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Dynamic behaviors of an obligate Gilpin-Ayala system

Danhong Wang*

*Correspondence:
dingdongwdh@126.com
Department of Mathematics,
Minjiang University, Fuzhou, Fujian
350108, P.R. China

Abstract

In this paper, a nonautonomous obligate Gilpin-Ayala system is proposed and studied. The persistence and extinction of the system are discussed by using the comparison theorem of differential equations. The results show that, depending on the cooperation intensity between the species, the first species will be driven extinct or be permanent. After that, by using the Lyapunov function method, series of sufficient conditions are obtained which ensure the global attractivity of the system. Finally, two examples are given to illustrate the feasibility of the main results.

MSC: 34C25; 92D25; 34D20; 34D40

Keywords: obligate Gilpin-Ayala system; extinction; permanence; Lyapunov function; global attractivity

1 Introduction

During the last decade, the dynamic behaviors of the mutualism model have been extensively investigated [1–8] and many excellent results were obtained, which concern the persistence, existence of a positive periodic solution, and stability of the system. However, there are only a few scholars to study the commensal symbiosis model.

In 2003, Sun and Wei [6] proposed the following system to describe the interspecific commensal relationship:

$$\begin{aligned} \frac{dx}{dt} &= r_1 x\left(1 - \frac{x}{k_1} + \frac{\alpha y}{k_1}\right), \\ \frac{dy}{dt} &= r_2 y\left(1 - \frac{y}{k_2}\right). \end{aligned} \tag{1.1}$$

They investigated the local stability of all equilibrium points and showed that there was only one local stable equilibrium point in the system.

Zhu *et al.* [7], probably because of the work of [6], proposed the following obligate Lotka-Volterra model:

$$\begin{aligned} \frac{dx}{dt} &= x(a_1 + b_1 x + c_1 y), \\ \frac{dy}{dt} &= y(a_2 + c_2 y), \end{aligned} \tag{1.2}$$

where $a_1 < 0$, $a_2 > 0$, $b_1 < 0$, $c_1 > 0$, $c_2 < 0$ are all constants. They conducted a qualitative analysis on the model, and studied the thresholds of persistence and extinction of two species in a polluted environment.

As is well known, the living environment of the species is constantly changing over time, so the nonautonomous model is more realistic. Yang *et al.* [8] proposed the following system:

$$\begin{aligned} \frac{dx_1}{dt} &= x_1\bigl(-a_1(t) - b_1(t)x_1 + c_1(t)x_2\bigr), \\ \frac{dx_2}{dt} &= x_2\bigl(a_2(t) - b_2(t)x_2\bigr), \end{aligned} \tag{1.3}$$

where $a_i(t)$, $b_i(t)$, $i = 1, 2$, $c_1(t)$, are continuous and positive functions with upper and lower bound. They investigated the persistence, extinction, and global attractivity of the system. For more work on the commensal symbiosis system, one could refer to [6–12].

On the other hand, Ayala *et al.* [13] conducted experiments on fruit fly dynamics to test the validity of competitions, and one of the models accounting best for the experimental results is given by

$$\begin{aligned} \frac{dx_1}{dt} &= r_1x_1\left(1 - \left(\frac{x_1}{k_1}\right)^{\theta_1} - \alpha_{12}\frac{x_2}{k_1}\right), \\ \frac{dx_2}{dt} &= r_2x_2\left(1 - \alpha_{21}\frac{x_1}{k_2} - \left(\frac{x_2}{k_2}\right)^{\theta_2}\right), \end{aligned} \tag{1.4}$$

where r_i is the intrinsic rate of growth of species, k_i is the carrying capacity of species i, θ_i provides a non-linear measure of interspecific interference, and α_{ij} provides a measure of interspecific interference. The Gilpin-Ayala competition system also has been studied from different aspects by many researchers (see [13–19]). Chen [17] proved that system (1.4) is global stable while $\theta_i \geq 1$ and $\theta_i < 1$ ($i = 1, 2$). Chen *et al.* [18] proposed a two dimensional Gilpin-Ayala competition system with infinite delay. They completely discussed the structure of equilibria and proved the global stability of the system. After that, Wang *et al.* [19] discussed the extinction to the same system as studied in [18], and their results supplemented the previous work.

