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A new inequality of $\mathcal{L}$-operator and its application to stochastic non-autonomous impulsive neural networks with delays

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Abstract

In this paper, based on the properties of $\mathcal{L}$-operator and $\mathcal{M}$-matrix, we develop a new inequality of $\mathcal{L}$-operator to be effective for non-autonomous stochastic systems. From the new inequality obtained above, we derive the sufficient conditions ensuring the global exponential stability of the stochastic non-autonomous impulsive cellular neural networks with delays. Our conclusions generalize some works published before. One example is provided to illustrate the superiority of the proposed results.

MSC: 34D23; 34K20

Keywords: exponential stability; inequality; non-autonomous; stochastic; delays; neural networks; impulses

1 Introduction

Recently, the dynamical behaviors for cellular neural networks have been popular with many researchers because of their extensive applications in signal processing, pattern recognition, optimization problems and many other fields. The stability of networks is one of the crucial properties in such applications. Delay and impulsive effects exist widely in many practical models such as population models and neural networks. There are many works on the dynamic behaviors of various kinds of neural networks with delays or impulses [1–9]. Furthermore, some real systems are usually affected by external disturbance with great uncertainty which may be treated as random. In real nervous systems and in the implementation of artificial networks, Haykin [10] has pointed out that the synaptic transmission is a noisy process which is caused by random fluctuations from the release of neurotransmitters and other probabilistic factors. Hence, noise must be taken into consideration in the model construction. Among them, stability analysis of different stochastic systems has been a focused research subject in the literature. Stochastic perturbation is the main factor that affect the stability of systems including neural networks in performing the computation. In addition, the results in [11] suggested that certain stochastic inputs can stabilize or destabilize one neural network. This implies that the stability analysis of stochastic neural networks has primary significance in its design and applications. Therefore, some results on the stability of neural networks with stochastic perturbations have been reported [11–21]. In addition, when we consider long-time dynamic behaviors of a system, the parameters of the system frequently vary with time due to the environmental disturbances. In this case, a non-autonomous neural network model is the best choice for accurately depicting evolutionary processes of networks. Therefore, it is of great significance to study the dynamic behaviors of non-autonomous neural networks [22–38].

By using a Lyapunov function, the authors of [34, 35] have investigated the stability of non-autonomous systems without impulses and obtained the determinant conditions for asymptotic stability or exponential stability of the corresponding system. However, the conditions are true only for all $t \geq 0$. Besides, many authors used the linear matrix inequality (LMI) and the Lyapunov-Krasovskii functional to study the dynamic behaviors of various kinds of neural networks and obtained many interesting new results [6, 16, 21, 25]. However, the results given in LMI form are commonly dependent on the delays. Particularly, for time-varying delays system [6, 16], one must require constraint conditions such as the differentiability of delay functions. The authors of [25] considered the stability and existence of periodic solution to bidirectional associative memory non-autonomous neural networks with delay and obtained some new results which are given in a function matrix inequality, but it is not easy to compute by Matlab LMI Control Toolbox. Thus, LMI technique is ineffective for dealing with the non-autonomous system. In addition to the methods mentioned before, a differential inequality is also a very useful tool for studying the dynamic behaviors of differential dynamical systems [24, 30, 32, 37–44], but many of the inequalities obtained before cannot be used to investigate the non-autonomous systems. The authors in [32] considered the periodic attractor and dissipativity of non-autonomous cellular neural networks with delays. The authors of [30] investigated the invariant and attracting sets of neural networks with reaction-diffusion terms. However, the results in [30, 32] require the time-varying coefficients to have a common factor. In practical applications, this condition is very strict and not easy to meet. The authors of [38] developed an inequality to investigate the stability of non-autonomous cellular neural networks with impulse and time-varying delays, but this inequality cannot handle stochastic non-autonomous neural networks. The authors of [37] investigated the exponential p-stability of stochastic Takagi-Sugeno non-autonomous neural networks with impulses and time-varying delays, but the conditions imposed on the diffusion coefficient matrix are very strict. As far as we know, there are no results on the stability of non-autonomous stochastic neural networks with time delays and impulses except for [37].

Motivated by the previous analysis, in this paper, applying the properties of $\mathcal{L}$-operator and $\mathcal{M}$-matrix, we develop a new inequality of $\mathcal{L}$-operator that is effective for stochastic non-autonomous system. Based on the new inequality of $\mathcal{L}$-operator, we study the stochastic non-autonomous impulsive cellular neural networks with time-varying delays and obtain the sufficient conditions for the pth moment exponential stability of the corresponding systems. Our main results do not require common factors of the coefficients of the system, relax the conditions imposed on the diffusion coefficient matrix, and generalize some early results. One example is provided to demonstrate the effectiveness of the proposed results.

2 Preliminaries

Let $R^{m\times n}$ be the set of $m \times n$ real matrices. Usually E denotes an $n \times n$ unit matrix. R^n denotes the space of n-dimensional real column vectors, $|\cdot|$ denotes Euclidean norm, $\mathcal{N} \triangleq \{1,2,\ldots,n\}$, $\mathbb{N} \triangleq \{1,2,\ldots\}$, $R_+ \triangleq [0,+\infty)$. For $M,N \in R^{m\times n}$ or $M,N \in R^n$, the notation

$M \geq N$ ($M > N$) indicates that each pair of corresponding elements of M and N satisfies the inequality '$\geq$ ($>$)'. Particularly, $M \in R^{m\times n}$ is called a non-negative matrix if $M \geq 0$, and $x \in R^n$ is called a positive vector if $x > 0$. Let $\rho(M)$ denote the spectral radius of square matrix M.

$L^1(R_+, R_+)$ denotes the family of all continuous functions $h : R_+ \to R_+$ satisfying $\int_0^{+\infty} h(t)\,dt < \infty$. $C[X, Y]$ denotes the space of continuous mappings from X to Y. In particular, let $\mathcal{C} \triangleq C[[-\tau, 0], R^n]$ denote the family of all R^n-valued functions φ which is bounded continuous and defined on $[-\tau, 0]$. The norm of $\mathcal{C}$ is defined by $\|\varphi\| = \sup_{-\tau\leq\theta\leq 0} |\varphi(\theta)|$.

$PC[J, R^n] = \{\phi : J \to R^n | \phi(\nu)$ is continuous for all but at most countable points $\nu \in J$ and at these points $\nu \in J, \phi(\nu^+)$ and $\phi(\nu^-)$ exist and $\phi(\nu) = \phi(\nu^+)\}$, where $\phi(\nu^-)$ and $\phi(\nu^+)$ denote the left-hand and right-hand limits of the function $\phi(\nu)$ at time ν, respectively, and $J \subset R$ is an interval. Especially, let $\mathcal{PC} \triangleq PC[[-\tau, 0], R^n]$.

For any $x \in R^n$, $\phi \in \mathcal{C}$ or $\phi \in \mathcal{PC}$, $p > 0$, we define

$$[x]^{+p} = \bigl(|x_1|^p, \ldots, |x_n|^p\bigr)^T, \qquad \bigl[\phi(t)\bigr]_\tau = \bigl(\bigl[\phi_1(t)\bigr]_\tau, \ldots, \bigl[\phi_n(t)\bigr]_\tau\bigr)^T,$$
$$\bigl[\phi_i(t)\bigr]_\tau = \sup_{-\tau\leq s\leq 0} \bigl|\phi_i(t+s)\bigr|, \quad i \in \mathcal{N},$$

and $D^+\phi(t)$ denotes the upper-right-hand derivative of $\phi(t)$ at time t.

$(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, P)$ denotes a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions (*i.e.*, it is right continuous and $\mathscr{F}_0$ contains all P-null sets). Let $C^b_{\mathscr{F}_0}[[t_0 - \tau, t_0], R^n]$ ($C^b_{\mathscr{F}_t}[[t_0 - \tau, t_0], R^n]$) denote the family of all bounded $\mathscr{F}_0(\mathscr{F}_t)$-measurable, $C[[t_0 - \tau, t_0], R^n]$-value random variables ϕ, let $PC^b_{\mathscr{F}_0}[[t_0 - \tau, t_0], R^n]$ ($PC^b_{\mathscr{F}_t}[[t_0 - \tau, t_0], R^n]$) denote the family of all bounded $\mathscr{F}_0(\mathscr{F}_t)$-measurable, $PC[[t_0 - \tau, t_0], R^n]$-value random variables ϕ, satisfying $\|\phi\|^p_{Lp} = \sup_{t_0-\tau\leq\theta\leq t_0} \mathbb{E}|\phi(\theta)|^p < \infty$ for $p > 0$, where $\mathbb{E}$ denotes the expectation of stochastic process.

