Jang et al. *Advances in Difference Equations* (2017) 2017:206
DOI 10.1186/s13662-017-1265-4

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

# Some identities of degenerate Daehee numbers arising from nonlinear differential equation

Gwan-Woo Jang[1], Jongkyum Kwon[2] and Jeong Gon Lee[3*]

*Correspondence:
jukolee@wku.ac.kr
[3]Division of Mathematics and Informational Statistics, Nanoscale Science and Technology Institute, Wonkwang University, Iksan, 570-749, Republic of Korea
Full list of author information is available at the end of the article

**Abstract**
Recently, Kim and Kim introduced some identities of degenerate Daehee numbers which are derived from nonlinear differential equations (see (Kim and Kim in J. Nonlinear Sci. Appl. 10:744-751, 2017)). From the viewpoint of inversion formula, we study the degenerate Daehee number arising from a nonlinear differential equation. In this paper, we obtain the explicit expression of degenerate Daehee numbers from the inversion formula of (Kim and Kim in J. Nonlinear Sci. Appl. 10:744-751, 2017) using the generating function and nonlinear differential equations.

**MSC:** 11B68; 11S40; 11S80

**Keywords:** differential equations; degenerate Daehee numbers; Stirling numbers

## 1 Introduction

The Daehee polynomials are defined by the generating function to be

$$\frac{\log(1+t)}{t}(1+t)^x=\sum_{n=0}^{\infty}D_n(x)\frac{t^n}{n!} \tag{1.1}$$

(see [2–7]).

For $x=0$, $D_n=D_n(0)$ are called the Daehee numbers.

In [1], Kim and Kim introduced the degenerate Daehee numbers which are given by the generating function:

$$\frac{\lambda\log(1+\frac{1}{\lambda}\log(1+\lambda t))}{\log(1+\lambda t)}=\sum_{n=0}^{\infty}D_{n,\lambda}\frac{t^n}{n!}. \tag{1.2}$$

For $x=0$, $D_{n,\lambda}=D_{n,\lambda}(0)$ are called the degenerate Daehee numbers.

We observe here that $D_{n,\lambda}\longrightarrow D_n$ as $\lambda\longrightarrow 0$.

The Stirling numbers of the first kind are given by

$$(x)_n=x(x-1)\cdots(x-n+1)=\sum_{l=0}^{n}S_1(n,l)x^l\quad(x\geq 0), \tag{1.3}$$

and the Stirling numbers of the first kind are defined by the generating function to be

$$\big(\log(x+1)\big)^n = n!\sum_{l=n}^{\infty} S_1(l,n)\frac{x^l}{l!} \quad (n \geq 0)$$

(see [1, 3]).

Recently, many researchers have studied nonlinear differential equations arising from the generating functions of various special polynomials (see [1–6, 8–20]). They also investigated some identities and explicit expression of these polynomials from the solution of nonlinear differential equations. In [1], Kim and Kim have studied some results of degenerate Daehee numbers which are derived from nonlinear differential equations. From the viewpoint of the inversion formula, we study the degenerate Daehee number arising from a nonlinear differential equation. In this paper, by using the generating function and nonlinear differential equations, we deduce the explicit expression of degenerate Daehee numbers as the inversion formula of [1].

## 2 Some identities of degenerate Daehee numbers arising from nonlinear differential equations

Let

$$F = F(t) = \log\left(1 + \frac{1}{\lambda}\log(1+\lambda t)\right). \tag{2.1}$$

Then, by taking the derivative with respect to $t$ of (2.1), we get

$$\begin{aligned} F^{(1)} = \frac{d}{dt}F(t) &= \left(1 + \frac{1}{\lambda}\log(1+\lambda t)\right)^{-1}\frac{1}{1+\lambda t} \\ &= \frac{1}{1+\lambda t}e^{-\log(1+\frac{1}{\lambda}\log(1+\lambda t))} \\ &= \frac{1}{1+\lambda t}e^{-F}. \end{aligned} \tag{2.2}$$

