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Global attractivity of a discrete competition model

Baoguo Chen*

*Correspondence:
chenbaoguo2016@163.com
Research Center for Science Technology and Society, Fuzhou University of International Studies and Trade, Fuzhou, Fujian 350202, P.R. China

Abstract

We study a discrete competition model of the form

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\left\{r_1\left[\frac{K_1+\alpha_1 x_2(k)}{1+x_2(k)} - x_1(k)\right]\right\}, \\ x_2(k+1) &= x_2(k)\exp\left\{r_2\left[\frac{K_2+\alpha_2 x_1(k)}{1+x_1(k)} - x_2(k)\right]\right\}, \end{aligned}$$

where $x_i(k)$ ($i = 1, 2$) are the population density of the ith species at k-generation. r_i, K_i, $i = 1, 2$, are all positive constants such that $K_i > \alpha_i$, $i = 1, 2$. By using the iterative method and the comparison principle of difference equations we obtain sufficient conditions that ensure the global attractivity of the interior equilibrium of the system. Our result supplements and complements the main result of Yang et al. (Abstr. Appl. Anal. 2014:709124, 2014).

MSC: 34C05; 92D25; 34D20; 34D40

Keywords: competition system; discrete; global stability

1 Introduction

Yang et al. [1] studied the dynamic behavior of the following autonomous discrete cooperative system (1.1):

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\left\{r_1\left[\frac{K_1+\alpha_1 x_2(k)}{1+x_2(k)} - x_1(k)\right]\right\}, \\ x_2(k+1) &= x_2(k)\exp\left\{r_2\left[\frac{K_2+\alpha_2 x_1(k)}{1+x_1(k)} - x_2(k)\right]\right\}, \end{aligned} \tag{1.1}$$

where $x_i(k)$ ($i = 1, 2$) are the population density of the ith species at k-generation. They showed that if

(H_1) r_i, K_i, α_i ($i = 1, 2$) are all positive constants, and $\alpha_i > K_i$ ($i = 1, 2$),

and

(H_2) $r_i\alpha_i \le 1$ ($i = 1, 2$),

then system (1.1) admits a unique positive equilibrium (x_1^*, x_2^*), which is globally asymptotically stable.

It brought to our attention that the main results of [1] deeply depend on the assumption $\alpha_i > K_i$, $i = 1, 2$. Now, an interesting issue is proposed: Is it possible for us to investigate the stability property of system (1.1) under the assumption $K_i > \alpha_i$, $i = 1, 2$?

Note that in this case, the first equation of system (1.1) can be rewritten as

$$\begin{aligned} &x_1(k+1) \\ &\quad = x_1(k)\exp\left\{r_1\left[\frac{K_1+\alpha_1 x_2(k)}{1+x_2(k)} - x_1(k)\right]\right\} \\ &\quad = x_1(k)\exp\left\{r_1\left[\frac{K_1(1+x_2(k))}{1+x_2(k)} - x_1(k) - \frac{(K_1-\alpha_1)x_2(k)}{1+x_2(k)}\right]\right\} \\ &\quad = x_1(k)\exp\left\{r_1\left[K_1 - x_1(k) - \frac{(K_1-\alpha_1)x_2(k)}{1+x_2(k)}\right]\right\}. \end{aligned} \tag{1.2}$$

Similarly to the above analysis, the second equation of system (1.1) can be rewritten as

$$x_2(k+1) = x_2(k)\exp\left\{r_2\left[K_2 - x_2(k) - \frac{(K_2-\alpha_2)x_1(k)}{1+x_1(k)}\right]\right\}. \tag{1.3}$$

