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Exponential stability of a class of networked control systems with disturbed controllers

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Abstract

This paper studies the exponential stability problem for a class of networked control systems (NCSs) with time delays and packet dropouts. By considering the disturbed state-feedback controller, the closed-loop NCS is modeled as a new discrete-time switched system. A sufficient condition is established for the exponential stability of the NCS under a packet-dropout rate. The state-feedback controller gain is obtained through the cone complementarity linearization approach. A numerical example is provided to show the effectiveness of the proposed method.

Keywords: networked control system; time delay; packet dropout; exponential stability; disturbed controller

1 Introduction

With the fast development of network technology, the network is being applied to the control field by researchers. Networked control systems (NCSs) whose control loops are connected via communication networks offer many advantages such as low cost of installation, ease of maintenance, high resource utilization, simple installation. Hence, NCSs have received increasing attention [1–3].

In the meantime, the introduction of network makes the analysis and design of system complexity. Due to the sampling data transmission through the network, the time delay and packet dropout are always inevitable, which often cause deterioration of system performance and instability of system. It is well known that the stability is one of the most important problems in the controller design. Therefore, the stability of systems has attracted much research interest [4–9]. It should be pointed out that the results in the aforesaid references do not apply to the stability of NCS.

Compared to the NCSs with only packet dropouts or time delays [10–12], it is more difficult to analyze the NCSs with both time delays and packet dropouts. When considering the packet-dropout problem, it is significant to establish the quantitative relation between the packet-dropout rate and the stability of the NCS. On this topic, only a few results have been presented [13–16]. In [13], the plant considered is a discrete-time one, therefore, the result of the paper does not be applied to the NCS when the plant is a continuous-time system. In [14, 15], the NCS with a continuous-time plant is studied. It should be pointed out that the delay is a constant or takes values in a finite set. However, the results are invalid

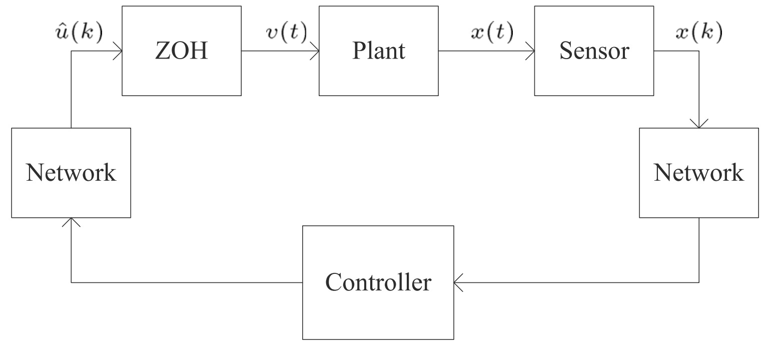

Figure 1 The structure of the NCS.

when one encounters infinite possible values for the delay. In [16], the delay considered can be an arbitrary value in a finite interval. A sufficient condition is obtained for the exponential stability of the closed-loop NCS with the plant being a continuous-time one. It is worthwhile noting that the controller gain is unchanged.

In practice, due to interference (for example, machine aging and measurement error), there is a certain change in the controller parameters. This change may destroy the stable performance of the closed-loop systems. At this time, if considering the constant controller gain in the analysis of the system, the system may exhibit a high degree of vulnerability, which motivates the present research.

This paper studies a class of NCSs with disturbed controllers. The delay can arbitrarily take values in a finite interval which is smaller than the sampling period. A new discrete-time switched NCS model is proposed. A sufficient condition is obtained for the exponential stability of the closed-loop NCS under the maximum packet-dropout rate. The discrete-time feedback controller gain is derived by solving a set of linear matrix inequalities with inversion constraints. A numerical example verifies the developed theory.

2 Model for networked control system

The structure of the NCS is shown in Figure 1. The plant is a continuous-time linear system described by

$$\dot{x} = A_p x(t) + B_p v(t), \tag{1}$$

where $x(t) \in R^n$ is the state, $v(t) \in R^m$ is the plant input, A_p and B_p are constant matrices of appropriate dimensions.

In the NCS, the discrete-time state-feedback controller is event-driven, the sensor is time-driven and the sampling period is T. The zeroth-order hold device does not update the output value until the new value arrives. The network-induced delay τ_k satisfies $0 \leq \tau_{\min} \leq \tau_k \leq \tau_{\max} < T$. $x(k)$ is the value of $x(t)$ at the sampling instant kT.

