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Absolute ruin problems in a compound Poisson risk model with constant dividend barrier and liquid reserves

Dan Peng[1*], Donghai Liu[1] and Zhenting Hou[2]

*Correspondence: danpengdanpeng@126.com
[1]School of Mathematics and Computational Science, Hunan University of Science and Technology, Xiangtan, Hunan 411201, P.R. China
Full list of author information is available at the end of the article

Abstract

In this paper, we consider a compound Poisson surplus model with constant dividend barrier and liquid reserves under absolute ruin. When the surplus is negative, the insurer is allowed to borrow money at a debit interest rate to continue the business; when the surplus is below a fixed level $\triangle$, the surplus is kept as liquid reserves, which do not earn interest; when the surplus attains the level $\triangle$, the excess of the surplus over the level receives interest at a constant rate; when the surplus reaches a higher level b, the excess of the surplus above b is all paid out as dividends to shareholders of the insurer. We first derive the integro-differential equations satisfied by the moment-generating function and moment of the discounted dividend payments until absolute ruin. Then, applying these results, we get explicit expressions of them for exponential claims and discuss the impact of the model parameters on the expected dividend payments by numerical examples.

MSC: 91B30

Keywords: absolute ruin; dividend payments; liquid reserve; moment-generating function; interest

1 Introduction

In the classical compound Poisson surplus model, $U(t)$ is given by

$$U(t) = u + ct - S(t) = u + ct - \sum_{i=1}^{N(t)} X_i, \quad t \geq 0,$$

where $U(0) = u$ is the initial surplus, $c > 0$ is the premium rate, $\{N(t), t \geq 0\}$ is a Poisson process with intensity $\lambda > 0$, which denotes the claim numbers in the interval $[0, t]$, and $\{X_i, i \geq 1\}$ (representing the sizes of claims and independent of $\{N(t), t \geq 0\}$) is a sequence of independent and identically distributed nonnegative random variables with common distribution function $F(x) = 1 - \overline{F}(x)$, which satisfies $F(0) = 0$ and has a mean $\mu = \int_0^\infty \overline{F}(x)\,dx > 0$.

In the recent study of risk theory, the classical compound Poisson surplus model has been modified to adopt economic and financial factors such as interest and dividends. The feature of debit interest assumes that the insurer is allowed to borrow money at a

debit interest rate $\beta > 0$ to pay claims when the surplus turns negative. As the insurer pays the debts from its premium income, the negative surplus may return to a positive level. When the premium income is not enough to pay the debit interest (that is, the surplus falls below $-\frac{c}{\beta}$), the absolute ruin is said to occur. In recent years, the issue of absolute ruin has received considerable attentions in the actuarial literature. See, for example, Embrechts and Schmidli [1], Cai [2], Yuen *et al.* [3], Wang *et al.* [4], and Yin and Wang [5]. For example, Yin and Wang [5] study absolute ruin questions for the perturbed compound Poisson risk process with investment and debit interests by the expected discounted penalty function at absolute ruin. On the other hand, even if an insurer invests all his positive surplus into a risk-free asset, in certain condition, only the excess of the surplus over a certain level can receive interest. To adopt a more flexible and tractable model, Embrechts and Schmidli [1] investigated the absolute ruin probability for a more complicated risk model. They assumed that the company can borrow money when the surplus is negative and receive interest for capital above a certain level. Furthermore, Cai *et al.* [6] considered the following special model of Embrechts and Schmidli [1]:

$$\begin{cases} dU(t) = c\,dt + r(U(t) - \triangle)\,dt - dS(t), & U(t) \geq \triangle, \\ dU(t) = a\,dt - dS(t), & 0 < U(t) < \triangle. \end{cases} \tag{1.1}$$

In (1.1), an insurer's surplus is below a certain level $\triangle > 0$ and is kept as liquid reserves. As the surplus attains the level $\triangle$, the excess of the surplus above $\triangle$ will earn interest at a constant interest force $r > 0$. They studied the Gerber-Shiu function and discussed the impact of interest and liquid reserves on the ruin probability.

On the other hand, the surplus of the insurer with a certain dividend strategy has also been receiving more and more attention, including [1, 7–9]. For instance, de Finetti [10] studied the dividend strategy in a discrete process. Lin *et al.* [9] investigated the classical risk model with constant dividend barrier and analyzed the Gerber-Shiu discounted penalty function at ruin. Albrecher *et al.* [11] considered the distribution of dividend payments in the Sparre Andersen model with constant dividend barrier. Cai *et al.* [12] considered a more general model that incorporates the notion of threshold strategy. Based on the model (1.1), they assume that if the surplus continues to surpass a higher level $b \geq \triangle$, then the excess of the surplus above b is paid out as dividends to the insurer's shareholders at a constant dividend rate, and no interest is earned on the surplus over the threshold level b, and they discuss the interactions of the liquid reserve level, the interest rate, and the threshold level in the proposed risk model by studying the expected discounted penalty function and the expected present value of dividends paid up to the time of ruin. More specifically, they assume that the portion of the surplus is below a present level $\triangle$ is liquid, and the amount in excess of this level is invested under a deterministic interest rate. Instead of implementing a threshold in Cai *et al.* [12], Sendova and Zhang [13] consider a percentage of the current surplus of the insurer and also study the expected discounted penalty function at ruin.

Motivated by these works, based on the model (1.1), we consider a more general model that incorporates the dividend strategy and debit interest. We assume that if the surplus is negative, then the insurer can borrow money at a debit interest $\beta > r$. If the surplus surpass a higher level $b \geq \triangle$, then the excess of the surplus above b is paid out as dividends to the

shareholders at a constant dividend rate $c + r(b - \triangle)$. The resulting surplus process $U_b(t)$ can be described by

$$dU_b(t) = \begin{cases} -dS(t), & U_b(t) > b, \\ (c + r(U_b(t) - \triangle))\,dt - dS(t), & \triangle \le U_b(t) \le b, \\ c\,dt - dS(t), & 0 \le U_b(t) < \triangle, \\ (c + \beta U_b(t))\,dt - dS(t), & -\frac{c}{\beta} < U_b(t) < 0, \end{cases} \tag{1.2}$$

where $U_b(0) = u$ is the initial surplus, b is the constant level of dividend barrier, β is the debit interest rate, r is the credit interest, c is the premium rate, and $S(t) = \sum_{i=1}^{N(t)} X_i$ is the aggregate Poisson claim-amount process.

Define $T_u^b = \inf\{t : U_b(t) \le -\frac{c}{\beta}\}$ as the time of absolute ruin ($T_u^b = \infty$ if $U_b(t) > -\frac{c}{\beta}$ for all $t > 0$). Let $D(t)$ be the cumulative amount of dividends up to time t, and $\alpha > 0$ be the force of interest. Then

$$D_{u,b} = \int_0^{T_u^b} e^{-\alpha t}\,dD(t)$$

is the present value of all dividends until T_u^b.

In the sequel, we consider the moment-generating function

$$M(u,y,b) = E\bigl[e^{yD_{u,b}}\bigr], \quad -\frac{c}{\beta} < u \le b,$$

where y is such that $M(u,y,b)$ exists. We denote the nth moment of the discounted dividends by

$$V_n(u,b) = E\bigl[D_{u,b}^n\bigr], \quad -\frac{c}{\beta} < u \le b, n \in N.$$

Note that $V_0(u,b) \equiv 1$ and, when $n = 1$, $V_1(u,b) = V(u,b)$ is the expectation of $D_{u,b}$. We will always assume that $M(u,y,b)$ and $V_n(u,b)$ are sufficiently smooth functions in u and y, respectively.

