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Explicit solutions and conservation laws of the logarithmic-KP equation

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Abstract
In this paper, we study the logarithmic-KP equation. The analysis depends mainly on the Lie symmetry method. The corresponding vector fields and symmetry reductions are derived. Furthermore, the conservation laws of the equation are constructed.

Keywords: logarithmic-KP equation; Lie symmetry method; explicit solutions; conservation laws

1 Introduction

Nonlinear evolution equations (NLEEs) have been used in many science fields, such as physics, chemistry, engineering, and other fields. The investigation of the explicit solutions of NLEEs gave rise to much research work. A great many of systematic and effective methods are used for investigating NLEEs. Some of the methods are the inverse scattering method [1], the Hirota bilinear method [2, 3], the Bäklund transformation method [4–6], Darboux transformation [7, 8], the Painlevé analysis [9], the Lie group method [10–28], the solitary wave ansatz method [29–36], and others.

Conservation laws (CLs) provide a important tool to investigate many problems involving mathematical physics. A systematic method for the determination of conservation laws is the famous Noether theorem [37]. Recently, the direct method was given in [11], a new method to construct the conservation laws was provided in [38].

The generalized KP equation is the so-called logarithmic-KP (log-KP) equation given by [39]

$$\left(v_t + \left(v \ln |v|\right)_x + v_{xxx}\right)_x + v_{yy} = 0, \tag{1}$$

where $v(x,t)$ represents the wave profile. In [39], the authors studied the Gaussian solitary waves of the log-KP equation. More explications of the log-KP equation and its applications can be found in [39] and references therein. In [25], the authors studied the log-KdV equation. The KP equation appears in many important fields, such as water waves, ferromagnetic media, and so on. Kadomtsev and Petviashvili first derived the famous KP equation [40]. There are many papers dealing with these types of equations [31–36, 39, 40]. We first employ the following transformation [39]:

$$v = e^u, \tag{2}$$

we get

$$\begin{aligned} &u_{xt} + u_{xx} + u_{yy} + u_{xxxx} + 3u_{xx}^2 + u_y^2 \\ &\quad + 3u_x u_{xxx} + u_x^2 + uu_{xx} + 3u_x^2 u_{xx} = 0. \end{aligned} \tag{3}$$

In this paper, we use the Lie group method to deal with (3). The outline of the paper is as follows: In Section 2, the vectors fields are derived. In Section 3, symmetry reductions and explicit solutions are constructed. In Section 4, conservation laws are presented using the new conservation law theorem. The conclusions are presented in the final section.

2 Lie symmetry analysis

Suppose that (3) is invariant via the one-parameter Lie group

$$\begin{aligned} &t^* = t + \varepsilon\xi_t(x,y,t,u) + O(\varepsilon^2), \\ &x^* = x + \varepsilon\xi_x(x,y,t,u) + O(\varepsilon^2), \\ &y^* = y + \varepsilon\xi_y(x,y,t,u) + O(\varepsilon^2), \\ &u^* = u + \varepsilon\eta(x,y,t,u) + O(\varepsilon^2), \end{aligned} \tag{4}$$

where ε is the group parameter, and the vector fields are

$$V = \xi_t(x,y,t,u)\frac{\partial}{\partial t} + \xi_x(x,y,t,u)\frac{\partial}{\partial x} + \xi_y(x,y,t,u)\frac{\partial}{\partial y} + \eta(x,y,t,u)\frac{\partial}{\partial u}. \tag{5}$$

Here

$$\begin{aligned} &\xi_t(x,y,t,u) = \frac{dt^*}{d\varepsilon}\bigg|_{\varepsilon=0}, \qquad \xi_x(x,y,t,u) = \frac{dx^*}{d\varepsilon}\bigg|_{\varepsilon=0}, \\ &\xi_y(x,y,t,u) = \frac{dy^*}{d\varepsilon}\bigg|_{\varepsilon=0}, \qquad \eta(x,y,t,u) = \frac{du^*}{d\varepsilon}\bigg|_{\varepsilon=0}. \end{aligned} \tag{6}$$