The output value of the disturbed state-feedback controller corresponding to $x(k)$ is denoted by $u(k)$,

$$\begin{aligned} u(k) &:= \widetilde{K}(k)x(k), \\ \widetilde{K}(k) &= K + \triangle(k), \end{aligned} \tag{2}$$

where K is the designed feedback gain and $\triangle(k)$ is the controller gain perturbation that satisfies

$$\triangle^{\mathrm{T}}(k)\triangle(k) \le \delta^2 I.$$

By considering the network-induced delay, the plant input is

$$v(t) = \begin{cases} \hat{u}(k-1), & \text{if } kT < t \le kT + \tau_k, \\ \hat{u}(k), & \text{if } kT + \tau_k < t \le (k+1)T, \end{cases} \tag{3}$$

where

$$\hat{u}(k) = \begin{cases} u(k), & \text{if } u(k) \text{ and } x(k) \text{ is successfully transmitted}, \\ \hat{u}(k-1), & \text{if } u(k) \text{ or } x(k) \text{ is lost during transmission}. \end{cases}$$

From (3), system (1) with sampling period T is discretized to

$$x(k+1) = Ax(k) + B_1\hat{u}(k-1) + B_0\hat{u}(k), \tag{4}$$

where

$$A = \exp\{A_p T\}, \qquad B_1 = \int_{T-\tau_k}^{T} \exp\{A_p s\}\,\mathrm{d}sB_p, \qquad B_0 = \int_0^{T-\tau_k} \exp\{A_p s\}\,\mathrm{d}sB_p. \tag{5}$$

During each sampling period, two cases may arise, which can be listed as follows:

- Packet dropout happens; (4) can be written as

$$\tilde{x}(k+1) = \tilde{A}_0(k)\tilde{x}(k), \tag{6}$$

where

$$\tilde{x}(k) = \begin{bmatrix} x(k) \\ \hat{u}(k-1) \end{bmatrix}, \qquad \tilde{A}_0(k) = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix}, \quad B = \int_0^T \exp\{A_p s\}\,\mathrm{d}sB_p. \tag{7}$$

- No packet dropout happens; (4) can be written as

$$\tilde{x}(k+1) = \tilde{A}_1(k)\tilde{x}(k), \tag{8}$$

where

$$\begin{aligned} \tilde{A}_1(k) &= \begin{bmatrix} A + B_0(\tau_k)\widetilde{K}(k) & B_1(\tau_k) \\ \widetilde{K}(k) & 0 \end{bmatrix} \\ &= \begin{bmatrix} A + (B_{00} + DF(\tau_k)E)\widetilde{K}(k) & B_{11} - DF(\tau_k)E \\ \widetilde{K}(k) & 0 \end{bmatrix}, \end{aligned} \tag{9}$$

B_{00}, B_{11}, D, and E are constant matrices, $F(\tau_k)$ satisfies $F^{\mathrm{T}}(\tau_k)F(\tau_k) \le I$.

If A_p has non-zero mutually different eigenvalues $\lambda_1,\dots,\lambda_n$ and Λ is the corresponding eigenvector matrix, choose $a_1,\dots,a_n$ satisfying $\lambda_1(T-a_1-\tau_k)<0,\dots,\lambda_n(T-a_n-\tau_k)<0$, then B_{00}, B_{11}, D, E, and $F(\tau_k)$ can be represented as

$$
\begin{aligned}
E &= \Lambda^{-1}B_p,\\
B_{00} &= \Lambda\operatorname{diag}\left(-\frac{1}{\lambda_1},\dots,-\frac{1}{\lambda_n}\right)\Lambda^{-1}B_p,\\
B_{11} &= \Lambda\operatorname{diag}\left(\frac{1}{\lambda_1}\exp\{\lambda_1 T\},\dots,\frac{1}{\lambda_n}\exp\{\lambda_n T\}\right)\Lambda^{-1}B_p,\\
D &= \Lambda\operatorname{diag}\left(\frac{1}{\lambda_1}\exp\{\lambda_1 a_1\},\dots,\frac{1}{\lambda_n}\exp\{\lambda_n a_n\}\right),\\
F(\tau_k) &= \operatorname{diag}\bigl(\exp\bigl\{\lambda_1(T-a_1-\tau_k)\bigr\},\dots,\exp\bigl\{\lambda_n(T-a_n-\tau_k)\bigr\}\bigr).
\end{aligned}
$$