The Gilpin-Ayala competition system has been extensively studied, and many excellent results as regards the system have been obtained. However, to this day, still no scholar considered the Gilpin-Ayala system with commensalism symbiosis.

Simulated by the work of [8], in this paper, we propose the following commensalism system:

$$\begin{aligned} \frac{dx_1(t)}{dt} &= r_1(t)x_1(t)\left(-1 - \left(\frac{x_1(t)}{k_1(t)}\right)^{\theta_1} + \alpha_{12}(t)\frac{x_2(t)}{k_1(t)}\right), \\ \frac{dx_2(t)}{dt} &= r_2(t)x_2(t)\left(1 - \left(\frac{x_2(t)}{k_2(t)}\right)^{\theta_2}\right), \end{aligned} \tag{1.5}$$

where $x_i(t)$ is the population density of the ith species. In this system, the second species is favorable to the first species; while the first species has no influence on the second one, the first species cannot live without the second one. The typical relationship like epiphyte and plants with epiphyte can be described by this system.

The aim of the paper is, by using Lemma 2.3 and developing the analysis technique of Chen [20] to obtain a set of sufficient conditions to ensure the extinction, persistence, and global attractivity of system (1.5).

The paper is organized as follows: In Section 2, the necessary preliminaries are presented. In Section 3, the dynamic behaviors such as the permanence, extinction, and the globally attractivity of the system are investigated. In Section 4, some examples are given to illustrate the feasibility of main results. In the end, we finish this paper by a brief conclusion.

Throughout the paper, we let $f^l = \inf_{t\in R} f(t)$ and $f^u = \sup_{t\in R} f(t)$, where $f(t)$ is a continuous and bounded function. In system (1.5), we always assume that θ_i ($i = 1,2$) are positive constants, $r_i(t)$, $k_i(t)$, $\alpha_{ij}(t)$ ($i = 1,2$) are continuous and strictly positive functions, which satisfy

$$\min\{r_i^l, k_i^l, \alpha_{ij}^l\} > 0, \qquad \max\{r_i^u, k_i^u, \alpha_{ij}^u\} < +\infty.$$

2 Preliminaries

Now we will state three lemmas which will be useful in proving the main theorems.

Lemma 2.1 *If $a > 0$, $b > 0$ and $\frac{dx(t)}{dt} \geq x(t)(b - ax^{\alpha}(t))$, where α is a positive constant, when $t \geq 0$, $x(0) > 0$, we have*

$$\liminf_{t\to+\infty} x(t) \geq \left(\frac{b}{a}\right)^{\frac{1}{\alpha}}.$$

If $a > 0$, $b > 0$ and $\frac{dx(t)}{dt} \leq x(t)(b - ax^{\alpha}(t))$, where α is a positive constant, and when $t \geq 0$, $x(0) > 0$, we have

$$\limsup_{t\to+\infty} x(t) \leq \left(\frac{b}{a}\right)^{\frac{1}{\alpha}}.$$

Lemma 2.1 is a direct corollary of Lemma 2.2 of Chen [20].

Lemma 2.2 *Let h be a real number and f be a nonnegative function defined on $[h;+\infty)$ such that f is integrable on $[h;+\infty)$ and is uniformly continuous on $[h;+\infty)$, then $\lim_{t\to+\infty} f(t) = 0$.*

Lemma 2.2 is Lemma 2.4 of Chen [20].