From (2.2), we get

$$e^{-F} = (1+\lambda t)F^{(1)}. \tag{2.3}$$

From (2.3), we note that

$$\big(-F^{(1)}\big)e^{-F} = \lambda F^{(1)} + (1+\lambda t)F^{(2)}. \tag{2.4}$$

Thus, by multiplying $(1+\lambda t)$ on both sides of (2.4), we get

$$(1+\lambda t)F^{(1)}e^{-F} = -\lambda(1+\lambda t)F^{(1)} - (1+\lambda t)^2F^{(2)}. \tag{2.5}$$

From (2.3) and (2.5), we get

$$e^{-2F} = -\lambda(1+\lambda t)F^{(1)} - (1+\lambda t)^2F^{(2)}. \tag{2.6}$$

From (2.6), we have

$$-2F^{(1)}e^{-2F} = -\lambda^2 F^{(1)} - 3\lambda(1+\lambda t)F^{(2)} - (1+\lambda t)^2 F^{(3)}. \tag{2.7}$$

Multiplying $(1+\lambda t)$ on both sides of (2.7), we get

$$\begin{aligned} 2!(1+\lambda t)F^{(1)}e^{-2F} &= (-1)^2\lambda^2(1+\lambda t)F^{(1)} + (-1)^2 3\lambda(1+\lambda t)^2 F^{(2)} \\ &\quad + (-1)^2(1+\lambda t)^3 F^{(3)}. \end{aligned} \tag{2.8}$$

From (2.3) and (2.8), we get

$$2!e^{-3F} = (-1)^2\lambda^2(1+\lambda t)F^{(1)} + (-1)^2 3\lambda(1+\lambda t)^2 F^{(2)} + (-1)^2(1+\lambda t)^3 F^{(3)}. \tag{2.9}$$

From (2.9), we have

$$\begin{aligned} 2!\big(-3F^{(1)}\big)e^{-3F} &= (-1)^2\lambda^3 F^{(1)} + (-1)^2 7\lambda^2(1+\lambda t)F^{(2)} \\ &\quad + (-1)^2 6\lambda(1+\lambda t)^2 F^{(3)} + (-1)^2(1+\lambda t)^3 F^{(4)}. \end{aligned} \tag{2.10}$$

Multiplying $(1+\lambda t)$ on both sides of (2.10), we get

$$\begin{aligned} 3!(1+\lambda t)F^{(1)}e^{-3F} &= (-1)^3\lambda^3(1+\lambda t)F^{(1)} + (-1)^3 7\lambda^2(1+\lambda t)^2 F^{(2)} \\ &\quad + (-1)^3 6\lambda(1+\lambda t)^3 F^{(3)} + (-1)^3(1+\lambda t)^4 F^{(4)}. \end{aligned} \tag{2.11}$$

From (2.3) and (2.11), we get

$$\begin{aligned} 3!e^{-4F} &= (-1)^3\lambda^3(1+\lambda t)F^{(1)} + (-1)^3 7\lambda^2(1+\lambda t)^2 F^{(2)} \\ &\quad + (-1)^3 6\lambda(1+\lambda t)^3 F^{(3)} + (-1)^3(1+\lambda t)^4 F^{(4)}. \end{aligned} \tag{2.12}$$

Continuing this process, we get

$$(N-1)!e^{-NF} = (-1)^{N-1}\sum_{k=1}^{N}\lambda^{N-k}(1+\lambda t)^k a_k(N)F^{(k)}. \tag{2.13}$$

Let us take the derivative on both sides of (2.13) with respect to $t$. Then we have

$$\begin{aligned} (N-1)!\big(-NF^{(1)}\big)e^{-NF} &= (-1)^{N-1}\sum_{k=1}^{N}\lambda^{N-k}a_k(N)\big\{k\lambda(1+\lambda t)^{k-1} \\ &\quad \times F^{(k)} + (1+\lambda t)^k F^{(k+1)}\big\}. \end{aligned} \tag{2.14}$$