Under the assumption of the infinitesimal invariance criterion, one gets

$$pr^{(3)}V(\Delta)|_{\Delta=0} = 0. \tag{7}$$

According to the Lie group theory, one has

$$\begin{aligned} &\eta^{xt} + \eta^{xx} + \eta^{yy} + \eta^{xxxx} + 6\eta^{xx}u_{xx} + 2u_y\eta^y + 3\eta^x u_{xxx} \\ &\quad + 3u_x\eta^{xxx} + 2u_x\eta^x + \eta u_{xx} + u\eta^{xx} + 6u_x u_{xx}\eta^x + 3u_x^2\eta^{xx} = 0. \end{aligned} \tag{8}$$

Putting (4) into (8), and letting all of the powers of derivatives of u be zero, one can obtain overdetermined systems. Solving the systems, one can get

$$\xi_t = c_1, \qquad \xi_x = c_3F(t), \qquad \xi_y = c_2, \qquad \eta = c_3F_t, \tag{9}$$

where c_1, c_2, and c_3 are arbitrary constants, and F is a smooth function of t. Consequently, we have

$$V_1 = \frac{\partial}{\partial t}, \qquad V_2 = \frac{\partial}{\partial y}, \qquad V_3 = F\frac{\partial}{\partial x} + F_t\frac{\partial}{\partial u}. \tag{10}$$

In addition, solving the Lie equation

$$\begin{aligned}
&\frac{d(\bar{x}(\varepsilon))}{d\varepsilon} = \xi_x\bigl(\bar{x}(\varepsilon), \bar{y}(\varepsilon), \bar{t}(\varepsilon), \bar{u}(\varepsilon)\bigr), \qquad \bar{x}(0) = x,\\
&\frac{d(\bar{y}(\varepsilon))}{d\varepsilon} = \xi_y\bigl(\bar{x}(\varepsilon), \bar{y}(\varepsilon), \bar{t}(\varepsilon), \bar{u}(\varepsilon)\bigr), \qquad \bar{y}(0) = y,\\
&\frac{d(\bar{t}(\varepsilon))}{d\varepsilon} = \xi_t\bigl(\bar{x}(\varepsilon), \bar{y}(\varepsilon), \bar{t}(\varepsilon), \bar{u}(\varepsilon)\bigr), \qquad \bar{t}(0) = t,\\
&\frac{d(\bar{u}(\varepsilon))}{d\varepsilon} = \eta\bigl(\bar{x}(\varepsilon), \bar{y}(\varepsilon), \bar{t}(\varepsilon), \bar{u}(\varepsilon)\bigr), \qquad \bar{u}(0) = u,
\end{aligned} \tag{11}$$

where ε is a group parameter, we get the Lie symmetry group,

$$g : (x, y, t, u) \to (\bar{x}, \bar{y}, \bar{t}, \bar{u}). \tag{12}$$

The associated one-parameter groups $g_i(\varepsilon)$ generated by V_i for $i = 1, 2, 3$ are

$$\begin{aligned}
&g_1 : (x, y, t, u) \mapsto (x, t + \varepsilon, y, u),\\
&g_2 : (x, y, t, u) \mapsto (x, t, y + \varepsilon, y, u),\\
&g_3 : (x, y, t, u) \mapsto (x + F\varepsilon, y, t, u + F_t\varepsilon).
\end{aligned} \tag{13}$$

In addition, we get the following associated theorem.

Theorem 1 *If $u = f(x, y, t)$ is a solution of the logarithmic-KP equation, the functions*

$$\begin{aligned}
&g_1(\varepsilon) \cdot f(x, y, t) = f(x, y, t - \varepsilon),\\
&g_2(\varepsilon) \cdot f(x, y, t) = f(x, y - \varepsilon, t),\\
&g_3(\varepsilon) \cdot f(x, y, t) = f(x - F\varepsilon, y, t) + F_t\varepsilon,
\end{aligned} \tag{14}$$

are also solutions of (3).

