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Finite-time control and synchronization for memristor-based chaotic system via impulsive adaptive strategy

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Abstract

This paper investigates the stabilizing and synchronization problems of a memristor-based Chua chaotic system in a finite time. A lemma concerning the finite-time stability for an impulsive system is proposed by extending the finite-time stability theory. Then some finite-time stabilizing and synchronization criterion are presented which guarantee the finite-time stabilization and synchronization for the model considered. Finally, the efficiency of the control scheme is further demonstrated by the simulation examples.

Keywords: finite time; control and synchronization; memristor; impulsive adaptive strategy

1 Introduction

In 1971, the memristor which is considered to be the missing fourth passive circuit element was postulated [1]. However, this important postulation has not caused attention in almost 40 years. Until in 2008, Hewlett-Packard Labs announced the development of a memristor based on nanotechnology [2]. As we know, the memristor takes its place along with the other three existing elements: the resistor, the capacitor, and the inductor. Increasing focus was put on the memristor for its potential applications in programmable logic, signal processing, neural networks, and so on [3].

Moreover, as the novel element, the circuit based on the memristor shares many interesting phenomenon. Recently, the research memristor chaotic circuits have become a focal topic [4–10]. In [4], the author presented a novel fourth-order memristor-based Chua oscillator by replacing Chua's diode with an active two-terminal circuit. The stabilization problem of a memristor-based chaotic system was investigated in [10]. As the most important phenomenon, the synchronization was also discussed [9]. In [9], the adaptive synchronization problem of memristor-based Chua circuits was investigated.

As time goes on, more and more researchers began to realize the important role of the synchronization time. To attain a high convergence speed, many effective methods have been introduced and finite-time control is one of them. Finite-time synchronization means the optimality in convergence time. Much research work has been done on chaos synchronization based on finite time (see for instance [11–22] and the references

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Figure 1 Chua's memristor-based chaotic circuits with a flux-controlled memristor.
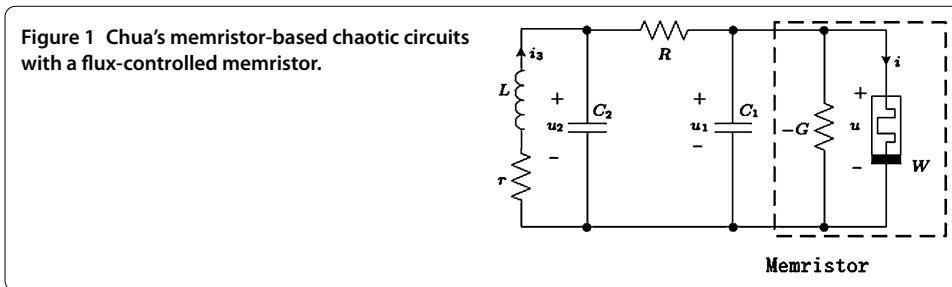
therein). However, the finite-time synchronization problem has not been fully investigated in the literature, and it still remains open. Motivated by the above discussion, we investigate the finite-time synchronization problem for a memristor-based Chua circuit. Based on the finite-time stability theory, a novel lemma which guarantees the impulsive system is finite-time stable is presented. Then the impulsive [16–18] adaptive control law is proposed to realize finite-time synchronization of the model considered. Numerical simulations demonstrate the effectiveness and correctness of this results.

The paper is organized as follows. Some preliminaries are presented in the next section. Section 3 proposes the main results of this paper. In Section 4, the numerical simulations are presented, which is followed by the conclusion in Section 5.

2 Preliminaries

In [6], the author proposed a novel nonlinear circuits, with a flux-controlled memristor which replaces the Chua diode. Figure 1 shows a memristor-based Chua oscillator with a flux-controlled memristor.

Applying a Kirchhoff voltage, the current law, and the volt-ampère relationship of the components, the state equation of a Chua memristor-based chaotic system can be described as follows:

$$\begin{cases} \dot{u}_1 = \frac{1}{C_1}(u_2 - u_1 + Gu_1 - W(\phi)u_1), \\ \dot{u}_2 = \frac{1}{C_2}(u_1 - u_2) + Ri_3, \\ \dot{i}_3 = -\frac{1}{L}u_2 + \frac{r}{L}i_3, \\ \dot{\phi} = u_1. \end{cases} \tag{1}$$