From (1.2) and (1.3) we can easily see that both species have negative effect to the other species, that is, under the assumption $K_i > \alpha_i$, $i = 1, 2$, the relationship between two species is competition. Also, we mention here that under the assumption $K_i > \alpha_i$, $i = 1, 2$, system (1.1) admits a unique positive equilibrium (x_1^*, x_2^*). Indeed, the positive equilibrium of system (1.1) satisfies

$$\begin{cases} \frac{K_1+\alpha_1 x_2}{1+x_2} - x_1 = 0, \\ \frac{K_2+\alpha_2 x_1}{1+x_1} - x_2 = 0, \end{cases} \tag{1.4}$$

which is equivalent to

$$\begin{cases} A_1 x_1^2 + A_2 x_1 + A_3 = 0, \\ B_1 x_2^2 + B_2 x_2 + B_3 = 0, \end{cases} \tag{1.5}$$

where

$$\begin{aligned} &A_1 = \alpha_2 + 1, \qquad A_2 = -\alpha_1\alpha_2 - K_1 + K_2 + 1, \qquad A_3 = -K_2\alpha_1 - K_1, \\ &B_1 = \alpha_1 + 1, \qquad B_2 = -\alpha_1\alpha_2 + K_1 - K_2 + 1, \qquad B_3 = -K_1\alpha_2 - K_1. \end{aligned} \tag{1.6}$$

From $A_1 > 0$, $A_3 < 0$, $B_1 > 0$, $B_3 < 0$ we can easily see that system (1.5) admits a unique positive solution

$$\begin{aligned} x_1^* &= \frac{-A_2 + \sqrt{A_2^2 - 4A_1A_3}}{2A_1}, \\ x_2^* &= \frac{-B_2 + \sqrt{B_2^2 - 4B_1B_3}}{2B_1}. \end{aligned} \tag{1.7}$$

Since the relationship between two species is competition, and the system admits a unique positive equilibrium, it is natural to seek some suitable conditions that ensure the global attractivity of the positive equilibrium. Since the existence of stable positive equilibrium represents the stable coexistence of the two species, establishing some similar result as that of [1] becomes a challenging problem.

The aim of this paper is to obtain a set of sufficient conditions to ensure the global attractivity of the interior equilibrium of system (1.1). More precisely, we will prove the following result.

Theorem 1.1 *Assume that*

(A_1) r_i, K_i, α_i $(i = 1, 2)$ *are all positive constants, and* $K_i > \alpha_i$ $(i = 1, 2)$,

and

(A_2) $r_i K_i \leq 1$ $(i = 1, 2)$.

Then system (1.1) *admits a unique positive equilibrium* (x_1^*, x_2^*), *which is globally attractive*.

The rest of the paper is arranged as follows. With the help of several useful lemmas, we will prove Theorem 1.1 in Section 2. An example, together with its numeric simulations, is presented in Section 3 to show the feasibility of our results. We end this paper by a brief discussion. For more work on cooperative or competitive systems, we refer to [1–33] and the references therein.

2 Global attractivity

Before we prove Theorem 1.1, we need to introduce several useful lemmas.

Lemma 2.1 ([12]) *Let* $f(u) = u\exp(\alpha - \beta u)$, *where* α *and* β *are positive constants. Then* $f(u)$ *is nondecreasing for* $u \in (0, \frac{1}{\beta}]$.

Lemma 2.2 ([12]) *Assume that a sequence* $\{u(k)\}$ *satisfies*

$$u(k+1) = u(k)\exp\bigl(\alpha - \beta u(k)\bigr), \quad k = 1, 2, \ldots,$$

where α *and* β *are positive constants, and* $u(0) > 0$. *Then*

(i) *if* $\alpha < 2$, *then* $\lim_{k\to+\infty} u(k) = \frac{\alpha}{\beta}$;

(ii) *if* $\alpha \leq 1$, *then* $u(k) \leq \frac{1}{\beta}$, $k = 2, 3, \ldots$.