Taking the following Gaussian solitary wave solution [30]:

$$u(x, t) = \frac{c}{k} + \frac{1}{2} - \frac{3k^2 + 2r^2}{12k^4}(kx + ry - ct)^2, \tag{15}$$

we can derive a new explicit solution of (3) using g_3,

$$u(x, t) = \frac{c}{k} + \frac{1}{2} - \frac{3k^2 + 2r^2}{12k^4}\bigl(k(x - F\varepsilon) + ry - ct\bigr)^2 + F_t\varepsilon. \tag{16}$$

Hence, one can get new solutions of (1),

$$v(x, t) = e^{\frac{c}{k} + \frac{1}{2} - \frac{1}{4k^2}(k(x - F\varepsilon) + ry - ct)^2 + F_t\varepsilon}. \tag{17}$$

In particular, letting $F(t) = t^2$, one can get

$$v(x,t) = e^{\frac{c}{k}+\frac{1}{2}-\frac{1}{4k^2}(k(x-t^2\varepsilon)+ry-ct)^2+2t\varepsilon}, \tag{18}$$

setting $F(t) = \sin t$, one has

$$v(x,t) = e^{\frac{c}{k}+\frac{1}{2}-\frac{1}{4k^2}(k(x-\sin t\varepsilon)+ry-ct)^2+\cos t\varepsilon}, \tag{19}$$

and setting $F(t) = \tanh t$, one obtains

$$v(x,t) = e^{\frac{c}{k}+\frac{1}{2}-\frac{1}{4k^2}(k(x-\tanh t\varepsilon)+ry-ct)^2+(1-\tanh t^2)\varepsilon}, \tag{20}$$

setting $F(t) = e^t$, one can arrive at

$$v(x,t) = e^{\frac{c}{k}+\frac{1}{2}-\frac{1}{4k^2}(k(x-e^t\varepsilon)+ry-ct)^2+e^t\varepsilon}, \tag{21}$$

setting $F(t) = \sin(e^t)$, one can lead to

$$v(x,t) = e^{\frac{c}{k}+\frac{1}{2}-\frac{1}{4k^2}(k(x-\sin(e^t)\varepsilon)+ry-ct)^2+\cos(e^t)e^t\varepsilon}, \tag{22}$$

setting $F(t) = \ln(t)$, one can have

$$v(x,t) = e^{\frac{c}{k}+\frac{1}{2}-\frac{1}{4k^2}(k(x-\ln(t)\varepsilon)+ry-ct)^2+\frac{1}{t}\varepsilon}. \tag{23}$$

Remark 1 Many new explicit solutions can be derived via the solutions obtained [30].

3 Symmetry reductions and explicit solutions

3.1 Symmetry reductions

In the present subsection, we will present symmetry reductions and explicit solutions of (3).

(1) V_1.

For the generator V_1, we have

$$f_{xx} + f_{yy} + f_{xxxx} + 3f_{xx}^2 + f_y^2 + 3f_xf_{xxx} + f_x^2 + ff_{xx} + 3f_x^2f_{xx} = 0. \tag{24}$$

For this equation, we found that it also is a PDE. In order to reduce this equation, once again, we use the Lie group method to deal with this equation. As in the previous step, one can get the corresponding vectors,

$$\Upsilon_1 = \frac{\partial}{\partial x}, \qquad \Upsilon_2 = \frac{\partial}{\partial y}. \tag{25}$$

(1.1) Υ_1.

For Υ_1, we have

$$g_{yy} + g_y^2 = 0. \tag{26}$$

Solving this equation, one can get

$$g = \ln(c_1 y + c_2). \tag{27}$$

That is to say,

$$u = \ln(c_1 y + c_2). \tag{28}$$

Also, one can get

$$v = c_1 y + c_2. \tag{29}$$

(1.2) Υ_2.
For Υ_2, we get

$$g_{xx} + g_{xxxx} + 3g_{xx}^2 + 3g_x g_{xxx} + g_x^2 + g g_{xx} + 3g_x^2 g_{xx} = 0. \tag{30}$$

(2) V_2.
In the case of V_2, we get the group-invariant solution,

$$u = f(x,t), \tag{31}$$

$$f_{xt} + f_{xx} + f_{xxxx} + 3f_{xx}^2 + 3f_x f_{xxx} + f_x^2 + f f_{xx} + 3f_x^2 f_{xx} = 0. \tag{32}$$

As in the previous step, we get the associated vectors,

$$\Gamma_1 = \frac{\partial}{\partial x}, \qquad \Gamma_2 = \frac{\partial}{\partial t}. \tag{33}$$

