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Passivity analysis and passive control for uncertain discrete switched time-delay systems via a simple switching signal design

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Abstract

In this paper, we consider the problem for passivity analysis and passive control of uncertain discrete switched systems with interval time-varying delay and linear fractional perturbations via a simple switching signal design. A new Lyapunov-Krasovskii functional is used to propose some LMI conditions that design the switching signal to guarantee the passivity and passive switching control of discrete switched time-delay systems. Jensen and Park inequalities combined with delay-partitioning approach are investigated to improve the conservativeness of the obtained results. Finally, some numerical examples and a water quality model illustrate the main proposed results.

Keywords: switching signal; passivity analysis; passive switching control; internal stability; discrete switched systems; interval time-varying delay; delay-partitioning approach

1 Introduction

The linear control systems and the complex or uncertain feedback systems can be bridged by the framework of switched linear systems [1]. The switched system is an important class of hybrid systems, which consists of some subsystems and a switching signal. The switching signal will handle the switching among subsystems. Many complicate nonlinear system behaviors can be produced under switching, such as multiple limit cycles and chaos [1, 2]. Switched systems are often encountered in many practical systems including automated highway systems, automotive engine control systems, chemical process, constrained robotics, mutlirate control, power systems and power electronics, robot manufacture, stepper motors, and water quality control [1–4]. It is also well known that instability or bad performance may be introduced by the existence of delay in a system [5–7]. Time-delay phenomena are often confronted in many practical engineering systems, such as chemical engineering systems, hydraulic systems, inferred grinding models, neural network circuits, nuclear reactor, and rolling mill systems. Hence, the problems of stability and stabilization for discrete switched systems with time delay have been investigated in recent years [1, 2, 8–17].

In the recent years, there are two interesting and well-known issues investigated for switched systems. (1) The stability property for each subsystem cannot guarantee that the overall system is stable under arbitrary switching [1, 2, 10, 14–16]. (2) The stability of a switched system may be achieved by selecting the switching signal even when each subsystem is unstable [1, 2, 11, 13, 18–23]. In [13], a design of switching signal is proposed to ensure the stability and stabilization of discrete switched systems with interval time-varying delay. In [16], the switching is identified to guarantee the stability of discrete switched time-delay system. In recent years, the passivity property was provided in diverse aspects, such as complexity analysis [24], fuzzy control [25], time-delay systems [1, 26–29], neural networks systems [29, 30], and signal processing [31]. The passivity theory was first introduced in circuit analysis, which is a promising approach to keep the internal stability of systems. Furthermore, linear fractional perturbations in [11, 18, 19, 32, 33] are more generalized than parameter ones in [10, 14, 15, 29, 30]. In this paper, a simple design scheme for switching signal in passivity analysis and passive control is proposed for discrete switched systems with interval time-varying delay and linear fractional perturbations. In [34], a delay-partitioning approach was proposed to improve the conservativeness of the developed results. In this paper, a new delay-partitioning approach is investigated to make more accurate evaluation for the allowable upper bound of interval time-varying delay. In [35], Jensen and Park inequalities are used to improve the main proposed results. To the best knowledge of the authors, results that combine the delay-partitioning approach and Jensen and Park inequalities are not reported in the past. Some numerical examples are made to demonstrate the obtained results. The practical application of a water quality model is also provided to illustrate the proposed results. From the simulation result, our proposed approach in this paper provides less conservative results. The main contribution of this paper can be highlighted as follows:

(1) The less conservative passivity analysis and passive switching control for discrete switched systems with linear fractional perturbations and interval time-varying delay via a switching signal design are considered.
(2) The proposed approach provides less LMI variables and a shorter program running time than some previous ones in the past.
(3) Jensen and Park inequalities combined with the delay-partitioning approach are used to improve the conservativeness of the developed results.
(4) The proposed design scheme for switching signal is more flexible than those in [12–14]. The proposed approach for switching signal design scheme can be easily applied to continuous switched time-delay systems.

In the past, some new relevant results and approaches had been proposed to achieve the performance of switched systems [36–39]. In [36], the Borne-Gentina practical stability of continuous switched systems is considered by the aggregation techniques. In [37], the stability and H_∞ control of switched systems with interval time-varying delay are guaranteed by input-output scaled small gain theorem approach. In [38], the stability for discrete switched nonlinear systems with unstable subsystems is considered by the T-S fuzzy model approach. In [39], the passivity of discrete switched nonlinear systems is studied by multiple storage functions and multiple supply rates approach. The proposed results in this paper can be considered and improved by the above developed approaches.

The notation used throughout this paper is as follows. For a matrix A, we denote the transpose by A^T, symmetric positive (negative) definite by $A > 0$ ($A < 0$); $A \leq B$ ($A < B$) means that $B - A$ is a symmetric positive semidefinite (definite) matrix; I denotes the identity matrix. Define $\overline{N} = \{1, 2, \ldots, N\}$, $A \backslash B = \{x | x \in A \text{ and } x \notin B\}$, $L_2(0, \infty) = \{w(k) | \sum_{k=0}^{\infty} w^T(k)w(k) < \infty\}$.

2 Problem statement and preliminaries

In this paper, we consider the following uncertain discrete switched system with time delay:

$$
\begin{aligned}
x(k+1) &= \left[A_\sigma + \Delta A_\sigma(k)\right]x(k) + \left[B_\sigma + \Delta B_\sigma(k)\right]x\bigl(k - r(k)\bigr) \\
&\quad + \left[D_\sigma + \Delta D_\sigma(k)\right]w(k), \qquad (1a) \\
z(k) &= \left[A_{z\sigma} + \Delta A_{z\sigma}(k)\right]x(k) + \left[B_{z\sigma} + \Delta B_{z\sigma}(k)\right]x\bigl(k - r(k)\bigr) \\
&\quad + \left[D_{z\sigma} + \Delta D_{z\sigma}(k)\right]w(k), \qquad (1b) \\
x(\theta) &= \varphi(\theta), \quad \theta = -r_M, -r_M + 1, \ldots, 0, \qquad (1c)
\end{aligned}
$$

where $x(k) \in \Re^n$, x_k is the state defined by $x_k(\theta) := x(k + \theta)$, $\forall \theta \in \{-r_M, -r_M + 1, \ldots, 0\}$, $w(k) \in \Re^p$ is a disturbance input, $z(k) \in \Re^p$ is a regulated output, σ is a switching signal in the finite set $\{1, 2, \ldots, N\}$ and will be selected to preserve the performance of the system, $\varphi(k) \in \Re^n$ denotes the initial function, the time-varying delay $r(k)$ is a function from $\{0, 1, 2, 3, \ldots\}$ to $\{1, 2, 3, \ldots\}$ such that $0 < r_m \le r(k) \le r_M$, where r_m and r_M are two given positive integers. The matrices A_i, B_i, D_i, A_{zi}, B_{zi}, D_{zi}, $i = 1, 2, \ldots, N$, are given constant matrices of appropriate dimensions; $\Delta A_i(k)$, $\Delta B_i(k)$, $\Delta D_i(k)$, $\Delta A_{zi}(k)$, $\Delta B_{zi}(k)$, and $\Delta D_{zi}(k)$ are some perturbed matrices satisfying the following conditions:

$$
\begin{aligned}
&\left[\Delta A_i(k) \quad \Delta B_i(k) \quad \Delta D_i(k)\right] = M_i \cdot \Delta_i(k) \cdot \left[N_{Ai} \quad N_{Bi} \quad N_{Di}\right], \qquad (1d) \\
&\left[\Delta A_{zi}(k) \quad \Delta B_{zi}(k) \quad \Delta D_{zi}(k)\right] = M_{zi} \cdot \Delta_{zi}(k) \cdot \left[N_{ZAi} \quad N_{ZBi} \quad N_{ZDi}\right], \qquad (1e) \\
&\Delta_i(k) = \left[I - \Gamma_i(k)\Xi_i\right]^{-1}\Gamma_i(k), \qquad \Xi_i \Xi_i^T < I, \qquad (1f) \\
&\Delta_{zi}(k) = \left[I - \Gamma_{zi}(k)\Xi_{zi}\right]^{-1}\Gamma_{zi}(k), \qquad \Xi_{zi}\Xi_{zi}^T < I, \qquad (1g)
\end{aligned}
$$

where $M_i \in \Re^{n \times q}$, $M_{zi} \in \Re^{p \times q}$, N_{Ai}, N_{Bi}, N_{Di}, N_{ZAi}, N_{ZBi}, and N_{ZDi}, $i = 1, 2, \ldots, N$, Ξ_i and Ξ_{zi} are some given constant matrices of appropriate dimensions; and $\Gamma_i(k)$ and $\Gamma_{zi}(k)$ are some unknown matrices satisfying

$$
\Gamma_i^T(k)\Gamma_i(k) \le I, \qquad \Gamma_{zi}^T(k)\Gamma_{zi}(k) \le I. \qquad (1h)
$$

Now we propose the following switching domains:

$$
\Omega_i(U_i) = \left\{x \in \Re^n : x^T U_i x \ge 0\right\}, \quad i = 1, 2, \ldots, N, \qquad (2a)
$$

where the matrices $U_i > 0$, $i = 1, 2, \ldots, N$, will be selected from our developed results, and

$$
\begin{aligned}
&\overline{\Omega}_1 = \Omega_1, \qquad \overline{\Omega}_2 = \Omega_2 \backslash \overline{\Omega}_1, \qquad \overline{\Omega}_3 = \Omega_3 \backslash \overline{\Omega}_1 \backslash \overline{\Omega}_2, \qquad \ldots, \\
&\overline{\Omega}_N = \Omega_N \backslash \overline{\Omega}_1 \backslash \cdots \backslash \overline{\Omega}_{N-1}.
\end{aligned} \qquad (2b)
$$

From the above definition of the domains, the switching signal can be selected by

$$\sigma\big(x(k)\big) = i, \quad \forall x(k) \in \overline{\Omega}_i, \tag{2c}$$

where $\overline{\Omega}_i$ is defined in (2b).

The following lemmas will be used to obtain the main proposed result.

Lemma 1 *If there exist some constants* $0 \le \alpha_i \le 1$, $i \in \overline{N}$, $\sum_{i=1}^{N} \alpha_i = 1$, *some matrices* $U_i > 0$, $i \in \overline{N}$, *such that*

$$\sum_{i=1}^{N} \alpha_i \cdot U_i > 0,$$

we have

$$\bigcup_{i=1}^{N} \overline{\Omega}_i = \Re^n \quad \textit{and} \quad \overline{\Omega}_i \cap \overline{\Omega}_j = \Phi, \quad \forall i \neq j,$$

where Φ *is the empty set, and* $\overline{\Omega}_i$ *is defined in* (2b).