We study the following stochastic non-autonomous impulsive cellular neural networks with delays:

$$\begin{cases} dx_i(t) = [-a_i(t)x_i(t) + \sum_{j=1}^n b_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^n c_{ij}(t)g_j(x_j(t - \tau_{ij}(t))) + I_i(t)]\,dt \\ \qquad\qquad + \sum_{j=1}^n \sigma_{ij}(t, x(t), x(t - \tau(t)))\,d\omega_j(t), \quad t \geq t_0, t \neq t_k, \\ x_i(t_k) = I_{ik}(x(t_k^-)), \quad t = t_k, \\ x_i(s) = \phi_i(s), \quad t_0 - \tau \leq s \leq t_0, \end{cases} \tag{2.1}$$

where $i \in \mathcal{N}$, and n is the number of units in a neural network; $x_i(t)$ is the state variable of the ith unit at time t; $f_j(\cdot)$ and $g_j(\cdot)$ are the activation functions of the jth unit at time t and $t - \tau_{ij}(t)$, respectively; $\tau_{ij}(t)$ is the time-varying delay satisfying $0 \leq \tau_{ij}(t) \leq \tau$ and $\tau > 0$ at time t; $a_i(t) > 0$ is the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs; $b_{ij}(t)$, $c_{ij}(t)$ denote the strengths of the jth neuron on the ith unit at time t and $t - \tau_{ij}(t)$, respectively; $I_i(t)$ denotes the bias of the ith unit at time t; $\sigma(t, x(t), x(t - \tau(t))) = (\sigma_{ij}(t, x(t), x(t - \tau(t))))_{n\times n}$ is the diffusion coefficient matrix, and $\omega(t) = (\omega_1(t), \ldots, \omega_n(t))^T$ is an n-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, P)$; $\phi(s) = (\phi_1(s), \phi_2(s), \ldots, \phi_n(s))^T \in PC^b_{\mathscr{F}_0}[[t_0 - \tau, t_0], R^n]$ is the initial function vector. The impulsive function $I_k = (I_{1k}, \ldots, I_{nk})^T \in C[R^n, R^n]$, and the fixed impulsive moments t_k ($k \in \mathbb{N}$) satisfy $t_0 < t_1 < t_2 < \cdots$ and $\lim_{k\to\infty} t_k = \infty$.

Throughout this paper, we assume that $f_j(\cdot)$, $g_j(\cdot)$, $\sigma_{ij}(t,\cdot,\cdot)$ satisfy the linear growth condition and are Lipschitz continuous as well. Therefore, we can know that system (2.1) has a unique global solution denoted by $x(t) = (x_1(t),\ldots,x_n(t))^T$ on $t \geq t_0$ and $\mathbb{E}(\sup_{t_0\leq s\leq t}|x(t)|^r) < \infty$ for all $t \geq t_0$ and $r > 0$.

Definition 2.1 System (2.1) is called globally and exponentially p-stable, if there exist constants $M \geq 1$ and $\lambda > 0$ such that for any two solutions $x(t,\phi)$ and $x(t,\psi)$ with $\phi,\psi \in PC^b_{\mathscr{F}_0}[[t_0-\tau,t_0],R^n]$, respectively, one has

$$\mathbb{E}\bigl|x(t,\phi)-x(t,\psi)\bigr|^p \leq M\|\phi-\psi\|^p_{L_p}e^{-\lambda(t-t_0)}, \quad t \geq t_0.$$

Furthermore, if x^* is an equilibrium point of system (2.1), then we call the equilibrium point x^* exponentially p-stable.

Definition 2.2 ([45]) Let matrix $D = (d_{ij})_{n\times n}$ satisfy $d_{ij} \leq 0$, $i \neq j$, then the statement 'D is a nonsingular $\mathcal{M}$-matrix' is equivalent to one of the following conditions.

(i) $D = B - M$ and $\rho(B^{-1}M) < 1$, where $M \geq 0$, $B = \operatorname{diag}\{b_1,\ldots,b_n\}$.

(ii) All the leading principal minors of D are positive.

(iii) The diagonal elements of D are all positive and there exists a positive vector d such that $Dd > 0$ or $D^Td > 0$.

For a $\mathcal{M}$-matrix D, from (iii) of Definition 2.1, we know $\Omega_{\mathcal{M}}(D) \overset{\Delta}{=} \{z \in R^n | Dz > 0, z > 0\} \neq \phi$ and satisfies $k_1z_1 + k_2z_2 \in \Omega_{\mathcal{M}}(D)$ for any vectors $z_1, z_2 \in \Omega_{\mathcal{M}}(D)$ and scalars $k_1, k_2 > 0$.

For $A \in R^{n\times n}$ and $|A| \neq 0$, we denote $\Omega_\rho(A) \overset{\Delta}{=} \{z \in R^n | Az = \rho(A)z\}$, where $\rho(A)$ is an eigenvalue of A. Then $\Omega_\rho(A)$ includes all positive eigenvectors of A provided that the matrix A has at least one positive eigenvector (see Ref. [46]).

Lemma 2.1 ([47]) *For $\alpha_i > 0$ and $\sum_{i=1}^n \alpha_i = 1$, $x_i \geq 0$, we have*

$$\prod_{i=1}^n x_i^{\alpha_i} \leq \sum_{i=1}^n \alpha_i x_i,$$

where the sign of equality holds if and only if $x_i = x_j$ for all $i,j \in \mathcal{N}$.

Lemma 2.2 ([12]) *For $a_i \geq 0$, $x_i \geq 0$, $i \in \mathcal{N}$ and any integral number $p > 0$, we have*

$$\left(\sum_{i=1}^n a_ix_i\right)^p \leq \left(\sum_{i=1}^n a_i\right)^{p-1}\sum_{i=1}^n a_ix_i^p.$$

Lemma 2.3 ([12]) *For an integral number $p \geq 2$, there exists $e_p(n) > 0$ such that*

$$e_p(n)\left(\sum_{i=1}^n |x_i|^2\right)^{\frac{p}{2}} \leq \sum_{i=1}^n |x_i|^p, \quad \forall x = (x_1,\ldots,x_n)^T \in R^n.$$

3 A new $\mathcal{L}$-operator inequality

Let $C^{2,1}[R^n \times R_+; R_+]$ denote the family of non-negative functions $V(x,t)$ on $R^n \times R_+$ which are once continuously differentiable in t and twice continuously differentiable in x. Associated with the system (2.1), for each $V(x,t) \in C^{2,1}[R^n \times R_+; R_+]$, we define an operator $\mathcal{L}V$ from $R^n \times R^n \times R_+$ to R by

$$\begin{aligned}\mathcal{L}V(x,t) &= V_t(x,t) + V_x(x,t)\bigl[-A(t)x(t) + B(t)f\bigl(x(t)\bigr) + C(t)g(y) + I(t)\bigr] \\ &\quad + \frac{1}{2}\operatorname{trace}\bigl[\sigma^T(t,x,y)V_{xx}\sigma(t,x,y)\bigr], \\ y &= x\bigl(t-\tau(t)\bigr), \qquad V_t(x,t) = \frac{\partial V(x,t)}{\partial t}, \\ V_x(x,t) &= \left(\frac{\partial V(x,t)}{\partial x_1}, \dots, \frac{\partial V(x,t)}{\partial x_n}\right), \qquad V_{xx} = \left(\frac{\partial V^2(x,t)}{\partial x_i \partial x_j}\right)_{n\times n}.\end{aligned}$$

The differential inequality is the main tool for investigating differential equations. Therefore, by using the properties of the $\mathcal{L}$-operator and the $\mathcal{M}$-matrix, we introduce a new inequality of the $\mathcal{L}$-operator that is effective for a stochastic non-autonomous system.

Theorem 3.1 *Let $\sigma < b \le +\infty$, $P = (p_{ij})_{n\times n}$, $p_{ij} \ge 0$ for $i \ne j$, $Q = (q_{ij})_{n\times n} \ge 0$, $\alpha(t) = (\alpha_{ij}(t))_{n\times n} \ge 0$, $\beta(t) = (\beta_{ij}(t))_{n\times n} \ge 0$, $r(t) = (r_1(t), \dots, r_n(t))^T \ge 0$ and for any $t \ge \sigma$, there exists a constant $\delta > 0$ such that $e^{\delta(t-\sigma)}r_i(t)$, $\alpha_{ij}(t)$, $\beta_{ij}(t)$, $i,j \in \mathcal{N}$ are integrable functions on $[\sigma, t]$. The functions $V_i(x) \in C^2[R^n, R_+]$ satisfy*

$$\begin{aligned}&\mathcal{L}V_i(x) \le \sum_{j=1}^{n}\bigl[\bigl(p_{ij} + \alpha_{ij}(t)\bigr)V_j\bigl(x(t)\bigr) + \bigl(q_{ij} + \beta_{ij}(t)\bigr)V_j\bigl(x\bigl(t-\tau(t)\bigr)\bigr)\bigr] + r_i(t), \\ &\quad t \in [\sigma, b), \forall i \in \mathcal{N}.\end{aligned} \tag{3.1}$$

Suppose that $\Pi = -(P+Q)$ is an $\mathcal{M}$-matrix, we obtain

$$\mathbb{E}V_i\bigl(x(t)\bigr) \le z_i e^{-\lambda(t-\sigma)}e^{\int_\sigma^t \theta(s)\,ds}, \quad t \in [\sigma, b), \forall i \in \mathcal{N}, \tag{3.2}$$

provided that $\mathbb{E}V_i(x(t)) < \infty$ for all $t \in [\sigma, b)$ and

$$\mathbb{E}V_i\bigl(x(t)\bigr) \le z_i e^{-\lambda(t-\sigma)}, \quad t \in [\sigma - \tau, \sigma], \forall i \in \mathcal{N}, \tag{3.3}$$

where $\lambda \in (0, \delta]$, $z \in \Omega_{\mathcal{M}}(\Pi)$ with $z_i \ge 1$, $\forall i \in \mathcal{N}$, satisfy

$$\bigl(\lambda E + P + Qe^{\lambda\tau}\bigr)z < 0, \tag{3.4}$$

and $\theta(s) = \max_{1\le i\le n}\{\sum_{j=1}^{n}(\alpha_{ij}(s) + \beta_{ij}(s)e^{\lambda\tau})\frac{z_j}{z_i} + e^{\lambda(s-\sigma)}r_i(s)\}$.