(2.1) Γ_1.
For Γ_1, we get the trivial solution,

$$u = c_1. \tag{34}$$

(2.2) Γ_2.
For Γ_2, we arrive at

$$g_{xx} + g_{xxxx} + 3g_{xx}^2 + 3g_x g_{xxx} + g_x^2 + g g_{xx} + 3g_x^2 g_{xx} = 0. \tag{35}$$

(3) V_3.
For this case, we get

$$u = \frac{g(y,t)}{F} + \frac{F_t x}{F}. \tag{36}$$

Plugging (36) into (3), one arrives at

$$\frac{1}{F^2}\left(FF_{tt} + Fg_{yy} + g_y^2\right) = 0. \tag{37}$$

By solving this equation, one obtains

$$g = -\frac{1}{2}\ln\left(\frac{F_{tt}F}{(F_1\sin(\frac{\sqrt{F_{tt}}y}{\sqrt{F}}) - F_2\cos(\frac{\sqrt{F_{tt}}y}{\sqrt{F}}))^2}\right)F. \tag{38}$$

In this way, we get

$$u = -\frac{1}{2}\ln\left(\frac{F_{tt}F}{(F_1\sin(\frac{\sqrt{F_{tt}}y}{\sqrt{F}}) - F_2\cos(\frac{\sqrt{F_{tt}}y}{\sqrt{F}}))^2}\right) + \frac{F_t x}{F}. \tag{39}$$

Thus, one can get the explicit solution of (1),

$$\begin{aligned} v = e^u &= e^{-\frac{1}{2}\ln(\frac{F_{tt}F}{(F_1\sin(\frac{\sqrt{F_{tt}}y}{\sqrt{F}}) - F_2\cos(\frac{\sqrt{F_{tt}}y}{\sqrt{F}}))^2}) + \frac{F_t x}{F}} \\ &= \left(\frac{F_{tt}F}{(F_1\sin(\frac{\sqrt{F_{tt}}y}{\sqrt{F}}) - F_2\cos(\frac{\sqrt{F_{tt}}y}{\sqrt{F}}))^2}\right)^{-\frac{1}{2}} e^{\frac{F_t x}{F}}, \end{aligned} \tag{40}$$

where $F_{tt} \neq 0$, F_1 and F_2 are functions of t.

In particular, if we set $F = t$, we have

$$u = \frac{g(y,t)}{t} + \frac{x}{t}. \tag{41}$$

Substituting (41) into (3), we obtain

$$\frac{1}{t^2}\left(tg_{yy} + g_y^2\right) = 0. \tag{42}$$

Solving this equation, one can get

$$g = t\ln\left(\frac{F_1(t)y + F_2(t)}{t}\right). \tag{43}$$

Therefore, one gets

$$u = \ln\left(\frac{F_1(t)y + F_2(t)}{t}\right) + \frac{x}{t}. \tag{44}$$

Therefore, one can derive the explicit solution of (1),

$$v = e^u = \left(\frac{F_1(t)y + F_2(t)}{t}\right)e^{\frac{x}{t}}. \tag{45}$$

(3) $V_1 + V_2$.

For this case, we get

$$u = f(\xi,\tau), \qquad \xi = x, \qquad \tau = y - t. \tag{46}$$

Plugging (46) into (3), one arrives at

$$-f_{\xi\tau} + f_{\xi\xi} + f_{\tau\tau} + f_{\xi\xi\xi\xi} + 3f_{\xi\xi}^2 + f_\tau^2 + 3f_\xi f_{\xi\xi\xi} + f_\xi^2 + ff_{\xi\xi} + 3f_\xi^2 f_{\xi\xi} = 0. \tag{47}$$

As in the previous step, we obtain the associated vectors,

$$\Gamma_1 = \frac{\partial}{\partial \xi}, \qquad \Gamma_2 = \frac{\partial}{\partial \tau}. \tag{48}$$

(3.1) Γ_1.

For Γ_1, we get

$$f = g(\tau) \tag{49}$$

and

$$g_\tau^2 + g_{\tau\tau} = 0. \tag{50}$$

(3.2) Γ_2.

For Γ_2, we have

$$g_{\xi\xi} + g_{\xi\xi\xi\xi} + 3g_{\xi\xi}^2 + 3g_\xi g_{\xi\xi\xi} + g_\xi^2 + gg_{\xi\xi} + 3g_\xi^2 g_{\xi\xi} = 0. \tag{51}$$

(3.3) $\Gamma_2 + \lambda\Gamma_1$ (traveling wave transformation).