Proof By the definition of $\overline{\Omega}_i$ in (2b), $\overline{\Omega}_i \cap \overline{\Omega}_j = \Phi$ is trivial. For any $x \in \Re^n$ with $\sum_{i=1}^{N} \alpha_i \cdot U_i > 0$, we have

$$x^T \Bigg[\sum_{i=1}^{N} \alpha_i \cdot U_i\Bigg] x = \sum_{i=1}^{N} \alpha_i \cdot x^T U_i x \ge 0, \quad \forall x \in \Re^n.$$

This condition implies

$$x^T U_i x \ge 0 \quad \text{for some } i \in \overline{N}.$$

From (2a) and (2b) we have

$$x \in \Omega_i(U_i) \quad \text{for some } i \in \overline{N},$$
$$x \in \overline{\Omega}_j(U_j) \quad \text{for some } i, j \in \overline{N}, i \ge j.$$

The proof is completed. □

Remark 1 In the recent years, some proposed switching domains are shown as follows:

(a) In [13], the switching domains are selected as

$$\Omega_i(P, U, A_i) = \big\{x \in \Re^n : x^T\big[(r_M - r_m) \cdot U - A_i^T P - PA_i\big]x < 0\big\}, \quad i = 1, 2, \ldots, N,$$

where the matrices $P > 0$, $U > 0$, $\overline{\Omega}_1 = \Omega_1, \overline{\Omega}_2 = \Omega_2 \backslash \overline{\Omega}_1, \ldots, \overline{\Omega}_N = \Omega_N \backslash (\bigcup_{i=1}^{N-1} \overline{\Omega}_i)$.

(b) In [18], the switching domains are selected as

$$\Omega_i(P, U, A_i) = \big\{x \in \Re^n : x^T\big(A_i^T PA_i\big)x \le x^T Ux\big\}, \quad i = 1, 2, \ldots, N,$$

where the matrices $P > 0$, $U > 0$, $\overline{\Omega}_1 = \Omega_1, \overline{\Omega}_2 = \Omega_2 \backslash \overline{\Omega}_1, \ldots, \overline{\Omega}_N = \Omega_N \backslash \overline{\Omega}_1 \backslash \cdots \backslash \overline{\Omega}_{N-1}$.

(c) In [19], the switching domains are selected as

$$\Omega_i(P,U,A_i) = \{x \in \Re^n : x^T(A_i^T P A_i)x \le x^T U_i x\}, \quad i = 1,2,\ldots,N,$$

where the matrices $P > 0$, $U_i > 0$, $i = 1,2,\ldots,N$, and $\overline{\Omega}_i$ is defined in (b).

In this paper, the switching domains are defined in (2a)-(2b) with

$$\Omega_i(U_i) = \{x \in \Re^n : x^T U_i x \ge 0\},$$

where the matrices $U_i = U_i^T$, $i = 1,2,\ldots,N$. The proposed approach in this paper does not depend on the system parameters A_i, which shows that the proposed scheme is more simple and flexible than the above results. It can be easily extended to continuous switched systems to design the switching signal under consideration.

Lemma 2 [40] *For a given symmetric matrix* $S = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}$, *the following conditions are equivalent*:

(1) $S < 0$,

(2) $S_{22} < 0$, $S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$.

Lemma 3 [32] *Suppose that* $\Delta_i(k)$ *is defined in* (1f) *and satisfies* (1h). *Then for real matrices* U_i, W_i, *and* X_i *with* $X_i = X_i^T$, *the following conditions are equivalent*:

(I) *The following inequality is satisfied*:

$$X_i + U_i\Delta_i(k)W_i + W_i^T\Delta_i^T(k)U_i^T < 0.$$

(II) *There exists a scalar* $\varepsilon_i > 0$ *such that*

$$\begin{bmatrix} X_i & U_i & \varepsilon_i \cdot W_i^T \\ * & -\varepsilon_i \cdot I & \varepsilon_i \cdot \Xi_i^T \\ * & * & -\varepsilon_i \cdot I \end{bmatrix} < 0,$$

where the matrix Ξ_i *is defined in* (1f).

Lemma 4 (Discrete Jensen inequality [35]) *For any matrix* $R > 0$, *integers* $r_1 < r_2$, *and a vector function* $\omega(i) \in \Re^n$, *the following inequality is satisfied*:

$$-(r_2 - r_1) \cdot \sum_{i=k-r_2}^{k-r_1-1} \omega^T(i)R\omega(i) \le -\left[\sum_{i=k-r_2}^{k-r_1-1} \omega(i)\right]^T R \left[\sum_{i=k-r_2}^{k-r_1-1} \omega(i)\right].$$

Lemma 5 (Park inequality in [35]) *For any matrices* $V \in \Re^{n\times n} > 0$, $M_1, M_2 \in \Re^{n\times m}$, *a positive real number* $0 < \alpha < 1$, *and a vector* $\omega \in \Re^m$, *there exists a matrix* $X \in \Re^{n\times n}$ *such that*

$$\begin{bmatrix} V & X \\ * & V \end{bmatrix} > 0.$$

Then the following inequality is satisfied:

$$-\left[\frac{1}{\alpha}\omega^T M_1^T V M_1\omega + \frac{1}{1-\alpha}\omega^T M_2^T V M_2\omega\right] \leq -\omega^T \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}^T \begin{bmatrix} V & X \\ * & V \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \omega.$$

Definition 1 The discrete switched system of (1a)-(1h) with the switching signal in (2c) is called passive if there exists a constant $\gamma > 0$ such that

$$-\gamma \cdot \sum_{k=0}^{\ell} w^T(k)w(k) \leq 2 \cdot \sum_{k=0}^{\ell} z^T(k)w(k)$$

for all $\ell > 0$ and zero initial condition $x(\theta) = 0$, $\theta = -r_M, -r_M + 1, \ldots, 0$. If the parameter ℓ is selected as ∞, then the disturbance input w belongs to $L_2(0, \infty)$.

In this paper, the following partitions are selected:

$$0 < r_m = r_0 < r_1 < r_2 < \cdots < r_{p-1} < r_p = r_M,$$

where p is the partition number, and r_i, $i = 1, 2, \ldots, p$, are some positive integers.

Theorem 1 *For some selected integers* $0 < r_m = r_0 < r_1 < r_2 < \cdots < r_{p-1} < r_p = r_M$, *constants* $0 \leq \alpha_j \leq 1$, $j = 1, 2, \ldots, N$, *and* $\sum_{i=1}^N \alpha_i = 1$, *system* (1a)-(1h) *is passive by the designed switching signal in* (2c) *if there exist some* $n \times n$ *symmetric matrices* $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$, $R_3 > 0$, $S > 0$, $T > 0$, $V_1 > 0$, $V_2 > 0$, U_j, W_1, W_2, *an* $n \times n$ *matrix* X, *and constants* $\varepsilon_j > 0$, $j = 1, 2, \ldots, N$ *such that the following LMI conditions are satisfied*:

$$R_2 + W_1 > 0, \qquad R_2 + W_2 > 0, \tag{3a}$$

$$\begin{bmatrix} Q_1 & W_1 \\ * & Q_2 \end{bmatrix} > 0, \qquad \begin{bmatrix} Q_1 & W_2 \\ * & Q_2 \end{bmatrix} > 0, \qquad \begin{bmatrix} V_1 & X \\ * & V_1 \end{bmatrix} > 0, \tag{3b}$$

$$\Sigma_j^i = \begin{bmatrix} \Sigma_{1j}^i & \Sigma_{2j} \\ * & \Sigma_{3j} \end{bmatrix} < 0, \quad i = 1, 2, \ldots, p, j = 1, 2, \ldots, N, \tag{3c}$$

$$\sum_{i=1}^N \alpha_i \cdot U_i > 0, \tag{3d}$$

where Σ_{1j}^i, Σ_{2j}, Σ_{3j}, Σ_{11j}, Σ_{13j}, Σ_{14j} *are defined by*

$$\Sigma_{1j}^i = \begin{bmatrix} \Sigma_{11j} & 0 & \Sigma_{13j} & \Sigma_{14j} & \Sigma_{15j} \\ * & \Sigma_{22j}^i & \Sigma_{23j}^i & \Sigma_{24j}^i & \Sigma_{25j} \\ * & * & \Sigma_{33j}^i & \Sigma_{34j} & 0 \\ * & * & * & \Sigma_{44j}^i & 0 \\ * & * & * & * & \Sigma_{55j} \end{bmatrix},$$

$$\Sigma_{2j} = \begin{bmatrix} \Sigma_{16j} & \Sigma_{17j} & 0 & 0 & \Sigma_{110j} & \Sigma_{111j} \\ \Sigma_{26j} & \Sigma_{27j} & 0 & 0 & \Sigma_{210j} & \Sigma_{211j} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \Sigma_{56j} & \Sigma_{57j} & 0 & \Sigma_{59j} & \Sigma_{510j} & \Sigma_{511j} \end{bmatrix},$$

$$\Sigma_{3j} = \begin{bmatrix} \Sigma_{66j} & 0 & \Sigma_{68j} & 0 & 0 & 0 \\ * & \Sigma_{77j} & \Sigma_{78j} & 0 & 0 & 0 \\ * & * & \Sigma_{88j} & 0 & \Sigma_{810j} & 0 \\ * & * & * & \Sigma_{99j} & 0 & \Sigma_{911j} \\ * & * & * & * & \Sigma_{1010j} & 0 \\ * & * & * & * & * & \Sigma_{1111j} \end{bmatrix},$$

$$i = 1, 2, \ldots, p, j = 1, 2, \ldots, N, \tag{3e}$$

$$\Sigma_{11j} = -P + T + (r_M - r_m)^2 \cdot Q_1 - R_1 - R_3 + U_j,$$

$$\Sigma_{13j} = R_1, \qquad \Sigma_{14j} = R_3, \qquad \Sigma_{15j} = -A_{zj}^T,$$

$$\Theta = r_M^2 \cdot R_1 + (r_M - r_m)^2 (Q_2 + R_2 + V_1 + V_2) + r_m^2 \cdot R_3,$$

$$\Sigma_{16j} = (A_j - I)^T \Theta, \qquad \Sigma_{17j} = A_j^T P, \qquad \Sigma_{110} = \varepsilon_j \cdot N_{Aj}^T,$$