For convenience, letting $x_1 = u_1$, $x_2 = u_2$, $x_3 = i_3$, $x_4 = \phi$, $\alpha = 1/C_1$, $\beta = 1/L_1$, $\gamma = r/L$, $\xi = G$, $C_2 = 1$, and $R = 1$, then the model can be rewritten as the following equation:

$$\begin{cases} \dot{x}_1 = \alpha(x_2 - x_1 + \xi x_1 - W(x_4)x_1), \\ \dot{x}_2 = x_1 - x_2 + x_3, \\ \dot{x}_3 = -\beta x_2 + \gamma x_3, \\ \dot{x}_4 = x_1. \end{cases} \tag{2}$$

If we set $\alpha = 10$, $\beta = 100/7$, $\gamma = 0.1$, $\xi = 9/7$, $a = 1/7$ and $b = 2/7$, and the initial values are $(10^{-10}, 0, 0, -0.515)$, then system (2) is a chaotic system and the chaotic attractor is shown in Figure 2.

In order to compute simply, letting $x = [x_1, x_2, x_3, x_4]^T$, then system (2) can be described as follows:

$$\dot{x} = Ax + \phi(x), \tag{3}$$

Figure 2 The chaotic attractor of the memristor-based Chua circuit.
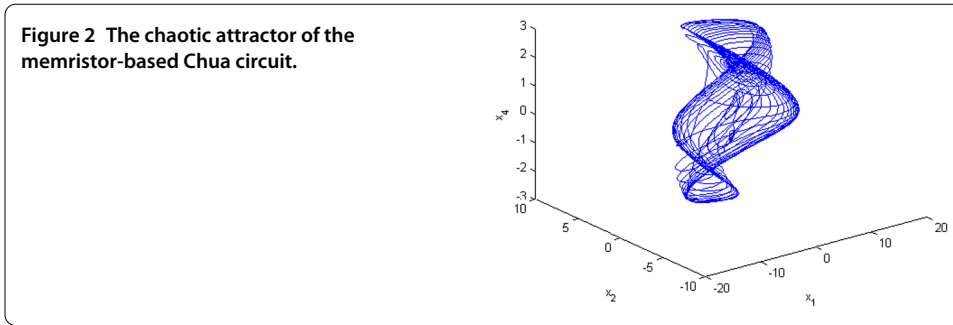
where

$$
A = \begin{bmatrix} \alpha(\xi - 1) & \alpha & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & -\beta & \gamma & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \qquad \phi(x) = \begin{bmatrix} -\alpha W(x_4) x_1 \\ 0 \\ 0 \\ 0 \end{bmatrix},
$$

and $W(x_4) = a + 3b x_4^2$, a, b, γ, ξ, α, and β are positive constants.

Similar to [7], the nonlinear functions $q(\phi)$, $W(\phi)$ are given by

$$
q(\phi) = a\phi + b\phi^3, \tag{4}
$$

$$
W(\phi) = \frac{dq(\phi)}{d\phi} = a + 3b\phi^2. \tag{5}
$$

Throughout this paper, the following assumption and lemma are necessary for our main results.

Assumption 1 System (2) is a chaotic system, namely, the state is bound, we assume that the following assumptions hold:

$$
|x_1| \leq M_1, \qquad |x_4| \leq M_2,
$$

where M_1, M_2 are real constants.

Lemma 1 *Suppose the function is continuous and non-negative when $t \in [0, \infty)$ and satisfies the following conditions*:

$$
\begin{cases} \dot{V}(t) \leq -\rho V^\eta(t), & t \neq t_k, \\ V(t_k^+) \leq \delta V(t_k), & k = 1, 2, \ldots, \end{cases} \tag{6}
$$

where $\rho > 0$, $0 < \eta < 1$, $0 < \delta < 1$, $k = \{1, 2, \ldots, m\}$, is a finite natural number set and m is a positive integer, then the following inequality holds:

$$
V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - \rho(1 - \eta)(t - t_0), \quad t_0 \leq t \leq T, \tag{7}
$$

where T is a constant which represents the setting time.

Proof Without loss of generality, let $t_0 = 0$. In order to prove (7) holds, the following function $H(t)$ is constructed:

$$H(t) = V^{1-\eta}(t) - V^{1-\eta}(0) + \rho(1-\eta)t. \tag{8}$$

Clearly, if the function $H(t)$ satisfies $H(t) \leq 0$, then the equality (7) holds.

