said to have *sensitive dependence on initial conditions* if there is a constant $\delta > 0$ such that for any nonempty open set U of X, there exist points $x, y \in U$ such that $d(f^n(x), f^n(y)) > \delta$ for $n \in \mathbb{N}^+$.

In [5], Blanchard introduced overall properties and partial properties. For example, sensitive dependence on initial conditions, Devaney chaos (see [6]), weak mixing, mixing and more belong to overall properties; Li-Yorke chaos (see [7]) and positive entropy (see [1, 8]) belong to partial properties. Weak mixing is an overall property, it is stable under semi-conjugate maps and implies Li-Yorke chaos. We find that a weakly mixing system always contains a dense uncountable scrambled set (see [9]). In [10], Blanchard and Huang introduced the concepts of a weakly mixing set, derived from a result given by Xiong and Yang [11] and showed 'partial weak mixing implies Li-Yorke chaos' and 'Li-Yorke chaos cannot imply partial weak mixing'.

Motivated by the idea of Blanchard and Huang's notion of a 'weakly mixing subset', Oprocha and Zhang [12] extended the notion of a weakly mixing set and gave the concept of a transitive set and discussed its basic properties. In this paper we give the concept of 's-set' for topological dynamical systems and investigate the relationship among transitive subsets, weakly mixing sets, and s-sets. We find that a TDS to have a weakly mixing set implies it has an s-set, and if periodic points are dense in the transitive set, then the transitive set is an s-set. In particular, a transitive set is an s-set for interval maps. The properties of transitivity, weak mixing, and sensitivity on initial conditions for a set-valued discrete dynamical system were discussed (see [13–18]). Also, we continue to discuss s-sets for set-valued discrete dynamical systems and investigate the relationship between a set-valued discrete system and an original system on an s-set. More precisely, a set-valued discrete system has an s-set, which implies that an original system has an s-set.

2 Preliminaries

A TDS (X,f) is *point transitive* if there exists a point $x_0 \in X$ with dense orbit *i.e.*, $\overline{\mathrm{orb}(x_0)} = X$. Such a point x_0 is called a transitive point of (X,f). In general, transitive and point transitive are independent (see [2]). A TDS (X,f) is *minimal* if $\overline{\mathrm{orb}(x,f)} = X$ for every $x \in X$, *i.e.*, every point is transitive point. A point x is called minimal if the subsystem $(\overline{\mathrm{orb}(x,f)},f)$ is minimal. Denote by $P(f)$ the set of all periodic points.

A Li-Yorke or scrambled pair (x,y) of (X,f) is a pair of points of X such that

(1) $\lim_{n\to\infty} \inf d(f^n(x),f^n(y)) = 0$, and

(2) $\lim_{n\to\infty} \sup d(f^n(x),f^n(y)) > 0$.

A set $S \subseteq X$ is said to be a *scrambled set* if for all $x,y \in S$, $x \neq y$, the (x,y) is a *scrambled pair*. A TDS (X,f) is called *Li-Yorke chaotic* if it contains an uncountable scrambled set.

A map f is said to be *Devaney chaotic* if f satisfies the following conditions:

(1) f is transitive,

(2) f is periodically dense; *i.e.*, the set of periodic points of f is dense in X, and

(3) f is sensitive dependent on initial conditions.

Definition 2.1 [10] Let (X,f) be a TDS and A be a closed subset of X with at least two elements. A is said to be *weakly mixing* if for any $k \in \mathbb{N}$, any choice of nonempty open subsets $V_1, V_2, \dots, V_k$ of A and nonempty open subsets $U_1, U_2, \dots, U_k$ of X with $A \cap U_i \neq \emptyset$, $i = 1, 2, \dots, k$, there exists an $m \in \mathbb{N}$ such that $f^m(V_i) \cap U_i \neq \emptyset$ for $1 \le i \le k$. (X,f) is called *partial weak mixing* if X contains a weakly mixing subset.

Definition 2.2 [12] Let (X,f) be a TDS and A be a nonempty subset of X. A is called a transitive set of (X,f) if, for any choice of nonempty open subset V^A of A and nonempty open subset U of X with $A \cap U \neq \emptyset$, there exists an $n \in \mathbb{N}$ such that $f^n(V^A) \cap U \neq \emptyset$.