Lemma 2.3 ([24]) *Suppose that functions* $f, g : Z_+ \times [0, \infty) \to [0, \infty)$ *satisfy* $f(k, x) \leq g(k, x)$ $(f(k, x) \geq g(k, x))$ *for* $k \in Z_+$ *and* $x \in [0, \infty)$ *and* $g(k, x)$ *is nondecreasing with respect to* x. *If* $\{x(k)\}$ *and* $\{u(k)\}$ *are the nonnegative solutions of the difference equations*

$$x(k+1) = f\bigl(k, x(k)\bigr) \quad \text{and} \quad u(k+1) = g\bigl(k, u(k)\bigr),$$

respectively, and $x(0) \leq u(0)$ $(x(0) \geq u(0))$. *Then*

$$x(k) \leq u(k) \qquad \bigl(x(k) \geq u(k)\bigr) \quad \text{for all } k \geq 0.$$

Proof of Theorem 1.1 Let $(x_1(k), x_2(k))$ be an arbitrary solution of system (1.1) with $x_1(0) > 0$ and $x_2(0) > 0$. Denote

$$U_1 = \limsup_{k\to+\infty} x_1(k), \qquad V_1 = \liminf_{k\to+\infty} x_1(k),$$
$$U_2 = \limsup_{k\to+\infty} x_2(k), \qquad V_2 = \liminf_{k\to+\infty} x_2(k).$$

We claim that $U_1 = V_1 = \overline{x}_1$ and $U_2 = V_2 = \overline{x}_2$.

From (1.2) we obtain

$$x_1(k+1) \le x_1(k)\exp\{r_1K_1 - r_1x_1(k)\}, \quad k = 0,1,2,\dots. \tag{2.1}$$

Consider the auxiliary equation

$$u(k+1) = u(k)\exp\{r_1K_1 - r_1u(k)\}, \quad k = 0,1,2,\dots. \tag{2.2}$$

Because of $0 < r_1K_1 \le 1$, according to (ii) of Lemma 2.2, we can obtain $u(k) \le \frac{1}{r_1}$ for all $k \ge 2$, where $u(k)$ is an arbitrary positive solution of (2.2) with initial value $u(0) > 0$. By Lemma 2.1, $f(u) = u\exp(r_1K_1 - r_1u)$ is nondecreasing for $u \in (0, \frac{1}{r_1}]$. According to Lemma 2.3, we obtain $x_1(k) \le u(k)$ for all $k \ge 2$, where $u(k)$ is the solution of (2.2) with initial value $u(2) = x_1(2)$. According to (i) of Lemma 2.2, we obtain

$$U_1 = \limsup_{k\to+\infty} x_1(k) \le \lim_{k\to+\infty} u(k) = K_1 \stackrel{\text{def}}{=} M_1^{x_1}. \tag{2.3}$$

From (1.3) we obtain

$$x_2(k+1) \le x_2(k)\exp\{r_2K_2 - r_2x_2(k)\}, \quad k = 0,1,2,\dots. \tag{2.4}$$

Similarly to the above analysis, we have

$$U_2 = \limsup_{k\to+\infty} x_2(k) \le \lim_{k\to+\infty} u(k) = K_2 \stackrel{\text{def}}{=} M_1^{x_2}. \tag{2.5}$$

Then, for a sufficiently small constant $\varepsilon > 0$, there is an integer $k_1 > 2$ such that

$$x_1(k) < M_1^{x_1} + \varepsilon, \qquad x_2(k) < M_1^{x_2} + \varepsilon \quad \text{for all } k > k_1. \tag{2.6}$$

According to (1.2), we obtain

$$\begin{aligned}
&x_1(k+1)\\
&\quad = x_1(k)\exp\left\{r_1\left[K_1 - x_1(k) - \frac{(K_1-\alpha_1)x_2(k)}{1+x_2(k)}\right]\right\}\\
&\quad \ge x_1(k)\exp\left\{r_1\left[K_1 - x_1(k) - \frac{(K_1-\alpha_1)(M_1^{x_2}+\varepsilon)}{1+(M_1^{x_2}+\varepsilon)}\right]\right\}\\
&\quad \ge x_1(k)\exp\left\{r_1\left[K_1 - x_1(k) - \frac{(K_1-\alpha_1)(M_1^{x_2}+\varepsilon)}{M_1^{x_2}+\varepsilon}\right]\right\}
\end{aligned}$$

$$\begin{aligned} &\geq x_1(k)\exp\bigl\{r_1\bigl[K_1 - x_1(k) - (K_1-\alpha_1)\bigr]\bigr\} \\ &= x_1(k)\exp\bigl\{r_1\alpha_1 - r_1x_1(k)\bigr\}. \end{aligned} \tag{2.7}$$