For this case, we get

$$f = g(\pi), \qquad \pi = \xi - \tau = x - \lambda(y - t) \tag{52}$$

and

$$\begin{aligned} &\lambda g_{\pi\pi} + g_{\pi\pi} + \lambda^2 g_{\pi\pi} + g_{\pi\pi\pi\pi} + 3g_{\pi\pi}^2 + \lambda^2 g_\pi^2 + 3g_\pi g_{\pi\pi\pi} \\ &\quad + g_\pi^2 + gg_{\pi\pi} + 3g_\pi^2 g_{\pi\pi} = 0. \end{aligned} \tag{53}$$

4 Conservation laws

In the present section, we derive the conservation laws of the logarithmic-KP equation.

4.1 Necessary preliminaries

For a conserved vector the following conservation equation holds:

$$D_t(C^t) + D_x(C^x) + D_y(C^y) = 0, \tag{54}$$

where $C^t = C^t(t,x,y,u,\ldots)$, $C^x = C^x(t,x,y,u,\ldots)$, $C^y = C^y(t,x,y,u,\ldots)$.

A formal Lagrangian for (3) is

$$\begin{aligned} L = {} & p(x,y,t)\bigl[u_{xt} + u_{xx} + u_{yy} + u_{xxxx} + 3u_{xx}^2 \\ & + u_y^2 + 3u_x u_{xxx} + u_x^2 + uu_{xx} + 3u_x^2 u_{xx}\bigr]. \end{aligned} \tag{55}$$

Here $p(x,y,t)$ is a new dependent variable.

Theorem 2 [29] *Every Lie point, Lie-Bäcklund, and nonlocal symmetry of equation* (3) *provides a conservation law for this equation and the adjoint equation. Then the elements of conservation vector are given by the following formula*:

$$C^i = \xi^i L + W^\alpha \left[\frac{\partial L}{\partial u_i^\alpha} - D_j\left(\frac{\partial L}{\partial u_{ij}^\alpha}\right) + D_j D_k \left(\frac{\partial L}{\partial u_{ijk}^\alpha}\right) - \cdots \right] + D_j\left(W^\alpha\right)\left[\left(\frac{\partial L}{\partial u_{ij}^\alpha}\right) - D_k\left(\left(\frac{\partial L}{\partial u_{ijk}^\alpha}\right)\right) + \cdots \right] + D_j D_k\left(W^\alpha\right)\left[\frac{\partial L}{\partial u_{ijk}^\alpha} - \cdots\right], \tag{56}$$

where $W^\alpha = \eta^\alpha - \xi^j u_j^\alpha$.

4.2 Conservation laws

The adjoint equation of (3) has the form

$$F = -2u_y p_y - 2pu_{yy} + p_{yy} + p_{xt} + 6u_x p_x u_{xx} + p_{xx} + up_{xx} + 3u_x^2 p_{xx} - 3u_{xx}p_{xx} - 3u_x p_{xxx} + p_{xxxx} = 0. \tag{57}$$

It is easily found that on substituting u instead of p in equation (57), equation (3) is not recovered. Thus, equation (3) is not self-adjoint. The Lagrangian is

$$L = p\left[u_{xt} + u_{xx} + u_{yy} + u_{xxxx} + 3u_{xx}^2 + u_y^2 + 3u_x u_{xxx} + u_x^2 + uu_{xx} + 3u_x^2 u_{xx}\right]. \tag{58}$$

From (56), one gets

$$C^t = \xi_t L + W\left(-D_x \frac{\partial L}{\partial u_{tx}}\right) + D_x(W)\frac{\partial L}{\partial u_{tx}}, \tag{59}$$

$$C^x = \xi L + W\left(\frac{\partial L}{\partial u_x} - D_x\frac{\partial L}{\partial u_{xx}} - D_t\frac{\partial L}{\partial u_{xt}} + D_x^2\frac{\partial L}{\partial u_{xxx}} - D_x^3(W)\frac{\partial L}{\partial u_{xxxx}}\right) + D_x(W)\left(\frac{\partial L}{\partial u_{xx}} - D_x\frac{\partial L}{\partial u_{xxx}} + D_x^2\frac{\partial L}{\partial u_{xxxx}}\right) + D_x^2(W)\left(\frac{\partial L}{\partial u_{xxx}} - D_x(W)\frac{\partial L}{\partial u_{xxxx}}\right), \tag{60}$$