Lemma 2.3 *If $m \leq x, y \leq M$, where $m \leq M$ are positive constants, when $\theta \geq 1$, we have $\frac{1}{\theta M^{\theta-1}}|x^\theta - y^\theta| \leq |x - y| \leq \frac{1}{\theta m^{\theta-1}}|x^\theta - y^\theta|$, and when $0 \leq \theta < 1$, we have $\frac{1}{\theta m^{\theta-1}}|x^\theta - y^\theta| \leq |x - y| \leq \frac{1}{\theta M^{\theta-1}}|x^\theta - y^\theta|$.*

Proof Let $f(t) = t^\theta$, $t \in [m, M]$, for any $m \leq x, y \leq M$, by the Lagrange theorem, we can obtain

$$x^\theta - y^\theta = \theta\xi^{\theta-1}(x - y),$$

where ξ is between x and y.

It easily follows that

when $\theta \geq 1$, we have $\frac{1}{\theta M^{\theta-1}}|x^\theta - y^\theta| \leq |x-y| \leq \frac{1}{\theta m^{\theta-1}}|x^\theta - y^\theta|$,

when $0 \leq \theta < 1$, we have $\frac{1}{\theta m^{\theta-1}}|x^\theta - y^\theta| \leq |x-y| \leq \frac{1}{\theta M^{\theta-1}}|x^\theta - y^\theta|$.

This ends the proof of Lemma 2.3. □

3 Main results

Theorem 3.1 *If*

$$(\mathrm{H}_1) \quad {\alpha_{12}}^u \cdot \frac{{k_2}^u}{k_1^l} < 1$$

holds, for any positive solution $(x_1(t), x_2(t))$ *of system* (1.5), *we have*

$$\lim_{t\to+\infty} x_1(t) = 0, \qquad \lim_{t\to+\infty} x_2(t) = x_2^*(t),$$

where $x_2^*(t)$ *is the unique positive solution of system* (1.5).

Proof From the second equation of system (1.5), we have

$$\frac{dx_2(t)}{dt} \leq r_2^u x_2(t)\left(1 - \left(\frac{x_2(t)}{k_2^u}\right)^{\theta_2}\right).$$

From Lemma 2.1, we can obtain

$$\limsup_{t\to+\infty} x_2(t) \leq k_2^u \stackrel{\text{def}}{=} M_2. \tag{3.1}$$

For any positive constant ε small enough, it follows from (3.1) that there exists a large enough $T_1 > 0$ such that

$$x_2(t) \leq k_2^u + \varepsilon, \quad t \geq T_1. \tag{3.2}$$

From the above equality and the first equation of system (1.5), we have

$$\frac{dx_1(t)}{dt} \leq r_1^u x_1(t)\left(-1 + \alpha_{12}^u \cdot \frac{({k_2}^u + \varepsilon)}{k_1^l}\right).$$

Let $\varepsilon \to 0$, by simple calculation we have

$$x_1(t) \leq x_1(0)\exp\left(r_1^u\left(-1 + \alpha_{12}^u \cdot \frac{{k_2}^u}{k_1^l}\right)\right).$$

From condition (H_1), we can obtain $-1 + {\alpha_{12}}^u \cdot \frac{{k_2}^u}{k_1^l} < 0$, it follows that

$$\lim_{t\to+\infty} x_1(t) = 0.$$

Now let us construct a Lyapunov function

$$V_2(t) = \left|\ln x_2(t) - \ln x_2^*(t)\right|.$$

Calculating the upper right derivative of $V_2(t)$ along the solution of system (1.5), it follows that

$$\begin{aligned} D^+V_2(t) &= \operatorname{sgn}\bigl(x_2(t) - x_2^*(t)\bigr) r_2(t)\left(-\left(\frac{x_2(t)}{k_2(t)}\right)^{\theta_2} + \left(\frac{x_2^*(t)}{k_2(t)}\right)^{\theta_2}\right) \\ &= -\frac{r_2(t)}{k_2^{\theta_2}(t)}\bigl|x_2^{\theta_2}(t) - x_2^{*\theta_2}(t)\bigr| \\ &\le -\frac{r_2^l}{(k_2^u)^{\theta_2}}\bigl|x_2^{\theta_2}(t) - x_2^{*\theta_2}(t)\bigr|. \end{aligned}$$