The rest of the paper is organized as follows. In Section 2, we get the integro-differential equations for the moment-generating function and the nth moment of the discounted dividends. In Section 3, we find their explicit expressions for exponential claims and discuss the impact of the model parameters on the expected dividend payments by numerical examples.

2 Integro-differential equations

In this section, we study the moment-generating function $M(u,y,b)$, which has been discussed in various surplus processes; for example, see Albrecher *et al.* [11], Cheung (2008) *etc.* Similarly, we can analyze the moments of $D(u,b)$ through $M(u,y,b)$; since $M(u,y,b)$ has different paths for $-\frac{c}{\beta} < u \le b$, we define

$$M(u,y,b) = \begin{cases} M_1(u,y,b), & -\frac{c}{\beta} < u < 0, \\ M_2(u,y,b), & 0 \le u < \triangle, \\ M_3(u,y,b), & \triangle \le u \le b. \end{cases}$$

Theorem 2.1 *$M_1(u,y,b)$, $M_2(u,y,b)$, and $M_3(u,y,b)$ satisfy the following system of integro-differential equations*:

$$\begin{aligned}&(\beta u+c)\frac{\partial M_1}{\partial u}(u,y,b)-\alpha y\frac{\partial M_1}{\partial y}(u,y,b)-\lambda M_1(u,y,b)+\lambda\overline{F}\left(u+\frac{c}{\beta}\right)\\&\quad+\lambda\int_0^{u+\frac{c}{\beta}}M_1(u-x,y,b)\,dF(x)=0,\quad -\frac{c}{\beta}<u<0,\end{aligned}\tag{2.1}$$

$$\begin{aligned}&c\frac{\partial M_2}{\partial u}(u,y,b)-\alpha y\frac{\partial M_2}{\partial y}(u,y,b)-\lambda M_2(u,y,b)+\lambda\int_0^{u}M_2(u-x,y,b)\,dF(x)\\&\quad+\lambda\int_u^{u+\frac{c}{\beta}}M_1(u-x,y,b)\,dF(x)+\lambda\overline{F}\left(u+\frac{c}{\beta}\right)=0,\quad 0\le u<\triangle,\end{aligned}\tag{2.2}$$

$$\begin{aligned}&\bigl[r(u-\triangle)+c\bigr]\frac{\partial M_3}{\partial u}(u,y,b)-\alpha y\frac{\partial M_3}{\partial y}(u,y,b)-\lambda M_3(u,y,b)\\&\quad+\lambda\int_0^{u-\triangle}M_3(u-x,y,b)\,dF(x)+\lambda\int_{u-\triangle}^{u}M_2(u-x,y,b)\,dF(x)\\&\quad+\lambda\int_u^{u+\frac{c}{\beta}}M_1(u-x,y,b)\,dF(x)+\lambda\overline{F}\left(u+\frac{c}{\beta}\right)=0,\quad \triangle\le u\le b,\end{aligned}\tag{2.3}$$

with boundary conditions:

$$M_1\left(-\frac{c}{\beta},y,b\right)=1,\tag{2.4}$$

$$\left.\frac{\partial M_3(u,y,b)}{\partial u}\right|_{u=b}=yM_3(b,y,b),\tag{2.5}$$

$$M_1(0-,y,b)=M_2(0,y,b),\qquad M_2(\triangle-,y,b)=M_3(\triangle,y,b),\tag{2.6}$$

$$\begin{aligned}&\left.\left(c\frac{\partial M_1(u,y,b)}{\partial u}-\alpha y\frac{\partial M_1(u,y,b)}{\partial y}\right)\right|_{u=0-}\\&\quad=\left.\left(c\frac{\partial M_2(u,y,b)}{\partial u}-\alpha y\frac{\partial M_2(u,y,b)}{\partial y}\right)\right|_{u=0},\\&\left.\left(c\frac{\partial M_2(u,y,b)}{\partial u}-\alpha y\frac{\partial M_2(u,y,b)}{\partial y}\right)\right|_{u=\triangle-}\\&\quad=\left.\left(c\frac{\partial M_3(u,y,b)}{\partial u}-\alpha y\frac{\partial M_2(u,y,b)}{\partial y}\right)\right|_{u=\triangle}.\end{aligned}\tag{2.7}$$

Proof When $-\frac{c}{\beta}<u<0$, we consider the infinitesimal time from 0 to t, and three distinct events can happen: no claim in $(0,t)$, a claim in $(0,t)$ without occurring ruin, a claim in $(0,t)$ with occurring ruin. Conditioning on the time and amount of the first claim, we obtain that

$$\begin{aligned}M_1(u,y,b)&=(1-\lambda t)M_1\left(ue^{\beta t}+c\frac{e^{\beta t}-1}{\beta},ye^{-\alpha t},b\right)+\lambda t\int_{ue^{\beta t}+c\frac{e^{\beta t}-1}{\beta}+\frac{c}{\beta}}^{\infty}dF(x)\\&\quad+\lambda t\int_0^{ue^{\beta t}+c\frac{e^{\beta t}-1}{\beta}+\frac{c}{\beta}}M_1\left(ue^{\beta t}+c\frac{e^{\beta t}-1}{\beta}-x,ye^{-\alpha t},b\right)dF(x)+o(t).\end{aligned}\tag{2.8}$$

Letting $h_\beta(t,u) = ue^{\beta t} + c\frac{e^{\beta t}-1}{\beta} - u = (c+\beta u)\frac{e^{\beta t}-1}{\beta}$, we observe that $h_\beta(t,u) \to 0$ as $t \to 0$. By Taylor expansion we have

$$\begin{aligned} &M_1\left(ue^{\beta t} + c\frac{e^{\beta t}-1}{\beta}, ye^{-\alpha t}, b\right) \\ &\quad = M_1(u,y,b) + (\beta u + c)t\frac{\partial M_1(u,y,b)}{\partial u} - \alpha yt\frac{\partial M_1(u,y,b)}{\partial y} + o(t). \end{aligned}$$

Substituting this expression into (2.8), dividing both sides of (2.8) by t, and letting $t \to 0$, we get (2.1). Similarly, we obtain (2.2) and (2.3).

When $u = -\frac{c}{\beta}$, absolute ruin is immediate, namely, no dividend is paid, and we obtain (2.4).