Proof By using the Itô formula, for the solution process $x(t)$ of (2.1) and $V_i(x) \in C^2[R^n, R_+]$, we can obtain

$$\begin{aligned}V_i\bigl(x(t)\bigr) &= V_i\bigl(x(\sigma)\bigr) + \int_\sigma^t \mathcal{L}V_i\bigl(x(s)\bigr)\,ds \\ &\quad + \int_\sigma^t \frac{\partial V_i(x(s))}{\partial x}\sigma\bigl(s, x(s), x\bigl(s-\tau(s)\bigr)\bigr)\,d\omega(s), \quad t \ge \sigma, \forall i \in \mathcal{N}.\end{aligned} \tag{3.5}$$

Then we get

$$\mathbb{E}V_i\big(x(t)\big) = \mathbb{E}V_i\big(x(\sigma)\big) + \int_\sigma^t \mathbb{E}\mathcal{L}V_i\big(x(s)\big)\,ds, \quad t \geq \sigma, \forall i \in \mathcal{N}. \tag{3.6}$$

For small enough $\triangle t > 0$, we have

$$\mathbb{E}V_i\big(x(t+\triangle t)\big) = \mathbb{E}V_i\big(x(\sigma)\big) + \int_\sigma^{t+\triangle t} \mathbb{E}\mathcal{L}V_i\big(x(s)\big)\,ds, \quad t \geq \sigma, \forall i \in \mathcal{N}. \tag{3.7}$$

Therefore, from (3.1), (3.6), and (3.7), we have

$$\begin{aligned}
&\mathbb{E}V_i\big(x(t+\triangle t)\big) - \mathbb{E}V_i\big(x(t)\big) \\
&\quad = \int_t^{t+\triangle t} \mathbb{E}\mathcal{L}V_i\big(x(s)\big)\,ds \\
&\quad \leq \int_t^{t+\triangle t} \Bigg[\sum_{j=1}^n \big(p_{ij} + \alpha_{ij}(s)\big)\mathbb{E}V_j\big(x(s)\big) + \sum_{j=1}^n \big(q_{ij} + \beta_{ij}(s)\big)\mathbb{E}V_j\big(x\big(s-\tau(s)\big)\big) + r_i(s)\Bigg]\,ds \\
&\quad \leq \int_t^{t+\triangle t} \Bigg[\sum_{j=1}^n \big(p_{ij} + \alpha_{ij}(s)\big)\mathbb{E}V_j\big(x(s)\big) \\
&\qquad + \sum_{j=1}^n \big(q_{ij} + \beta_{ij}(s)\big)\big[\mathbb{E}V_j\big(x(s)\big)\big]_\tau + r_i(s)\Bigg]\,ds, \quad t \geq \sigma, \forall i \in \mathcal{N}.
\end{aligned} \tag{3.8}$$

Because $\mathbb{E}V_i(x(t)) < \infty$ for all $t \in [\sigma, b)$, we know $\mathbb{E}V_i(x(t))$ is continuous. Thus, from (3.8), we can obtain

$$\begin{aligned}
&D^+\mathbb{E}V_i\big(x(t)\big) \leq \sum_{j=1}^n \big[\big(p_{ij} + \alpha_{ij}(t)\big)\mathbb{E}V_j\big(x(t)\big) + \big(q_{ij} + \beta_{ij}(t)\big)\big[\mathbb{E}V_j\big(x(t)\big)\big]_\tau\big] + r_i(t), \\
&\quad t \in [\sigma, b), \forall i \in \mathcal{N}.
\end{aligned} \tag{3.9}$$

Let $v_i(t) = \mathbb{E}V_i(x(t))$. For proving Theorem 3.1, we only need to prove

$$v_i(t) \leq z_i e^{-\lambda(t-\sigma)} e^{\int_\sigma^t \theta(s)\,ds}, \quad t \in [\sigma, b), \forall i \in \mathcal{N}, \tag{3.10}$$

provided that

$$v_i(t) \leq z_i e^{-\lambda(t-\sigma)}, \quad t \in [\sigma - \tau, \sigma], \forall i \in \mathcal{N}, \tag{3.11}$$

holds.

Because Π is an $\mathcal{M}$-matrix, we can get a vector $z \in \Omega_{\mathcal{M}}(\Pi)$ with $z_i \geq 1$, $i \in \mathcal{N}$ and $\Pi z > 0$, that is, $(P + Q)z < 0$. From the continuity of the function, we know there exists a constant $\lambda \in (0, \delta]$ satisfying (3.4).

For proving (3.10), we first of all prove, for any given $\epsilon > 0$,

$$v_i(t) < (1+\epsilon) z e^{-\lambda(t-\sigma)} e^{\int_\sigma^t \theta(s)\,ds} \triangleq \xi_i(t), \quad t \in [\sigma, b), \forall i \in \mathcal{N}. \tag{3.12}$$

If (3.12) is not true, given that $v_i(t)$ is continuous on $[\sigma, b)$ and the fact (3.11) holds, then there must be a constant $t^* \in (\sigma, b)$ and $m \in \mathcal{N}$ such that

$$v_m(t^*) = \xi_m(t^*), \qquad D^+ v_m(t^*) \ge \xi_m'(t^*), \tag{3.13}$$

$$v_i(t) \le \xi_i(t), \quad t \in [\sigma - \tau, t^*], \forall i \in \mathcal{N}. \tag{3.14}$$

By using (3.9), (3.11)-(3.14), $z_m \ge 1$, $p_{ij} \ge 0$ ($i \ne j$), and $Q \ge 0$, we can get

$$\begin{aligned}
D^+ v_m(t^*) &\le \sum_{j=1}^n \bigl[\bigl(p_{mj} + \alpha_{mj}(t^*)\bigr) v_j(t^*) + \bigl(q_{mj} + \beta_{mj}(t^*)\bigr)\bigl[v_j(t^*)\bigr]_\tau\bigr] + r_m(t^*) \\
&\le \sum_{j=1}^n \bigl[\bigl(p_{mj} + \alpha_{mj}(t^*)\bigr)(1+\epsilon) z_j e^{-\lambda(t^*-\sigma)} e^{\int_\sigma^{t^*} \theta(s)\,ds} \\
&\quad + \bigl(q_{mj} + \beta_{mj}(t^*)\bigr) e^{\lambda\tau}(1+\epsilon) z_j e^{-\lambda(t^*-\sigma)} e^{\int_\sigma^{t^*} \theta(s)\,ds}\bigr] + r_m(t^*) \\
&\le \sum_{j=1}^n \bigl(p_{mj} + q_{mj} e^{\lambda\tau}\bigr)(1+\epsilon) z_j e^{-\lambda(t^*-\sigma)} e^{\int_\sigma^{t^*} \theta(s)\,ds} \\
&\quad + \sum_{j=1}^n \bigl(\alpha_{mj}(t^*) + \beta_{mj}(t^*) e^{\lambda\tau}\bigr) \frac{z_j}{z_m} (1+\epsilon) z_m e^{-\lambda(t^*-\sigma)} e^{\int_\sigma^{t^*} \theta(s)\,ds} \\
&\quad + e^{\lambda(t^*-\sigma)} r_m(t^*)(1+\epsilon) z_m e^{-\lambda(t^*-\sigma)} e^{\int_\sigma^{t^*} \theta(s)\,ds} + r_m(t^*)\bigl[1 - (1+\epsilon) z_m e^{\int_\sigma^{t^*} \theta(s)\,ds}\bigr] \\
&< -\lambda(1+\epsilon) z_m e^{-\lambda(t^*-\sigma)} e^{\int_\sigma^{t^*} \theta(s)\,ds} + \theta(t^*)(1+\epsilon) z_m e^{-\lambda(t^*-\sigma)} e^{\int_\sigma^{t^*} \theta(s)\,ds} \\
&= \xi_m'(t^*),
\end{aligned} \tag{3.15}$$

which contradicts the second inequality in (3.13). Thus, (3.12) holds. Letting $\epsilon \to 0^+$ in (3.12), we obtain (3.10). □

Remark 3.1 If $\alpha(t) \equiv 0$ and $\beta(t) \equiv 0$ in (3.1), we can get Theorem 1 in [44].