Consider the auxiliary equation

$$u(k+1) = u(k)\exp\bigl\{r_1\alpha_1 - r_1u(k)\bigr\}. \tag{2.8}$$

Since $0 < r_1\alpha_1 \leq r_1K_1 \leq 1$, according to (ii) of Lemma 2.2, we obtain $u(k) \leq \frac{1}{r_1}$ for all $k \geq 2$, where $u(k)$ is an arbitrary positive solution of (2.8) with initial value $u(0) > 0$. By Lemma 2.1, $f(u) = u\exp(r_1\alpha_1 - r_1u)$ is nondecreasing for $u \in (0, \frac{1}{r_1}]$. According to Lemma 2.3, we obtain $x_1(k) \geq u(k)$ for all $k \geq 2$, where $u(k)$ is the solution of (2.8) with initial value $u(k_1) = x_1(k_1)$. According to (i) of Lemma 2.2, we have

$$V_1 = \liminf_{k\to+\infty} x_1(k) \geq \lim_{k\to+\infty} u(k) = \alpha_1 \stackrel{\text{def}}{=} m_1^{x_1}. \tag{2.9}$$

From (1.3) we obtain

$$x_2(k+1) \geq x_2(k)\exp\bigl\{r_2\alpha_2 - r_2x_2(k)\bigr\}. \tag{2.10}$$

Similarly to the analysis of (2.7)-(2.9), we have

$$V_2 = \liminf_{k\to+\infty} x_2(k) \geq \lim_{k\to+\infty} u(k) = \alpha_2 \stackrel{\text{def}}{=} m_1^{x_2}. \tag{2.11}$$

Then, for a sufficiently small constant $\varepsilon > 0$ (without loss of generality, we may assume that $\varepsilon < \frac{1}{2}\{\alpha_1, \alpha_2\}$), there exists an integer $k_2 > k_1$ such that

$$x_1(k) > m_1^{x_1} - \varepsilon, \qquad x_2(k) > m_1^{x_2} - \varepsilon \quad \text{for all } k > k_2. \tag{2.12}$$

The second inequality in (2.12), combined with (1.2), leads to

$$\begin{aligned} &x_1(k+1) \\ &\quad = x_1(k)\exp\left\{r_1\left[K_1 - x_1(k) - \frac{(K_1-\alpha_1)x_2(k)}{1+x_2(k)}\right]\right\} \\ &\quad \leq x_1(k)\exp\left\{r_1\left[K_1 - x_1(k) - \frac{(K_1-\alpha_1)(m_1^{x_2}-\varepsilon)}{1+(m_1^{x_2}-\varepsilon)}\right]\right\}. \end{aligned} \tag{2.13}$$

Noting that

$$\begin{aligned} 0 < r_1\alpha_1 &= r_1\left[K_1 - \frac{(K_1-\alpha_1)(m_1^{x_2}-\varepsilon)}{(m_1^{x_2}-\varepsilon)}\right] \\ &< r_1\left[K_1 - \frac{(K_1-\alpha_1)(m_1^{x_2}-\varepsilon)}{1+(m_1^{x_2}-\varepsilon)}\right] \leq r_1K_1 \leq 1, \end{aligned}$$

similarly to the analysis of (2.1)-(2.3), we have

$$U_1 = \limsup_{k\to+\infty} x_1(k) \leq K_1 - \frac{(K_1-\alpha_1)(m_1^{x_2}-\varepsilon)}{1+(m_1^{x_2}-\varepsilon)} \stackrel{\text{def}}{=} M_2^{x_1}. \tag{2.14}$$