If A_p has zero eigenvalues or multiple eigenvalues, for example, A_p has one zero eigenvalue, one r multiple eigenvalue λ_*, and non-zero mutually different eigenvalues $\lambda_2,\dots,\lambda_{n-r}$, in this case, A_p can be represented as

$$
A_p = \Lambda\operatorname{diag}(0,J_1,J_2)\Lambda^{-1},
$$

where J_1 is a diagonal matrix corresponding to $\lambda_2,\dots,\lambda_{n-r}$, J_2 is the Jordan block corresponding to λ_*. Choose a_i, $i=1,2,\dots,n-r$, satisfying $a_1>\tau_k$, $\lambda_l(T-\tau_k-a_l)<0$, $l=2,3,\dots,n-r$. Then we have

$$
\begin{aligned}
B_{00} &= \Lambda\operatorname{diag}(T,\tilde{J}_1,\tilde{J}_2)\Lambda^{-1}B_p,\\
B_{11} &= \Lambda\operatorname{diag}(0,\hat{J}_1,\hat{J}_2)\Lambda^{-1}B_p,\\
D &= \Lambda\operatorname{diag}\left(a_1,\frac{1}{\lambda_2}\exp\{\lambda_2 a_2\},\dots,\frac{1}{\lambda_{n-r}}\exp\{\lambda_{n-r}a_{n-r}\},P_2\right),\\
F(\tau_k) &= \operatorname{diag}\left(-\frac{\tau_k}{a_1},\exp\bigl\{\lambda_2(T-a_2-\tau_k)\bigr\},\dots,\exp\bigl\{\lambda_{n-r}(T-a_{n-r}-\tau_k)\bigr\},P_2^{-1}\bar{J}_2\right),
\end{aligned}
$$

where P_2 is a invertible diagonal matrix and satisfies $\|P_2^{-1}\bar{J}_2\|<1$, and

$$
\begin{aligned}
\tilde{J}_1 &= \begin{bmatrix} -\frac{1}{\lambda_2} & & \\ & \ddots & \\ & & -\frac{1}{\lambda_{n-r}} \end{bmatrix},\\
\hat{J}_1 &= \begin{bmatrix} -\frac{1}{\lambda_2}\exp\{\lambda_2 T\} & & \\ & \ddots & \\ & & -\frac{1}{\lambda_{n-r}}\exp\{\lambda_{n-r}T\} \end{bmatrix}.
\end{aligned}
$$

When $\lambda_* \neq 0$,

$$\hat{J}_2 = \begin{bmatrix} \frac{1}{\lambda_*}\exp\{\lambda_* T\} & & & \\ \frac{1}{\lambda_*^2}(\lambda_* T - 1)\exp\{\lambda_* T\} & \ddots & & \\ \vdots & \ddots & & \\ \frac{1}{\lambda_*^r}\sum_{k=1}^{r}(-1)^{k-1}\frac{(\lambda_* T)^{r-k}}{(r-k)!}\exp\{\lambda_* T\} & \cdots & \frac{1}{\lambda_*^2}(\lambda_* T - 1)\exp\{\lambda_* T\} & \frac{1}{\lambda_*}\exp\{\lambda_* T\} \end{bmatrix},$$

$$\bar{J}_2 = \begin{bmatrix} \frac{1}{\lambda_*}\exp\{x\} & & & \\ \frac{1}{\lambda_*^2}(x - 1)\exp\{x\} & \ddots & & \\ \vdots & \ddots & & \\ \frac{1}{\lambda_*^r}\sum_{k=1}^{r}(-1)^{k-1}\frac{x^{r-k}}{(r-k)!}\exp\{x\} & \cdots & \frac{1}{\lambda_*^2}(x - 1)\exp\{\lambda_* x\} & \frac{1}{\lambda_*}\exp\{x\} \end{bmatrix},$$

$$x = \lambda_*(T - \tau_k),$$

$$\tilde{J}_2 = \begin{bmatrix} -\frac{1}{\lambda_*} & & & \\ -\frac{1}{\lambda_*^2} & \ddots & & \\ \vdots & \ddots & & \\ (-1)^r\frac{1}{\lambda_*^r} & \cdots & -\frac{1}{\lambda_*^2} & -\frac{1}{\lambda_*} \end{bmatrix}.$$