Integrating the above inequality from T to t produces

$$V_2(t) + \frac{r_2^l}{(k_2^u)^{\theta_2}}\int_T^t \bigl|x_2^{\theta_2}(t) - x_2^{*\theta_2}(t)\bigr|\,ds \le V_2(T) < +\infty, \quad t \ge T.$$

Therefore

$$\int_T^t \bigl|x_2^{\theta_2}(t) - x_2^{*\theta_2}(t)\bigr|\,ds < +\infty, \quad t \ge T.$$

Hence,

$$\bigl|x_2^{\theta_2}(t) - x_2^{*\theta_2}(t)\bigr| \in L^1\bigl([T, +\infty)\bigr).$$

From equality (3.1), we know that $x_2(t)$ and $x_2^*(t)$ all have bounded derivatives for $t \ge T$. So it follows that $|x_2^{\theta_2}(t) - x_2^{*\theta_2}(t)|$ is uniformly continuous on $[T, +\infty)$. By Lemma 2.2, we have

$$\lim_{t\to+\infty}\bigl|x_2^{\theta_2}(t) - x_2^{*\theta_2}(t)\bigr| = 0.$$

From this, it easily follows that

$$\lim_{t\to+\infty} x_2(t) = x_2^*(t).$$

This ends the proof of Theorem 3.1. □

Theorem 3.2 *If*

$$(\mathrm{H}_2) \quad {\alpha_{12}}^l \cdot \frac{{k_2}^l}{k_1^u} > 1$$

holds, then the system (1.5) *is persistent. That is, for any positive solution* $(x_1(t), x_2(t))$ *of system* (1.5), *we have*

$$m_i \le \liminf_{t\to+\infty} x_i(t) \le \limsup_{t\to+\infty} x_i(t) \le M_i, \quad i = 1, 2,$$

where m_i, M_i, $i = 1, 2$, *are positive constants.*

Proof From the first equation of system (1.5) and equality (3.2), for ε and T_1 in equality (3.2), we have

$$\frac{dx_1(t)}{dt} \le r_1^u x_1(t)\left(-1-\left(\frac{x_1(t)}{k_1^u}\right)^{\theta_1} + \alpha_{12}^u \cdot \frac{(k_2^u+\varepsilon)}{k_1^l}\right), \quad t \ge T_1.$$

From (H_2), for the above ε, we can easily get

$$-1 + {\alpha_{12}}^u \cdot \frac{({k_2}^u+\varepsilon)}{k_1^l} > 0.$$

Let $\varepsilon \to 0$, by Lemma 2.1, we have

$$\limsup_{t\to+\infty} x_1(t) \le k_1^u \cdot \left(-1+\alpha_{12}^u \cdot \frac{{k_2}^u}{k_1^l}\right)^{\frac{1}{\theta_1}} \stackrel{\text{def}}{=} M_1. \tag{3.3}$$

From the second equation of system (1.5), we have

$$\frac{dx_2(t)}{dt} \ge r_2^l x_2(t)\left(1-\left(\frac{x_2(t)}{k_2^l}\right)^{\theta_2}\right).$$

From Lemma 2.1, we can obtain

$$\liminf_{t\to+\infty} x_2(t) \ge k_2^l \stackrel{\text{def}}{=} m_2. \tag{3.4}$$

From the above equality and the above ε, there exist $T_2 > T_1 > 0$, such that

$$x_2(t) \ge k_2^l - \varepsilon, \quad t \ge T_2.$$

From the first equation of system (1.5), we have

$$\frac{dx_1}{dt} \ge r_1^l x_1(t)\left(-1-\left(\frac{x_1(t)}{k_1^l}\right)^{\theta_1} + \alpha_{12}^l \cdot \frac{(k_2^l-\varepsilon)}{k_1^u}\right), \quad t \ge T_2.$$

From (H_2) and Lemma 2.1, we can obtain

$$\liminf_{t\to+\infty} x_1(t) \ge k_1^l \cdot \left(\alpha_{12}^l \cdot \frac{{k_2}^l}{k_1^u} - 1\right)^{\frac{1}{\theta_1}} \stackrel{\text{def}}{=} m_1. \tag{3.5}$$