Multiplying $(1+\lambda t)$ on both sides of (2.14), we get

$$\begin{aligned} N!(1+\lambda t)F^{(1)}e^{-NF} &= (-1)^{N}\sum_{k=1}^{N}\lambda^{N-k}a_k(N)\big\{k\lambda(1+\lambda t)^{k} \\ &\quad \times F^{(k)} + (1+\lambda t)^{k+1} F^{(k+1)}\big\}. \end{aligned} \tag{2.15}$$

Then, by (2.3) and (2.15), we get

$$\begin{aligned} N!e^{-(N+1)F} &= (-1)^N \sum_{k=1}^{N} \lambda^{N-K} a_k(N)\big\{k\lambda(1+\lambda t)^k \\ &\quad \times F^{(k)} + (1+\lambda t)^{k+1}F^{(k+1)}\big\} \\ &= (-1)^N \sum_{k=1}^{N} \lambda^{N-k+1}(1+\lambda t)^k k a_k(N) F^{(k)} \\ &\quad + (-1)^N \sum_{k=1}^{N} \lambda^{N-k}(1+\lambda t)^{k+1} a_k(N) F^{(k+1)} \\ &= (-1)^N \sum_{k=1}^{N} \lambda^{N-k+1}(1+\lambda t)^k k a_k(N) F^{(k)} \\ &\quad + (-1)^N \sum_{k=2}^{N+1} \lambda^{N-k+1}(1+\lambda t)^k a_{k-1}(N) F^{(k)} \\ &= (-1)^N \lambda^N (1+\lambda t) a_1(N) F^{(1)} + (-1)^N (1+1\lambda t)^{N+1} \\ &\quad \times a_N(N) F^{(N+1)} + (-1)^N \sum_{k=2}^{N} \lambda^{N-k+1}(1+\lambda t)^k \\ &\quad \times \big(k a_k(N) + a_{k-1}(N)\big) F^{(k)}. \end{aligned} \tag{2.16}$$

By replacing $N$ by $N+1$ in (2.13), we get

$$\begin{aligned} N!e^{-(N+1)F} &= (-1)^N \sum_{k=1}^{N+1} \lambda^{N-k+1}(1+\lambda t)^k a_k(N+1) F^{(k)} \\ &= (-1)^N \lambda^N (1+\lambda t) a_1(N+1) F^{(1)} + (-1)^N (1+\lambda t)^{N+1} \\ &\quad \times a_{N+1}(N+1) F^{(N+1)} + (-1)^N \sum_{k=2}^{N} \lambda^{N-k+1}(1+\lambda t)^k \\ &\quad \times a_k(N+1) F^k. \end{aligned} \tag{2.17}$$

Comparing the coefficients on both sides of (2.16) and (2.17), we have

$$a_1(N+1) = a_1(N), \qquad a_{N+1}(N+1) = a_N(N), \tag{2.18}$$

and

$$a_k(N+1) = k a_k(N) + a_{k-1}(N), \quad \text{for } 2 \le k \le N. \tag{2.19}$$

From (2.3) and (2.13), we have

$$e^{-F} = (1+\lambda t)F^{(1)} = (1+\lambda t) a_1(1) F^{(1)}. \tag{2.20}$$

By (2.20), we get

$$a_1(1) = 1. \tag{2.21}$$

Thus, by (2.18) and (2.21), we have

$$a_1(N+1) = a_1(N) = a_1(N-1) = \cdots = a_1(1) = 1 \tag{2.22}$$

and

$$a_{N+1}(N+1) = a_N(N) = a_{N-1}(N-1) = \cdots = a_1(1) = 1. \tag{2.23}$$

For $k = 2$ in (2.19), we have

$$\begin{aligned} a_2(N+1) &= 2a_2(N) + a_1(N) \\ &= 2\bigl(2a_2(N-1) + a_1(N-1)\bigr) + a_1(N) \\ &= 2^2 a_2(N-1) + 2a_1(N-1) + a_1(N) \\ &= \cdots \\ &= 2^{N-1}a_2(2) + 2^{N-2}a_1(2) + \cdots + a_1(N). \end{aligned} \tag{2.24}$$