When $\lambda_* = 0$,

$$\hat{J}_2 = \begin{bmatrix} 0 & \cdots & & 0 \\ \frac{T^2}{2} & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \\ \frac{T^r}{r!} & \cdots & \frac{T^2}{2} & 0 \end{bmatrix},$$

$$\bar{J}_2 = \begin{bmatrix} -\tau_k & & & \\ \frac{(T-\tau_k)^2}{2} & & \ddots & \\ \vdots & \ddots & & \\ \frac{(T-\tau_k)^r}{r!} & \cdots & \frac{(T-\tau_k)^2}{2} & -\tau_k \end{bmatrix},$$

$$\tilde{J} = \operatorname{diag}(T, T, \ldots, T).$$

Combining (6) and (8) one obtains the following discrete-time switched system model:

$$\tilde{x}(k+1) = \tilde{A}_{\sigma(k)}(k)\tilde{x}(k). \tag{10}$$

$\sigma(k)$ is called a switching signal. $\sigma(k) = 1$ implies there is no packet dropout, while $\sigma(k) = 0$ implies packet dropout.

3 Exponential stability analysis

Lemma 1 [17] *For constant matrices M, N, a symmetric matrix W, and scalar $\varepsilon > 0$, the following inequality holds*:

$$W + MFN + N^{\mathrm{T}}F^{\mathrm{T}}M^{\mathrm{T}} < 0,$$

where F satisfies $F^{\mathrm{T}}F \le I$, if and only if that there exists a matrix $\varepsilon > 0$

$$W + \varepsilon MM^{\mathrm{T}} + \varepsilon^{-1}N^{\mathrm{T}}N < 0.$$

Lemma 2 [18] *For the system* (10), *if there exist a Lyapunov function $V(\tilde{x}(k)) = \tilde{x}(k)^{\mathrm{T}}\tilde{P}\tilde{x}(k)$ and positive scalars α_0, α_1, such that*

$$\alpha_0^{r_0}\alpha_1^{1-r_0} > 1, \tag{11}$$

$$\begin{bmatrix} -\tilde{P}^{-1} & \alpha_0\tilde{A}_0 \\ * & -\tilde{P} \end{bmatrix} \le 0, \tag{12}$$

$$\begin{bmatrix} -\tilde{P}^{-1} & \alpha_1\tilde{A}_1 \\ * & -\tilde{P} \end{bmatrix} \le 0, \tag{13}$$

then the system is exponentially stable with the packet-dropout rate $r \le r_0$.

Theorem 3.1 *For given positive scalars r_0, α_0, α_1, if there exist positive definite matrices P, Q, such that*

$$\alpha_0^{r_0}\alpha_1^{1-r_0} > 1, \tag{14}$$

$$\begin{bmatrix} -P^{-1} & 0 & \alpha_0 A & \alpha_0 B \\ * & -Q^{-1} & 0 & \alpha_0 I \\ * & * & -P & 0 \\ * & * & * & -Q \end{bmatrix} \le 0, \tag{15}$$

$$\begin{bmatrix} -P^{-1} & 0 & \alpha_1 A + \alpha_1 B_{00}K & \alpha_1 B_{11} & \varepsilon_1\alpha_1 D & 0 & \varepsilon_2\sqrt{\delta}\alpha_1 B_{00} & 0 & 0 & 0 \\ * & -Q^{-1} & \alpha_1 K & 0 & 0 & 0 & \varepsilon_2\sqrt{\delta}\alpha_1 I & 0 & 0 & 0 \\ * & * & -P & 0 & 0 & K^{\mathrm{T}}E^{\mathrm{T}} & 0 & \sqrt{\delta}I & \varepsilon_3\sqrt{\delta}I & 0 \\ * & * & * & -Q & 0 & -E^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 & E \\ * & * & * & * & * & * & -\varepsilon_2 I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_3 I \end{bmatrix} < 0, \tag{16}$$

then the system is exponentially stable with the packet-dropout rate $r \le r_0$.

Proof For the system (10), construct the Lyapunov function:

$$V(k) = \tilde{x}^{\mathrm{T}}(k)\tilde{P}\tilde{x}(k)$$

$$\tilde{P} = \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix},$$

where P, Q are positive definite matrices.