Obviously,

$$M_2^{x_1} = K_1 - \frac{(K_1 - \alpha_1)(m_1^{x_2} - \varepsilon)}{1 + (m_1^{x_2} - \varepsilon)} < K_1 = M_1^{x_1}. \tag{2.15}$$

For $k > k_2$, the second inequality in (2.12), combined with (1.3), leads to

$$x_2(k+1) \le x_2(k)\exp\left\{r_2\left[K_2 - x_2(k) - \frac{(K_2 - \alpha_2)(m_1^{x_1} - \varepsilon)}{1 + (m_1^{x_1} - \varepsilon)}\right]\right\}. \tag{2.16}$$

By applying Lemma 2.3 to the above inequality we obtain

$$U_2 = \limsup_{k\to+\infty} x_2(k) \le K_2 - \frac{(K_2 - \alpha_2)(m_1^{x_1} - \varepsilon)}{1 + (m_1^{x_1} - \varepsilon)} \stackrel{\text{def}}{=} M_2^{x_2}. \tag{2.17}$$

Obviously, we have

$$M_2^{x_2} = K_2 - \frac{(K_2 - \alpha_2)(m_1^{x_1} - \varepsilon)}{1 + (m_1^{x_1} - \varepsilon)} < K_2 = M_1^{x_2}. \tag{2.18}$$

Then, for a sufficiently small constant $\varepsilon > 0$, there is an integer $k_3 > k_2$ such that

$$x_1(k) < M_2^{x_1} + \varepsilon, \qquad x_2(k) < M_2^{x_2} + \varepsilon \quad \text{for all } k > k_3. \tag{2.19}$$

The second inequality in (2.19), combined with (1.2), leads to

$$\begin{aligned} &x_1(k+1) \\ &\quad = x_1(k)\exp\left\{r_1\left[K_1 - x_1(k) - \frac{(K_1 - \alpha_1)x_2(k)}{1 + x_2(k)}\right]\right\} \\ &\quad \ge x_1(k)\exp\left\{r_1\left[K_1 - x_1(k) - \frac{(K_1 - \alpha_1)(M_2^{x_2} + \varepsilon)}{1 + (M_2^{x_2} + \varepsilon)}\right]\right\}. \end{aligned} \tag{2.20}$$

Noting that

$$\begin{aligned} 0 < r_1\alpha_1 &= r_1\left[K_1 - \frac{(K_1 - \alpha_1)(M_2^{x_2} + \varepsilon)}{(M_2^{x_2} + \varepsilon)}\right] \\ &< r_1\left[K_1 - \frac{(K_1 - \alpha_1)(M_2^{x_2} + \varepsilon)}{1 + (M_2^{x_2} + \varepsilon)}\right] \le r_1K_1 \le 1, \end{aligned}$$

from this we finally obtain

$$V_1 = \liminf_{k\to+\infty} x_1(k) \ge K_1 - \frac{(K_1 - \alpha_1)(M_2^{x_2} + \varepsilon)}{1 + (M_2^{x_2} + \varepsilon)} \stackrel{\text{def}}{=} m_2^{x_1}. \tag{2.21}$$

From the monotonicity of the function $g(x) = \frac{x}{1+x}$ and (2.18) we have

$$m_2^{x_1} > m_1^{x_1}. \tag{2.22}$$

The first inequality in (2.19), combined with (1.3), leads to

$$\begin{aligned} &x_2(k+1) \\ &\quad = x_2(k)\exp\left\{r_2\left[K_2 - x_2(k) - \frac{(K_2-\alpha_2)x_1(k)}{1+x_1(k)}\right]\right\} \\ &\quad \geq x_2(k)\exp\left\{r_2\left[K_2 - x_2(k) - \frac{(K_2-\alpha_2)(M_2^{x_1}+\varepsilon)}{1+(M_2^{x_1}+\varepsilon)}\right]\right\}. \end{aligned} \tag{2.23}$$