Then by (2.22), (2.23) and (2.24), we get

$$\begin{aligned} a_2(N+1) &= 2^{N-1}a_2(2) + 2^{N-2}a_1(2) + \cdots + a_1(N) \\ &= 2^{N-1}a_1(1) + 2^{N-2}a_1(2) + \cdots + a_1(N) \\ &= 2^{N-1} + 2^{N-2} + \cdots + 1 \\ &= \sum_{i_1=0}^{N-1} 2^{i_1}. \end{aligned} \tag{2.25}$$

For $k = 3$ in (2.19), we have

$$\begin{aligned} a_3(N+1) &= 3a_3(N) + a_2(N) \\ &= 3\bigl(3a_3(N-1) + a_2(N-1)\bigr) + a_2(N) \\ &= 3^2 a_3(N-1) + 3a_2(N-1) + a_2(N) \\ &= \cdots \\ &= 3^{N-2}a_3(3) + 3^{N-3}a_2(3) + \cdots + a_2(N) \\ &= 3^{N-2}a_2(2) + 3^{N-3}a_2(3) + \cdots + a_2(N) \\ &= \sum_{i_2=0}^{N-2} 3^{i_2} a_2(N-i_2). \end{aligned} \tag{2.26}$$

Then by (2.25) and (2.26), we get

$$\begin{aligned} a_3(N+1) &= \sum_{i_2=0}^{N-2} 3^{i_2} a_2(N-i_2) \\ &= \sum_{i_2=0}^{N-2} 3^{i_2} \sum_{i_1=0}^{N-2-i_2} 2^{i_1} \\ &= \sum_{i_2=0}^{N-2} \sum_{i_1=0}^{N-2-i_2} 3^{i_2} 2^{i_1}. \end{aligned} \tag{2.27}$$

For $k = 4$ in (2.19), we have

$$\begin{aligned} a_4(N+1) &= 4a_4(N) + a_3(N) \\ &= 4\bigl(4a_4(N-1) + a_3(N-1)\bigr) + a_3(N) \\ &= 4^2 a_4(N-1) + 4a_3(N-1) + a_3(N) \\ &= \cdots \\ &= 4^{N-3} a_4(4) + 4^{N-4} a_3(4) + \cdots + a_3(N) \\ &= 4^{N-3} a_3(3) + 4^{N-4} a_3(4) + \cdots + a_3(N) \\ &= \sum_{i_3=0}^{N-3} 4^{i_3} a_3(N-i_3). \end{aligned} \tag{2.28}$$

By (2.27) and (2.28), we have

$$\begin{aligned} a_4(N+1) &= \sum_{i_3=0}^{N-3} 4^{i_3} a_3(N-i_3) \\ &= \sum_{i_3=0}^{N-3} 4^{i_3} \sum_{i_2=0}^{N-3-i_3} \sum_{i_1=0}^{N-3-i_3-i_2} 3^{i_2} 2^{i_1} \\ &= \sum_{i_3=0}^{N-3} \sum_{i_2=0}^{N-3-i_3} \sum_{i_1=0}^{N-3-i_3-i_2} 4^{i_3} 3^{i_2} 2^{i_1}. \end{aligned} \tag{2.29}$$

Continuing this process, for $2 \le k \le N$, we have

$$a_k(N+1) = \sum_{i_{k-1}=0}^{N-k+1} \sum_{i_{k-2}=0}^{N-k+1-i_{k-1}} \cdots \sum_{i_1=0}^{N-k+1-i_{k-1}-\cdots-i_2} k^{i_{k-1}} \cdots 2^{i_1}. \tag{2.30}$$

Therefore, we obtain the following differential equations.