Then (12) can be written as (15) and (13) can be written as follows:

$$\begin{bmatrix} -P^{-1} & 0 & \alpha_1 A + \alpha_1(B_{00} + DF(\tau_k)E)\widetilde{K}(k) & \alpha_1 B_{11} - \alpha_1 DF(\tau_k)E) \\ * & -Q^{-1} & \alpha_1\widetilde{K}(k) & 0 \\ * & * & -P & 0 \\ * & * & * & -Q \end{bmatrix} \le 0, \tag{17}$$

which can be described as

$$\begin{bmatrix} -P^{-1} & 0 & \alpha_1 A + \alpha_1 B_{00}\widetilde{K}(k) & \alpha_1 B_{11} \\ * & -Q^{-1} & \alpha_1 \widetilde{K}(k) & 0 \\ * & * & -P & 0 \\ * & * & * & -Q \end{bmatrix} + \begin{bmatrix} \alpha_1 D \\ 0 \\ 0 \\ 0 \end{bmatrix} F(\tau_k) \begin{bmatrix} 0 \\ 0 \\ \widetilde{K}(k)^{\mathrm{T}} E^{\mathrm{T}} \\ -E^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$$
$$+ \begin{bmatrix} 0 \\ 0 \\ \widetilde{K}(k)^{\mathrm{T}} E^{\mathrm{T}} \\ -E^{\mathrm{T}} \end{bmatrix} F(\tau_k)^{\mathrm{T}} \begin{bmatrix} \alpha_1 D \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}} \leq 0. \tag{18}$$

From Lemma 1, inequality (18) is true if and only if there exists a scalar $\varepsilon_1 > 0$ such that the following inequality holds:

$$\begin{bmatrix} -P^{-1} & 0 & \alpha_1 A + \alpha_1 B_{00}\widetilde{K}(k) & \alpha_1 B_{11} \\ * & -Q^{-1} & \alpha_1 \widetilde{K}(k) & 0 \\ * & * & -P & 0 \\ * & * & * & -Q \end{bmatrix} + \varepsilon_1 \begin{bmatrix} \alpha_1 D \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \alpha_1 D \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}}$$
$$+ \varepsilon_1^{-1} \begin{bmatrix} 0 \\ 0 \\ \widetilde{K}(k)^{\mathrm{T}} E^{\mathrm{T}} \\ -E^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \widetilde{K}(k)^{\mathrm{T}} E^{\mathrm{T}} \\ -E^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \leq 0. \tag{19}$$

It then follows from the Schur complement that inequality (19) is equivalent to the following matrix inequality:

$$\begin{bmatrix} -P^{-1} & 0 & \alpha_1 A + \alpha_1 B_{00}\widetilde{K}(k) & \alpha_1 B_{11} & \varepsilon_1 \alpha_1 D & 0 \\ * & -Q^{-1} & \alpha_1 \widetilde{K}(k) & 0 & 0 & 0 \\ * & * & -P & 0 & 0 & \widetilde{K}(k)^{\mathrm{T}} E^{\mathrm{T}} \\ * & * & * & -Q & 0 & -E^{\mathrm{T}} \\ * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & -\varepsilon_1 I \end{bmatrix} < 0,$$

which can be written as

$$\begin{bmatrix} -P^{-1} & 0 & \alpha_1 A + \alpha_1 B_{00} K & \alpha_1 B_{11} & \varepsilon_1 \alpha_1 D & 0 \\ * & -Q^{-1} & \alpha_1 K & 0 & 0 & 0 \\ * & * & -P & 0 & 0 & K^{\mathrm{T}} E^{\mathrm{T}} \\ * & * & * & -Q & 0 & -E^{\mathrm{T}} \\ * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & -\varepsilon_1 I \end{bmatrix} + \begin{bmatrix} \alpha_1 B_{00} \\ \alpha_1 I \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \triangle(k) \begin{bmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}}$$

$$\times \begin{bmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \\ 0 \end{bmatrix} \triangle^{\mathrm{T}}(k) \begin{bmatrix} \alpha_1 B_{00} \\ \alpha_1 I \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \\ 0 \end{bmatrix} \triangle^{\mathrm{T}}(k) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E \end{bmatrix} \triangle(k) \begin{bmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq 0. \tag{20}$$