$$\Sigma_{111j} = \varepsilon_j \cdot N_{ZAj}^T, \qquad \gamma_{23}^i = \frac{(r_M - r_m)}{(r_M - r_{i-1})}, \qquad \gamma_{24}^i = \frac{(r_M - r_m)}{(r_i - r_m)},$$

$$\Sigma_{22j}^i = -\gamma_{23}^i \cdot (R_2 + W_2) - \gamma_{24}^i \cdot (R_2 + W_1) - (r_M - r_m) \cdot (W_1 - W_2) - 2V_1 + X + X^T,$$

$$\Sigma_{23j}^i = \gamma_{23}^i \cdot (R_2 + W_2) + V_1 - X^T,$$

$$\Sigma_{24}^i = \gamma_{24}^i \cdot (R_2 + W_1) - X + V_1, \qquad \Sigma_{25j} = -B_{zj}^T,$$

$$\Sigma_{26j} = B_j^T \Theta, \qquad \Sigma_{27j} = B_j^T P, \qquad \Sigma_{210j} = \varepsilon_j \cdot N_{Bj}^T, \qquad \Sigma_{211j} = \varepsilon_{1j} \cdot N_{ZBj}^T,$$

$$\Sigma_{33j}^i = -S - R_1 - \gamma_{23}^i \cdot (R_2 + W_2) - (r_M - r_m) \cdot W_2 - V_1 - V_2, \qquad \Sigma_{34j} = V_2 + X,$$

$$\Sigma_{44j}^i = -(T - S) - \gamma_{24}^i \cdot (R_2 + W_1) - R_3 + (r_M - r_m) \cdot W_1 - V_1 - V_2,$$

$$\Sigma_{55j} = -D_{zj} - D_{zj}^T - \gamma \cdot I, \qquad \Sigma_{56j} = D_j^T \Theta, \qquad \Sigma_{57j} = D_j^T P, \qquad \Sigma_{59j} = -M_{zj},$$

$$\Sigma_{510j} = \varepsilon_j \cdot N_{Dj}^T, \qquad \Pi_{511j} = \varepsilon_{2j} \cdot N_{ZDj}^T, \qquad \Sigma_{66j} = -\Theta, \qquad \Sigma_{68j} = \Theta^T M_j,$$

$$\Sigma_{77j} = -P, \qquad \Sigma_{78j} = PM_j, \qquad \Sigma_{88j} = -\varepsilon_j \cdot I, \qquad \Sigma_{810j} = \varepsilon_j \cdot \Xi_j^T,$$

$$\Sigma_{99j} = -\varepsilon_j \cdot I, \qquad \Sigma_{911j} = \varepsilon_{1j} \cdot \Xi_{zj}^T, \qquad \Sigma_{1010j} = -\varepsilon_j \cdot I, \qquad \Sigma_{1111j} = -\varepsilon_j \cdot I. \tag{3f}$$

Proof Define the Lyapunov-Krasovskii functional by

$$\begin{aligned} V(x_k) &= x^T(k)Px(k) + (r_M - r_m) \cdot \sum_{j=-r_M+1}^{-r_m} \sum_{i=k-1+j}^{k-1} z^T(i)\hat{Q}z(i) \\ &\quad + r_M \cdot \sum_{j=-r_M+1}^{0} \sum_{i=k-1+j}^{k-1} y^T(i)R_1 y(i) \\ &\quad + (r_M - r_m) \cdot \sum_{j=-r_M+1}^{-r_m} \sum_{i=k-1+j}^{k-1} y^T(i)[R_2 + V_1 + V_2]y(i) \\ &\quad + r_m \cdot \sum_{j=-r_m+1}^{0} \sum_{i=k-1+j}^{k-1} y^T(i)R_3 y(i) + \sum_{i=k-r_M}^{k-1-r_m} x^T(i)Sx(i) + \sum_{i=k-r_m}^{k-1} x^T(i)Tx(i), \end{aligned} \tag{4}$$

where $P > 0$, $\hat{Q} = \mathrm{diag}[Q_1\ Q_2] > 0$, $R_1 > 0$, $R_2 > 0$, $R_3 > 0$, $V_1 > 0$, $S > 0$, $T > 0$, $y(i) = x(i+1) - x(i)$, and $z(i) = [x(i)^T\ y(i)^T]^T$. The difference of functional (4) along the solutions of system

(1a)-(1h) has the form

$$\begin{aligned}
\Delta V(x_k) &= V(x_{k+1}) - V(x_k) \\
&= \bigl[x^T(k+1)Px(k+1) - x^T(k)Px(k)\bigr] + (r_M - r_m)^2 \cdot z^T(k)\hat{Q}z(k) \\
&\quad - (r_M - r_m) \cdot \sum_{i=k-r_M}^{k-r_m-1} z^T(i)\hat{Q}z(i) + r_M^2 \cdot y^T(k)R_1y(k) \\
&\quad - r_M \cdot \sum_{i=k-r_M}^{k-1} y^T(i)R_1y(i) + (r_M - r_m)^2 \cdot y^T(k)[R_2 + V_1 + V_2]y(k) \\
&\quad - (r_M - r_m) \cdot \sum_{i=k-r_M}^{k-r_m-1} y^T(i)[R_2 + V_1 + V_2]y(i) + r_m^2 \cdot y^T(k)R_3y(k) \\
&\quad - r_m \cdot \sum_{i=k-r_m}^{k-1} y^T(i)R_3y(i) + x^T(k-r_m)Sx(k-r_m) \\
&\quad - x^T(k-r_M)Sx(k-r_M) + \bigl[x^T(k)Tx(k) - x^T(k-r_m)Tx(k-r_m)\bigr].
\end{aligned} \tag{5}$$

By the definitions $y(i) = x(i+1) - x(i)$ and $z(i) = [x(i)^T \ y(i)^T]^T$ we have

$$\begin{aligned}
-\sum_{i=k-r_M}^{k-r_m-1} z^T(i)\hat{Q}z(i) &= -\sum_{i=k-r_M}^{k-r(k)-1} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}^T \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix} \\
&\quad - \sum_{i=k-r(k)}^{k-r_m-1} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}^T \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix},
\end{aligned} \tag{6a}$$

$$\begin{aligned}
\lambda_1 &= \bigl[x^T(k-r_m)W_1x(k-r_m) - x^T\bigl(k-r(k)\bigr)W_1x\bigl(k-r(k)\bigr)\bigr] \\
&\quad - \sum_{i=k-r(k)}^{k-r_m-1} \bigl[y^T(i)W_1y(i) + 2x^T(i)W_1y(i)\bigr] = 0,
\end{aligned} \tag{6b}$$

$$\begin{aligned}
\lambda_2 &= \bigl[x^T\bigl(k-r(k)\bigr)W_2x\bigl(k-r(k)\bigr) - x^T(k-r_M)W_2x(k-r_M)\bigr] \\
&\quad - \sum_{i=k-r_M}^{k-r(k)-1} \bigl[y^T(i)W_2y(i) + 2x^T(i)W_2y(i)\bigr] = 0.
\end{aligned} \tag{6c}$$

From the previous derivations we obtain the following result:

$$\begin{aligned}
&\Delta V(x_k) + (r_M - r_m) \cdot (\lambda_1 + \lambda_2) + \bigl[-2z^T(k)w(k) - \gamma \cdot w^T(k)w(k)\bigr] \\
&\quad = x^T(k+1)Px(k+1) - x^T(k)[P - T]x(k) + (r_M - r_m)^2 \cdot x^T(k)Q_1x(k) \\
&\qquad - (r_M - r_m) \cdot x^T\bigl(k-r(k)\bigr)[W_1 - W_2]x\bigl(k-r(k)\bigr) \\
&\qquad + \bigl[x(k+1) - x(k)\bigr]^T\bigl[r_M^2 \cdot R_1 + (r_M - r_m)^2(Q_2 + R_2 + V_1 + V) + r_m^2 \cdot R_3\bigr] \\
&\qquad \cdot \bigl[x(k+1) - x(k)\bigr] - r_M \cdot \sum_{i=k-r_M}^{k-1} y^T(i)R_1y(i)
\end{aligned}$$

$$
\begin{aligned}
&- (r_M - r_m) \cdot \left[\sum_{i=k-r_M}^{k-r(k)-1} y^T(i)[R_2 + W_2]y(i) + \sum_{i=k-r(k)}^{k-r_m-1} y^T(i)[R_2 + W_1]y(i) \right] \\
&- (r_M - r_m) \cdot \left\{ \sum_{i=k-r_M}^{k-r_m-1} y^T(i) V_1 y(i) + \sum_{i=k-r_M}^{k-r_m-1} y^T(i) V_2 y(i) \right\} \\
&- r_m \cdot \left[\sum_{i=k-r_m}^{k-1} y^T(i) R_3 y(i) \right] - x^T(k - r_m)\big[T - (r_M - r_m) \cdot W_1 - S\big]x(k - r_m) \\
&- x^T(k - r_M)\big[S + (r_M - r_m) \cdot W_2\big]x(k - r_M) \\
&- (r_M - r_m) \cdot \sum_{i=k-r_M}^{k-r(k)-1} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}^T \begin{bmatrix} Q_1 & W_2 \\ * & Q_2 \end{bmatrix} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix} \\
&- (r_M - r_m) \cdot \sum_{i=k-r(k)}^{k-r_m-1} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}^T \begin{bmatrix} Q_1 & W_1 \\ * & Q_2 \end{bmatrix} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix} \\
&+ \big[-2z^T(k)w(k) - \gamma \cdot w^T(k)w(k)\big]. \qquad (6d)
\end{aligned}
$$