**Theorem 2.1** *Let $N \in \mathbb{N}$. Then the differential equations*

$$(N-1)! e^{-NF} = (-1)^{N-1} \sum_{k=1}^{N} \lambda^{N-k} (1+\lambda t)^k a_k(N) F^{(k)}$$

*have a solution* $F = F(t) = \log(1 + \frac{1}{\lambda}\log(1+\lambda t))$, *where*

$$a_N(N) = 1, \qquad a_1(N) = 1$$

*and*

$$a_k(N) = \sum_{i_{k-1}=0}^{N-k} \sum_{i_{k-2}=0}^{N-k-i_{k-1}} \cdots \sum_{i_1=0}^{N-k-i_{k-1}-\cdots-i_2} k^{i_{k-1}} \cdots 2^{i_1}.$$

From (2.1), we easily get

$$\begin{aligned} F &= \log\left(1 + \frac{1}{\lambda}\log(1+\lambda t)\right) \\ &= \frac{\lambda \log(1 + \frac{1}{\lambda}\log(1+\lambda t))}{\log(1+\lambda t)} \cdot \frac{\log(1+\lambda t)}{\lambda} \\ &= \left(\sum_{l_1=0}^{\infty} D_{l_1,\lambda} \frac{t^{l_1}}{l_1!}\right)\left(\frac{1}{\lambda}\sum_{l_2=1}^{\infty} \frac{(-1)^{l_2-1}\lambda^{l_2}}{l_2} t^{l_2}\right) \\ &= \left(\sum_{l_1=0}^{\infty} D_{l_1,\lambda} \frac{t^{l_1}}{l_1!}\right)\left(\sum_{l_2=1}^{\infty} \frac{(-\lambda)^{l_2-1}}{l_2} t^{l_2}\right) \\ &= \sum_{l_3=1}^{\infty}\left(\sum_{l_1=0}^{l_3-1} \frac{D_{l_1,\lambda}}{l_1!} \cdot \frac{(-\lambda)^{l_3-l_1-1}}{(l_3-l_1)}\right) t^{l_3}. \end{aligned} \tag{2.31}$$

From (2.31), we get

$$\begin{aligned} F^{(k)} &= \left(\frac{d}{dt}\right)^k \left\{\sum_{l_3=1}^{\infty}\left(\sum_{l_1=0}^{l_3-1} \frac{D_{l_1,\lambda}}{l_1!} \cdot \frac{(-\lambda)^{l_3-l_1-1}}{(l_3-l_1)}\right) t^{l_3}\right\} \\ &= \sum_{l_3=k}^{\infty}\left(\sum_{l_1=0}^{l_3-1} \frac{D_{l_1,\lambda}}{l_1!} \cdot \frac{(-\lambda)^{l_3-l_1-1}}{(l_3-l_1)}\right)(l_3)_k t^{l_3-k} \\ &= \sum_{l_3=0}^{\infty}\left(\sum_{l_1=0}^{l_3+k-1} \frac{D_{l_1,\lambda}}{l_1!} \cdot \frac{(-\lambda)^{l_3+k-l_1-1}}{(l_3+k-l_1)}\right)(l_3+k)_k t^{l_3}. \end{aligned} \tag{2.32}$$

From (2.32), we get

$$\begin{aligned} (1+\lambda t)^k F^{(k)} &= \left(\sum_{l=0}^{\infty}\binom{k}{l}\lambda^l t^l\right)\left\{\sum_{l_3=0}^{\infty}\left(\sum_{l_1=0}^{l_3+k-1} \frac{D_{l_1,\lambda}}{l_1!} \cdot \frac{(-\lambda)^{l_3+k-l_1-1}}{(l_3+k-l_1)}\right)(l_3+k)_k t^{l_3}\right\} \\ &= \left(\sum_{l=0}^{\infty}(k)_l \lambda^l \frac{t^l}{l!}\right)\left\{\sum_{l_3=0}^{\infty}\left(\sum_{l_1=0}^{l_3+k-1} \frac{D_{l_1,\lambda}}{l_1!} \cdot \frac{(-\lambda)^{l_3+k-l_1-1}}{(l_3+k-l_1)}\right)(l_3+k)! \frac{t^{l_3}}{l_3!}\right\} \\ &= \sum_{n=0}^{\infty}\left(\sum_{l_3=0}^{n}\sum_{l_1=0}^{l_3+k-1} \frac{(l_3+k)!\binom{n}{l_3}(k)_{n-l_3}}{l_1!(l_3+k-l_1)} D_{l_1,\lambda}\right. \\ &\quad \left. \times (-1)^{l_3+k-l_1-1}\lambda^{n+k-l_1-1}\right)\frac{t^n}{n!} \end{aligned} \tag{2.33}$$