Remark 2.1

(1) (X,f) is topologically transitive if and only if X is a transitive set of (X,f).

(2) By [12], A is a transitive set if and only if $\overline{A}$ is a transitive set, where $\overline{A}$ denotes the closure of A.

According to the definitions of transitive set and weakly mixing subset, we have the following results.

Result 1. If A is a weakly mixing set of (X,f), then A is a transitive set of (X,f).
Result 2. If $a \in X$ is a transitive point of (X,f), then $\{a\}$ is a transitive set of (X,f).
Result 3. If $A = \operatorname{orb}(x,f)$ is a periodic orbit of (X,f) for some $x \in X$, then A is a transitive set of (X,f).

Definition 2.3 A nonempty subset A is called an s-set of (X,f) if there exists a $\delta > 0$ such that for any $x \in A$ and $\varepsilon > 0$, there exist a $y \in B(x,\varepsilon) \cap A$ and an $n \in \mathbb{N}^+$ satisfying $d(f^n(x),f^n(y)) > \delta$.

Remark 2.2 The s-set is dense in itself, *i.e.*, it contains no isolated points. X is an s-set of (X,f) if and only if (X,f) is sensitive dependent on initial conditions.

Let (X,f) and (Y,g) be two TDSs. Then (X,f) is an extension of (Y,g), or (Y,g) is a factor of (X,f) if there exists a surjective continuous map $h: X \to Y$ (called a factor map) such that $h \circ f(x) = g \circ h(x)$ for every $x \in X$. If further h is a homeomorphism, then (X,f) and (Y,g) are said to be topologically conjugate and the homeomorphism h is called a conjugated map.

Theorem 2.1 *Let A be a weakly mixing set of (X,f). Then A is an s-set of (X,f).*

Proof Let A be a weakly mixing set of (X,f). Then A contains at least two points. Pick up two distinct points $x_1, x_2 \in A$ and set $\delta = \frac{1}{4}d(x_1,x_2) > 0$. For any $x \in A$ and $\varepsilon > 0$, we see that $B(x,\varepsilon)$ is a nonempty open subset of X.

We consider an open subset $B(x,\varepsilon) \cap A$ of A and two open subsets $B(x_1,\delta)$, $B(x_2,\delta)$ of X. Since A is a weakly mixing set and $B(x_1,\delta) \cap A \neq \emptyset$, $B(x_2,\delta) \cap A \neq \emptyset$, there exists an $n \in \mathbb{N}$ such that $f^n(B(x,\varepsilon) \cap A) \cap B(x_1,\delta) \neq \emptyset$ and $f^n(B(x,\varepsilon) \cap A) \cap B(x_2,\delta) \neq \emptyset$. Furthermore, there exist $y_1, y_2 \in B(x,\varepsilon) \cap A$ such that $f^n(y_1) \in B(x_1,\delta)$ and $f^n(y_2) \in B(x_2,\delta)$. Moreover,

$$d\bigl(f^n(y_1),f^n(y_2)\bigr) \geq d(x_1,x_2) - d\bigl(x_1,f^n(y_1)\bigr) - d\bigl(x_2,f^n(y_2)\bigr) > 4\delta - \delta - \delta = 2\delta.$$

Therefore, either $d(f^n(x),f^n(y_1)) > \delta$ or $d(f^n(x),f^n(y_2)) > \delta$. This shows that A is an s-set of (X,f). □

3 Characterizing s-sets

In this section, we discuss the properties of s-sets of (X,f). For a TDS (X,f) and two nonempty subsets $U, V \subseteq X$, we use the following notation:

$$N(U,V) = \bigl\{n \in \mathbb{N} : f^n(U) \cap V \neq \emptyset\bigr\}.$$

Lemma 3.1 *Let A be an infinite subset of X and $P(f)$ be dense in A. Then $P(f) \cap A$ is infinite.*