Without loss of generality, assuming that $0 \le r_m < r(k) < r_M$ and $r_{i-1} \le r(k) \le r_i$, $i = 1, 2, \ldots, p$, for some k, by Lemma 4 we have the following results:

$$
\begin{aligned}
-r_M \cdot \sum_{i=k-r_M}^{k-1} y^T(i) R_1 y(i) &\le -\left[\sum_{i=k-r_M}^{k-1} y^T(i) \right]^T R_1 \left[\sum_{i=k-r_M}^{k-1} y^T(i) \right] \\
&= -\big[x(k) - x(k - r_M)\big]^T R_1 \big[x(k) - x(k - r_M)\big], \qquad (7a)
\end{aligned}
$$

$$
\begin{aligned}
&-(r_M - r_m) \cdot \left[\sum_{i=k-r_M}^{k-r(k)-1} y^T(i)[R_2 + W_2]y(i) + \sum_{i=k-r(k)}^{k-r_m-1} y^T(i)[R_2 + W_1]y(i) \right] \\
&\quad \le -\frac{(r_M - r_m)}{(r_M - r(k))} \cdot \xi_1 - \frac{(r_M - r_m)}{(r(k) - r_m)} \cdot \xi_2 \le -\gamma_{23}^i \cdot \xi_1 - \gamma_{24}^i \cdot \xi_2, \qquad (7b)
\end{aligned}
$$

$$
-r_m \cdot \sum_{i=k-r_m}^{k-1} y^T(i) R_3 y(i) \le -\big[x(k) - x(k - r_m)\big]^T R_3 \big[x(k) - x(k - r_m)\big], \qquad (7c)
$$

$$
\begin{aligned}
&-(r_M - r_m) \cdot \sum_{i=k-r_M}^{k-r_m-1} y^T(i) V_1 y(i) \\
&\quad = -(r_M - r_m) \cdot \left[\sum_{i=k-r_M}^{k-r(k)-1} y^T(i) V_1 y(i) + \sum_{i=k-r(k)}^{k-r_m-1} y^T(i) V_1 y(i) \right] \\
&\quad \le -\frac{(r_M - r_m)}{(r_M - r(k))} \cdot \big[\big(x\big(k - r(k)\big) - x(k - r_M)\big)^T V_1 \big(x\big(k - r(k)\big) - x(k - r_M)\big)\big] \\
&\qquad - \frac{(r_M - r_m)}{(r(k) - r_m)} \cdot \big[\big(x(k - r_m) - x\big(k - r(k)\big)\big)^T V_1 \big(x(k - r_m) - x\big(k - r(k)\big)\big)\big], \qquad (7d)
\end{aligned}
$$

$$
\begin{aligned}
&-(r_M - r_m) \cdot \sum_{i=k-r_M}^{k-r_m-1} y^T(i) V_2 y(i) \\
&\quad \le -\big[x(k - r_m) - x(k - r_M)\big]^T V_2 \big[x(k - r_m) - x(k - r_M)\big], \qquad (7e)
\end{aligned}
$$

where

$$\begin{aligned} \xi_1 &= \bigl[x\bigl(k-r(k)\bigr)-x(k-r_M)\bigr]^T(R_2+W_2)\bigl[x\bigl(k-r(k)\bigr)-x(k-r_M)\bigr], \\ \xi_2 &= \bigl[x(k-r_m)-x\bigl(k-r(k)\bigr)\bigr]^T(R_2+W_1)\bigl[x(k-r_m)-x\bigl(k-r(k)\bigr)\bigr]. \end{aligned} \tag{7f}$$

Define

$$Z^T(k)=\bigl[x^T(k) \quad x^T(k-r(k)) \quad x^T(k-r_M) \quad x^T(k-r_m) \quad w^T(k)\bigr]. \tag{8a}$$

By Park inequality in Lemma 5 with (3b) and (7d) we have

$$\begin{aligned} &-(r_M-r_m)\cdot\sum_{i=k-r_M}^{k-r_m-1}y^T(i)U_1y(i) \\ &\quad\le -Z^T(k)\begin{bmatrix}M_1\\M_2\end{bmatrix}^T\begin{bmatrix}V_1 & X\\ * & V_1\end{bmatrix}\begin{bmatrix}M_1\\M_2\end{bmatrix}Z(k) \\ &\quad= Z^T(k)\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ * & -2V_1+X+X^T & V_1-X^T & -X+V_1 & 0\\ * & * & -V_1 & X & 0\\ * & * & * & -V_1 & 0\\ * & * & * & * & 0\end{bmatrix}Z(k), \end{aligned} \tag{8b}$$

where $M_1=[0\ I\ -I\ 0\ 0]$ and $M_2=[0\ -I\ 0\ I\ 0]$. Assuming that $\sigma(x(k))=j\in\overline{N}$, from (3a)-(3c), (6d), and (8a)-(8b) we derive the following result:

$$\Delta V(x_k)+\bigl[-2z^T(k)w(k)-\gamma\cdot w^T(k)w(k)\bigr]\le -x^T(k)U_jx(k)+Z^T(k)\cdot\hat{\Sigma}_j^i\cdot Z(k), \tag{9a}$$

where

$$\begin{aligned} \hat{\Sigma}_j^i &= \Sigma_{1j}^i-\begin{bmatrix}\overline{\Sigma}_{16j}\\ \overline{\Sigma}_{26j}\\ 0\\ 0\\ \overline{\Sigma}_{56j}\end{bmatrix}\Sigma_{66j}^{-1}\begin{bmatrix}\overline{\Sigma}_{16j}\\ \overline{\Sigma}_{26j}\\ 0\\ 0\\ \overline{\Sigma}_{56j}\end{bmatrix}^T-\begin{bmatrix}\overline{\Sigma}_{17j}\\ \overline{\Sigma}_{27j}\\ 0\\ 0\\ \overline{\Sigma}_{57j}\end{bmatrix}\Sigma_{77j}^{-1}\begin{bmatrix}\overline{\Sigma}_{17j}\\ \overline{\Sigma}_{27j}\\ 0\\ 0\\ \overline{\Sigma}_{57j}\end{bmatrix}^T, \\ \Sigma_{1j}^i &= \begin{bmatrix}\Sigma_{11j} & 0 & \Sigma_{13j} & \Sigma_{14j} & \overline{\Sigma}_{15j}\\ * & \Sigma_{22j}^i & \Sigma_{23j}^i & \Sigma_{24j}^i & \overline{\Sigma}_{25j}\\ * & * & \Sigma_{33j}^i & 0 & 0\\ * & * & * & \Sigma_{44j}^i & 0\\ * & * & * & * & \overline{\Sigma}_{55j}\end{bmatrix}, \\ \overline{\Sigma}_{15j} &= -(A_{zj}+\Delta A_{zj})^T, \qquad \overline{\Sigma}_{16j}=(A_j+\Delta A_j-I)^T\Theta, \qquad \overline{\Sigma}_{17j}=(A_j+\Delta A_j)^TP, \\ \overline{\Sigma}_{25j} &= -(B_{zj}+\Delta B_{zj})^T, \qquad \overline{\Sigma}_{26j}=(B_j+\Delta B_j)^T\Theta, \qquad \overline{\Sigma}_{27j}=(B_j+\Delta B_j)^TP, \\ \overline{\Sigma}_{55j} &= -(D_{zj}+\Delta D_{zj})-(D_{zj}+\Delta D_{zj})^T-\gamma\cdot I, \\ \overline{\Sigma}_{56j} &= (D_j+\Delta D_j)^T\Theta, \qquad \overline{\Sigma}_{57j}=(D_j+\Delta D_j)^TP, \end{aligned} \tag{9b}$$

Σ_{klj} and Σ_{klj}^i, $k,l=1,2,\ldots,7$, and Θ are defined in (3f).

Define

$$\overline{\Sigma}_j^i = \begin{bmatrix} \Sigma_{1j}^i & \overline{\Sigma}_{2j} \\ * & \Sigma_{3j} \end{bmatrix} = \begin{bmatrix} \Sigma_{1j}^i & \Sigma_{2j} \\ * & \Sigma_{3j} \end{bmatrix} + \Pi_j \overline{\Delta}_j(k) \Psi_j^T + \Psi_j \overline{\Delta}_j^T(k) \Pi_j^T, \tag{9c}$$

where

$$\overline{\Sigma}_{2j} = \begin{bmatrix} \overline{\Sigma}_{16j} & \overline{\Sigma}_{17j} \\ \overline{\Sigma}_{26j} & \overline{\Sigma}_{27j} \\ 0 & 0 \\ 0 & 0 \\ \overline{\Sigma}_{56j} & \overline{\Sigma}_{57j} \end{bmatrix}, \qquad \Sigma_{2j} = \begin{bmatrix} \Sigma_{16j} & \Sigma_{17j} \\ \Sigma_{26j} & \Sigma_{27j} \\ 0 & 0 \\ 0 & 0 \\ \Sigma_{56j} & \Sigma_{57j} \end{bmatrix},$$

$$\Sigma_{3j} = \begin{bmatrix} \Sigma_{66j} & 0 \\ 0 & \Sigma_{77j} \end{bmatrix}, \qquad \Pi_j = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \Sigma_{68j}^T & \Sigma_{78j}^T \\ 0 & 0 & 0 & 0 & \Sigma_{59j}^T & 0 & 0 \end{bmatrix}^T,$$

$$\begin{aligned} \overline{\Delta}_j(k) &= \begin{bmatrix} \Delta_j(k) & 0 \\ 0 & \Delta_{zj}(k) \end{bmatrix} = \begin{bmatrix} I - \Gamma_j(k)\Xi_j & 0 \\ 0 & I - \Gamma_{zj}(k)\Xi_{zj} \end{bmatrix}^{-1} \begin{bmatrix} \Gamma_i(k) & 0 \\ 0 & \Gamma_{zi}(k) \end{bmatrix} \\ &= \left\{ \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} \Gamma_j(k) & 0 \\ 0 & \Gamma_{zj}(k) \end{bmatrix} \begin{bmatrix} \Xi_j & 0 \\ 0 & \Xi_{zj} \end{bmatrix} \right\}^{-1} \begin{bmatrix} \Gamma_i(k) & 0 \\ 0 & \Gamma_{zi}(k) \end{bmatrix}, \end{aligned} \tag{9d}$$

$$\Psi_j = \begin{bmatrix} N_{Aj} & N_{Bj} & 0 & 0 & N_{Dj} & 0 & 0 \\ N_{ZAj} & N_{ZBj} & 0 & 0 & N_{ZDj} & 0 & 0 \end{bmatrix}^T.$$

By (3d) with Lemma 1 and switching signal in (2c) we have

$$x^T(k) U_j x(k) \geq 0, \quad \forall x(k) \in \overline{\Omega}_j. \tag{10}$$

By Lemmas 2 and 3 with (9d), the condition $\Sigma_j^i < 0$ in (3c) implies $\overline{\Sigma}_j^i < 0$ in (9c), which also implies $\hat{\Sigma}_j^i < 0$ in (9b). From (10) by summing (9a) from 0 to ℓ we derive the following condition:

$$V(x_\ell) - V(\varphi_0) + \sum_{k=0}^{\ell} \left[-2 \cdot z^T(k) w(k) - \gamma \cdot w^T(k) w(k) \right] \leq 0.$$

With zero initial condition ($\varphi(k) = 0, -r_M \leq k \leq 0$) we have

$$V(\varphi_0) = 0.$$

By the definition of the functional $V(x_k)$ in (4) we have

$$V(x_\ell) \geq 0.$$

From the previous derivations the following condition is guaranteed:

$$-\gamma \cdot \sum_{k=0}^{\ell} \left[w^T(k) w(k) \right] \leq 2 \cdot \sum_{k=0}^{\ell} \left[z^T(k) w(k) \right].$$

By Definition 1 the system (1a)-(1h) is passive by the switching signal designed in (2c). This completes this proof. □

Remark 2 In view of (9a) in the proof of Theorem 1, we have $\Delta V(x_k) < 0$ when $w(k) = 0$. The internal stability of system (1a)-(1h) can be achieved by the proposed results.