When $u = b$,

$$\begin{aligned} M_3(b,y,b) = {} & (1-\lambda t)e^{y[c+r(b-\triangle)]t}M_3\bigl(b, ye^{-\alpha t}, b\bigr) \\ & + \lambda t\int_0^{b-\triangle} M_3\bigl(b-x, ye^{-\alpha t}, b\bigr)\,dF(x) \\ & + \lambda t\int_{b-\triangle}^{b} M_2\bigl(b-x, ye^{-\alpha t}, b\bigr)\,dF(x) \\ & + \lambda t\int_{b}^{b+\frac{c}{\beta}} M_1\bigl(b-x, ye^{-\alpha t}, b\bigr)\,dF(x) \\ & + \lambda t\int_{b+\frac{c}{\beta}}^{\infty} dF(x) + o(t). \end{aligned} \tag{2.9}$$

By similar methods we obtain the following equation from (2.9):

$$\begin{aligned} \alpha y\frac{\partial M_3(u,y,b)}{\partial y} = {} & \bigl[y\bigl(r(b-\triangle)+c\bigr) - \lambda\bigr]M_3(b,y,b) + \lambda\overline{F}\left(b+\frac{c}{\beta}\right) \\ & + \lambda\int_0^{b-\triangle} M_3(b-x,y,b)\,dF(x) + \lambda\int_{b-\triangle}^{b} M_2(b-x,y,b)\,dF(x) \\ & + \lambda\int_{b}^{b+\frac{c}{\beta}} M_1(b-x,y,b)\,dF(x). \end{aligned} \tag{2.10}$$

Letting $u \uparrow b$ in (2.3) and comparing it with (2.10), we obtain (2.5).

Next, we prove condition (2.6). Here we only prove $M_2(\triangle-,y,b) = M_3(\triangle,y,b)$. For $0 \le u < \triangle$, let $\tau_\triangle$ be the time that the surplus reaches $\triangle$ for the first time from $0 \le u < \triangle$. As before, we know that t_1 is the time that the surplus reaches $\triangle$ for the first time from $0 \le u < \triangle$ with no claims. Then, by the Markov property of $U_b(t)$,

$$\begin{aligned} M_2(u,y,b) &= E\bigl[I\bigl(\tau_\triangle < T_u^b\bigr)e^{yD_{u,b}}\bigr] + E\bigl[I\bigl(\tau_\triangle \ge T_u^b\bigr)e^{yD_{u,b}}\bigr] \\ &= E\bigl[I\bigl(\tau_\triangle < T_u^b\bigr)M_3\bigl(\triangle, ye^{-\alpha\tau_\triangle}, b\bigr)\bigr] + P\bigl(\tau_\triangle \ge T_u^b\bigr) \\ &= M_3(\triangle,y,b)E\bigl[e^{-\alpha\tau_\triangle}I\bigl(\tau_\triangle < T_u^b\bigr)\bigr] + P\bigl(\tau_\triangle \ge T_u^b\bigr) \\ &\le M_3(\triangle,y,b) + P\bigl(\tau_\triangle \ge T_u^b\bigr). \end{aligned} \tag{2.11}$$

On the other hand, we have

$$\begin{aligned} M_2(u,y,b) &\geq E\big[I\big(\tau_\triangle < T_u^b, \tau_\triangle = t_1\big)e^{yD_{u,b}}\big] + E\big[I\big(\tau_\triangle < T_u^b\big)e^{yD_{u,b}}\big] \\ &= E\big[I\big(\tau_\triangle < T_u^b, \tau_\triangle = t_1\big)M_3\big(\triangle, ye^{-\alpha t_1}, b\big)\big] + P\big(\tau_\triangle \geq T_u^b\big) \\ &= M_3(\triangle, y, b)e^{-\alpha t_1}P(T_1 > t_1) + P\big(\tau_\triangle \geq T_u^b\big) \\ &\geq e^{-(\lambda+\alpha)t_1}M_3(\triangle, y, b) + P\big(\tau_\triangle \geq T_u^b\big), \end{aligned} \tag{2.12}$$

where T_1 is the first claim time. As $u \uparrow \triangle$, $\tau_\triangle$ and t_1 both tend to zero, and $\lim_{u\uparrow\triangle} P(\tau_\triangle \geq T_u^b) = 0$; letting $u \uparrow \triangle$ in (2.11) and (2.12), we get $M_2(\triangle-, y, b) = M_3(\triangle, y, b)$.

Further, letting $u \uparrow 0$ in (2.1), $u \downarrow 0$ in (2.2), and using (2.6), and then letting $u \uparrow \triangle$ in (2.2), $u \downarrow \triangle$ in (2.3), and using (2.6), we get (2.7). □

Remark 2.1 When $\triangle = 0$, the conclusions are consistent with Wang *et al.* [4]. Write

$$V_n(u,y,b) = \begin{cases} V_{n1}(u,y,b), & -\frac{c}{\beta} < u < 0, \\ V_{n2}(u,y,b), & 0 \leq u < \triangle, \\ V_{n3}(u,y,b), & \triangle \leq u \leq b. \end{cases}$$

Theorem 2.2 *The moment of the discounted dividend payments until absolute ruin satisfies the following integro-differential equations*:

$$\begin{aligned} &(\beta u + c)V_{n1}'(u,b) - (\lambda + n\alpha)V_{n1}(u,b) \\ &\quad + \lambda \int_0^{u+\frac{c}{\beta}} V_{n1}(u-x,b)\,dF(x) = 0, \quad -\frac{c}{\beta} < u < 0, \end{aligned} \tag{2.13}$$

$$\begin{aligned} &cV_{n2}'(u,b) - (\lambda + n\alpha)V_{n2}(u,b) + \lambda \int_0^u V_{n2}(u-x,b)\,dF(x) \\ &\quad + \lambda \int_u^{u+\frac{c}{\beta}} V_{n1}(u-x,b)\,dF(x) = 0 \end{aligned} \tag{2.14}$$

for $0 \leq u < \triangle$, *and, for* $\triangle \leq u \leq b$,

$$\begin{aligned} &\big[r(u-\triangle) + c\big]V_{n3}'(u,b) - (\lambda + n\alpha)V_{n3}(u,b) + \lambda \int_0^{u-\triangle} V_{n3}(u-x,b)\,dF(x) \\ &\quad + \lambda \int_{u-\triangle}^u V_{n2}(u-x,b)\,dF(x) + \lambda \int_u^{u+\frac{c}{\beta}} V_{n1}(u-x,b)\,dF(x) = 0, \end{aligned} \tag{2.15}$$

with the following conditions:

$$V_{n1}\left(-\frac{c}{\beta}, b\right) = 0, \tag{2.16}$$

$$V_{n3}'(u,b)|_{u=b} = nV_{(n-1)3}(b,b), \tag{2.17}$$

$$V_{n1}(0-,b) = V_{n2}(0,b), \qquad V_{n2}(\triangle-,b) = V_{n3}(\triangle,b), \tag{2.18}$$

$$V_{n1}'(0-,b) = V_{n2}'(0,b), \qquad V_{n2}'(\triangle-,b) = V_{n3}'(\triangle,b). \tag{2.19}$$

Proof The proof is obvious and we omit it here. □

Corollary 2.1 *For $n = 1$, we retain the risk process, and indeed* (2.13), (2.14), *and* (2.15) *can be simplified to*

$$(\beta u + c)V_{11}'(u,b) - (\lambda + \alpha)V_{11}(u,b) + \lambda \int_0^{u+\frac{c}{\beta}} V_{11}(u-x,b)\,dF(x) = 0, \quad -\frac{c}{\beta} < u < 0, \tag{2.20}$$

$$cV_{12}'(u,b) - (\lambda + \alpha)V_{12}(u,b) + \lambda \int_0^{u} V_{12}(u-x,b)\,dF(x) + \lambda \int_u^{u+\frac{c}{\beta}} V_{11}(u-x,b)\,dF(x) = 0 \tag{2.21}$$

for $0 \le u < \triangle$, and for $\triangle \le u \le b$,

$$\bigl[r(u-\triangle) + c\bigr]V_{13}'(u,b) - (\lambda + \alpha)V_{13}(u,b) + \lambda \int_0^{u-\triangle} V_{13}(u-x,b)\,dF(x) + \lambda \int_{u-\triangle}^{u} V_{12}(u-x,b)\,dF(x) + \lambda \int_u^{u+\frac{c}{\beta}} V_{11}(u-x,b)\,dF(x) = 0. \tag{2.22}$$