4 Application to neural networks

For system (2.1), some assumptions are given in the following:

(A_1) For $i, j \in \mathcal{N}$, $a_i(t) > 0$, $b_{ij}(t)$, $c_{ij}(t)$ and $I_i(t)$ are bounded continuous functions defined on R_+.

(A_2) There are positive constants l_j and k_j, $j \in \mathcal{N}$ such that

$$\bigl|f_j(r) - f_j(s)\bigr| \le l_j|r - s|, \qquad \bigl|g_j(r) - g_j(s)\bigr| \le k_j|r - s|, \quad \forall r, s \in R.$$

(A_3) There exist non-negative bounded functions $m_{ij}(t)$, $n_{ij}(t)$, and $h_i(t)$, $i, j \in \mathcal{N}$ such that

$$\begin{aligned}
&\bigl[\sigma_{ij}(t, v, u) - \sigma_{ij}(t, \bar{v}, \bar{u})\bigr]^2 \le m_{ij}(t)(v_j - \bar{v}_j)^2 + n_{ij}(t)(u_j - \bar{u}_j)^2 + h_i(t), \\
&\quad \forall u, \bar{u}, v, \bar{v} \in R^n, t \ge t_0.
\end{aligned}$$

(A_4) There exist non-negative integrable functions $\hat{\alpha}_{ij}(t)$, $\hat{\beta}_{ij}(t)$ on $[t_0, t]$ such that

$$P(t) \le \widehat{P} + \hat{\alpha}(t), \qquad Q(t) \le \widehat{Q} + \hat{\beta}(t) \quad \text{and}$$
$$\widehat{\Pi} = -(\widehat{P} + \widehat{Q}) \text{ is a nonsingular } \mathcal{M}\text{-matrix,}$$

where $P(t) = (p_{ij}(t))_{n\times n}$, $p_{ii}(t) = -pa_i(t) + (p-1)\sum_{j=1}^{n}(|b_{ij}(t)|l_j + |c_{ij}(t)|k_j) + \frac{1}{2}(p-1)(p-2)\sum_{j=1}^{n}(m_{ij}(t) + n_{ij}(t)) + |b_{ii}(t)|l_i + (p-1)m_{ii}(t) + \frac{1}{2}(p-1)(p-2)$, $p_{ij}(t) = |b_{ij}(t)|l_j + (p-1)m_{ij}(t)$, $i \ne j$, $Q(t) = (q_{ij}(t))_{n\times n}$, $q_{ij}(t) = |c_{ij}(t)|k_j + (p-1)n_{ij}(t)$, $\hat{\alpha}(t) = (\hat{\alpha}_{ij}(t))_{n\times n}$, $\hat{\beta}(t) = (\hat{\beta}_{ij}(t))_{n\times n}$, $\widehat{P} = (\hat{p}_{ij})_{n\times n}$, $\hat{p}_{ij} \ge 0$, $i \ne j$, $\widehat{Q} = (\hat{q}_{ij})_{n\times n} \ge 0$, $i, j \in \mathcal{N}$, $p \ge 2$. Let $\hat{r}_i(t) = (p-1)(nh_i(t))^{\frac{p}{2}}$, $i \in \mathcal{N}$, and there exists a constant $\hat{\delta} > 0$ such that $e^{\hat{\delta}(t-t_0)}\hat{r}_i(t)$ is an integrable function on $[t_0, t]$.

(A_5) For any $u, v \in R^n$, there exist matrices $R_k = (r_{ij}^{(k)})_{n\times n} \ge 0$ such that

$$\bigl[I_k(u) - I_k(v)\bigr]^+ \le R_k[u - v]^+, \quad k \in \mathbb{N}.$$

Let $\hat{R}_k = (\hat{r}_{ij}^{(k)})_{n\times n}$, $\hat{r}_{ij}^{(k)} \ge r_{ij}^{(k)}(\sum_{j=1}^{n} r_{ij}^{(k)})^{p-1}$.

(A_6) The set $\Omega = \bigcap_{k=1}^{\infty}(\Omega_\rho(\hat{R}_k)) \cap \Omega_{\mathcal{M}}(\widehat{\Pi})$ is nonempty (*i.e.*, $\Omega \ne \emptyset$), for a given $z \in \Omega$, the scalar $\lambda \in (0, \hat{\delta}]$ satisfies

$$\bigl(\lambda E + \widehat{P} + \widehat{Q}e^{\lambda\tau}\bigr)z < 0. \tag{4.1}$$

(A_7) There are constants $0 \le \mu < \lambda$ and $b \ge 0$ which satisfy

$$\int_{t_0}^{t} \hat{\theta}(s)\,ds \le \mu(t - t_0) + b, \tag{4.2}$$

where $\hat{\theta}(s) = \max_{1\le i\le n}\{\sum_{j=1}^{n}(\hat{\alpha}_{ij}(s) + \hat{\beta}_{ij}(s)e^{\lambda\tau})\frac{z_j}{z_i} + e^{\lambda(s-t_0)}\hat{r}_i(s)\}$.

(A_8) There exists a constant γ such that

$$\frac{\ln \gamma_k}{t_k - t_{k-1}} \le \gamma < \lambda - \mu, \quad k \in \mathbb{N}, \tag{4.3}$$

where $\gamma_k \ge \max\{1, \rho(\hat{R}_k)\}$.

Theorem 4.1 *Assume that* (A_1)-(A_8) *are all true. Then we know system* (2.1) *is exponentially p-stable and the exponential convergent rate is no less than* $\lambda - \mu - \gamma$.

Proof For any two solutions $x(t)$ and $y(t)$ of system (2.1) corresponding to initial values $\phi(s), \varphi(s) \in PC_{\mathscr{F}_0}^{b}[[t_0 - \tau, t_0], R^n]$, respectively. Let $z_i(t) = x_i(t) - y_i(t)$, $i \in \mathcal{N}$. Then from (2.1), we get

$$\begin{cases} dz_i(t) = [-a_i(t)z_i(t) + \sum_{j=1}^{n} b_{ij}(t)(f_j(x_j(t)) - f_j(y_j(t))) \\ \qquad + \sum_{j=1}^{n} c_{ij}(t)(g_j(x_j(t - \tau_{ij}(t))) - g_j(y_j(t - \tau_{ij}(t))))]\,dt \\ \qquad + \sum_{j=1}^{n}(\sigma_{ij}(t, x(t), x(t - \tau(t))) - \sigma_{ij}(t, y(t), y(t - \tau(t))))\,d\omega_j(t), \\ \qquad t \ge t_0, t \ne t_k, \\ z_i(t_k) = x_i(t_k) - y_i(t_k) = I_{ik}(x(t_k^-)) - I_{ik}(y(t_k^-)), \quad t = t_k, \\ z_i(s) = \phi_i(s) - \varphi_i(s), \quad t_0 - \tau \le s \le t_0. \end{cases} \tag{4.4}$$

Let $V_i(z(t)) = |z_i(t)|^p$, $p \geq 2$, $i \in \mathcal{N}$. Then we get

$$\frac{\partial V_i(z)}{\partial z_i} = p|z_i|^{p-1}\operatorname{sgn}(z_i) = p|z_i|^{p-2}z_i, \qquad \frac{\partial^2 V_i(z)}{\partial z_i^2} = p(p-1)|z_i|^{p-2},$$

where sgn(·) denotes sign function. Therefore, from (A_1)-(A_4), Lemma 2.1, and (4.4), we get