Remark 3 The matrix perturbations in (1d)-(1h) are usually called linear fractional perturbations [11, 18, 19, 32, 33]. The parametric perturbations in [10, 14, 15, 29, 30] are the special conditions of system under consideration with $\Xi_i = 0$, $\Xi_{zi} = 0$, $i \in \overline{N}$.

Remark 4 The Lyapunov functional proposed in (4) is different from our previous ones in [18, 19]. In this paper, upper and lower bounds of delay are used instead of interval time-varying $r(k)$ in [18, 19]. Hence, some complicated derivations can be ignored in this paper. On the other hand, the discrete Jensen inequality approach is used instead of the nonnegative inequality approach in [18, 19]. Since the number and dimension of variables have been reduced, the LMI program can be formulated easily. In this paper, we also use a delay-partitioning approach to improve the conservativeness of the developed results. Uniform or nonuniform partitions can be performed by our proposed results.

3 Passive switching control for uncertain discrete switched system

Next, we consider the passive switching control of the following system:

$$x(k+1) = \big[A_\sigma + \Delta A_\sigma(k)\big]x(k) + \big[B_\sigma + \Delta B_\sigma(k)\big]x\big(k - r(k)\big) + \big[D_{w\sigma} + \Delta D_{w\sigma}(k)\big]w(k) + \big[D_{u\sigma} + \Delta D_{u\sigma}(k)\big]u(k), \quad k = 0,1,2,\ldots, \tag{11a}$$

$$z(k) = \big[A_{z\sigma} + \Delta A_{z\sigma}(k)\big]x(k) + \big[B_{z\sigma} + \Delta B_{z\sigma}(k)\big]x\big(k - r(k)\big) + \big[D_{zw\sigma} + \Delta D_{zw\sigma}(k)\big]w(k), \quad k = 0,1,2,\ldots, \tag{11b}$$

$$x(\theta) = \varphi(\theta), \quad \theta = -r_M, -r_M + 1, \ldots, 0, \tag{11c}$$

where $u(k) \in \Re^{\upsilon}$ is the control input, D_{ui}, $i = 1,2,\ldots,N$, are some given constant matrices of appropriate dimensions. Other definitions are shown in system (1a)-(1h). $\Delta A_i(k)$, $\Delta B_i(k)$, $\Delta D_{wi}(k)$, $\Delta D_{ui}(k)$, $\Delta A_{zi}(k)$, $\Delta B_{zi}(k)$, and $\Delta D_{zwi}(k)$ are some perturbed matrices satisfying the following conditions:

$$\begin{bmatrix}\Delta A_i(k) & \Delta B_i(k) & \Delta D_{wi}(k) & \Delta D_{ui}(k)\end{bmatrix} = M_i \cdot \Delta_i(k) \cdot \begin{bmatrix}N_{Ai} & N_{Bi} & N_{DWi} & N_{DUi}\end{bmatrix}, \tag{11d}$$

$$\begin{bmatrix}\Delta A_{zi}(k) & \Delta B_{zi}(k) & \Delta D_{zwi}(k)\end{bmatrix} = M_{Zi} \cdot \Delta_{Zi}(k) \cdot \begin{bmatrix}N_{ZAi} & N_{ZBi} & N_{ZWi}\end{bmatrix}, \tag{11e}$$

$$\Delta_i(k) = \big[I - \Gamma_i(k)\Xi_i\big]^{-1}\Gamma_i(k), \qquad \Xi_i\Xi_i^T < I, \tag{11f}$$

$$\Delta_{Zi}(k) = \big[I - \Gamma_{zi}(k)\Xi_{zi}\big]^{-1}\Gamma_{zi}(k), \qquad \Xi_{zi}\Xi_{zi}^T < I, \tag{11g}$$

where $M_i \in \Re^{n\times q}$, $M_{Zi} \in \Re^{p\times q}$, N_{Ai}, N_{Bi}, N_{DWi}, N_{DUi}, N_{ZAi}, N_{ZBi}, N_{ZWi}, Ξ_i, and Ξ_{zi}, $i = 1,2,\ldots,N$, are some given constant matrices with appropriate dimensions, $\Gamma_i(k)$ and $\Gamma_{zi}(k)$ are some unknown matrices satisfying

$$\Gamma_i^T(k)\Gamma_i(k) \le I, \qquad \Gamma_{zi}^T(k)\Gamma_{zi}(k) \le I. \tag{11h}$$

The switching domains are also defined by

$$\Omega_i(U_i) = \{x \in \Re^n : x^T U_i x \geq 0\}, \quad i = 1, 2, \ldots, N, \tag{12a}$$

where the matrices $U_i > 0$, $i = 1, 2, \ldots, N$, will be selected from our developed results, and

$$\begin{aligned} &\overline{\Omega}_1 = \Omega_1, \qquad \overline{\Omega}_2 = \Omega_2 \backslash \overline{\Omega}_1, \qquad \overline{\Omega}_3 = \Omega_3 \backslash \overline{\Omega}_1 \backslash \overline{\Omega}_2, \qquad \ldots, \\ &\overline{\Omega}_N = \Omega_N \backslash \overline{\Omega}_1 \backslash \cdots \backslash \overline{\Omega}_{N-1}. \end{aligned} \tag{12b}$$

By the definition of domains the switching signal can be designed by

$$\sigma(x(k)) = i, \quad \forall x(k) \in \overline{\Omega}_i, \tag{12c}$$

where $\overline{\Omega}_i$ is defined in (12b). Now we define the state feedback switching control to achieve the stabilization and passivity for the switched system in (11a)-(11h):

$$u(k) = -K_i x(k), \quad \text{when } \sigma(x(k)) = i, \tag{13}$$

where the state feedback gain $K_i \in \Re^{\upsilon \times n}$ will be selected from our developed result.

Lemma 6 [41] *For matrices X, Y, and Z with $X = X^T$ and $Z = Z^T$, the following statements are equivalent*:

(a)

$$S = \begin{bmatrix} X & Y \\ * & -Z^{-1} \end{bmatrix} < 0.$$

(b) *There exists a scalar $\eta > 0$ such that*

$$\begin{bmatrix} X & \eta \cdot Y & 0 \\ * & -2\eta \cdot I & Z \\ * & * & -Z \end{bmatrix} < 0.$$

Lemma 7 [42] *Suppose that $\Delta_i(k)$ is defined in* (11f) *and satisfies* (11h). *Then for real matrices V_i, W_i, and X_i with $X_i = X_i^T$, the following statements are equivalent*:

(a)

$$X_i + V_i \Delta_i(k) W_i + W_i^T \Delta_i^T(k) V_i^T < 0.$$

(b) *There exists a scalar $\varepsilon_i > 0$ such that*

$$\begin{bmatrix} X_i & \varepsilon_i \cdot V_i & W_i^T \\ * & -\varepsilon_i \cdot I & \varepsilon_i \cdot \Xi_i^T \\ * & * & -\varepsilon_i \cdot I \end{bmatrix} < 0,$$

where the matrix Ξ_i is defined in (11f).

Definition 2 [43] The discrete switched system (11a)-(11h) with switching signal in (12c) and switching control in (13) is called passive if there exists a constant $\gamma > 0$ such that

$$-\gamma \cdot \sum_{k=0}^{\ell} w^T(k)w(k) \le 2 \cdot \sum_{k=0}^{\ell} z^T(k)w(k)$$

for all $\ell > 0$ and zero initial condition $x(\theta) = 0$, $\theta = -r_M, -r_M + 1, \ldots, 0$. If the parameter ℓ is selected as ∞, then the disturbance input w belongs to $L_2(0, \infty)$.

Theorem 2 *For some selected integers* $0 < r_m = r_0 < r_1 < r_2 < \cdots < r_{p-1} < r_p = r_M$, *constants* $\gamma > 0$, $0 \le \alpha_i \le 1$, $i \in \overline{N}$, *and* $\sum_{i=1}^{N} \alpha_i = 1$, *system* (11a)-(11h) *is passive by the designed switching signal in* (12c) *and switching control in* (13) *with control gains* $K_i = \hat{K}_i/\eta_i$ *if there exist* $n \times n$ *symmetric matrices* $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$, $R_3 > 0$, $S > 0$, $T > 0$, $V_1 > 0$, $V_2 > 0$, U_j, W_1, W_2, *matrices* $X \in \Re^{n\times n}$, $\hat{K}_j \in \Re^{\upsilon\times n}$, $j = 1, 2, \ldots, N$, *and constants* $\varepsilon_j > 0$, $\eta_j > 0$, $j = 1, 2, \ldots, N$, *such that the following LMI conditions are satisfied*:

$$R_2 + W_1 > 0, \qquad R_2 + W_2 > 0, \tag{14a}$$

$$\begin{bmatrix} Q_1 & W_1 \\ * & Q_2 \end{bmatrix} > 0, \qquad \begin{bmatrix} Q_1 & W_2 \\ * & Q_2 \end{bmatrix} > 0, \qquad \begin{bmatrix} V_1 & X \\ * & V_1 \end{bmatrix} > 0, \tag{14b}$$

$$\tilde{\Sigma}_j^i = \begin{bmatrix} \tilde{\Sigma}_{j1}^i & \tilde{\Sigma}_{j2} \\ * & \tilde{\Sigma}_{j3} \end{bmatrix} < 0, \quad i = 1, 2, \ldots, p, j = 1, 2, \ldots, N, \tag{14c}$$