From inequalities (3.1), (3.3), (3.4), (3.5), we have

$$m_i \le \liminf_{t\to+\infty} x_i(t) \le \limsup_{t\to+\infty} x_i(t) \le M_i, \quad i = 1, 2.$$

This ends the proof of Theorem 3.2. □

Theorem 3.3 *If condition* (H_2) *holds, we see that, when*

$$(\mathrm{H}_3) \quad \theta_2 \ge 1, \quad \frac{\theta_2 r_2^l k_1^l (k_2^l)^{\theta_2-1}}{\alpha_{12}^u r_1^u (k_2^u)^{\theta_2}} > 1$$

or

$$(\mathrm{H}_4)\quad 0 \le \theta_2 \le 1, \quad \frac{\theta_2 r_2^l k_1^l}{\alpha_{12}^u r_1^u k_2^u} > 1$$

hold, *system* (1.5) *is globally attractive*.

Proof Let $(x_1(t), x_2(t))$ with $x_1(0) > 0$, $x_2(0) > 0$ and $(x_1^*(t), x_2^*(t))$ with $x_1^*(0) > 0$, $x_2^*(0) > 0$ be any two positive solutions of (1.5).

Now let us construct a Lyapunov function

$$V(t) = \left|\ln x_1(t) - \ln x_1^*(t)\right| + \left|\ln x_2(t) - \ln x_2^*(t)\right|.$$

Calculating the upper right derivative of $V(t)$ along the solution of system (1.5), it follows that

$$\begin{aligned} D^+V(t) &= r_1(t)\operatorname{sgn}\bigl(x_1(t) - x_1^*(t)\bigr)\biggl[-\biggl(\biggl(\frac{x_1(t)}{k_1(t)}\biggr)^{\theta_1} - \biggl(\frac{x_1^*(t)}{k_1(t)}\biggr)^{\theta_1}\biggr) + \alpha_{12}(t)\biggl(\biggl(\frac{x_2(t)}{k_1(t)}\biggr) \\ &\quad - \biggl(\frac{x_2^*(t)}{k_1(t)}\biggr)\biggr)\biggr] - r_2(t)\operatorname{sgn}\bigl(x_2(t) - x_2^*(t)\bigr)\biggl(\biggl(\frac{x_2(t)}{k_2(t)}\biggr)^{\theta_2} - \biggl(\frac{x_2^*(t)}{k_2(t)}\biggr)^{\theta_2}\biggr) \\ &\le -\frac{r_1^l}{(k_1^u)^{\theta_1}}\bigl|x_1^{\theta_1}(t) - x_1^{*\theta_1}(t)\bigr| + \frac{r_1^u \alpha_{12}^u}{(k_1^l)}\bigl|x_2(t) - x_2^*(t)\bigr| \\ &\quad - \frac{r_2^l}{(k_2^u)^{\theta_2}}\bigl|x_2^{\theta_2}(t) - x_2^{*\theta_2}(t)\bigr|. \end{aligned} \tag{3.6}$$

From Theorem 3.2, we can see that there exists a large enough $T_3 > 0$ such that

$$k_2^l \le x_2(t), \qquad x_2^*(t) \le k_2^u, \quad i = 1, 2, t \ge T_3.$$

From Lemma 2.3, we see that, if $\theta_2 \ge 1$, then

$$\bigl|x_2(t) - x_2^*(t)\bigr| \le \frac{1}{\theta_2 (k_2^l)^{\theta_2 - 1}}\bigl|x_2(t)^{\theta_2} - x_2^*(t)^{\theta_2}\bigr|, \quad t \ge T_3.$$

So, from inequality (3.6) we have

$$D^+V(t) \le -\frac{r_1^l}{(k_1^u)^{\theta_1}}\bigl|x_1^{\theta_1}(t) - x_1^{*\theta_1}(t)\bigr| - \biggl(\frac{r_2^l}{(k_2^u)^{\theta_2}} - \frac{\alpha_{12}^u r_1^u}{\theta_2 k_1^l (k_2^l)^{\theta_2 - 1}}\biggr)\bigl|x_2^{\theta_2}(t) - x_2^{*\theta_2}(t)\bigr|.$$