One can easily observe that

$$H(0) = V^{1-\eta}(0) - V^{1-\eta}(0) \leq 0. \tag{9}$$

Next, we will prove that $H(t) \leq 0$ holds for $t \in [t_0, t_1]$. Otherwise, there exists t^* such that

$$H(t) > 0, \quad t \in (t^*, t_1), \qquad H(t) < 0, \qquad t \in (t_0, t^*), \tag{10}$$

$$H(t^*) = 0, \qquad \dot{H}(t^*) > 0. \tag{11}$$

Combing (10) with (11), one has

$$\begin{aligned}
\dot{H}(t^*) &= (1-\eta)V^{-\eta}(t^*)\dot{V}(t^*) + \rho(1-\eta) \\
&= (1-\eta)V^{-\eta}(t^*)\left(-\rho V^{-\eta}(t^*)\right) + \rho(1-\eta) \\
&= 0
\end{aligned} \tag{12}$$

which contradicts (11). Namely, $H(t) \leq 0$ holds for $t \in [t_0, t_1)$.

When $t = t_1$, we get

$$\begin{aligned}
H(t_1) - H(t_1^-) &= H(t_1^+) - H(t_1^-) \\
&= V^{1-\eta}(t_1^+) - V^{1-\eta}(t_1^-) = \left(\beta^{1-\eta} - 1\right)V^{1-\eta}(t_1^-) \\
&\leq 0.
\end{aligned} \tag{13}$$

It yields

$$H(t_1) \leq H(t_1^-) \leq 0. \tag{14}$$

Then we suppose that $H(t) \leq 0$ holds for $t \in [t_{k-1}, t_k]$. For $t \in [t_k, t_{k+1}]$, we have

$$\begin{aligned}
H(t_{k+1}) &= V^{1-\eta}(t_{k+1}) - V^{1-\eta}(0) + \alpha(1-\eta)t_{k+1} \\
&= \beta^{1-\eta}V^{1-\eta}(t_{k+1}^-) - V^{1-\eta}(0)\alpha(1-\eta)t_{k+1}^- \\
&= \left[\beta^{1-\eta} - 1\right]V^{1-\eta}(t_{k+1}^-) - H(t_{k+1}^-) \\
&\leq \left[\beta^{1-\eta} - 1\right]V^{1-\eta}(t_{k+1}^-) - H(t_k^+) \\
&\leq 0.
\end{aligned} \tag{15}$$

From (8)-(15), we know that $H(t) \leq 0$ holds. This completes the proof. □

3 Main results

In this section, the finite-time control and synchronization problems via an impulsive adaptive strategy are investigated, respectively. Taking the impulsive adaptive strategy into account in (3), one obtains

$$\begin{cases} \dot{x}(t) = Ax(t) + \phi(x(t)) + u(t), & t \neq t_k, \\ \Delta x(t_k) = I_k(x(t_k)) = -Bx(t_k), & t = t_k, k \in \ell, \end{cases} \tag{16}$$

where $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, $x(t_k^+) = \lim_{t \to t_k^+} x(t)$, $x(t_k^-) = \lim_{t \to t_k^-} x(t)$, $\ell = \{1, 2, \ldots, n, n_1, \ldots, n_k\}$, is a finite natural number set. For simplicity, it is assumed that $x(t_k^-) = x(t_k)$, which means that $x(t_k)$ is left continuous. Letting $u(t) = -k_1 x(t) - k_2 \operatorname{sign}(x(t))|x(t)|^\gamma$, we have the following theorem.

Theorem 1 *Suppose Assumption 1 holds. There exists a positive constant γ satisfying $0 < \gamma < 1$ such that the memristor-based chaotic system is finite-time stable if the following conditions hold*:

(i) $q = \lambda_{\max}[A^T + A - (2k_1 + 1)I] < 0$;

(ii) $d = \lambda_{\max}(I + B)^T(I + B) < 1$.

Proof Construct the following Lyapunov candidate function:

$$V(t, x) = x^T x. \tag{17}$$

Calculating the derivative along the trajectory of (16) we have

$$\begin{aligned} D^+ V(t, x) &= \dot{x}^T x + x^T \dot{x} \\ &= \big(Ax + \phi(x) + u(t)\big)^T x + x^T \big(Ax + \phi(x) + u(t)\big) \\ &= x^T \big(A + A^T - 2k_1 I\big)x + 2x^T \phi(t, x) - 2k_2 x^T \operatorname{sign}\big(x(t)\big)|x(t)|^\gamma. \end{aligned} \tag{18}$$

From Assumption 1, one has

$$\begin{aligned} 2x^T \phi(x) &\leq x^T x + \phi^T(x)\phi(x) \\ &\leq x^T x + \big[(a + bx_4^2)x_1\big]^2 \\ &\leq x^T x + x_1^2\big[a + bM_4^2\big]^2 \leq (I + J)x^T x, \end{aligned} \tag{19}$$

where $J = [[a + bM_4^2]^2, 0, 0, 0]^T$.