$$\sum_{i=1}^{N} \alpha_i \cdot U_i > 0, \tag{14d}$$

where

$$\tilde{\Sigma}_{j1}^i = \begin{bmatrix} \Sigma_{11j} & 0 & \Sigma_{13j} & \Sigma_{14j} & \Sigma_{15j} \\ * & \Sigma_{22j}^i & \Sigma_{23j}^i & \Sigma_{24j}^i & \Sigma_{25j} \\ * & * & \Sigma_{33j}^i & \Sigma_{34j} & 0 \\ * & * & * & \Sigma_{44j}^i & 0 \\ * & * & * & * & \Sigma_{55j} \end{bmatrix},$$

$$\tilde{\Sigma}_{j2} = \begin{bmatrix} \Sigma_{16j} & \Sigma_{17j} & 0 & 0 & 0 & 0 & \Sigma_{112j} & \Sigma_{113j} \\ \Sigma_{26j} & \Sigma_{27j} & 0 & 0 & 0 & 0 & \Sigma_{212j} & \Sigma_{213j} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \Sigma_{56j} & \Sigma_{57j} & 0 & 0 & 0 & \Sigma_{511j} & \Sigma_{512j} & \Sigma_{513j} \end{bmatrix},$$

$$\tilde{\Sigma}_{j3} = \begin{bmatrix} \Sigma_{66j} & 0 & \Sigma_{68j} & 0 & \Sigma_{610j} & 0 & 0 & 0 \\ * & \Sigma_{77j} & 0 & \Sigma_{79j} & \Sigma_{710j} & 0 & 0 & 0 \\ * & * & \Sigma_{88j} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Sigma_{99j} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Sigma_{1010j} & 0 & \Sigma_{1012j} & 0 \\ * & * & * & * & * & \Sigma_{1111j} & 0 & \Sigma_{1113j} \\ * & * & * & * & * & * & \Sigma_{1212j} & 0 \\ * & * & * & * & * & * & * & \Sigma_{1313j} \end{bmatrix},$$

$$\Sigma_{11j} = -P + T + (r_M - r_m)^2 \cdot Q_1 - R_1 - R_3 + U_j,$$

$$\Sigma_{13j} = R_1, \qquad \Sigma_{14j} = R_3, \qquad \Sigma_{15j} = -A_{zj}^T,$$

$$\Sigma_{16j} = \eta_j \cdot (A_j - I)^T - \hat{K}_j^T D_{uj}^T, \qquad \Sigma_{17j} = \eta_j \cdot A_j^T - \hat{K}_j^T D_{uj}^T,$$

$$\Sigma_{112j} = \eta_j \cdot N_{Aj}^T - \hat{K}_j^T N_{DUj}^T, \qquad \Sigma_{113j} = N_{ZAj}^T,$$

$$\gamma_{23}^i = \frac{(r_M - r_m)}{(r_M - r_{i-1})}, \qquad \gamma_{24}^i = \frac{(r_M - r_m)}{(r_i - r_m)},$$

$$\Sigma_{22j}^i = -\gamma_{23}^i \cdot (R_2 + W_2) - \gamma_{24}^i \cdot (R_2 + W_1) - (r_M - r_m) \cdot (W_1 - W_2) - 2V_1 + X + X^T,$$

$$\Sigma_{23j}^i = \gamma_{23}^i \cdot (R_2 + W_2) + V_1 - X^T, \qquad \Sigma_{24}^i = \gamma_{24}^i \cdot (R_2 + W_1) - X + V_1,$$

$$\Sigma_{25j} = -B_{zj}^T, \qquad \Sigma_{26j} = \eta_j \cdot B_j^T, \qquad \Sigma_{27j} = \eta_j \cdot B_j^T,$$

$$\Sigma_{212j} = \eta_j \cdot N_{Bj}^T, \qquad \Sigma_{213j} = N_{ZBj}^T,$$

$$\Sigma_{33j}^i = -S - R_1 - \gamma_{23}^i \cdot (R_2 + W_2) - (r_M - r_m) \cdot W_2 - V_1 - V_2, \qquad \Sigma_{34j} = V_2 + X,$$

$$\Sigma_{44j}^i = -(T - S) - \gamma_{24}^i \cdot (R_2 + W_1) - R_3 + (r_M - r_m) \cdot W_1 - V_1 - V_2,$$

$$\Sigma_{55j} = -D_{zwj} - D_{zwj}^T - \gamma \cdot I, \qquad \Sigma_{56j} = \eta_j \cdot D_{wj}^T, \qquad \Sigma_{57j} = \eta_j \cdot D_{wj}^T,$$

$$\Sigma_{511j} = -\varepsilon_j \cdot M_{Zj}, \qquad \Sigma_{512j} = \eta_j \cdot N_{DWj}^T, \qquad \Sigma_{513j} = N_{ZWj}^T, \qquad \Sigma_{66j} = \Sigma_{77j} = -2\eta_j \cdot I,$$

$$\Theta = r_M^2 \cdot R_1 + (r_M - r_m)^2 (Q_2 + R_2 + V_1 + V_2) + r_m^2 \cdot R_3,$$

$$\Sigma_{68j} = \Theta, \qquad \Sigma_{610j} = \varepsilon_j \cdot M_j, \qquad \Sigma_{79j} = P,$$

$$\Sigma_{710j} = \varepsilon_j \cdot M_j, \qquad \Sigma_{88j} = -\Theta, \qquad \Sigma_{99j} = -P,$$

$$\Sigma_{1010j} = \Sigma_{1111j} = \Sigma_{1212j} = \Sigma_{1313j} = -\varepsilon_j \cdot I,$$

$$\Sigma_{1012j} = \varepsilon_j \cdot \Xi_j^T, \qquad \Sigma_{1113j} = \varepsilon_j \cdot \Xi_{zj}^T.$$

Proof For the functional given in (4), the derivations in (4)-(7f) with (11a)-(13) can be formulated as

$$\begin{aligned} &\Delta V(x_k) + (r_M - r_m) \cdot (\lambda_1 + \lambda_2) + \left[-2z^T(k)w(k) - \gamma \cdot w^T(k)w(k)\right] \\ &\quad \leq -x^T(k)U_j x(k) + Z^T(k) \cdot \hat{\Sigma}_j^i \cdot Z(k), \end{aligned} \tag{15a}$$

where $Z(k)$ is defined in (8a), and

$$\hat{\Sigma}_j^i = \hat{\Sigma}_{1j}^i - \begin{bmatrix} \hat{\Sigma}_{16j} \\ \hat{\Sigma}_{26j} \\ 0 \\ 0 \\ \hat{\Sigma}_{56j} \end{bmatrix} \Sigma_{66j}^{-1} \begin{bmatrix} \hat{\Sigma}_{16j} \\ \hat{\Sigma}_{26j} \\ 0 \\ 0 \\ \hat{\Sigma}_{56j} \end{bmatrix}^T - \begin{bmatrix} \hat{\Sigma}_{17j} \\ \hat{\Sigma}_{27j} \\ 0 \\ 0 \\ \hat{\Sigma}_{57j} \end{bmatrix} \Sigma_{77j}^{-1} \begin{bmatrix} \hat{\Sigma}_{17j} \\ \hat{\Sigma}_{27j} \\ 0 \\ 0 \\ \hat{\Sigma}_{57j} \end{bmatrix}^T, \tag{15b}$$

$$\hat{\Sigma}_{1j}^i = \begin{bmatrix} \Sigma_{11j} & 0 & \Sigma_{13j} & \Sigma_{14j} & \hat{\Sigma}_{15j} \\ * & \Sigma_{22j}^i & \Sigma_{23j}^i & \Sigma_{24j}^i & \hat{\Sigma}_{25j} \\ * & * & \Sigma_{33j}^i & 0 & 0 \\ * & * & * & \Sigma_{44j}^i & 0 \\ * & * & * & * & \hat{\Sigma}_{55j} \end{bmatrix}, \tag{15c}$$

$$\hat{\Sigma}_{15j} = -(A_{zj} + \Delta A_{zj})^T, \qquad \hat{\Sigma}_{16j} = \bigl(A_j + \Delta A_j - (D_{uj} + \Delta D_{uj})K_j - I\bigr)^T,$$
$$\hat{\Sigma}_{17j} = \bigl(A_j + \Delta A_j - (D_{uj} + \Delta D_{uj})K_j\bigr)^T, \qquad \hat{\Sigma}_{25j} = -(B_{zj} + \Delta B_{zj})^T,$$
$$\hat{\Sigma}_{26j} = (B_j + \Delta B_j)^T, \qquad \hat{\Sigma}_{27j} = (B_j + \Delta B_j)^T,$$
$$\hat{\Sigma}_{55j} = -(D_{zwj} + \Delta D_{zwj}) - (D_{zwj} + \Delta D_{zwj})^T - \gamma \cdot I, \qquad \hat{\Sigma}_{56j} = (D_{wj} + \Delta D_{wj})^T,$$
$$\hat{\Sigma}_{57j} = (D_{wj} + \Delta D_{wj})^T, \qquad \Sigma_{66j} = -\Theta^{-1}, \qquad \Sigma_{77j} = -P^{-1},$$

Σ_{klj}, $k, l = 1, 2, \ldots, 4$ and Θ are defined in (14c).