From this inequality we obtain

$$V_2 = \liminf_{k\to+\infty} x_2(k) \geq K_2 - \frac{(K_2-\alpha_2)(M_2^{x_1}+\varepsilon)}{1+(M_2^{x_1}+\varepsilon)} \stackrel{\text{def}}{=} m_2^{x_2}. \tag{2.24}$$

From the monotonicity of the function $g(x) = \frac{x}{1+x}$ and (2.15) we have

$$m_2^{x_2} > m_1^{x_2}. \tag{2.25}$$

Then, for a sufficiently small constant $\varepsilon > 0$, there is an integer $k_4 > k_3$ such that

$$x_1(k) > m_2^{x_1} - \varepsilon, \qquad x_2(k) > m_2^{x_2} - \varepsilon \quad \text{for all } k > k_4. \tag{2.26}$$

Continuing the above steps, we get four sequences $\{M_k^{x_1}\}$, $\{M_k^{x_2}\}$, $\{m_k^{x_1}\}$, and $\{m_k^{x_2}\}$ such that

$$\begin{aligned} M_k^{x_1} &= K_1 - \frac{(K_1-\alpha_1)(m_{k-1}^{x_2}-\varepsilon)}{1+(m_{k-1}^{x_2}-\varepsilon)}, \\ M_k^{x_2} &= K_2 - \frac{(K_2-\alpha_2)(m_{k-1}^{x_1}-\varepsilon)}{1+(m_{k-1}^{x_1}-\varepsilon)}, \end{aligned} \tag{2.27}$$

$$\begin{aligned} m_k^{x_1} &= K_1 - \frac{(K_1-\alpha_1)(M_k^{x_2}+\varepsilon)}{1+(M_k^{x_2}+\varepsilon)}, \\ m_k^{x_2} &= K_2 - \frac{(K_2-\alpha_2)(M_k^{x_1}+\varepsilon)}{1+(M_k^{x_1}+\varepsilon)}. \end{aligned} \tag{2.28}$$

Clearly, we have

$$m_k^{x_1} < V_1 \leq U_1 < M_k^{x_1}, \qquad m_k^{x_2} < V_2 \leq U_2 < M_k^{x_2}, \quad k = 0,1,2,\ldots. \tag{2.29}$$

Now, we will prove by induction that $\{M_k^{x_i}\}$ $(i = 1,2)$ is monotonically decreasing and $\{m_k^{x_i}\}$ $(i = 1,2)$ is monotonically increasing.

First of all, it is clear that $M_2^{x_i} < M_1^{x_i}$ and $m_2^{x_i} > m_1^{x_i}$ $(i = 1,2)$. For $i \geq 2$, we assume that $M_i^{x_1} < M_{i-1}^{x_1}$ and $m_i^{x_1} > m_{i-1}^{x_1}$. Then, since the function $g(x) = \frac{x}{1+x}$ is increasing, we have

$$\begin{aligned} M_{i+1}^{x_2} &= K_2 - \frac{(K_2-\alpha_2)(m_i^{x_1}-\varepsilon)}{1+(m_i^{x_1}-\varepsilon)} \\ &< K_2 - \frac{(K_2-\alpha_2)(m_{i-1}^{x_1}-\varepsilon)}{1+(m_{i-1}^{x_1}-\varepsilon)} = M_i^{x_2}, \end{aligned} \tag{2.30}$$

$$\begin{aligned} M_{i+1}^{x_1} &= K_1 - \frac{(K_1-\alpha_1)(m_i^{x_2}-\varepsilon)}{1+(m_i^{x_2}-\varepsilon)} \\ &< K_1 - \frac{(K_1-\alpha_1)(m_{i-1}^{x_2}-\varepsilon)}{1+(m_{i-1}^{x_2}-\varepsilon)} = M_i^{x_1}, \end{aligned} \tag{2.31}$$