From (18) and (19), one obtains

$$\begin{aligned} D^+ V(t, x) &\leq x^T\big[A^T + A - (2k_1 + 1)I\big]x - 2k_2 x^T \operatorname{sign}\big(x(t)\big)|x(t)|^\gamma \\ &\leq qV(t) - 2k_2 \sum_{i=1}^4 |x_i|^{\gamma+1}, \end{aligned} \tag{20}$$

where $q = \lambda_{\max}[A^T + A - (2k_1 + 1)I]$. From the fact that $0 < \gamma < 1$, one obtains

$$\left(\sum_{i=1}^4 |x_i|^{\gamma+1}\right)^{\frac{1}{\gamma+1}} \geq \left(\sum_{i=1}^4 |x_i|^2\right)^{\frac{1}{2}}. \tag{21}$$

Also

$$\sum_{i=1}^{n} |x_i|^{\gamma+1} \geq \left(\sum_{i=1}^{n} |x_i|^2\right)^{\frac{\gamma+1}{2}} = \left(x^T x\right)^{\frac{\gamma+1}{2}}. \tag{22}$$

Then, from condition (1) of Theorem 1, we have for $t \neq t_k$

$$D^+ V(t, x) \leq -2k_2 V(x)^{\frac{\gamma+1}{2}}. \tag{23}$$

When $t = t_k$, one obtains

$$
\begin{aligned}
V\left(t_k^+\right) &= (x + Bx)^T (x + Bx) \\
&= x\left(I + B^T\right)(I + B)x \\
&\leq dV(t_k).
\end{aligned} \tag{24}
$$

Through condition in Theorem 1 and Lemma 1, we can easily see that the system (16) is finite-time stable, which complete the proof. $\qquad\square$

Next, we investigate the problem of finite-synchronization for a memristor-based chaotic system. Based on the drive-response synchronization concept, letting system (3) be the drive system, the response system with control input u is as follows:

$$\dot{y} = Ay + \phi(y) + u. \tag{25}$$

Taking the impulsive adaptive effects into account, the response system (25) is as follows:

$$
\begin{cases}
\dot{y} = Ay + \phi(y) + u(t), & t \neq t_k, \\
\Delta y = I_k(y) = -Be, & t = t_k, \\
y(t_0^+) = y_0, & t_0 \geq 0,
\end{cases} \tag{26}
$$

where $e = [x - y]^T = [x_1 - y_1, x_2 - y_2, x_3 - y_3, x_4 - y_4]^T$, t_k are the impulsive instants which satisfy $t_1 < t_2 < \cdots < t_{k-1} < t_k$ and $\lim_{k \to \infty} t_k = \infty$. Letting

$$u = -k_1 e(t) - k_2 \operatorname{sign}(e(t)) |e(t)|^{\gamma}, \tag{27}$$

where the constants k_1, k_2 are the control strength coefficients to be designed, the real number γ satisfies $0 \leq \gamma < 1$.

Combing (3) with (26), the error system can be described as follows:

$$
\begin{cases}
\dot{e} = Ae + \phi(e) + u(t), & t \neq t_k, \\
\Delta e = Be, & t = t_k, k = 1, 2, \dots,
\end{cases} \tag{28}
$$

where $\phi(e) = [\alpha W(y_4) y_1 - \alpha W(x_4) x_1, 0, 0, 0]^T$. Hence, we have the following theorem.

Theorem 2 *Suppose Assumption* 1 *holds. There exists a positive constant γ satisfying $0 < \gamma < 1$ such that the memristor-based Chua systems* (3) *and* (26) *can be synchronized under the impulsive adaptive strategy if the following conditions hold*:

(i) $A^T + A - 2k_1 I - 2abM_1 M_2 I < 0$;

(ii) $d = \lambda_{\max}(I + B)^T(I + B) < 1$.