$$\begin{aligned}
\mathcal{L}V_i(z) = {} & p\bigl|z_i(t)\bigr|^{p-2}z_i(t)\Biggl\{-a_i(t)z_i(t) + \sum_{j=1}^{n} b_{ij}(t)\bigl[f_j\bigl(x_j(t)\bigr) - f_j\bigl(y_j(t)\bigr)\bigr] \\
& + \sum_{j=1}^{n} c_{ij}(t)\bigl[g_j\bigl(x_j\bigl(t-\tau_{ij}(t)\bigr)\bigr) - g_j\bigl(y_j\bigl(t-\tau_{ij}(t)\bigr)\bigr)\bigr]\Biggr\} \\
& + \frac{1}{2}p(p-1)|z_i|^{p-2}\sum_{j=1}^{n}\bigl[\sigma_{ij}\bigl(t,x(t),x\bigl(t-\tau(t)\bigr)\bigr) - \sigma_{ij}\bigl(t,y(t),y\bigl(t-\tau(t)\bigr)\bigr)\bigr]^2 \\
\leq {} & -pa_i(t)\bigl|z_i(t)\bigr|^p + p\bigl|z_i(t)\bigr|^{p-1}\sum_{j=1}^{n}\bigl|b_{ij}(t)\bigr|l_j\bigl|z_j(t)\bigr| \\
& + p\bigl|z_i(t)\bigr|^{p-1}\sum_{j=1}^{n}\bigl|c_{ij}(t)\bigr|k_j\bigl|z_j\bigl(t-\tau_{ij}(t)\bigr)\bigr| \\
& + \frac{1}{2}p(p-1)\bigl|z_i(t)\bigr|^{p-2}\sum_{j=1}^{n}\bigl[m_{ij}(t)\bigl|z_j(t)\bigr|^2 + n_{ij}(t)\bigl|z_j\bigl(t-\tau_{ij}(t)\bigr)\bigr|^2 + h_i(t)\bigr] \\
\leq {} & -pa_i(t)\bigl|z_i(t)\bigr|^p + \sum_{j=1}^{n}|b_{ij}|l_j\bigl[(p-1)\bigl|z_i(t)\bigr|^p + \bigl|z_j(t)\bigr|^p\bigr] \\
& + \sum_{j=1}^{n}|c_{ij}|k_j\bigl[(p-1)\bigl|z_i(t)\bigr|^p + \bigl|z_j\bigl(t-\tau_{ij}(t)\bigr)\bigr|^p\bigr] \\
& + \frac{1}{2}(p-1)(p-2)\sum_{j=1}^{n}m_{ij}(t)\bigl|z_i(t)\bigr|^p \\
& + (p-1)\sum_{j=1}^{n}m_{ij}(t)\bigl|z_j(t)\bigr|^p + \frac{1}{2}(p-1)(p-2)\sum_{j=1}^{n}n_{ij}(t)\bigl|z_i(t)\bigr|^p \\
& + (p-1)\sum_{j=1}^{n}n_{ij}(t)\bigl|z_j\bigl(t-\tau_{ij}(t)\bigr)\bigr|^p + \frac{1}{2}(p-1)(p-2)\bigl|z_i(t)\bigr|^p + (p-1)\bigl(nh_i(t)\bigr)^{\frac{p}{2}} \\
\leq {} & \sum_{j=1}^{n}\bigl[p_{ij}(t)V_j\bigl(z(t)\bigr) + q_{ij}(t)V_j\bigl(z\bigl(t-\tau(t)\bigr)\bigr)\bigr] + \hat{r}_i(t) \\
\leq {} & \sum_{j=1}^{n}\bigl[\bigl(\hat{p}_{ij} + \hat{\alpha}_{ij}(t)\bigr)V_j\bigl(z(t)\bigr) + \bigl(\hat{q}_{ij} + \hat{\beta}_{ij}(t)\bigr)V_j\bigl(z\bigl(t-\tau(t)\bigr)\bigr)\bigr] + \hat{r}_i(t).
\end{aligned} \tag{4.5}$$

For the initial conditions $\phi(s), \varphi(s) \in PC^b_{\mathscr{F}_0}[[t_0-\tau, t_0], R^n]$, we know $z(s) = \phi(s) - \varphi(s) \in PC^b_{\mathscr{F}_0}[[t_0-\tau, t_0], R^n]$. From the assumption that, for any initial value in $PC^b_{\mathscr{F}_0}[[t_0-\tau, t_0], R^n]$, model (2.1) has a global solution satisfying $\mathbb{E}(\sup_{t_0\leq s\leq t}|x(t)|^r) < \infty$ for all $t \geq t_0$ and $r > 0$, we know $\mathbb{E}(\sup_{t_0\leq s\leq t}|z(t)|^r) < \infty$ for all $t \geq t_0$ and $r > 0$. Thus, we know

$\mathbb{E}V_i(z(t)) < \infty$. Since $\widehat{\Pi} = -(\widehat{P} + \widehat{Q})$ is an $\mathcal{M}$-matrix and Ω is nonempty, there must be a positive vector $z \in \Omega$ and a constant $\lambda \in (0, \hat{\delta}]$ such that (4.1) holds and

$$\mathbb{E}V_i\big(z(t)\big) \leq \frac{z_i}{\min_{1\leq j\leq n}\{z_j\}}\kappa\|\phi - \varphi\|_{L^p}^p e^{-\lambda(t-t_0)}, \quad t \in [t_0 - \tau, t_0], \tag{4.6}$$

where $\kappa > 0$ is a constant such that $\kappa\|\phi - \varphi\|_{L^p}^p \geq 1$.

From (A_4), (4.5), (4.6), and Theorem 3.1, we get

$$\mathbb{E}V_i\big(z(t)\big) \leq \frac{z_i}{\min_{1\leq j\leq n}\{z_j\}}\kappa\|\phi - \varphi\|_{L^p}^p e^{-\lambda(t-t_0)} e^{\int_{t_0}^t \hat{\theta}(s)\,ds}, \quad t \in [t_0, t_1). \tag{4.7}$$

Assume that the inequalities

$$\begin{aligned}\mathbb{E}V_i\big(z(t)\big) &\leq \gamma_0\gamma_1\cdots\gamma_{m-1}\frac{z_i}{\min_{1\leq j\leq n}\{z_j\}}\kappa\|\phi - \varphi\|_{L^p}^p e^{-\lambda(t-t_0)} e^{\int_{t_0}^t \hat{\theta}(s)\,ds}, \\ &t_{m-1} \leq t < t_m,\end{aligned} \tag{4.8}$$

hold for all $m = 1, 2, \ldots, k$, where $\gamma_0 = 1$. Then, from (4.8), (A_5), and Lemma 2.2, we obtain

$$\begin{aligned}\mathbb{E}V_i\big(z(t_k)\big) &= \mathbb{E}\big|x_i(t_k) - y_i(t_k)\big|^p \\ &= \mathbb{E}\big|I_{ik}\big(x(t_k^-)\big) - I_{ik}\big(y(t_k^-)\big)\big|^p \\ &\leq \mathbb{E}\left(\sum_{j=1}^n r_{ij}^{(k)}\big|z_j(t_k^-)\big|\right)^p \\ &\leq \left(\sum_{j=1}^n r_{ij}^{(k)}\right)^{p-1}\sum_{j=1}^n r_{ij}^{(k)}\mathbb{E}\big|z_j(t_k^-)\big|^p \\ &\leq \sum_{j=1}^n \hat{r}_{ij}^{(k)}\mathbb{E}\big|z_j(t_k^-)\big|^p = \sum_{j=1}^n \hat{r}_{ij}^{(k)}\mathbb{E}V_j\big(z(t_k^-)\big) \\ &\leq \gamma_0\gamma_1\cdots\gamma_{k-1}\sum_{j=1}^n \hat{r}_{ij}^{(k)}\frac{z_j}{\min_{1\leq j\leq n}\{z_j\}}\kappa\|\phi - \varphi\|_{L^p}^p e^{-\lambda(t_k-t_0)} e^{\int_{t_0}^{t_k}\hat{\theta}(s)\,ds} \\ &= \gamma_0\gamma_1\cdots\gamma_{k-1}\rho(\hat{R}_k)\frac{z_i}{\min_{1\leq j\leq n}\{z_j\}}\kappa\|\phi - \varphi\|_{L^p}^p e^{-\lambda(t_k-t_0)} e^{\int_{t_0}^{t_k}\hat{\theta}(s)\,ds} \\ &\leq \gamma_0\gamma_1\cdots\gamma_{k-1}\gamma_k\frac{z_i}{\min_{1\leq j\leq n}\{z_j\}}\kappa\|\phi - \varphi\|_{L^p}^p e^{-\lambda(t_k-t_0)} e^{\int_{t_0}^{t_k}\hat{\theta}(s)\,ds}.\end{aligned} \tag{4.9}$$

This, together with (4.8) and $\gamma_k \geq 1$, leads to

$$\begin{aligned}\mathbb{E}V_i\big(z(t)\big) &\leq \gamma_0\gamma_1\cdots\gamma_{k-1}\gamma_k\frac{z_i}{\min_{1\leq j\leq n}\{z_j\}}\kappa\|\phi - \varphi\|_{L^p}^p e^{-\lambda(t-t_0)} e^{\int_{t_0}^t \hat{\theta}(s)\,ds} \\ &= \gamma_0\gamma_1\cdots\gamma_k e^{\int_0^{t_k}\hat{\theta}(s)\,ds} e^{-\lambda(t_k-t_0)}\frac{z_i}{\min_{1\leq j\leq n}\{z_j\}}\kappa\|\phi - \varphi\|_{L^p}^p e^{-\lambda(t-t_k)}, \\ &t \in [t_k - \tau, t_k].\end{aligned} \tag{4.10}$$