and so,

$$\begin{aligned} m_{i+1}^{x_1} &= K_1 - \frac{(K_1-\alpha_1)(M_{i+1}^{x_2}+\varepsilon)}{1+(M_{i+1}^{x_2}+\varepsilon)} \\ &> K_1 - \frac{(K_1-\alpha_1)(M_i^{x_2}+\varepsilon)}{1+(M_i^{x_2}+\varepsilon)} = m_i^{x_1}, \end{aligned} \tag{2.32}$$

$$\begin{aligned} m_{i+1}^{x_2} &= K_2 - \frac{(K_2-\alpha_2)(M_{i+1}^{x_1}+\varepsilon)}{1+(M_{i+1}^{x_1}+\varepsilon)} \\ &> K_2 - \frac{(K_2-\alpha_2)(M_i^{x_1}+\varepsilon)}{1+(M_i^{x_1}+\varepsilon)} = m_i^{x_2}. \end{aligned} \tag{2.33}$$

Inequalities (2.30)-(2.33) show that $\{M_k^{x_1}\}$ and $\{M_k^{x_2}\}$ are decreasing and $\{m_k^{x_1}\}$ and $\{m_k^{x_2}\}$ are increasing. Consequently, $\lim_{k\to+\infty}\{M_k^{x_i}\}$ and $\lim_{k\to+\infty}\{m_k^{x_i}\}$ $(i=1,2)$ both exist. Let

$$\lim_{k\to+\infty} M_k^{x_i} = \overline{X}_i, \qquad \lim_{k\to+\infty} m_k^{x_i} = \underline{X}_i, \quad i=1,2. \tag{2.34}$$

From (2.27) and (2.28) we have

$$\overline{X}_1 = K_1 - \frac{(K_1-\alpha_1)\underline{X}_2}{1+\underline{X}_2}, \qquad \overline{X}_2 = K_2 - \frac{(K_2-\alpha_2)\underline{X}_1}{1+\underline{X}_1}; \tag{2.35}$$

$$\underline{X}_1 = K_1 - \frac{(K_1-\alpha_1)\overline{X}_2}{1+\overline{X}_2}, \qquad \underline{X}_2 = K_2 - \frac{(K_2-\alpha_2)\overline{X}_1}{1+\overline{X}_1}. \tag{2.36}$$

Equalities (2.35) and (2.36) are equivalent to

$$\overline{X}_1 = \frac{K_1+\alpha_1\underline{X}_2}{1+\underline{X}_2}, \qquad \overline{X}_2 = \frac{K_2+\alpha_2\underline{X}_1}{1+\underline{X}_1}; \tag{2.37}$$

$$\underline{X}_1 = \frac{K_1+\alpha_1\overline{X}_2}{1+\overline{X}_2}, \qquad \underline{X}_2 = \frac{K_2+\alpha_2\overline{X}_1}{1+\overline{X}_1}. \tag{2.38}$$

Equalities (2.37) and (2.38) show that $(\overline{X}_1, \underline{X}_2)$ and $(\underline{X}_1, \overline{X}_2)$ both are solutions of system (1.4). However, under the assumption of Theorem 1.1, system (1.4) has a unique positive solution (x_1^*, x_2^*). Therefore,

$$U_i = V_i = \lim_{k\to+\infty} x_i(k) = x_i^*, \quad i=1,2, \tag{2.39}$$

that is, $E_+(x_1^*, x_2^*)$ is globally attractive. This ends the proof of Theorem 1.1. □

3 Example

In this section, we give an example to illustrate the feasibility of the main result.