Correspondingly, the boundary condition can be simplified to

$$V_{11}\left(-\frac{c}{\beta}, b\right) = 0, \tag{2.23}$$

$$V_{13}'(u,b)|_{u=b} = 1, \tag{2.24}$$

$$V_{11}(0-,b) = V_{12}(0,b), \qquad V_{12}(\triangle-,b) = V_{13}(\triangle,b), \tag{2.25}$$

$$V_{11}'(0-,b) = V_{12}'(0,b), \qquad V_{12}'(\triangle-,b) = V_{13}'(\triangle,b). \tag{2.26}$$

Remark 2.2 When $\triangle = 0$, (2.20) and (2.22) are reduced to (2.1) and (2.2) of Yuen *et al.* [3], and (2.23)-(2.26) are reduced to (A_1)-(A_4) of Yuen *et al.* [3], respectively.

3 Explicit expressions for exponential claims

In this section, we assume that $F(x) = 1 - e^{-\frac{x}{\mu}}$, $x > 0$, $\mu > 0$, namely, the claim size distribution $F(x)$ is the exponential distribution with mean μ. We obtain explicit expressions of the moment-generating function and higher moments of the discounted dividends.

Substituting $F(x) = 1 - e^{-\frac{x}{\mu}}$ into (2.13), (2.14), and (2.15), we obtain:

$$(\beta u + c)V_{n1}'(u,b) = (\lambda + n\alpha)V_{n1}(u,b) - \frac{\lambda}{\mu}e^{-\frac{u}{\mu}} \int_{-\frac{c}{\beta}}^{u} V_{n1}(x,b)e^{\frac{x}{\mu}}\,dx, \quad -\frac{c}{\beta} < u < 0, \tag{3.1}$$

$$cV_{n2}'(u,b) = (\lambda + n\alpha)V_{n2}(u,b) - \frac{\lambda}{\mu}e^{-\frac{u}{\mu}} \left(\int_{-\frac{c}{\beta}}^{0} V_{n1}(x,b)e^{\frac{x}{\mu}}\,dx + \int_0^{u} V_{n2}(x,b)e^{\frac{x}{\mu}}\,dx \right) \tag{3.2}$$

for $0 \le u < \triangle$ and, for $\triangle \le u \le b$,

$$\begin{aligned}\big[r(u-\triangle)+c\big]V'_{n3}(u,b) &= (\lambda+n\alpha)V_{n3}(u,b) - \frac{\lambda}{\mu}e^{-\frac{u}{\mu}}\bigg(\int_{\triangle}^{u} V_{n3}(x,b)e^{\frac{x}{\mu}}\,dx \\ &\quad + \int_0^{\triangle} V_{n2}(x,b)e^{\frac{x}{\mu}}\,dx + \int_{-\frac{c}{\beta}}^{0} V_{n1}(x,b)e^{\frac{x}{\mu}}\,dx\bigg). \qquad (3.3)\end{aligned}$$

Applying the operator $\frac{d}{du}+\frac{1}{\mu}$ on (3.1), (3.2), and (3.3), respectively, we obtain:

$$(\beta u+c)V''_{n1}(u,b) + \bigg(\frac{\beta u+c}{\mu}+\beta-(\lambda+n\alpha)\bigg)V'_{n1}(u,b) - \frac{n\alpha}{\mu}V_{n1}(u,b) = 0, \qquad (3.4)$$

$$cV''_{n2}(u,b) + \bigg(\frac{c}{\mu}-(\lambda+n\alpha)\bigg)V'_{n2}(u,b) - \frac{n\alpha}{\mu}V_{n2}(u,b) = 0, \qquad (3.5)$$

$$\begin{aligned}&\big[r(u-\triangle)+c\big]V''_{n3}(u,b) + \bigg(\frac{r(u-\triangle)+c}{\mu}+r-(\lambda+n\alpha)\bigg)V'_{n3}(u,b) \\ &\quad - \frac{n\alpha}{\mu}V_{n3}(u,b) = 0. \qquad (3.6)\end{aligned}$$

Letting $V_{n1}(u,b) = g_n(z)$ and $z = -\frac{\beta u+c}{\beta\mu}$ for $-\frac{c}{\beta} < u < 0$, (3.4) is reduced to

$$zg''_n(z) + \bigg(1-\frac{\lambda+n\alpha}{\beta}-z\bigg)g'_n(z) + \frac{n\alpha}{\beta}g_n(z) = 0.$$

By Slater [14], p.5, the solution of this equation is of the form

$$\begin{aligned}g_n(z) &= a_{n1}z^{\frac{\lambda+n\alpha}{\beta}}e^z U\bigg(1+\frac{n\alpha}{\beta},1+\frac{\lambda+n\alpha}{\beta},-z\bigg) \\ &\quad + a_{n2}z^{\frac{\lambda+n\alpha}{\beta}}e^z M\bigg(1+\frac{n\alpha}{\beta},1+\frac{\lambda+n\alpha}{\beta},-z\bigg),\end{aligned}$$

where a_{n1} and a_{n2} are arbitrary constants,

$$\begin{aligned}M(a,v,x) &= \frac{\Gamma(v)}{\Gamma(v-a)\Gamma(a)}\int_0^1 e^{xt}t^{a-1}(1-t)^{v-a-1}\,dt, \quad v > a > 0, \\ U(a,v,x) &= \frac{1}{\Gamma(a)}\int_0^{\infty} e^{-xt}t^{a-1}(1+t)^{v-a-1}\,dt, \quad a > 0,\end{aligned}$$

are the confluent hypergeometric functions of the first and second kinds, respectively. Then

$$V_{n1}(u,b) = g_n\bigg(-\frac{\beta u+c}{\beta\mu}\bigg) = a_{n1}h_{n1}(u) + a_{n2}h_{n2}(u) \qquad (3.7)$$

with

$$\begin{aligned}h_{n1}(u) &= \bigg(-\frac{\beta u+c}{\beta\mu}\bigg)^{\frac{\lambda+n\alpha}{\beta}} e^{-\frac{\beta u+c}{\beta\mu}} U\bigg(1+\frac{n\alpha}{\beta},1+\frac{\lambda+n\alpha}{\beta},\frac{\beta u+c}{\beta\mu}\bigg), \\ h_{n2}(u) &= \bigg(-\frac{\beta u+c}{\beta\mu}\bigg)^{\frac{\lambda+n\alpha}{\beta}} e^{-\frac{\beta u+c}{\beta\mu}} M\bigg(1+\frac{n\alpha}{\beta},1+\frac{\lambda+n\alpha}{\beta},\frac{\beta u+c}{\beta\mu}\bigg).\end{aligned}$$

We know that

$$\lim_{u\downarrow -\frac{c}{\beta}} h_{n1}(u) = \frac{\Gamma(\frac{\lambda+n\alpha}{\beta})}{\Gamma(\frac{\beta+n\alpha}{\beta})}, \qquad \lim_{u\downarrow -\frac{c}{\beta}} h_{n2}(u) = 0. \tag{3.8}$$