Define

$$\tilde{\Sigma}_j^i = \begin{bmatrix} \hat{\Sigma}_{1j}^i & \hat{\Sigma}_{2j} \\ * & -\hat{\Sigma}_{4j}^{-1} \end{bmatrix}, \tag{16}$$

where $\hat{\Sigma}_{1j}^i$ is defined in (15c), and

$$\hat{\Sigma}_{2j} = \begin{bmatrix} \hat{\Sigma}_{16j} & \hat{\Sigma}_{17j} \\ \hat{\Sigma}_{26j} & \hat{\Sigma}_{27j} \\ 0 & 0 \\ 0 & 0 \\ \hat{\Sigma}_{56j} & \hat{\Sigma}_{57j} \end{bmatrix}, \qquad \hat{\Sigma}_{4j} = \begin{bmatrix} -\Sigma_{66j} & 0 \\ 0 & -\Sigma_{77j} \end{bmatrix} = \begin{bmatrix} \Theta & 0 \\ 0 & P \end{bmatrix}.$$

Consider the following matrices with constants $\eta_j > 0$, $j = 1, 2, \ldots, N$:

$$\tilde{\tilde{\Sigma}}_j^i = \begin{bmatrix} \hat{\Sigma}_{1j}^i & \eta_j \cdot \hat{\Sigma}_{2j} & 0 \\ * & \Sigma_{3j} & \hat{\Sigma}_{4j} \\ * & * & -\hat{\Sigma}_{4j} \end{bmatrix} = \begin{bmatrix} \Sigma_{1j}^i & \Sigma_{2j} & 0 \\ * & \Sigma_{3j} & \Sigma_{4j} \\ * & * & -\Sigma_{4j} \end{bmatrix} + \Pi_j \hat{\Delta}_j(k) \Psi_j^T + \Psi_j \hat{\Delta}_j^T(k) \Pi_j^T,$$

where Σ_{klj}, $k, l = 1, 2, \ldots, 7$, are defined in (14c),

$$\Sigma_{1j}^i = \begin{bmatrix} \Sigma_{11j} & 0 & \Sigma_{13j} & \Sigma_{14j} & \Sigma_{15j} \\ * & \Sigma_{22j}^i & \Sigma_{23j}^i & \Sigma_{24j}^i & \Sigma_{25j} \\ * & * & \Sigma_{33j}^i & 0 & 0 \\ * & * & * & \Sigma_{44j}^i & 0 \\ * & * & * & * & \Sigma_{55j} \end{bmatrix}, \qquad \Sigma_{2j} = \begin{bmatrix} \Sigma_{16j} & \Sigma_{17j} \\ \Sigma_{26j} & \Sigma_{27j} \\ 0 & 0 \\ 0 & 0 \\ \Sigma_{56j} & \Sigma_{57j} \end{bmatrix},$$
$$\Sigma_{3j} = \begin{bmatrix} -2 \cdot \eta_j \cdot I & 0 \\ 0 & -2 \cdot \eta_j \cdot I \end{bmatrix}, \qquad \Sigma_{4j} = \begin{bmatrix} \Theta & 0 \\ 0 & P \end{bmatrix},$$
$$\Pi_j = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & M_j^T & M_j^T & 0 & 0 \\ 0 & 0 & 0 & 0 & -M_{Zj}^T & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$
$$\hat{\Delta}_j(k) = \begin{bmatrix} \Delta_j(k) & 0 \\ 0 & \Delta_{zj}(k) \end{bmatrix} = \begin{bmatrix} I - \Gamma_j(k)\Xi_j & 0 \\ 0 & I - \Gamma_{zj}(k)\Xi_{zj} \end{bmatrix}^{-1} \begin{bmatrix} \Gamma_i(k) & 0 \\ 0 & \Gamma_{zi}(k) \end{bmatrix}$$
$$= \left\{ \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} \Gamma_j(k) & 0 \\ 0 & \Gamma_{zj}(k) \end{bmatrix} \begin{bmatrix} \Xi_j & 0 \\ 0 & \Xi_{zj} \end{bmatrix} \right\}^{-1} \begin{bmatrix} \Gamma_i(k) & 0 \\ 0 & \Gamma_{zi}(k) \end{bmatrix},$$

$$\Psi_j = \begin{bmatrix} \eta_j \cdot N_{Aj} - N_{DUj}\hat{K}_j & \eta_j \cdot N_{Bj} & 0 & 0 & \eta_j \cdot N_{DWj} & 0 & 0 & 0 & 0 \\ N_{ZAj} & N_{ZBj} & 0 & 0 & N_{ZWj} & 0 & 0 & 0 & 0 \end{bmatrix}^T, \quad \hat{K}_j = \eta_j \cdot K_j.$$

By Lemmas 6 and 7 the condition in (14c) should be imposed to achieve the passivity of system under consideration. □

4 Illustrative examples

Example 1 We consider system (1a)-(1h) with the following parameters:

$$\begin{aligned}
&A_1 = \begin{bmatrix} 1.01 & 0.01 \\ 0.01 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & 0.01 \\ 0.01 & 1.01 \end{bmatrix}, \\
&B_1 = \begin{bmatrix} -0.1 & 0.1 \\ 0 & -0.1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -0.1 & 0 \\ 0.1 & 0.1 \end{bmatrix}, \\
&D_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.1 \end{bmatrix}, \\
&A_{z1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad A_{z2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.05 \end{bmatrix}, \\
&B_{z1} = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.01 \end{bmatrix}, \qquad B_{z2} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix}, \\
&D_{z1} = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.2 \end{bmatrix}, \qquad D_{z2} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \\
&M_1 = M_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad M_{z1} = M_{z2} = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
&N_{A1} = N_{A2} = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.01 \end{bmatrix}, \qquad N_{B1} = N_{B2} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix}, \\
&N_{D1} = N_{D2} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \qquad N_{ZA1} = N_{ZA2} = N_{B1}, \\
&N_{ZB1} = N_{ZB2} = N_{A1}, \qquad N_{ZD1} = N_{ZD2} = N_{D1}, \qquad \Xi_1 = \Xi_2 = \Xi_{z1} = \Xi_{z2} = 0.01 \cdot I.
\end{aligned} \tag{17}$$

With $p = 2$, $r_m = r_0 = 1$, $r_1 = 3$, $r_M = r_2 = 4$, $\gamma = 0.5$, and $\alpha_1 = \alpha_2 = 0.5$, the LMI conditions in Theorem 1 have a feasible solution with (some matrix solutions for LMI variables are not listed here)

$$U_1 = \begin{bmatrix} -0.5249 & -0.0165 \\ -0.0165 & 0.7641 \end{bmatrix}, \qquad U_2 = \begin{bmatrix} 0.5225 & 0.0166 \\ 0.0166 & -0.7635 \end{bmatrix}.$$

System (1a)-(1h) with (17) is passive with $r_m = 1$, $r_M = 4$, $\gamma = 0.5$ by the switching signal designed by

$$\sigma = \begin{cases} 1, & x \in \overline{\Omega}_1, \\ 2, & x \in \Re^2 \backslash \overline{\Omega}_1, \end{cases} \tag{18}$$

Table 1 The obtained results for our proposed results

The delay upper bound and switching domains to guarantee the passivity property of systems		
Results		**Number for elements of LMI variables**
[10]	$r_m = 1$, $r_M = 3$ fail to guarantee stability even when $\Xi_i = 0$, $\Xi_{zi} = 0$, $i = 1, 2$	Fail
[13]	$r_m = 1$, $r_M = 3$ cannot design a switching signal to guarantee the stability even when $\Xi_i = 0$, $\Xi_{zi} = 0$, $i = 1, 2$	Fail
[14]	$r_m = 1$, $r_M = 3$ fail to guarantee the stability even when $\Xi_i = 0$, $\Xi_{zi} = 0$, $i = 1, 2$	Fail
[18]	$r_m = 1$, $r_M = 3$ $\overline{\Omega}_1 = \{[x_1\ x_2]^T : 1.0102x_1^2 + 0.0016x_1x_2 - 0.9057x_2^2 \le 0\}$, $\overline{\Omega}_2 = \Re^2 \backslash \overline{\Omega}_1$ $r_m = 2$, $r_M = 4$ $\overline{\Omega}_1 = \{[x_1\ x_2]^T : 0.3961x_1^2 - 0.0012x_1x_2 - 0.4515x_2^2 \le 0\}$, $\overline{\Omega}_2 = \Re^2 \backslash \overline{\Omega}_1$	338 (program running time about 2 minutes)
Results of this paper	$r_m = 1$, $r_M = 4$ ($p = 2$, $r_1 = 3$, $\alpha_1 = \alpha_2 = 0.5$) $\overline{\Omega}_1 = \{[x_1\ x_2]^T : -0.5249x_1^2 - 0.033x_1x_2 + 0.7641x_2^2 \ge 0\}$, $\overline{\Omega}_2 = \Re^2 \backslash \overline{\Omega}_1$	45 (program running time about 5 seconds)

where $\overline{\Omega}_1 = \{[x_1\ x_2]^T : -0.5249x_1^2 - 0.033x_1x_2 + 0.7641x_2^2 \ge 0\}$. Some delay upper bounds and switching domains in (18) that guarantee the passivity property ($\gamma = 0.5$) for system (1a)-(1h) with (17) are provided in Table 1 for $\alpha_1 = \alpha_2 = 0.5$.

Two issues about passivity analysis of switched systems:

1. Under arbitrary switching signal, passivity analysis and passive control can be investigated to guarantee the performance for uncertain discrete switched systems.
2. Design of a switching signal (and a switching control) guarantees the passivity property for uncertain discrete switched systems. This paper is focused on this issue.

Note that the matrices A_1 and A_2 in this example are not Hurwitz (have at least one eigenvalue greater than 1) and the results in [10, 13, 14] cannot be applied to find any feasible solution to guarantee the stability for discrete switching systems for any arbitrary switching.

Example 2 We consider system (11a)-(11h) with the following parameters:

$$
\begin{aligned}
&A_1 = \begin{bmatrix} 1.01 & 0.02 \\ 0.01 & 0.01 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0.01 & 0.01 \\ 0.02 & 1.01 \end{bmatrix}, \\
&B_1 = \begin{bmatrix} -0.2 & 0.1 \\ 0 & -0.2 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, \\
&D_{w1} = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \qquad D_{w2} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.1 \end{bmatrix}, \\
&D_{u1} = \begin{bmatrix} 1 & 0.2 \\ 0.1 & 1 \end{bmatrix}, \qquad D_{u2} = \begin{bmatrix} 1 & 0.1 \\ 0.2 & 1 \end{bmatrix}, \\
&A_{z1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad A_{z2} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.1 \end{bmatrix}, \\
&B_{z1} = \begin{bmatrix} 0.04 & 0 \\ 0 & 0.03 \end{bmatrix}, \qquad B_{z2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},
\end{aligned} \tag{19}
$$

$$D_{zw1} = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.2 \end{bmatrix}, \qquad D_{zw2} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad M_1 = M_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$M_{z1} = M_{z2} = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}, \qquad N_{A1} = N_{A2} = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.01 \end{bmatrix},$$
$$N_{B1} = N_{B2} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix}, \qquad N_{DW1} = N_{DW2} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix},$$
$$N_{DU1} = N_{DU2} = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.01 \end{bmatrix}, \qquad N_{ZA1} = N_{ZA2} = N_{A1}, \qquad N_{ZB1} = N_{ZB2} = N_{B1},$$
$$N_{ZW1} = N_{ZW2} = N_{DW1}, \qquad \Xi_1 = \Xi_2 = \Xi_{z1} = \Xi_{z2} = 0.01 \cdot I.$$