Proof Construct the following Lyapunov candidate function

$$V(e) = e^T e. \tag{29}$$

For $t \in [t_k, t_{k+1})$, the derivative of $V(t)$ along the trajectory of (28) is

$$D^+ V(e) = 2e^T \dot{e} = 2e^T(A - k_1 I)e(t) + 2e^T \phi(e) - 2k_2 e^T \, \text{sign}(e(t)) |e(t)|^\alpha. \tag{30}$$

From Assumption 1, we have

$$
\begin{aligned}
2e^T \phi(e) &= 2\alpha \left[(a + by_4^2)y_1 - (a + bx_4^2)x_1 \right] e_1 \\
&= -2\alpha e_1^2 + 2ab(y_4^2 y_1 - y_4^2 x_1 + y_4^2 x_1 - x_4^2 x_1) \\
&\le 2abM_1 M_2 e^T e.
\end{aligned} \tag{31}
$$

Combing (30) with (31), we have

$$D^+ V(e) \le e^T (A^T + A - 2k_1 I - 2abM_1 M_2 I)e - 2k_2 \sum_{i=1}^{4} |e_i|^{\gamma+1}. \tag{32}$$

From the fact that $0 < \gamma < 1$, one obtains

$$\left(\sum_{i=1}^{4} |e_i|^{\gamma+1} \right)^{\frac{1}{\gamma+1}} \ge \left(\sum_{i=1}^{4} |e_i|^2 \right)^{\frac{1}{2}}. \tag{33}$$
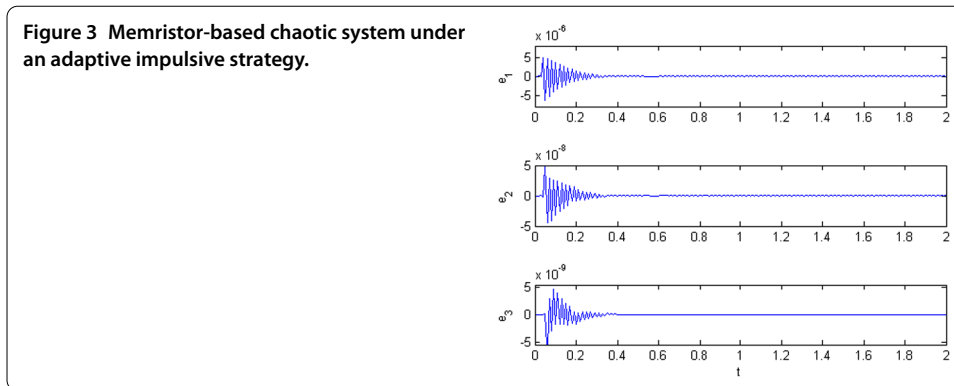
Also

$$\sum_{i=1}^{n} |e_i|^{\gamma+1} \ge \left(\sum_{i=1}^{n} |e_i|^2 \right)^{\frac{\gamma+1}{2}} = (e^T e)^{\frac{\gamma+1}{2}}. \tag{34}$$

Thus, based on the condition (i) in Theorem 2, we have for $t \ne t_k$

$$D^+ V(e) \le -2k_2 V(e)^{\frac{\gamma+1}{2}}. \tag{35}$$

When $t = t_k$, one obtains

$$
\begin{aligned}
V(t_k^+) &= (e + Be)^T(e + Be) \\
&= e(I + B^T)(I + B)e \\
&\le dV(t_k).
\end{aligned} \tag{36}
$$

Figure 3 Memristor-based chaotic system under an adaptive impulsive strategy.

Through Theorem 2 and Lemma 1, we can easily see that the system (28) is finite-time stable. This implies that the response system (26) is synchronized with the drive system (3) in a finite time. □

4 Simulation results

The numerical simulations are carried out using the fourth-order Runge-Kutta method. The initial states of the drive and response systems are $(10^{-10}, 0, 0, 0)$ and $(0, 0, 0, 0)$. The parameters of the drive systems are $\alpha = 10$, $\beta = 100/7$, $\gamma = 0.1$, $\xi = 9/7$, $a = 1/7$, and $b = 2/7$. Solving the inequality in Theorem 2, and choosing $B = \text{diag}(-0.9, -0.9, -0.9, -0.9)$, $k_1 = 0.02$, $k_2 = 0.01$, $\gamma = 0.3$, the response system synchronizes with the drive system as shown in Figure 3. It is easily shown that the state response curve of the error system is stable.

5 Conclusion

In this paper, the finite-time control and synchronization problems of memristor-based chaotic systems have been investigated. Some novel impulsive adaptive control laws which guarantee the memristor-based Chua circuits is stabilized and synchronized in finite time have been proposed. Moreover, simulation results were given to verify the effectiveness and feasibility of the method. Our future research topics mainly consider the time delay effects on the finite-time stability of the memristor-based nonlinear system.

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