Letting $u \downarrow -\frac{c}{\beta}$ on both sides of (3.7) and substituting (2.16) and (3.8) into it, we obtain $a_{n1} = 0$, and thus

$$V_{n1}(u,b) = a_{n2}h_{n2}(u). \tag{3.9}$$

It follows from (3.5) that $V_{n2}(u,b)$ takes the form

$$V_{n2}(u,b) = a_{n3}e^{s_{n1}u} + a_{n4}e^{s_{n2}u}, \tag{3.10}$$

where a_{n3} and a_{n4} are arbitrary constants, and s_{n1}, s_{n2} are the two roots of the following equation:

$$s^2 + \left(\frac{1}{\mu} - \frac{n\alpha+\lambda}{c}\right)s - \frac{n\alpha}{\mu c} = 0,$$

with $p_n = \frac{1}{\mu} - \frac{n\alpha+\lambda}{c}$, $q_n = -\frac{n\alpha}{\mu c}$, that is,

$$s_{n1} = \frac{-p_n + \sqrt{p_n^2 - 4q_n}}{2}, \qquad s_{n2} = \frac{-p_n - \sqrt{p_n^2 - 4q_n}}{2}.$$

On the other hand, let $V_{n3}(u,b) = j_n(y)$ and $y = -\frac{ru+c-r\triangle}{r\mu}$. Similarly to (3.4), we obtain the solution of (3.6):

$$\begin{aligned} j_n(y) &= a_{n5}y^{\frac{\lambda+n\alpha}{r}}e^y U\left(1+\frac{n\alpha}{r}, 1+\frac{\lambda+n\alpha}{r}, -y\right) \\ &\quad + a_{n6}y^{\frac{\lambda+n\alpha}{r}}e^y M\left(1+\frac{n\alpha}{r}, 1+\frac{\lambda+n\alpha}{r}, -y\right). \end{aligned}$$

So we get

$$V_{n3}(u,b) = j_n\left(-\frac{ru+c-r\triangle}{r\mu}\right) = a_{n5}h_{n3}(u) + a_{n6}h_{n4}(u), \tag{3.11}$$

where

$$\begin{aligned} h_{n3}(u) &= e^{-\frac{ru+c-r\triangle}{r\mu}}\left(-\frac{ru+c-r\triangle}{r\mu}\right)^{\frac{\lambda+n\alpha}{r}} U\left(1+\frac{n\alpha}{r}, 1+\frac{\lambda+n\alpha}{r}, \frac{ru+c-r\triangle}{r\mu}\right), \\ h_{n4}(u) &= e^{-\frac{ru+c-r\triangle}{r\mu}}\left(-\frac{ru+c-r\triangle}{r\mu}\right)^{\frac{\lambda+n\alpha}{r}} M\left(1+\frac{n\alpha}{r}, 1+\frac{\lambda+n\alpha}{r}, \frac{ru+c-r\triangle}{r\mu}\right), \end{aligned}$$

and a_{n5} and a_{n6} are arbitrary constants.

When $n = 1$, since $a_{11} = 0$, we can get explicit values of $a_{12}, a_{13}, \ldots, a_{16}$. By (2.23)-(2.26) we obtain the following equations:

$$\begin{cases} a_{15}h'_{13}(b) + a_{16}h'_{14}(b) = 1, \\ a_{12}h_{12}(0) = a_{13} + a_{14}, \\ a_{13}e^{s_{11}\triangle} + a_{14}e^{s_{12}\triangle} = a_{15}h_{13}(\triangle) + a_{16}h_{14}(\triangle), \\ a_{12}h'_{12}(0) = a_{13}s_{11} + a_{14}s_{12}, \\ a_{13}s_{11}e^{s_{11}\triangle} + a_{14}s_{12}e^{s_{12}\triangle} = a_{15}h'_{13}(\triangle) + a_{16}h'_{14}(\triangle). \end{cases}$$

Solving this system, we obtain:

$$\begin{cases} a_{12} = \frac{[h_{14}(\triangle)h_{13}(\triangle)C_{12}B_1 - h'_{14}(\triangle)h_{13}(\triangle)C_{12}A_1 + h^2_{14}(\triangle)C_{11}B_1 - h'_{14}(\triangle)h_{14}(\triangle)C_{11}A_1](s_{12}-s_{11})}{(h'_{12}(0)-s_{11}h_{12}(0))C_{11}C_{12}A_1}, \\ a_{13} = \frac{[h_{14}(\triangle)h_{13}(\triangle)C_{12}B_1 - h'_{14}(\triangle)h_{13}(\triangle)C_{12}A_1 + h^2_{14}(\triangle)C_{11}B_1 - h'_{14}(\triangle)h_{14}(\triangle)C_{11}A_1](s_{12}h_{12}(0)-h'_{12}(0))}{(h'_{12}(0)-s_{11}h_{12}(0))C_{11}C_{12}A_1}, \\ a_{14} = \frac{h_{14}(\triangle)h_{13}(\triangle)C_{12}B_1 - h'_{14}(\triangle)h_{13}(\triangle)C_{12}A_1 + h^2_{14}(\triangle)C_{11}B_1 - h'_{14}(\triangle)h_{14}(\triangle)C_{11}A_1}{C_{11}C_{12}A_1}, \\ a_{15} = \frac{h_{14}(\triangle)B_1 - h'_{14}(\triangle)A_1}{C_{11}}, \\ a_{16} = \frac{h_{13}(\triangle)B_1 - h'_{13}(\triangle)A_1}{C_{12}}, \end{cases} \tag{3.12}$$

where

$$\begin{aligned} A_1 &= \frac{(s_{12}h_{12}(0) - h'_{12}(0))e^{s_{11}\triangle} + (h'_{12}(0) - s_{11}h_{12}(0))e^{s_{12}\triangle}}{h'_{12}(0) - s_{11}h_{12}(0)}, \\ B_1 &= \frac{(s_{12}h_{12}(0) - h'_{12}(0))s_{11}e^{s_{11}\triangle} + (h'_{12}(0) - s_{11}h_{12}(0))s_{12}e^{s_{12}\triangle}}{h'_{12}(0) - s_{11}h_{12}(0)}, \\ C_{11} &= \left[h'_{13}(b)h_{14}(\triangle) - h_{13}(\triangle)h'_{14}(b)\right]B_1 + \left[h'_{13}(\triangle)h'_{14}(b) - h'_{13}(b)h'_{14}(\triangle)\right]A_1, \\ C_{12} &= \left[h'_{14}(b)h_{13}(\triangle) - h_{14}(\triangle)h'_{13}(b)\right]B_1 + \left[h'_{13}(b)h'_{14}(\triangle) - h'_{13}(\triangle)h'_{14}(b)\right]A_1. \end{aligned}$$

When $n \geq 2$, by (2.16)-(2.19) we obtain the following equations:

$$\begin{cases} a_{n5}h'_{n3}(b) + a_{n6}h'_{n4}(b) = nV'_{(n-1)3}(b,b), \\ a_{n2}h_{n2}(0) = a_{n3} + a_{n4}, \\ a_{n3}e^{s_{n1}\triangle} + a_{n4}e^{s_{n2}\triangle} = a_{n5}h_{n3}(\triangle) + a_{n6}h_{n4}(\triangle), \\ a_{n2}h'_{n2}(0) = a_{n3}s_{n1} + a_{n4}s_{n2}, \\ a_{n3}s_{n1}e^{s_{n1}\triangle} + a_{n4}s_{n2}e^{s_{n2}\triangle} = a_{n5}h'_{n3}(\triangle) + a_{n6}h'_{n4}(\triangle). \end{cases}$$