With $p = 2$, $r_m = 1$, $r_1 = 4$, $r_M = 8$, $\gamma = 1.5$, and $\alpha_1 = \alpha_2 = 0.5$, the LMI conditions in Theorem 2 have a feasible solution with (some matrix solutions for LMI variables are not listed here)

$$U_1 = \begin{bmatrix} 0.0056 & 0.0021 \\ 0.0021 & -0.0055 \end{bmatrix}, \qquad U_2 = \begin{bmatrix} -0.0036 & -0.0048 \\ -0.0048 & 0.0102 \end{bmatrix},$$
$$\hat{K}_1 = \begin{bmatrix} 0.8955 & 0.2065 \\ -0.0729 & -0.8734 \end{bmatrix}, \qquad \hat{K}_2 = \begin{bmatrix} -0.7817 & -0.0618 \\ 0.1892 & 0.929 \end{bmatrix},$$
$$\eta_1 = 1.7259, \qquad \eta_2 = 1.6971.$$

System (1a)-(1h) with (19) is passive with $r_m = 1$, $r_M = 10$, $\gamma = 1.5$ by the switching signal given in (12c) with

$$\sigma = \begin{cases} 1, & x \in \overline{\Omega}_1, \\ 2, & x \in \overline{\Omega}_2, \end{cases} \tag{20}$$

where

$$\overline{\Omega}_1 = \left\{ \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \in R^2 : 0.0056x_1^2 + 0.0042x_1x_2 - 0.0055x_2^2 \geq 0 \right\} \quad \text{and}$$
$$\overline{\Omega}_2 = \Re^2 \backslash \overline{\Omega}_1.$$

The proposed switching control gains in (13) are given by

$$K_1 = \hat{K}_1/\eta_1 = \begin{bmatrix} 0.5188 & 0.1196 \\ -0.0422 & -0.5061 \end{bmatrix}, \qquad K_2 = \hat{K}_2/\eta_2 = \begin{bmatrix} -0.4606 & -0.0364 \\ 0.1115 & 0.5474 \end{bmatrix}. \tag{21}$$

Under the disturbance inputs $w(k) = [10 \times (0.9)^k \ - 10 \times (0.4)^k]^T$ shown in Figure 1 and zero initial conditions, the regulated outputs $z(k) \in \Re^2$ of switched system (1a)-(1h) with (19)-(21) and no perturbations are shown in Figure 2. Under zero disturbance, the initial state function $\varphi(\theta) = [10 \ -5]^T$, $\theta = -8, -7, \ldots, -1, 0$, and no perturbations, the state trajectories $x(k) \in \Re^2$ of switched system (1a)-(1h) with (19)-(21) are shown in Figure 3. By Theorem 2 system (1a)-(1h) with (20) and $1 \leq r(k) \leq 8$ is passive by the proposed switching signal in (20) and switching control in (13) with control gains in (21). With $1 \leq r(k) \leq 8$,

Figure 1 **The disturbance inputs of switched system (solid line: $w_1(k)$, dashed line: $w_2(k)$).**

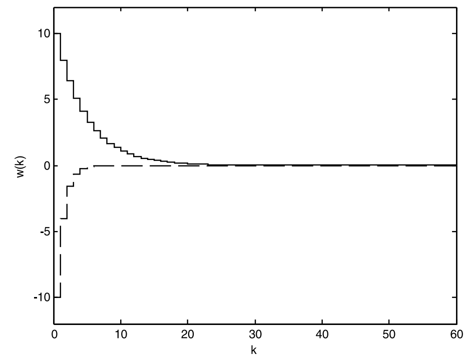


Figure 2 **Regulated outputs of switched system (solid line: $z_1(k)$, dashed line: $z_2(k)$).**

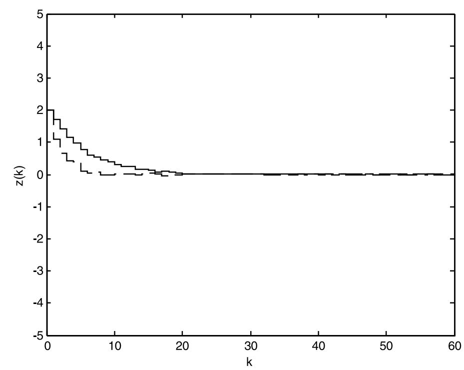
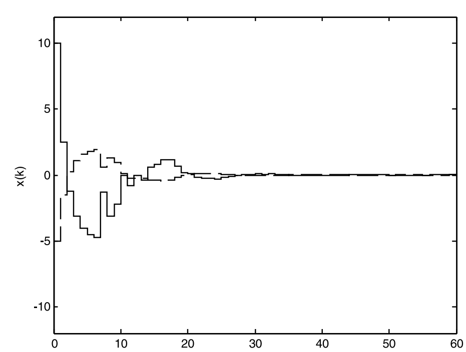


Figure 3 **State trajectories for switched system (solid line: $x_1(k)$, dashed line: $x_2(k)$).**



our previous results in [10, 11] cannot provide or guarantee any performance of uncertain discrete switched time-delay system. By using Theorem 2, the maximal delay upper bounds with respect to $\gamma = 1.5$ and $\gamma = 0.1$ that guarantee the passivity property for system (1a)-(1h) with (19) are provided in Table 2 for $\alpha_1 = \alpha_2 = 0.5$, $p = 2$, $r_1 = 4$, respectively. The obtained result for passive switching control in this paper is more efficient than our previous results in [11].

There are some major contributions:

(a) Less LMI variable elements are used in this proposed approach. Implementation of LMI program can be achieved easily.

(b) Under the same passivity requirement, a more spacious range of interval time-varying delay can be guaranteed by the proposed approach.

Table 2 The obtained results for our proposed results

The delay upper bound to guarantee the stability and passivity property of systems				
[10]	**[11]**	**(switching signal + switching control) $r_m = 1$**		
		Conditions	**[19]**	**Results of this paper**
Fail (stability for arbitrary switching)	$r_m = 1, r_M = 2$ (stability for switching signal design)	$\gamma = 1.5$	$r_M = 7$	$r_M = 8$
		$\gamma = 0.1$	$r_M = 6$	$r_M = 7$
		Number for elements of LMI variables	348	55

(c) The proposed new Lyapunov functional does not depend on time-varying delay $r(k)$, and some conservativeness can be reduced.
(d) A simple design scheme for the proposed switching signal can be easily generalized to continuous switched time-delay systems.
(e) Better passivity of switching systems can be achieved by switching signal design and switching state feedback control.
(f) The delay-partition approach and Jensen-Park inequalities are used to reduce the conservativeness of the proposed results.

Example 3 The following water quality model is presented in Chapter 11 of [1]:

$$x(k+1) = A_\sigma x(k) + B_\sigma x\bigl(k - r(k)\bigr) + D_\sigma w(k), \tag{22a}$$

$$z(k) = A_{z\sigma} x(k) + B_{z\sigma} x\bigl(k - r(k)\bigr) + D_{z\sigma} w(k), \tag{22b}$$

where the time delay $r(k) > 0$ is shown to reflect the mixing effect of biochemical constituents in time instance k, the state vector $x(k) \in \Re^n$ is the water-quality constituents (like algae, ammonia nitrogen, dissolved oxygen, biochemical oxygen demand), the output $z(k) \in \Re^q$ is the performance, the disturbance input $w(k) \in \Re^q$ is the irregular discharge of effluents and belongs to $L_2(0,\infty)$, and the switching signal σ is located in the finite set $\{1, 2, \ldots, N\}$. Typically, the switching rule σ is not known *a priori*, but we assume that its instantaneous value is available in real time for practical implementations by water pollution management. In this paper, the passivity problem defined in Definition 1 will be investigated. The time-varying delay is defined by $0 < r_m \le r(k) \le r_M$, where r_m and r_M present the extreme cases of light and heavy waste dump loadings, respectively. Consider the following parameters of the model ($N = 3$):

$$A_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.2 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.71 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0.2 & 0 \\ 0.1 & 0.1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -0.2 & 0.1 \\ 0 & -0.2 \end{bmatrix}, \qquad B_3 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.05 \\ 0.1 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}, \qquad D_3 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \tag{23}$$

$$A_{z1} = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}, \qquad A_{z2} = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix},$$
$$A_{z3} = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}, \qquad B_{z1} = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix},$$
$$B_{z2} = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, \qquad B_{z3} = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix},$$
$$D_{z1} = 0.1, \qquad D_{z2} = 0.1, \qquad D_{z3} = 0.1.$$

With $r_m = r_0 = 1$, $r_2 = 2$, $r_M = r_3 = 3$, and $\alpha_1 = \alpha_2 = 0.4$, $\alpha_3 = 0.2$, there is a feasible solution in Theorem 1 with (some matrix solutions for LMI variables are not listed here)

$$U_1 = \begin{bmatrix} 1.4679 & -0.6667 \\ -0.6667 & 2.7987 \end{bmatrix},$$
$$U_2 = \begin{bmatrix} -0.4749 & 1.2218 \\ 1.2218 & 1.0524 \end{bmatrix},$$
$$U_3 = \begin{bmatrix} 0.2111 & -1.3439 \\ -1.3439 & -7.0066 \end{bmatrix}.$$

Hence, we can conclude that the water quality model (22a)-(22b) with (23) is passive with $\gamma = 1$ by the switching signal designed by

$$\sigma = \begin{cases} 1, & x(k) \in \overline{\Omega}_1, \\ 2, & x(k) \in \overline{\Omega}_2, \\ 3, & x(k) \in \Re^2 \backslash \overline{\Omega}_1 \backslash \overline{\Omega}_2, \end{cases} \tag{24}$$

where

$$\overline{\Omega}_1 = \left\{ \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \in \Re^2 : 1.4679x_1^2 - 1.3334x_1x_2 + 2.7987x_2^2 \geq 0 \right\},$$
$$\overline{\Omega}_2 = \left\{ \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \in \Re^2 : -0.4749x_1^2 + 2.4436x_1x_2 + 1.0524x_2^2 \geq 0 \right\} \backslash \overline{\Omega}_1.$$

5 Conclusions

In this paper, a simple switching signal design scheme has been investigated to guarantee the passivity property and passive switching control for uncertain discrete switched systems with interval time-varying delay and linear fractional perturbations. A new Lyapunov functional is applied to guaranteed the obtained results. Jensen and Park inequalities combined with delay-partitioning approach are used to improve the conservativeness of the proposed results. The results proposed in this paper are shown to be less conservative than some recent reports from numerical examples. The passivity of a water quality model is guaranteed by selecting a suitable switching signal.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

The authors declare that the study was realized in collaboration with the same responsibility. All authors read and approved the final manuscript.

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