Proof Suppose that $P(f) \cap A$ is finite and let $\operatorname{card}(P(f) \cap A) = n$, where card represents the cardinality of a set. Since A is an infinite subset of X, there exists a pairwise disjoint open set V_i^A of A for $i = 1, 2, \ldots, n+1$, *i.e.*, $V_i^A \cap V_j^A = \emptyset$ for $i,j \in \{1,2,\ldots,n+1\}$ and $i \neq j$. Moreover, $P(f) \cap A$ is dense in A, which implies $\operatorname{card}(P(f) \cap A) \geq n+1$. This is a contradiction. Therefore, $P(f) \cap A$ is infinite. □

Theorem 3.1 *Let (X,f) be a TDS and A be an infinite subset of X. If A is a transitive set of (X,f) and $P(f)$ is dense in A, then A is an s-set of (X,f).*

Proof We first prove that there exists $\delta_0 > 0$ such that, for any $x \in A$, there exists $q \in P(f) \cap A$ satisfying

$$d\big(\mathrm{orb}(q), x\big) \geq \frac{\delta_0}{2}.$$

Indeed, by Lemma 3.1, $P(f) \cap A$ is an infinite set. Hence, we pick two different points $q_1, q_2 \in P(f) \cap A$ such that $\mathrm{orb}(q_1) \cap \mathrm{orb}(q_2) = \emptyset$. Let

$$\delta_0 = d\big(\mathrm{orb}(q_1), \mathrm{orb}(q_2)\big) > 0. \tag{3.1}$$

Then, for any $x \in A$, we have

$$d\big(\mathrm{orb}(q_1), x\big) > \frac{\delta_0}{2} \quad \text{or} \quad d\big(\mathrm{orb}(q_2), x\big) > \frac{\delta_0}{2}. \tag{3.2}$$

If (3.2) is false, then

$$d\big(\mathrm{orb}(q_1), x\big) \leq \frac{\delta_0}{2} \quad \text{and} \quad d\big(\mathrm{orb}(q_2), x\big) \leq \frac{\delta_0}{2}.$$

Hence, by the triangle inequality, we have

$$d\big(\mathrm{orb}(q_1), \mathrm{orb}(q_2)\big) < \delta_0.$$

This is a contradiction by (3.1).

Take $\delta = \frac{\delta_0}{8}$. For any $x \in A$ and $\varepsilon > 0$, without loss of generality, let $\varepsilon < \delta$. Since $P(f)$ is dense in A, we have $P(A) \cap (B(x, \varepsilon) \cap A) \neq \emptyset$. Furthermore, we can take $p \in B(x, \varepsilon) \cap A$ and let $f^n(p) = p$. Since $x \in A$, there exists $q \in P(f) \cap A$ such that

$$d\big(\mathrm{orb}(q), x\big) \geq 4\delta.$$

Let $U = \bigcap_{i=1}^n f^{-i}(B(f^i(q), \delta))$. Since $q \in U$, we have $q \in U \cap A$, which implies $U \cap A \neq \emptyset$. Moreover, A is a transitive set of (X,f), thus there exists a $k \in \mathbb{N}^+$ such that $B(x, \varepsilon) \cap A \cap f^{-k}(U) \neq \emptyset$. Take $y \in B(x, \varepsilon) \cap A \cap f^{-k}(U)$. Then $f^k(y) \in U$. Let $j = [\frac{k}{n} + 1]$. Then $1 \leq nj - k \leq n$. Furthermore, we have

$$f^{nj}(y) \in f^{nj-k}\big(f^k(y)\big) \in f^{nj-k}(U) \subseteq B\big(f^{nj-k}(q), \delta\big).$$

Since $f^n(p) = p$, we have $f^{nj}(p) = p$. Hence, by the triangle inequality,

$$d\big(f^{nj}(p), f^{nj}(y)\big) = d\big(p, f^{nj}(y)\big) \geq d\big(x, f^{nj-k}(q)\big) - d\big(f^{nj-k}(q), f^{nj}(y)\big) - d(p, x).$$

As $p \in B(x, \delta) \cap A$ and $f^{nj}(y) \in B(f^{nj-k}(q), \delta)$, so

$$d\big(f^{nj}(p), f^{nj}(y)\big) > 4\delta - \delta - \delta = 2\delta.$$

Again, by the triangle inequality, we have

$$d\big(f^{nj}(x),f^{nj}(y)\big) > \delta \quad \text{or} \quad d\big(f^{nj}(x),f^{nj}(p)\big) > \delta,$$

for some $p \in B(x,\varepsilon) \cap A$ and some $y \in B(x,\varepsilon) \cap A$. Therefore, A is an s-set of (X,f).