Solving this system of equations, we obtain:

$$\begin{cases} a_{n2} = \frac{nV_{(n-1)3}(b,b)[h_{n4}(\triangle)h_{n3}(\triangle)C_{n2}B_n - h'_{n4}(\triangle)h_{n3}(\triangle)C_{n2}A_n + h^2_{n4}(\triangle)C_{n1}B_n - h'_{n4}(\triangle)h_{n4}(\triangle)C_{n1}A_n](s_{n2}-s_{n1})}{(h'_{n2}(0)-s_{n1}h_{n2}(0))C_{n1}C_{n2}A_n}, \\ a_{n3} = \frac{nV_{(n-1)3}(b,b)[h_{n4}(\triangle)h_{n3}(\triangle)C_{n2}B_n - h'_{n4}(\triangle)h_{n3}(\triangle)C_{n2}A_n + h^2_{n4}(\triangle)C_{n1}B_n - h'_{n4}(\triangle)h_{n4}(\triangle)C_{n1}A_n]}{(h'_{n2}(0)-s_{n1}h_{n2}(0))C_{n1}C_{n2}A_n} \\ \qquad \times (s_{n2}h_{n2}(0) - h'_{n2}(0)), \\ a_{n4} = \frac{nV_{(n-1)3}(b,b)[h_{n4}(\triangle)h_{n3}(\triangle)C_{n2}B_n - h'_{n4}(\triangle)h_{n3}(\triangle)C_{n2}A_n + h^2_{n4}(\triangle)C_{n1}B_n - h'_{n4}(\triangle)h_{n4}(\triangle)C_{n1}A_n]}{C_{n1}C_{n2}A_n}, \\ a_{n5} = \frac{nV_{(n-1)3}(b,b)[h_{n4}(\triangle)B_n - h'_{n4}(\triangle)A_n]}{C_{n1}}, \\ a_{n6} = \frac{nV_{(n-1)3}(b,b)[h_{n3}(\triangle)B_n - h'_{n3}(\triangle)A_n]}{C_{n2}}, \end{cases} \tag{3.13}$$

where

$$A_n = \frac{(s_{n2}h_{n2}(0) - h'_{n2}(0))e^{s_{n1}\triangle} + (h'_{n2}(0) - s_{n1}h_{n2}(0))e^{s_{n2}\triangle}}{h'_{n2}(0) - s_{n1}h_{n2}(0)},$$

$$B_n = \frac{(s_{n2}h_{n2}(0) - h'_{n2}(0))s_{n1}e^{s_{n1}\triangle} + (h'_{n2}(0) - s_{n1}h_{n2}(0))s_{n2}e^{s_{n2}\triangle}}{h'_{n2}(0) - s_{n1}h_{n2}(0)},$$

$$C_{n1} = \left[h'_{n3}(b)h_{n4}(\triangle) - h_{n3}(\triangle)h'_{n4}(b)\right]B_n + \left[h'_{n3}(\triangle)h'_{n4}(b) - h'_{n3}(b)h'_{n4}(\triangle)\right]A_n,$$

$$C_{n2} = \left[h'_{n4}(b)h_{n3}(\triangle) - h_{n4}(\triangle)h'_{n3}(b)\right]B_n + \left[h'_{n3}(b)h'_{n4}(\triangle) - h'_{n3}(\triangle)h'_{n4}(b)\right]A_n.$$

From (3.11) and a_{15}, a_{16} in (3.12) we can obtain $V_{13}(b,b)$ see the following Examples 1-4. Recursively, we can obtain explicit expressions of $V_{n1}(u,b)$, $V_{n2}(u,b)$, and $V_{n3}(u,b)$.

We summarize the exact solution for $V_{n1}(u,b)$, $V_{n2}(u,b)$, and $V_{n3}(u,b)$ in the following theorem.

Theorem 3.1 *Suppose that the claim size distribution is the exponential distribution with* $F(x) = 1 - e^{-\frac{x}{\mu}}$. *Then* $V_n(u,b)$ *is given by*

$$\begin{aligned} V_{n1}(u,b) &= a_{n2}h_{n2}(u), \quad -\frac{c}{\beta} < u < 0, \\ V_{n2}(u,b) &= a_{n3}e^{s_{n1}u} + a_{n4}e^{s_{n2}u}, \quad 0 \le u < \triangle, \\ V_{n3}(u,b) &= a_{n5}h_{n3}(u) + a_{n6}h_{n4}(u), \quad \triangle \le u \le b. \end{aligned} \tag{3.14}$$

Here, $a_{12}, a_{13}, \ldots, a_{16}$ *are obtained in* (3.12) *for* $n = 1$; *otherwise, for* $n \ge 2$, $a_{n2}, a_{n3}, \ldots, a_{n6}$ *are obtained by* (3.13).

Remark 3.1 In fact, we can obtain the following explicit expressions of $M_1(u,y,b)$, $M_2(u,y,b)$, and $M_3(u,y,b)$:

$$\begin{aligned} M_1(u,y,b) &= 1 + \sum_{n=1}^{\infty} \frac{y^n}{n!} a_{n2}h_{n2}(u), \quad -\frac{c}{\beta} < u < 0, \\ M_2(u,y,b) &= 1 + \sum_{n=1}^{\infty} \frac{y^n}{n!} \left(a_{n3}e^{s_{n1}u} + a_{n4}e^{s_{n2}u}\right), \quad 0 \le u < \triangle, \\ M_3(u,y,b) &= 1 + \sum_{n=1}^{\infty} \frac{y^n}{n!} \left(a_{n5}h_{n3}(u) + a_{n6}h_{n4}(u)\right), \quad \triangle \le u \le b. \end{aligned}$$

At the end of this section, we use the following numerical examples to discuss the impact of the model parameters on the expected dividend payments.

Example 1 Table 1 provides numerical results for $V_{13}(u,b)$ for various u and r. We find out that $V_{13}(u,b)$ increases as the credit interest or the initial surplus increases.

Example 2 Table 2 provides numerical results for $V_{13}(u,b)$ for various u and β. We find out that $V_{13}(u,b)$ decreases as the force of debit interest increases but increases as the initial surplus increases.