From (H_3), we have

$$\biggl(\frac{r_2^l}{(k_2^u)^{\theta_2}} - \frac{\alpha_{12}^u r_1^u}{\theta_2 k_1^l (k_2^l)^{\theta_2 - 1}}\biggr) > 0.$$

So, it follows that there exists a positive constant $\alpha > 0$ and large enough $T_4 > T_3 > 0$ such that

$$D^+V(t) \le -\alpha\bigl(\bigl|x_1^{\theta_1}(t) - x_1^{*\theta_1}(t)\bigr| + \bigl|x_2^{\theta_2}(t) - x_2^{*\theta_2}(t)\bigr|\bigr), \quad t \ge T_4. \tag{3.7}$$

Similar to the analysis in Theorem 3.1, we can get

$$\lim_{t\to+\infty}\left(\left|x_1^{\theta_1}(t)-x_1^{*\theta_1}(t)\right|+\left|x_2^{\theta_2}(t)-x_2^{*\theta_2}(t)\right|\right)=0.$$

It easily follows that

$$\lim_{t\to+\infty}x_1(t)=x_1^*(t),\qquad \lim_{t\to+\infty}x_2(t)=x_2^*(t).$$

From Lemma 2.3, we see that, if $0\leq\theta_2<1$, then

$$\left|x_2(t)-x_2^*(t)\right|\leq\frac{1}{\theta_2(k_2^u)^{\theta_2-1}}\left|x_2(t)^{\theta_2}-x_2^*(t)^{\theta_2}\right|.$$

So, from equality (3.6) we have

$$D^+V(t)\leq-\frac{r_1^l}{(k_1^u)^{\theta_1}}\left|x_1^{\theta_1}(t)-x_1^{*\theta_1}(t)\right|-\left(\frac{r_2^l}{(k_2^u)^{\theta_2}}-\frac{\alpha_{12}^u r_1^u}{\theta_2 k_1^l(k_2^u)^{\theta_2-1}}\right)\left|x_2^{\theta_2}(t)-x_2^{*\theta_2}(t)\right|.$$

From (H_4), we have

$$\left(\frac{r_2^l}{(k_2^u)^{\theta_2}}-\frac{\alpha_{12}^u r_1^u}{\theta_2 k_1^l(k_2^u)^{\theta_2-1}}\right)>0.$$

So, it follows that there exists a positive constant $\beta>0$ and large enough $T_5>T_3>0$ such that

$$D^+V(t)\leq-\beta\left(\left|x_1^{\theta_1}(t)-x_1^{*\theta_1}(t)\right|+\left|x_2^{\theta_2}(t)-x_2^{*\theta_2}(t)\right|\right),\quad t\geq T_5. \tag{3.8}$$

Similar to the analysis in Theorem 3.1, we can get

$$\lim_{t\to+\infty}\left(\left|x_1^{\theta_1}(t)+x_1^{*\theta_1}(t)\right|+\left|x_2^{\theta_2}(t)-x_2^{*\theta_2}(t)\right|\right)=0.$$

It easily follows that

$$\lim_{t\to+\infty}x_1(t)=x_1^*(t),\qquad \lim_{t\to+\infty}x_2(t)=x_2^*(t).$$

This ends the proof of Theorem 3.3. □

4 Examples

The following two examples show the feasibility of our main results.