By [19], if X is a non-degenerate compact interval, $f : X \to X$ is a continuous map and f is transitive, then $P(f)$ is dense in X. We prove that if X is a non-degenerate compact interval, A is a non-degenerate closed interval, and A is a transitive set of (X,f), then $P(f)$ is dense in A. □

Lemma 3.2 [19] *Suppose that I is a non-degenerate interval and $f : I \to I$ is a continuous map. If $J \subseteq I$ is an interval which contains no periodic points of f and $z, f^m(z)$ and $f^n(z) \in J$ with $0 < m < n$, then either $z < f^m(z) < f^n(z)$ or $z > f^m(z) > f^n(z)$.*

Theorem 3.2 *Let I be a non-degenerate interval and $f : I \to I$ be a continuous map. If $J \subseteq I$ is a non-degenerate closed interval and J is a transitive set of (I,f), then $P(f)$ is dense in J.*

Proof Suppose that $P(f)$ is not dense in J. Then there exists a nonempty open set J_1 of J containing no periodic points, *i.e.*, $P(f) \cap J_1 = \emptyset$. Without loss of generality, let J_1 be an open interval of I and $J_1 \subseteq J$. Take an $x \in J_1$ which is not an endpoint of J_1, an open neighborhood $U \subsetneq J_1$ of x and an open interval $E \subseteq J_1 \setminus U$.

We consider open neighborhood U of J_1 and open interval E with $J_1 \cap E \neq \emptyset$. Since J is a transitive set of (I,f), there exists an $m \in \mathbb{N}$ such that $f^m(U) \cap E \neq \emptyset$. Furthermore, there exists a $y \in U \subseteq J_1$ such that $f^m(y) \in E \subseteq J_1$. Moreover, $P(f) \cap J_1 = \emptyset$, it means that $y \neq f^m(y)$. Since I is a Hausdorff space and f is continuous, there exists an open neighborhood V of y such that $f^m(V) \cap V = \emptyset$. Hence, we can take an open interval J_2 of I such that $J_2 \subseteq J_1$ and $y \in J_2 \subseteq V$, thus $f^m(J_2) \cap J_2 = \emptyset$. Since J_2 is an open interval of I and $J_2 \subseteq J_1$ and J is a transitive set, there exist $n > m$ and $z \in U$ such that $f^n(z) \in J_2$. Furthermore, we have $0 < m < n$ and $z, f^n(z) \in J_2$ while $f^m(z) \notin J_2$. This is a contradiction by Lemma 3.2. Therefore, $P(f)$ is dense in J. □

Example 3.1 We have the tent map (see Figures 1 and 2)

$$f(x) = \begin{cases} 2x, & \text{if } 0 \leq x \leq \frac{1}{2}, \\ 2(1-x), & \text{if } \frac{1}{2} \leq x \leq 1, \end{cases}$$

which is called Devaney chaos on $I = [0,1]$ by [6]. We will prove that $[\frac{1}{4}, \frac{3}{4}]$ is a transitive set of (I,f).

Let $S(f^k)$ denote the set of extreme value points of f^k for every $k \in \mathbb{N}^+$. Then $S(f^k) = \{\frac{1}{2^k}, \frac{2}{2^k}, \dots, \frac{2^k-1}{2^k}\}$. Since $S(f) = \{\frac{1}{2}\}, f(\frac{1}{2}) = 1, f(0) = 0$, and $f(1) = 0$, we have

$$f^k(x) = \begin{cases} 1, & \text{if } x = \frac{1}{2^k}, \frac{3}{2^k}, \dots, \frac{2^k-1}{2^k}, \\ 0, & \text{if } x = 0, \frac{2}{2^k}, \frac{4}{2^k}, \dots, \frac{2^k-2}{2^k}, 1. \end{cases}$$

Let $I_k^j = [\frac{j}{2^k}, \frac{j+1}{2^k}]$ for $0 \leq j \leq 2^k - 1$. Then $f^k(I_k^j) = [0,1]$. For any nonempty open set U of $[\frac{1}{4}, \frac{3}{4}]$, without loss of generality, we take $U = (x_0 - \varepsilon, x_0 + \varepsilon)$ for a given $\varepsilon > 0$ and

Figure 1 The tent map *f*.