Table 1 Influence of u and r on $V_{13}(u,b)$ with $b = 2.8$, $\Delta = 1.5$, $\alpha = 0.03$, $\mu = 1$, $\lambda = 1$, $c = 1.5$, $\beta = 0.09$

$u \setminus r$	0.03	0.04	0.05	0.06	0.07	0.08
1.6	27.8537	28.1717	28.4903	28.8095	29.1294	29.4498
1.7	27.9233	28.2420	28.5614	28.8813	29.2018	29.5230
1.8	27.9961	28.3155	28.6355	28.9562	29.2744	29.5992
1.9	28.0719	28.3920	28.7127	29.0340	29.3559	29.6784
2.0	28.1507	28.4714	28.7928	29.1147	29.4372	29.7603
2.1	28.2323	28.5537	28.8756	29.1981	29.5212	29.8449
2.2	28.3167	28.6386	28.9610	29.2840	29.6077	29.9318
2.3	28.4038	28.7261	29.0490	29.3725	29.6965	30.0212
2.4	28.4933	28.8161	29.1394	29.4632	29.7877	30.1127

Table 2 Influence of u and β on $V_{13}(u,b)$ with $b = 2.8$, $\Delta = 1.5$, $\alpha = 0.03$, $\mu = 1$, $\lambda = 1$, $c = 1.5$, $r = 0.04$

$u \setminus \beta$	0.09	0.10	0.11	0.12	0.13	0.14
1.6	28.1717	27.1050	26.1146	25.1984	24.3525	23.5719
1.7	28.2420	27.1783	26.1906	25.2769	24.4333	23.6549
1.8	28.3155	27.2544	26.2691	25.3577	24.5162	23.7396
1.9	28.3920	27.3332	26.3501	25.4407	24.6010	23.8262
2.0	28.4714	27.4147	26.4334	25.5258	24.6877	23.9144
2.1	28.5537	27.4987	26.5191	25.6129	24.7763	24.0042
2.2	28.6386	27.5851	26.6069	25.7020	24.8666	24.0956
2.3	28.7261	27.6738	29.6968	25.7930	24.9585	24.1884
2.4	28.8161	27.7648	26.7887	25.8857	25.0521	24.2827

Table 3 Influence of u and Δ on $V_{13}(u,b)$ with $b = 2.8$, $\alpha = 0.03$, $\mu = 1$, $\lambda = 1$, $c = 1.5$, $\beta = 0.09$, $r = 0.04$

$u \setminus \Delta$	0.9	1.1	1.3	1.5	1.7	1.9
1.6	20.8956	22.6238	24.9465	28.1717	32.8679	40.2129
1.7	20.9881	22.7109	25.0265	28.2420	32.9245	40.2483
1.8	21.0815	22.7994	25.1086	28.3155	32.9857	40.2906
1.9	21.1756	22.8892	25.1928	28.3920	33.0512	40.3394
2.0	21.2706	22.9803	25.2789	28.4714	33.1211	40.3944
2.1	21.3662	23.0727	25.3669	28.5537	33.1950	40.4556
2.2	21.4625	23.1662	25.4568	28.6386	33.2779	40.5256
2.3	21.5594	23.2608	25.5484	28.7261	33.3545	40.5952
2.4	21.6570	23.3565	25.6417	28.8161	33.4398	40.6732

Table 4 Influence of u and b on $V_{13}(u,b)$ with $\Delta = 1.5$, $\alpha = 0.03$, $\mu = 1$, $\lambda = 1$, $c = 1.5$, $\beta = 0.09$, $r = 0.04$

$u \setminus b$	2.5	2.6	2.7	2.8	2.9	3.0	3.1
1.6	30.1186	29.4128	28.7660	28.1717	27.6244	27.1195	26.6526
1.7	30.1938	29.4862	28.8378	28.2420	27.6934	27.1872	26.7192
1.8	30.2724	29.5630	28.9128	28.3155	27.7655	27.2579	26.7887
1.9	30.3542	29.6428	28.9909	28.3920	27.8405	27.3316	26.8611
2.0	30.4391	29.7258	29.0720	28.4714	27.9184	27.4080	26.9362
2.1	30.5270	29.8116	29.1560	28.5537	27.9990	27.4872	27.0140
2.2	30.6178	29.9003	29.2427	28.6386	28.0823	27.5689	27.0943
2.3	30.7113	29.9916	29.3321	28.7261	28.1681	27.6532	27.1771
2.4	30.8075	30.0856	29.4239	28.8161	28.2563	27.7398	27.2623

Example 3 Table 3 provides numerical results for $V_{13}(u,b)$ for various u and $\triangle$. We find out that $V_{13}(u,b)$ increases as the liquid reserve or the initial surplus increases.

Example 4 Table 4 provides numerical results for $V_{13}(u,b)$ for various u and b. We find out that $V_{13}(u,b)$ decreases as b increases but increases as the initial surplus increases.

We plot four figures (Figures 1-4) for the surfaces of the expected dividend payments with the help of Tables 1-4, from which we can see the influence of credit interest, debit interest, dividend barrier b, and liquid reserve $\triangle$ on the values of the expected dividend payments.

For $\triangle = 0$, our model is reduced to Wang and Yin [15]. For example, Figures 5 and 6 show that the expected dividend payments $V(u,b)$ decrease as the debit interest increases but increase as credit interest increases, which is also obtained by Wang and Yin [15], Tables 1 and 2.

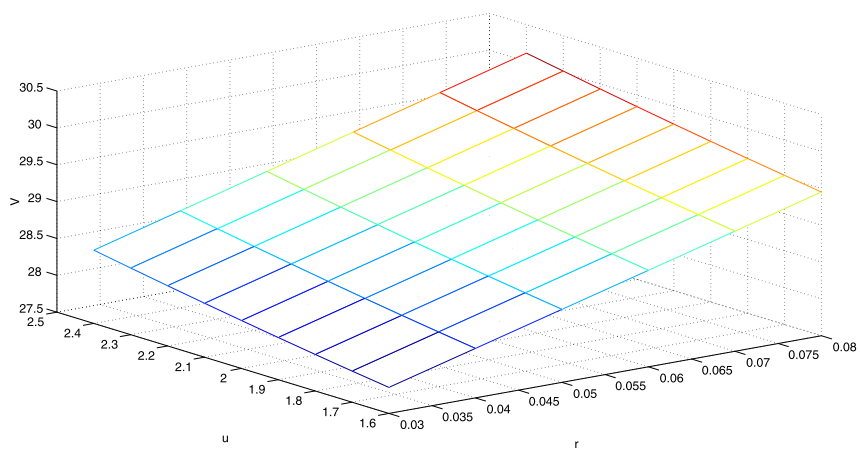

Figure 1 Surfaces of $V(u,r)$ when $b = 2.8$, $\triangle = 1.5$, $\alpha = 0.03$, $\mu = 1$, $\lambda = 1$, $c = 1.5$, $\beta = 0.09$.

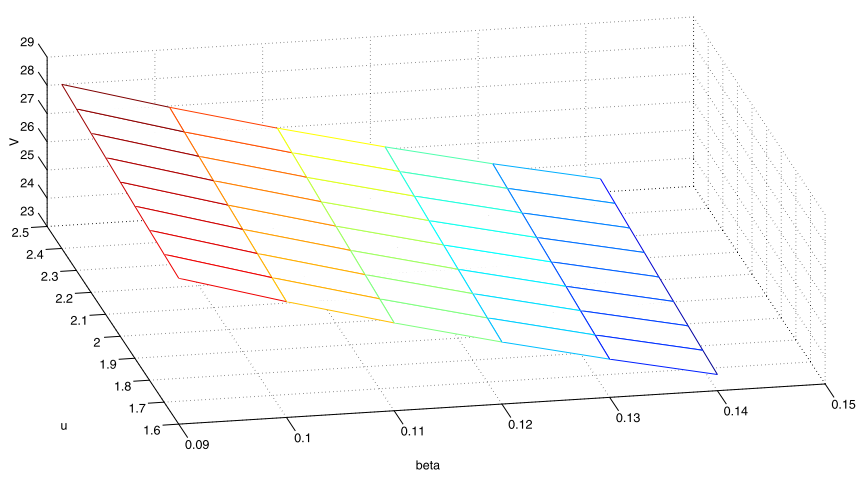

Figure 2 Surfaces of $V(u,\beta)$ when $b = 2.8$, $\triangle = 1.5$, $\alpha = 0.03$, $\mu = 1$, $\lambda = 1$, $c = 1.5$, $r = 0.04$.