$$C^y = \xi_y L + W\left(\frac{\partial L}{\partial u_y} - D_y\frac{\partial L}{\partial u_{yy}}\right) + D_y(W)\frac{\partial L}{\partial u_{yy}}, \tag{61}$$

which leads to

$$C^t = -v_x W + vW_x, \tag{62}$$

$$C^x = W\left(u_x v - v_x - v_x u - 3u_x^2 v_x - v_t - v_{xxx} + 3u_x v_{xx}\right) + (W_x)\left(v + 3vu_{xx} + uv + 3u_x^2 v - 3u_x v_x + v_{xx}\right) + W_{xx}(3u_x v - v_x), \tag{63}$$

$$C^y = 2vWu_y - Wv_y + vW_y. \tag{64}$$

In particular:

1. For the case $V_1 = \partial_y$, we get $W = -u_y$, and

$$C^t = v_x u_y - v u_{yx}, \tag{65}$$

$$\begin{aligned} C^x = & -u_y\left(u_x v - v_x - v_x u - 3u_x^2 v_x - v_t - v_{xxx} + 3u_x v_{xx}\right) \\ & - u_{xy}\left(v + 3v u_{xx} + uv + 3u_x^2 v - 3u_x v_x + v_{xx}\right) \\ & - u_{yxx}(3u_x v - v_x), \end{aligned} \tag{66}$$

$$C^y = -2v u_y^2 + u_y v_y - v u_{yy}. \tag{67}$$

2. For $V_2 = \partial_t$, one has $W = -u_t$, and

$$C^t = u_t v_x - v u_{tx}, \tag{68}$$

$$\begin{aligned} C^x = & -u_t\left(u_x v - v_x - v_x u - 3u_x^2 v_x - v_t - v_{xxx} + 3u_x v_{xx}\right) \\ & - (u_{tx})\left(v + 3v u_{xx} + uv + 3u_x^2 v - 3u_x v_x + v_{xx}\right) \\ & - u_{txx}(3u_x v - v_x), \end{aligned} \tag{69}$$

$$C^y = -2v u_t u_y + u_t v_y - v u_{ty}. \tag{70}$$

3. For the case $V_3 = F\partial_x + F_t\partial_u$, we have $W = F_t - Fu_x$, and we arrive at

$$\begin{aligned} C^t &= -v_x(F_t - Fu_x) + v(F_t - Fu_x)_x \\ &= Fv_x u_x - F_t v_x + F_{tx} v - F_x v u_x - Fv u_{xx}, \end{aligned} \tag{71}$$

$$\begin{aligned} C^x = {} & (F_t - Fu_x)\left(u_x v - v_x - v_x u - 3u_x^2 v_x - v_t - v_{xxx} + 3u_x v_{xx}\right) \\ & + (F_{tx} - F_x u_x - Fu_{xt})\left(v + 3v u_{xx} + uv + 3u_x^2 v - 3u_x v_x + v_{xx}\right) \\ & + (F_{txx} - F_{xx}u_x - F_x u_{xx} - F_x u_{xt} - Fu_{xxt})(3u_x v - v_x), \end{aligned} \tag{72}$$

$$\begin{aligned} C^y &= 2vWu_y - Wv_y + vW_y, \\ &= 2v u_y F_t - 2v u_y F u_x - F_t v_y + F u_x v_y + v F_{ty} - v F_y u_x - F v u_{xy}. \end{aligned} \tag{73}$$

5 Concluding remarks

In this paper, we studied the logarithmic-KP equation. The Lie group method was applied to conduct the analysis for this work. Symmetry reductions and explicit solutions were obtained. These solutions maybe explain some complex physical phenomena. It is to be noted that conservation laws were also constructed. We hope that the results obtained may be useful in further numerical analysis. Comparing with [30], it can be seen that our results are new. In the future work, we will try to employ more methods, such as nonclassical Lie groups, the nonlocal symmetry method, and other methods, to derive more novel exact solutions of the logarithmic types of equations.

Competing interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Authors' contributions

The authors wrote this paper together. The authors read and approved the final paper.

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