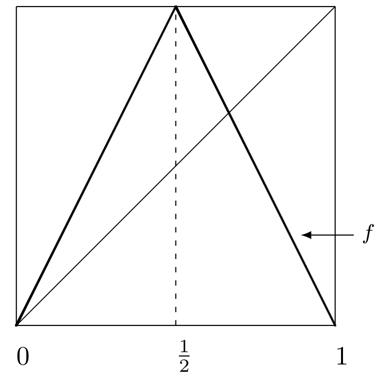
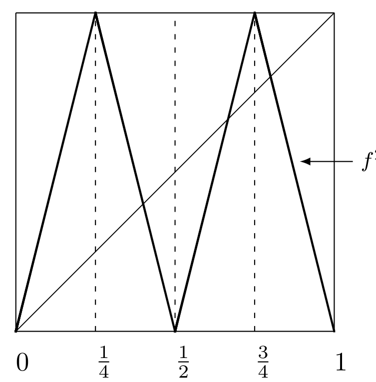


Figure 2 The 2nd iterate f^2 of the tent map *f*.



$x_0 \in \operatorname{int}[\frac{1}{4}, \frac{3}{4}]$, where $\operatorname{int}[\frac{1}{4}, \frac{3}{4}]$ denotes the interior of $[\frac{1}{4}, \frac{3}{4}]$. When $l \in \mathbb{N}$ and $l > \log_2 \frac{1}{\varepsilon}$, there exists $j, j \in \mathbb{N}$ and $0 \le j \le 2^l - 1$, such that $I_l^j \subseteq U$. Furthermore, we have $f^l(U) = [0,1]$. Thus, for any nonempty open set U of $[\frac{1}{4}, \frac{3}{4}]$ and nonempty open set V of $[0,1]$ with $V \cap [\frac{1}{4}, \frac{3}{4}] \neq \emptyset$, there exists a $k \in \mathbb{N}$ such that $f^k(U) \cap V \neq \emptyset$. This shows that $[\frac{1}{4}, \frac{3}{4}]$ is a transitive set of (I,f). Since $P(f)$ is dense in I, $P(f)$ is also dense in $[\frac{1}{4}, \frac{3}{4}]$. By Theorem 3.2, $[\frac{1}{4}, \frac{3}{4}]$ is an s-set of (I,f).

4 s-sets for set-valued discrete dynamical systems

The distance from a point x to a nonempty set A in X is defined by

$$d(x,A) = \inf_{a \in A} d(x,a).$$

Let $\kappa(X)$ be the family of all nonempty compact subsets of X. The Hausdorff metric on $\kappa(X)$ is defined by

$$d_H(A,B) = \max\left\{\sup_{a \in A} d(a,B), \sup_{b \in B} d(b,A)\right\} \quad \text{for every } A, B \in \kappa(X).$$

It follows from Michael [20] and Engelking [21] that $\kappa(X)$ is a compact metric space. The Vietoris topology τ_υ on $\kappa(X)$ is generated by the base

$$\upsilon(U_1, U_2, \ldots, U_n) = \left\{F \in \kappa(X) : F \subseteq \bigcup_{i=1}^{n} U_i \text{ and } F \cap U_i \neq \emptyset \text{ for all } i \le n\right\},$$

where $U_1, U_2, \ldots, U_n$ are open subsets of X. Let $\bar{f}$ be the induced set-valued map defined by

$$\bar{f}: \kappa(X) \to \kappa(X), \quad \bar{f}(F) = f(F), \quad \text{for every } F \in \kappa(X).$$

Then $\bar{f}$ is well defined. $(\kappa(X), \bar{f})$ is called a set-valued discrete dynamical system.

Let X be a T_1 space, that is, a single point set that is closed. Then $\kappa(A) = \{F \in \kappa(X) : F \subseteq A\}$ is a closed subset of $\kappa(X)$ for any nonempty closed subset A of X (see [20]).