Let

$$\tilde{z}_i = \gamma_0\gamma_1\cdots\gamma_k e^{\int_0^{t_k}\hat{\theta}(s)\,ds}\frac{z_i}{\min_{1\le j\le n}\{z_j\}}\kappa\|\phi-\varphi\|_{L^p}^p, \qquad \tilde{z} = (\tilde{z}_1,\ldots,\tilde{z}_n)^T,$$
$$U_i\bigl(z(t)\bigr) = e^{\lambda(t_k-t_0)}V_i\bigl(z(t)\bigr),$$

then we know the vector $\tilde{z}\in\Omega_{\mathcal{M}}(\widehat{\Pi})$ with $\tilde{z}_i\ge 1$, $i\in\mathcal{N}$. From (4.10), we get

$$\mathbb{E}U_i\bigl(z(t)\bigr)\le \tilde{z}_i e^{-\lambda(t-t_k)}, \quad t\in[t_k-\tau,t_k]. \tag{4.11}$$

From (4.5), we obtain

$$\mathcal{L}U_i\bigl(z(t)\bigr)\le\sum_{j=1}^n\bigl[\bigl(\hat{p}_{ij}+\hat{\alpha}_{ij}(t)\bigr)U_j\bigl(z(t)\bigr)+\bigl(\hat{q}_{ij}+\hat{\beta}_{ij}(t)\bigr)U_j\bigl(z\bigl(t-\tau(t)\bigr)\bigr)\bigr]+e^{\lambda(t_k-t_0)}\hat{r}_i(t). \tag{4.12}$$

Furthermore, we can easily get

$$\begin{aligned}\breve{\theta}(s) &= \max_{1\le i\le n}\left\{\sum_{j=1}^n\bigl(\hat{\alpha}_{ij}(s)+\hat{\beta}_{ij}(s)e^{\lambda\tau}\bigr)\frac{\tilde{z}_j}{\tilde{z}_i}+e^{\lambda(s-t_k)}e^{\lambda(t_k-t_0)}\hat{r}_i(s)\right\}\\ &= \max_{1\le i\le n}\left\{\sum_{j=1}^n\bigl(\hat{\alpha}_{ij}(s)+\hat{\beta}_{ij}(s)e^{\lambda\tau}\bigr)\frac{z_j}{z_i}+e^{\lambda(s-t_0)}\hat{r}_i(s)\right\}=\hat{\theta}(s).\end{aligned}$$

Therefore, from (A_4), (4.11), (4.12), and Theorem 3.1, we get

$$\mathbb{E}U_i\bigl(z(t)\bigr)\le\tilde{z}_i e^{-\lambda(t-t_k)}e^{\int_{t_k}^t\hat{\theta}(s)\,ds}, \quad t\in[t_k,t_{k+1}), \tag{4.13}$$

that is,

$$\begin{aligned}&\mathbb{E}V_i\bigl(z(t)\bigr)\le\gamma_0\gamma_1\cdots\gamma_{k-1}\gamma_k\frac{z_i}{\min_{1\le j\le n}\{z_j\}}\kappa\|\phi-\varphi\|_{L^p}^p e^{-\lambda(t-t_0)}e^{\int_{t_0}^t\hat{\theta}(s)\,ds},\\ &\quad t\in[t_k,t_{k+1}).\end{aligned} \tag{4.14}$$

By using the mathematical induction method, we know

$$\begin{aligned}&\mathbb{E}V_i\bigl(z(t)\bigr)\le\gamma_0\gamma_1\cdots\gamma_{k-1}\frac{z_i}{\min_{1\le j\le n}\{z_j\}}\kappa\|\phi-\varphi\|_{L^p}^p e^{-\lambda(t-t_0)}e^{\int_{t_0}^t\hat{\theta}(s)\,ds},\\ &\quad t\in[t_{k-1},t_k), k\in\mathbb{N}.\end{aligned} \tag{4.15}$$

From (4.3), we know $\gamma_k\le e^{\gamma(t_k-t_{k-1})}$. Then we can use (4.2) and (4.15) to get

$$\begin{aligned}&\bigl|x_i(t)-y_i(t)\bigr|^p\\ &\quad=\mathbb{E}V_i\bigl(z(t)\bigr)\le e^{\gamma(t_1-t_0)}\cdots e^{\gamma(t_{k-1}-t_{k-2})}\frac{z_i}{\min_{1\le j\le n}\{z_j\}}\kappa\|\phi-\varphi\|_{L^p}^p e^{-\lambda(t-t_0)}e^{\int_{t_0}^t\hat{\theta}(s)\,ds}\\ &\quad\le\frac{z_i}{\min_{1\le j\le n}\{z_j\}}\hat{\kappa}\|\phi-\varphi\|_{L^p}^p e^{-(\lambda-\mu-\gamma)(t-t_0)}, \quad\forall t\in[t_0,t_k), k\in\mathbb{N},\end{aligned} \tag{4.16}$$

where $\hat{\kappa}\ge\kappa$ is a proper constant.

From (4.16) and Lemma 2.3, we get

$$\begin{aligned}\mathbb{E}\big|x(t)-y(t)\big|^p &\le \frac{1}{e_p(n)}\sum_{i=1}^{n}\mathbb{E}\big|x_i(t)-y_i(t)\big|^p = \frac{1}{e_p(n)}\sum_{i=1}^{n}\mathbb{E}V_i\big(z(t)\big)\\ &\le \frac{1}{e_p(n)}\sum_{i=1}^{n}\frac{z_i}{\min_{1\le j\le n}\{z_j\}}\hat{\kappa}\,\|\phi-\varphi\|_{L^p}^p e^{-(\lambda-\mu-\gamma)(t-t_0)}\\ &\triangleq M\|\phi-\varphi\|_{L^p}^p e^{-(\lambda-\mu-\gamma)(t-t_0)}, \quad t\ge t_0. \end{aligned} \tag{4.17}$$

Therefore, the conclusion of Theorem 4.1 holds.

If $I_i(t)\equiv 0$, $\sigma_{ij}(t,0,0)\equiv 0$ for $t\ge t_0$, $I_{ik}(0)=0$, $f_j(0)=g_j(0)=0$, $i,j\in\mathcal{N}$, $k\in\mathbb{N}$ then the system (2.1) becomes the following system:

$$\begin{cases} dx_i(t) = [-a_i(t)x_i(t) + \sum_{j=1}^{n} b_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^{n} c_{ij}(t)g_j(x_j(t-\tau_{ij}(t)))]\,dt \\ \qquad\qquad + \sum_{j=1}^{n}\sigma_{ij}(t,x(t),x(t-\tau(t)))\,d\omega_j(t), \quad t\ge t_0, t\ne t_k, \\ x_i(t_k) = I_{ik}(x(t_k^-)), \quad t=t_k, \\ x_i(s)=\phi_i(s), \quad t_0-\tau\le s\le t_0, \end{cases} \tag{4.18}$$

with an equilibrium point $x^*=0$. From Theorem 4.1, we can get the following conclusion. □

Corollary 4.1 *Assume that the conditions* (A_1)-(A_8) *are all true. Then the zero solution* $x^*=0$ *of* (4.18) *is exponentially p-stable and the exponential convergent rate is no less than* $\lambda-\mu-\gamma$.

Remark 4.1 The authors in [24] obtained some new results on p-moment exponential stability of non-autonomous stochastic differential equation with delay. The model (4.18) without impulses is a special case of equation (2) in [24]. However, the results in [24] require the coefficients to have a common factor and $h_i(t)\equiv 0$ ($i\in\mathcal{N}$, $t\ge t_0$) in assumption (A_3) to be true.

If $I_{ik}(x)=x_i$, $i\in\mathcal{N}$, $k\in\mathbb{N}$ and $\phi(s)=(\phi_1(s),\ldots,\phi_n(s))^T\in C^b_{\mathscr{F}_0}[[t_0-\tau,t_0],R^n]$, from system (2.1), we can get the following model without impulses:

$$\begin{cases} dx_i(t) = [-a_i(t)x_i(t) + \sum_{j=1}^{n} b_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^{n} c_{ij}(t)g_j(x_j(t-\tau_{ij}(t))) + I_i(t)]\,dt \\ \qquad\qquad + \sum_{j=1}^{n}\sigma_{ij}(t,x(t),x(t-\tau(t)))\,d\omega_j(t), \quad t\ge t_0, \\ x_i(s)=\phi_i(s), \quad t_0-\tau\le s\le t_0. \end{cases} \tag{4.19}$$

Then we can get the following conclusion.

Theorem 4.2 *Assume that* (A_1)-(A_4) *hold,* (A_7) *holds for* $\lambda\in(0,\hat{\delta}]$ *which satisfies*

$$\big(\lambda E+\widehat{P}+\widehat{Q}e^{\lambda\tau}\big)z<0, \quad z\in\Omega_{\mathcal{M}}(\widehat{\Pi}). \tag{4.20}$$

Then the system (4.19) *is exponentially p-stable and the exponential convergent rate is no less than* $\lambda-\mu$.