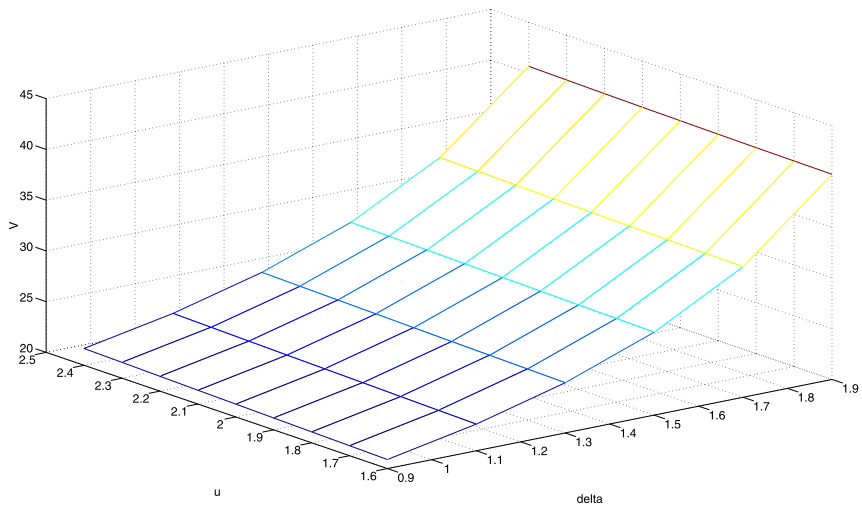

Figure 3 Surfaces of $V(u, \Delta)$ when $b = 2.8$, $\alpha = 0.03$, $\mu = 1$, $\lambda = 1$, $c = 1.5$, $\beta = 0.09$, $r = 0.04$.

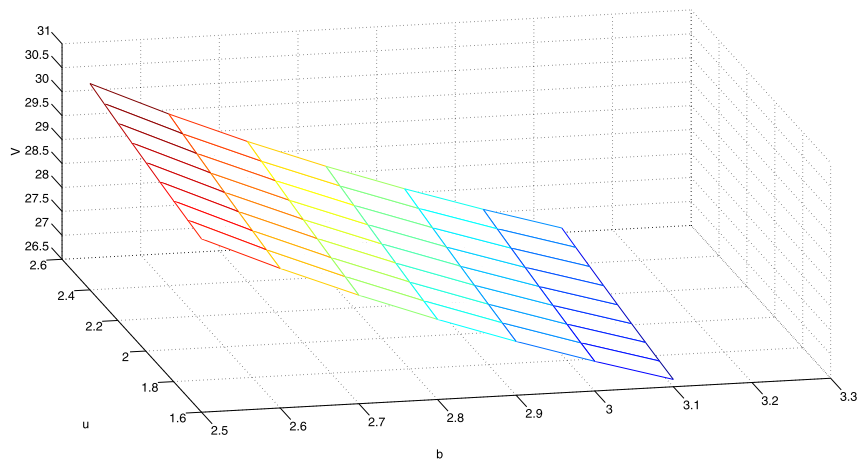

Figure 4 Surfaces of $V(u, b)$ when $\Delta = 1.5$, $\alpha = 0.03$, $\mu = 1$, $\lambda = 1$, $c = 1.5$, $\beta = 0.09$, $r = 0.04$.

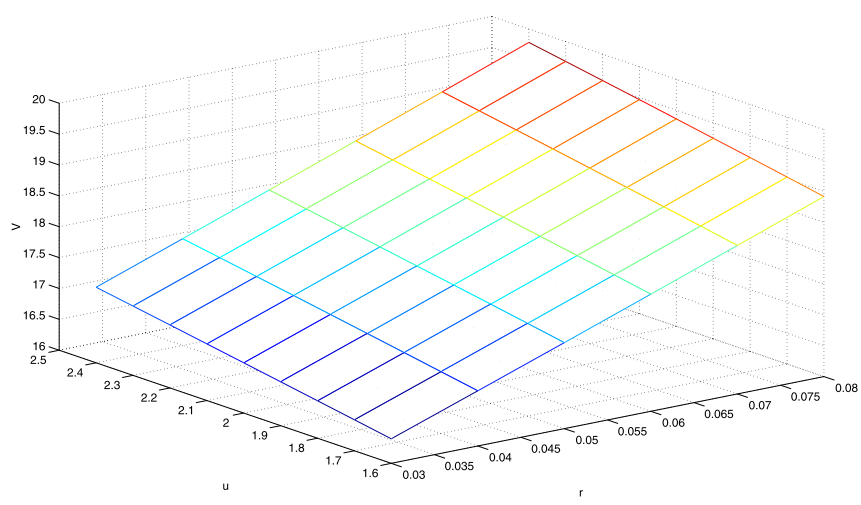

Figure 5 Surfaces of $V(u, b)$ when $\Delta = 0$, $\alpha = 0.03$, $\mu = 1$, $\lambda = 1$, $c = 1.5$, $\beta = 0.09$, $r = 0.04$.

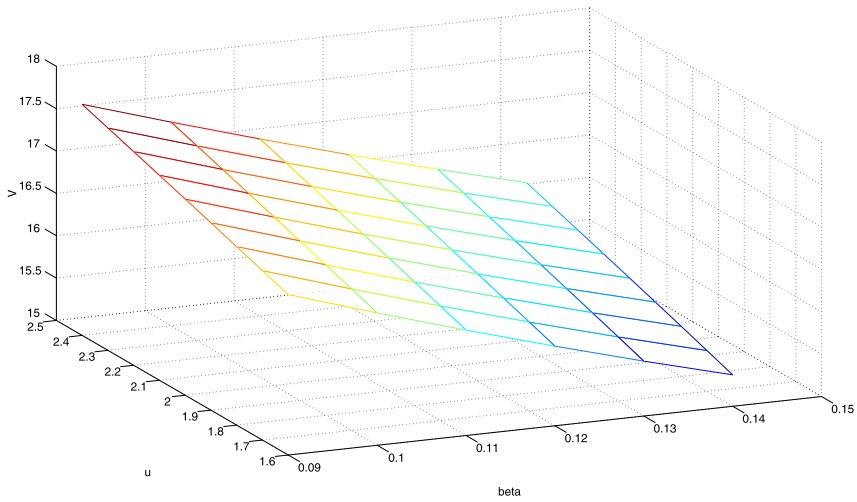

Figure 6 Surfaces of $V(u,b)$ when $\Delta = 0$, $\alpha = 0.03$, $\mu = 1$, $\lambda = 1$, $c = 1.5$, $\beta = 0.09$, $r = 0.04$.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
DL conceived of the study and drafted the manuscript, DP participated in the design of the study and made the contribution to simulations. ZH participated in its design and helped to draft the manuscript. All authors have read and approved the final manuscript.

Author details
[1]School of Mathematics and Computational Science, Hunan University of Science and Technology, Xiangtan, Hunan 411201, P.R. China. [2]School of Mathematics and Computational Science, Central South University, Changsha, Hunan 410075, P.R. China.

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