Theorem 4.1 *Let A be a nonempty closed subset of X. If $P(f)$ is dense in A, then $P(\bar{f})$ is dense in $\kappa(A)$.*

Proof Let $\mathcal{V}^{\kappa(A)}$ be a nonempty open subset of $\kappa(A)$. Then there exists an open set $\mathcal{V}$ of $\kappa(X)$ such that $\mathcal{V}^{\kappa(A)} = \mathcal{V} \cap \kappa(A)$. Without loss of generality, let $\mathcal{V} = \nu(V_1, V_2, \ldots, V_m)$. Take $F \in \mathcal{V} \cap \kappa(A)$. Then we have $F \subseteq A$, $F \subseteq \bigcup_{i=1}^{m} V_i$ and $F \cap V_i \neq \emptyset$ for each $i = 1, 2, \ldots, m$. Hence, $V_i \cap A \neq \emptyset$ for each $i = 1, 2, \ldots, m$. Since $P(f)$ is dense in A, it follows that $P(f) \cap (V_i \cap A) \neq \emptyset$ for each $i = 1, 2, \ldots, m$. Furthermore, there exist $y_i \in P(f) \cap (V_i \cap A)$ and $n_i \in \mathbb{N}^+$ such that $f^{n_i}(y_i) = y_i$ for each $i = 1, 2, \ldots, m$. Let $G = \{y_1, y_2, \ldots, y_m\}$. Then $G \in \mathcal{V}$ and $G \in \kappa(A)$, which implies $G \in \mathcal{V}^{\kappa(A)}$. Moreover, $f^{n_1 n_2 \cdots n_m}(y_i) = y_i$ for each $i = 1, 2, \ldots, m$. Therefore, $(\bar{f})^{n_1 n_2 \cdots n_m}(G) = f^{n_1 n_2 \cdots n_m}(G) = G$, it means that $P(\bar{f}) \cap \mathcal{V}^{\kappa(A)} \neq \emptyset$. This shows that $P(\bar{f})$ is dense in $\kappa(A)$. □

Theorem 4.2 *Let A be a nonempty closed subset of X. If $\kappa(A)$ is a sensitive set of $(\kappa(X), \bar{f})$, then A is an s-set of (X, f).*

Proof Since $\kappa(A)$ is an s-set of $(\kappa(X), \bar{f})$, there exists a constant $\delta > 0$ such that $K \in \kappa(A)$ and every $\varepsilon > 0$ there exist $G \in B(K, \varepsilon) \cap \kappa(A)$ and $n \in \mathbb{N}^+$ such that $d_H((\bar{f})^n(K), (\bar{f})^n(G)) > \delta$.

Let $x \in A$ and $\varepsilon > 0$. Take $K = \{x\} \in \kappa(A)$. Then there exist $G \in B(\{x\}, \varepsilon) \cap \kappa(A)$ and $n \in \mathbb{N}^+$ such that

$$d_H\big((\bar{f})^n(\{x\}), (\bar{f})^n(G)\big) = d_H\big(f^n(\{x\}), f^n(G)\big) \geq \delta.$$

Since $d_H(f^n(\{x\}), f^n(G)) = \sup_{y \in G} d(f^n(x), f^n(y))$, G is a compact subset of X and $f : X \to X$ is a continuous map, there exists $y_0 \in G$ such that

$$d_H\big(f^n(\{x\}), f^n(G)\big) = d\big(f^n(x), f^n(y_0)\big) > \delta.$$

Moreover, $G \in B(\{x\}, \varepsilon) \cap \kappa(A)$ implies $G \subseteq B(x, \varepsilon)$ and $G \subseteq A$, consequently, $y_0 \in B(x, \varepsilon) \cap A$. This shows that A is an s-set of (X, f). □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
LL and HL carried out the study of stronger forms of sensitivity for inverse limit dynamical systems and drafted the manuscript. JP helped to draft the manuscript. All authors read and approved the final manuscript.

Author details
[1]School of Mathematics and Information Science, Shangqiu Normal University, Shangqiu, Henan 476000, P.R. China.
[2]College of Computer and Information Technology, Shangqiu Normal University, Shangqiu, Henan 476000, P.R. China.
[3]Department of Mathematics, Shangqiu Normal College, Shangqiu, Henan 476000, P.R. China.

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