If $I_i(t) \equiv 0$, $\sigma_{ij}(t,0,0) \equiv 0$ for $t \geq t_0$, $f_j(0) = g_j(0) = 0$, $i,j \in \mathcal{N}$, then the system (4.19) becomes the following model:

$$\begin{cases} dx_i(t) = [-a_i(t)x_i(t) + \sum_{j=1}^n b_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^n c_{ij}(t)g_j(x_j(t-\tau_{ij}(t)))]\,dt \\ \qquad + \sum_{j=1}^n \sigma_{ij}(t,x(t),x(t-\tau(t)))\,d\omega_j(t), \quad t \geq t_0, \\ x_i(s) = \phi_i(s), \quad t_0 - \tau \leq s \leq t_0, \end{cases} \tag{4.21}$$

with an equilibrium point $x^* = 0$. From Theorem 4.2, we get the following corollary.

Corollary 4.2 *Assume that* (A_1)-(A_4) *hold*, (A_7) *holds for* $\lambda \in (0,\hat{\delta}]$ *which satisfies the inequality* (4.20). *Then the zero solution* $x^* = 0$ *of* (4.21) *is exponentially p-stable and the exponential convergent rate is no less than* $\lambda - \mu$.

Remark 4.2 The models investigated in [14, 21] can be considered as special cases of model (4.21), but they require the differentiability of delay functions and $\sup_{t\geq t_0} \dot{\tau}_{ij}(t) < 1$. In addition, combining $\sigma_{ij}(t,0,0) \equiv 0$ for $t \geq t_0$ with (A_3), we can get

$$\begin{aligned} \operatorname{trace}\bigl[\sigma^T(t,v,u)\sigma(t,v,u)\bigr] &= \sum_{i=1}^n\sum_{j=1}^n \sigma_{ij}^2(t,v,u) \\ &\leq \sum_{j=1}^n \Biggl[\Biggl(\sum_{i=1}^n m_{ij}(t)\Biggr)v_j^2 + \Biggl(\sum_{i=1}^n n_{ij}(t)\Biggr)u_j^2 + \sum_{i=1}^n h_i(t)\Biggr]. \end{aligned}$$

However, the authors of [14, 21] require that $h_i(t) \equiv 0$ ($i \in \mathcal{N}$, $t \geq t_0$) in assumption (A_3) is true.

Remark 4.3 The authors in [23] used the methods in [34, 35] to study the p-moment exponential stability of non-autonomous stochastic Cohen-Grossberg neural networks and obtained some new results. It is well known that the model (4.21) is a special case of equation (3) in [23], however, the condition (12) in [23] is equivalent to requiring that $-(P(t) + Q(t))$ is a nonsingular $\mathcal{M}$-matrix for all $t \geq t_0$. In addition, the results in [23] require that $h_i(t) \equiv 0$ ($i \in \mathcal{N}$, $t \geq t_0$) in assumption (A_3) is true.

5 Examples

Example 5.1 Consider the following system:

$$\begin{cases} dx_1(t) = [-(3 - \frac{3}{8}|\cos t|)x_1(t) + (1 + \sin te^{-0.2t})f_1(x_1(t)) + (\frac{3}{5} + e^{-0.1t})f_2(x_2(t)) \\ \qquad + (\frac{1}{2} + 2e^{-0.1t})g_1(x_1(t - 0.1|\sin 20t|)) \\ \qquad + (\frac{1}{2} + e^{-0.1t})g_2(x_2(t - 0.1|\sin 30t|))]\,dt \\ \qquad + (\frac{\sqrt{5}}{10}x_1(t) + e^{-t}\sin^2 x_1(t))\,d\omega_1(t) + (\frac{\sqrt{5}}{10}x_2(t) + e^{-t}\sin^2 x_2(t))\,d\omega_2(t), \\ dx_2(t) = [-(\frac{57}{20} - \frac{3}{8}|\cos t|)x_2(t) + (\frac{4}{15} + e^{-0.2t})f_1(x_1(t)) + (1 + \cos te^{-0.2t}))f_2(x_2(t)) \\ \qquad + (\frac{7}{12} + e^{-0.2t})g_1(x_1(t - 0.1|\sin 30t|)) \\ \qquad + (\frac{1}{2} + e^{-0.2t})g_2(x_2(t - 0.1|\sin 40t|))]\,dt \\ \qquad + (\frac{\sqrt{5}}{10}x_1(t) + e^{-t}\sin^2 x_1(t))\,d\omega_1(t) + (\frac{\sqrt{5}}{10}x_2(t) + e^{-t}\sin^2 x_2(t))\,d\omega_2(t), \\ x_i(s) = \phi_i(s), \quad -0.1 \leq s \leq 0, \end{cases} \tag{5.1}$$

where $f_1(s) = f_2(s) = \frac{1}{2}(|s+1| - |s-1|)$, $g_1(s) = g_2(s) = s$. We can easily know $\tau = 0.1$, $l_1 = l_2 = k_1 = k_2 = 1$, $f_1(0) = f_2(0) = g_1(0) = g_2(0) = 0$, $\sigma_{ij}(t,0,0) \equiv 0$, $i,j = 1,2$. Evidently, model (5.1) has an equilibrium point zero.

For $\sigma_{ij}(t,x(t),x(t-\tau(t))) = \frac{\sqrt{5}}{10}x_j(t) + e^{-t}\sin^2 x_j(t)$, $i,j = 1,2$, we can derive $|\sigma_{ij}(t,x(t),x(t-\tau(t)))|^2 \leq \frac{1}{10}x_j^2(t) + 2e^{-2t}$, that is, $m_{ij}(t) \equiv \frac{1}{10}$, $n_{ij}(t) \equiv 0$, $h_i(t) = 2e^{-2t}$, $\hat{r}_i(t) = 4e^{-2t}$, $i,j = 1,2$.

Case 1. Let $p = 2$, by simple computation, we can get the parameters of (A_4) as follows:

$$P(t) = \begin{pmatrix} -\frac{23}{10} + \frac{3}{4}|\cos t| + 2|\sin t|e^{-0.2t} + 4e^{-0.1t} & \frac{7}{10} + e^{-0.1t} \\ \frac{11}{30} + e^{-0.2t} & -\frac{45}{20} + \frac{3}{4}|\cos t| + 2|\cos t|e^{-0.2t} + 3e^{-0.2t} \end{pmatrix},$$

$$Q(t) = \begin{pmatrix} \frac{1}{2} + 2e^{-0.1t} & \frac{1}{2} + e^{-0.1t} \\ \frac{7}{12} + e^{-0.2t} & \frac{1}{2} + 2e^{-0.2t} \end{pmatrix}, \qquad \widehat{P} = \begin{pmatrix} -\frac{23}{10} & \frac{7}{10} \\ \frac{11}{30} & -\frac{45}{20} \end{pmatrix},$$

$$\widehat{Q} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{7}{12} & \frac{1}{2} \end{pmatrix}, \qquad \widehat{\Pi} = -(\widehat{P} + \widehat{Q}) = \begin{pmatrix} 1.8 & -1.2 \\ -0.95 & 1.75 \end{pmatrix},$$

$$\hat{\alpha}(t) = \big(\hat{\alpha}_{ij}^1(t)\big)_{2\times 2} + \big(\hat{\alpha}_{ij}^2(t)\big)_{2\times 2} = \begin{pmatrix} \frac{3}{4}|\cos t| & 0 \\ 0 & \frac{3}{4}|\cos t| \end{pmatrix} + \begin{pmatrix} 2|\sin t|e^{-0.2t} + 4e^{-0.1t} & e^{-0.1t} \\ e^{-0.2t} & 2|\cos t|e^{-0.2t} + 3e^{-0.2t} \end{pmatrix},$$

$$\hat{\beta}(t) = \big(\hat{\beta}_{ij}(t)\big)_{2\times 2} = \begin{pmatrix} 2e^{-0.1t} & e^{-0.1t} \\ e^{-0.2t} & 2e^{-0.2t} \end{pmatrix}.$$

We can easily know that $\widehat{\Pi}$ is a nonsingular $\mathcal{M}$-matrix, and we obtain

$$\Omega_{\mathcal{M}}(\widehat{\Pi}) = \left\{(z_1,z_2)^T > 0 \,\middle|\, \frac{19}{35}z_1 < z_2 < \frac{3}{2}z_1\right\}.$$

Apparently, $z = (1,1)^T \in \Omega_{\mathcal{M}}(\widehat{\Pi})$, and $\lambda = 0.54$ satisfies

$$\big(\lambda E + \widehat{P} + \widehat{Q}e^{\lambda\tau}\big)z = (-0.0045, -0.1999)^T < (0,0)^T.$$