Example 4.1 Consider the following equations:

$$\begin{aligned}\frac{dx_1(t)}{dt}&=x_1(t)\left(-1-\left(\frac{x_1(t)}{2+\sin(\sqrt{5}t)}\right)^2+\frac{1}{5}\left(\frac{x_2(t)}{2+\sin(\sqrt{5}t)}\right)\right),\\ \frac{dx_2(t)}{dt}&=2x_2(t)\left(1-\left(\frac{x_2(t)}{2+\cos(\sqrt{5}t)}\right)^{\frac{1}{2}}\right).\end{aligned}\tag{4.1}$$

Figure 1 Dynamic behavior of the solution $(x_1(t), x_2(t))$ to system (4.1) with the initial conditions $(x_1(0), x_2(0)) = (0.7, 1.8), (1.2, 0.3)$, and $(2.2, 2.8)$, respectively.

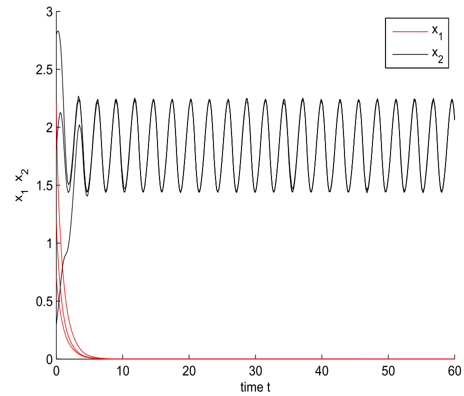
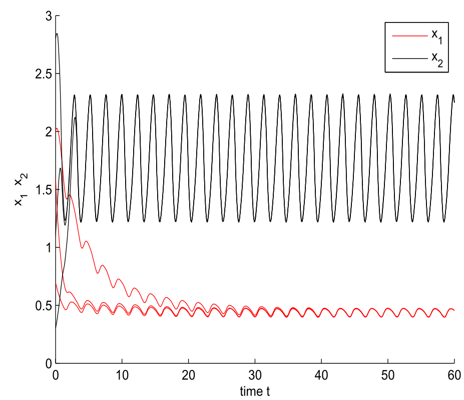


Figure 2 Dynamic behavior of the solution $(x_1(t), x_2(t))$ to system (4.2) with the initial conditions $(x_1(0), x_2(0)) = (0.7, 1.1), (1.6, 0.3)$, and $(2.2, 2.8)$, respectively.



By simple calculation, we see that (H_1) holds, by Theorem 3.1, for any positive solution $(x_1(t), x_2(t))$ of system (4.1),

$$\lim_{t\to+\infty} x_1(t) = 0, \qquad \lim_{t\to+\infty} x_2(t) = x_2^*(t).$$

Figure 1 shows the dynamic behavior of system (4.1).

Example 4.2 Consider the following system:

$$\begin{aligned} \frac{dx_1(t)}{dt} &= \frac{1}{2}x_1(t)\left(-1 - \left(\frac{x_1(t)}{2+\sin(\sqrt{7}t)}\right)^{\frac{2}{5}} + \frac{5}{3}\left(\frac{x_2(t)}{2+\sin(\sqrt{7}t)}\right)\right), \\ \frac{dx_2(t)}{dt} &= x_2(t)\left(1 - \left(\frac{x_2(t)}{2+\cos(\sqrt{7}t)}\right)^2\right). \end{aligned} \tag{4.2}$$

By simple calculation, (H_2) and (H_3) hold, and by Theorem 3.3, system (4.2) is globally attractive. Figure 2 shows the dynamic behavior of system (4.2).

5 Conclusion

In this paper, a nonautonomous obligate Gilpin-Ayala system is considered. Some results as regards extinction, persistence, and global attractivity of the system are obtained. Theorem 3.1 and Theorem 3.2 show that, when (H_1) holds, that is, $\frac{\alpha_{12}^u}{k_1^l} < \frac{1}{k_2^u}$, the cooperation between the species is small, then the first species could be driven extinct, while the second species has stability; when (H_2) holds, that is, $\frac{\alpha_{12}^l}{k_1^u} > \frac{1}{k_2^l}$, the cooperation between species is big, then the two species could be permanent. These results show that the extinction or permanence of the first species is depending on the cooperation intensity between the two species.

Competing interests
The author declares that they have no competing interests.

Author's contributions
Only the author contributed to the writing of this paper. The author read and approved the final manuscript.

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