Example Considering the following competition system:

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\left\{2\left[\frac{0.3+0.1x_2(k)}{1+x_2(k)} - x_1(k)\right]\right\}, \\ x_2(k+1) &= x_2(k)\exp\left\{3\left[\frac{0.2+0.1x_1(k)}{1+x_1(k)} - x_2(k)\right]\right\}. \end{aligned} \tag{3.1}$$

For system (1.1), we have $r_1 = 2$, $r_2 = 3$, $K_1 = 0.3$, $K_2 = 0.2$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$. By calculation we have that the positive equilibrium $E_+(x_1^*, x_2^*) \approx (0.27, 0.18)$, $r_1K_1 = 0.6 < 1$, $r_2K_2 = 0.6 < 1$, $K_i > \alpha_i$ $(i = 1, 2)$, and the coefficients of system (3.1) satisfy (A_1) and (A_2) in Theorem 1.1. By Theorem 1.1 the unique positive equilibrium $E_+(x_1^*, x_2^*)$ is globally attractive. Numeric simulations also support our finding (see Figures 1 and 2).

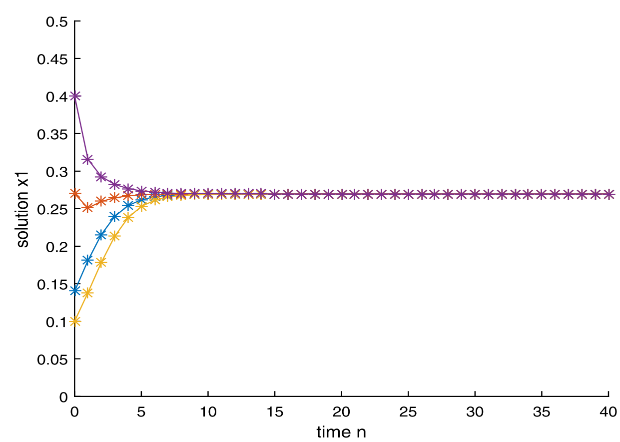

Figure 1 Dynamic behavior of the first component of the solution $(x_1(k), x_2(k))$ of system (3.1) with initial conditions $(x_1(0), x_2(0)) = (0.14, 0.19)$, $(0.27, 0.69)$, $(0.4, 0.1)$, and $(0.10, 0.24)$, respectively.

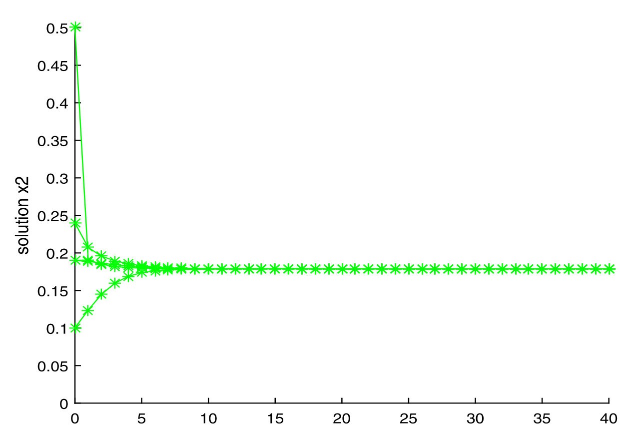

Figure 2 Dynamic behavior of the second component of the solution $(x_1(k), x_2(k))$ of system (3.1), with initial conditions $(x_1(0), x_2(0)) = (0.14, 0.19)$, $(0.27, 0.69)$, $(0.4, 0.1)$, and $(0.10, 0.24)$, respectively.

4 Discussion

Yang et al. [1] proposed system (1.1) under the assumption $\alpha_i > K_i$, $i = 1,2$, and showed that if $r_i\alpha_i \leq 1$, then the mutualism model admits a unique globally asymptotically stable positive equilibrium.

In this paper, we try to deal with the case $K_i > \alpha_i$. We first show that under this assumption, the relationship between two species is competition, and then we show that if $r_iK_i \leq 1$, then two species can coexist in a stable state.

We mention here that a more suitable model should consider the past state of the species, and this leads to the competition system with delay; indeed, delay is one of the most important factors to influence the dynamic behavior of the competition system ([31–37]). It seems interesting to incorporate the time delay to system (1.1) and investigate the dynamic behavior of the system. We leave this for future study.

Competing interests
The author declares that there is no conflict of interests.

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