We compute

$$J(t) \triangleq \int_0^t |\cos s|\, ds.$$

For any $t \geq 0$, there must be an integer $n \geq 0$ satisfying $n\pi - \frac{\pi}{2} \leq t < n\pi + \frac{\pi}{2}$. Let $t = n\pi - \frac{\pi}{2} + u$, $0 \leq u < \pi$, then we get

$$\begin{aligned} J(t) &\triangleq \int_0^t |\cos s|\, ds \\ &= \int_0^{\frac{\pi}{2}} \cos s\, ds + \sum_{k=1}^{n-1}(-1)^k \int_{k\pi-\frac{\pi}{2}}^{k\pi+\frac{\pi}{2}} \cos s\, ds + (-1)^n \int_{n\pi-\frac{\pi}{2}}^{t} \cos s\, ds \\ &= 1 + \sum_{k=1}^{n-1}(-1)^k\left(\sin\left(k\pi - \frac{\pi}{2}\right) - \sin\left(k\pi + \frac{\pi}{2}\right)\right) + (-1)^n\left(\sin t - \sin\left(n\pi - \frac{\pi}{2}\right)\right) \end{aligned}$$

$$
\begin{aligned}
&= 1 + \sum_{k=1}^{n-1}\left((-1)^{2k} - (-1)^{2k-1}\right) + (-1)^n\left(\sin\left(n\pi - \frac{\pi}{2} + u\right) - (-1)^{n-1}\right) \\
&= (2n - \cos u) = \frac{2}{\pi}\left(n\pi - \frac{\pi}{2}\right) + 1 - \cos u \\
&\le \frac{2}{\pi}t + (1 - \cos u), \quad 0 \le u < \pi. \qquad (5.2)
\end{aligned}
$$

Since $\hat{\theta}(s) = \frac{3}{4}|\cos s| + \max_{1\le i\le 2}\{\sum_{j=1}^{2}(\hat{\alpha}_{ij}^2(s) + \hat{\beta}_{ij}(s)e^{\lambda\tau}) + e^{0.54s}\hat{r}_i(s)\} \triangleq \frac{3}{4}|\cos s| + \hat{\theta}^*(s)$ and $\hat{\alpha}_{ij}^2, \hat{\beta}_{ij} \in L^1[R_+, R_+]$, $e^{0.54s}\hat{r}_i(s) = 4e^{-1.46s} \in L^1[R_+, R_+]$, $i, j = 1, 2$, we easily know $\hat{\theta}^*(s) \in L^1[R_+, R_+]$. Combined with (5.2), we obtain

$$
\begin{aligned}
e^{\int_0^t \hat{\theta}(s)\,ds} &= e^{\int_0^t \hat{\theta}^*(s)\,ds} e^{\frac{3}{4}\int_0^t |\cos s|\,ds} \\
&\le e^{\int_0^t \hat{\theta}^*(s)\,ds} e^{\frac{3}{2\pi}t} e^{\frac{3}{4}(1-\cos u)} \\
&\le M e^{\frac{3}{2\pi}t}, \qquad (5.3)
\end{aligned}
$$

where $M \ge 1$ is a constant.

Thus, from Corollary 4.2, we know the zero solution $x^* = 0$ of (5.1) is exponentially 2-stable (see Figure 1), the exponential convergent rate is no less than $0.54 - \frac{3}{2\pi} = 0.0625$.

Remark 5.1 Apparently, $-(P(t) + Q(t))$ is not a nonsingular $\mathcal{M}$-matrix for all $t \ge 0$, and $h_i(t) = 2e^{-2t} \equiv 0$ is not true for all $t \ge 0$, thus the results in [23, 24] are invalid for (5.1). In addition, the delay functions $\tau_{ij}(t)$ do not satisfy $\sup_{t\ge 0}\dot{\tau}_{ij}(t) < 1$, therefore, when model (5.1) is autonomous, the results in [14, 21] are invalid for it.

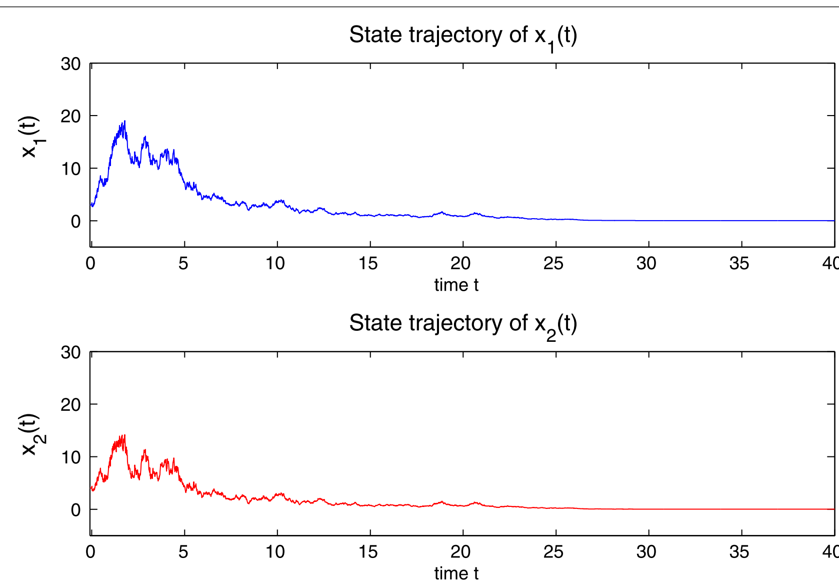

Figure 1 The state trajectories of model (5.1) without impulses.

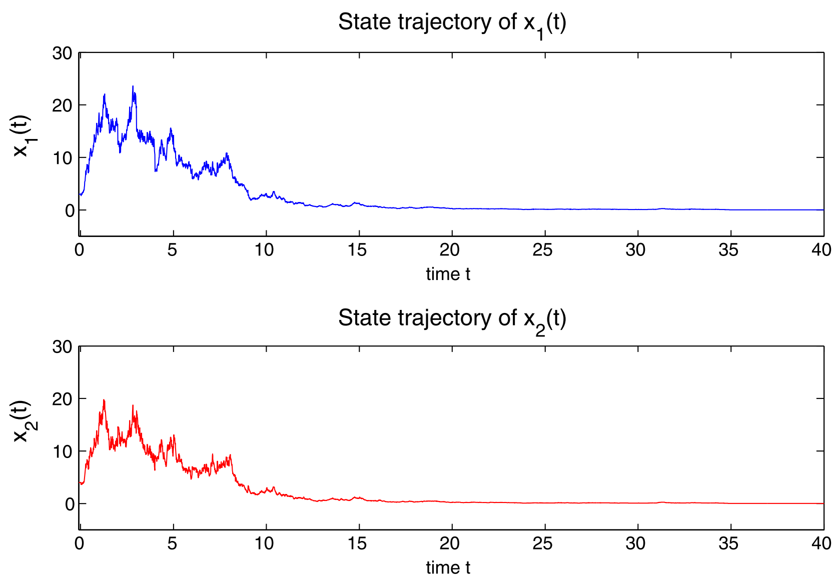

Figure 2 The state trajectories of model (5.1) with impulses (5.2).

Case 2. If

$$\begin{aligned} x_1(t_k) &= I_{1k}\big(x(t_k^-)\big) = 0.2e^{0.02}x_1(t_k^-) + 0.8e^{0.02}x_2(t_k^-), \\ x_2(t_k) &= I_{2k}\big(x(t_k^-)\big) = 0.6e^{0.02}x_1(t_k^-) + 0.4e^{0.02}x_2(t_k^-), \quad t_k - t_{k-1} = 1, k \in \mathbb{N}, \end{aligned} \tag{5.4}$$

then we can get the following parameters of (A_5), (A_6), (A_8):

$$\hat{R}_k = e^{0.04}\begin{pmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{pmatrix}, \qquad \rho(\hat{R}_k) = e^{0.04}, \qquad \Omega_\rho(\hat{R}_k) = \big\{(z_1,z_2) > 0 | z_1 = z_2\big\}.$$

Therefore, $\Omega = \bigcap_{k=1}^{\infty}(\Omega_\rho(\hat{R}_k)) \cap \Omega_{\mathcal{M}}(\widehat{\Pi}) = \{(z_1,z_2) > 0 | z_1 = z_2\}$ is not empty. Let $z = (1,1)^T \in \Omega$ and $\gamma_k = e^{0.04}$, we can obtain for $k \in \mathbb{N}$

$$\frac{\ln \gamma_k}{t_k - t_{k-1}} = \frac{\ln e^{0.04}}{1} = 0.04 = \gamma < \lambda - \mu = 0.0625.$$

From Corollary 4.1, we know the zero solution to (5.1) with impulses (5.4) is exponentially 2-stable (see Figure 2).

6 Conclusion

In this paper, we have analyzed the stochastic non-autonomous impulsive cellular neural networks with delays. Based on the properties of $\mathcal{L}$-operator and $\mathcal{M}$-matrix, we have developed a new inequality of $\mathcal{L}$-operator. We have applied the new inequality to stochastic non-autonomous neural networks and derived the sufficient conditions for the pth moment exponential stability of the considered system without impulses or with impulses. Our results do not require differentiability of the delay functions and have relaxed the conditions imposed on the diffusion coefficient matrix. The results obtained have generalized some early works.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors have made equal and significant contributions in writing this paper. All authors read and approved the final manuscript.

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