From Lemma 1, inequality (20) is true if and only if there exists a scalar $\varepsilon_2 > 0$, $\varepsilon_3 > 0$ such that the following inequality holds:

$$\begin{bmatrix} -P^{-1} & 0 & \alpha_1 A + \alpha_1 B_{00} K & \alpha_1 B_{11} & \varepsilon_1 \alpha_1 D & 0 \\ * & -Q^{-1} & \alpha_1 K & 0 & 0 & 0 \\ * & * & -P & 0 & 0 & K^{\mathrm{T}} E^{\mathrm{T}} \\ * & * & * & -Q & 0 & -E^{\mathrm{T}} \\ * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & -\varepsilon_1 I \end{bmatrix}$$

$$+ \varepsilon_2 \sqrt{\delta} \begin{bmatrix} \alpha_1 B_{00} \\ \alpha_1 I \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \sqrt{\delta} \begin{bmatrix} \alpha_1 B_{00} \\ \alpha_1 I \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}} + \varepsilon_2^{-1} \sqrt{\delta} \begin{bmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \\ 0 \end{bmatrix} \sqrt{\delta} \begin{bmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}}$$

$$+ \varepsilon_3 \sqrt{\delta} \begin{bmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \\ 0 \end{bmatrix} \sqrt{\delta} \begin{bmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}} + \varepsilon_3^{-1} \sqrt{\delta} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E \end{bmatrix} \sqrt{\delta} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E \end{bmatrix}^{\mathrm{T}} \leq 0. \tag{21}$$

From the Schur complement, we see that (21) is equivalent to (16).

From Lemma 2, we see that if (14), (15), and (16) hold, then the system (10) has exponential stability.

The conditions in Theorem 3.1 are a set of LMIs with inversion constraints. K can be solved by an iterative LMI approach which is called cone complementarity linearization [19, 20]. □

4 Numerical example

Consider the following system from [16]:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} v(t). \tag{22}$$

Choose the sampling period $T = 0.3$ s, and suppose $0 \leq \tau_k \leq 0.1$ s, $r_0 = 0.1$, $\delta = 0.1$. Using Theorem 3.1, we get

$$K = [-2.8706 \quad -11.5619].$$

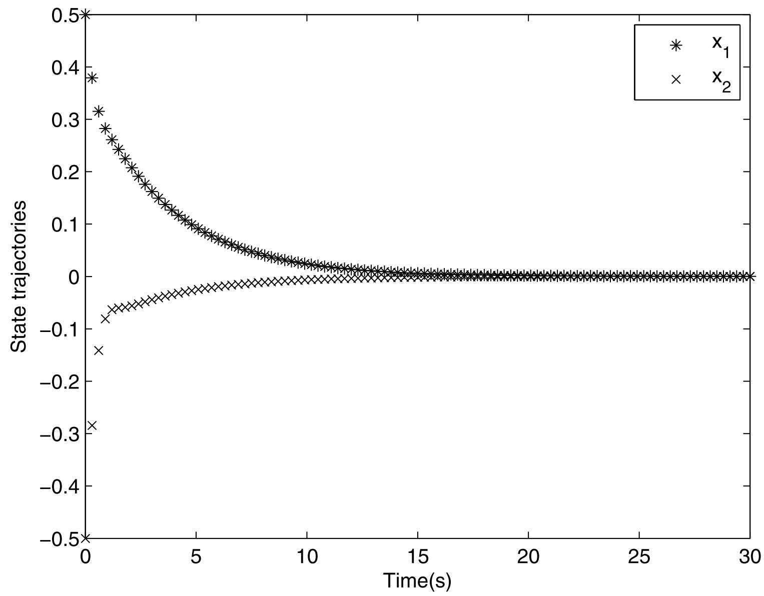

Figure 2 State trajectories of the NCS.

Suppose that the initial condition is $x^{\mathrm{T}}(0) = [0.5\ \ -0.5]$. Figure 2 shows the state trajectories of the NCS. We can see that the networked control system is exponentially stable.

In [16], the designed state-feedback gain is unchanged. When the feedback controller is disturbed, it cannot guarantee the system stability. However, from Theorem 3.1, the designed-state feedback gain subject to a certain additive interference still can make the system stable.

5 Conclusions

In this paper, a new discrete-time switched NCSs model that can deal simultaneously with packet dropout and time delay is presented. The criterion for the exponential stability of the system is derived. The gain of the disturbed state-feedback controller can be solved by the proposed method.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Both authors contributed equally to this work and read and approved the final manuscript.

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