and also

$$\begin{aligned}
e^{-NF} &= \sum_{m_1=0}^{\infty} \frac{(-N)^{m_1}}{m_1!} F^{m_1} \\
&= \sum_{m_1=0}^{\infty} \frac{(-N)^{m_1}}{m_1!} \left( \log\left(1 + \frac{1}{\lambda} \log(1+\lambda t)\right)\right)^{m_1} \\
&= \sum_{m_1=0}^{\infty} (-N)^{m_1} \sum_{m_2=m_1}^{\infty} S_1(m_2, m_1) \frac{1}{m_2!} \left(\frac{1}{\lambda} \log(1+\lambda t)\right)^{m_2} \\
&= \sum_{m_2=0}^{\infty} \left( \sum_{m_1=0}^{m_2} (-N)^{m_1} S_1(m_2, m_1) \lambda^{-m_2} \right) \frac{1}{m_2!} \bigl(\log(1+\lambda t)\bigr)^{m_2} \\
&= \sum_{m_2=0}^{\infty} \left( \sum_{m_1=0}^{m_2} (-N)^{m_1} S_1(m_2, m_1) \lambda^{-m_2} \right) \left( \sum_{n=m_2}^{\infty} S_1(n, m_2) \frac{\lambda^n t^n}{n!} \right) \\
&= \sum_{n=0}^{\infty} \left( \sum_{m_2=0}^{n} \sum_{m_1=0}^{m_2} (-N)^{m_1} S_1(m_2, m_1) S_1(n, m_2) \lambda^{n-m_2} \right) \frac{t^n}{n!}. \qquad (2.34)
\end{aligned}$$

Here $S_1(n,k)$ is the Stirling number of the first kind.

Thus, by (2.13) and (2.33), we get

$$\begin{aligned}
&(N-1)! e^{-NF} \\
&= (-1)^{N-1} \sum_{k=1}^{N} \lambda^{N-k} a_k(N) \\
&\quad \times \sum_{n=0}^{\infty} \left( \sum_{l_3=0}^{n} \sum_{l_1=0}^{l_3+k-1} \frac{(l_3+k)! \binom{n}{l_3} (k)_{n-l_3}}{l_1! (l_3+k-l_1)} D_{l_1,\lambda} (-1)^{l_3+k-l_1-1} \lambda^{n+k-l_1-1} \right) \frac{t^n}{n!} \\
&= \sum_{n=0}^{\infty} \left( (-1)^{N-1} \sum_{k=1}^{N} \lambda^{N-k} a_k(N) \sum_{l_3=0}^{n} \sum_{l_1=0}^{l_3+k-1} \frac{(l_3+k)! \binom{n}{l_3} (k)_{n-l_3}}{l_1! (l_3+k-l_1)} \right. \qquad (2.35) \\
&\quad \left. \times D_{l_1,\lambda} (-1)^{l_3+k-l_1-1} \lambda^{n+k-l_1-1} \right) \frac{t^n}{n!} \\
&= \sum_{n=0}^{\infty} \left( \sum_{k=1}^{N} \sum_{l_3=0}^{n} \sum_{l_1=0}^{l_3+k-1} (-1)^{N+l_3+k-l_1} \lambda^{N+n-l_1-1} a_k(N) \right. \\
&\quad \left. \times \frac{(l_3+k)! \binom{n}{l_3} (k)_{n-l_3}}{l_1! (l_3+k-l_1)} D_{l_1,\lambda} \right) \frac{t^n}{n!}. \qquad (2.36)
\end{aligned}$$

By (2.34) and (2.36), we get

$$\begin{aligned}
&(N-1)! \sum_{n=0}^{\infty} \left( \sum_{m_2=0}^{n} \sum_{m_1=0}^{m_2} (-N)^{m_1} S_1(m_2, m_1) S_1(n, m_2) \lambda^{n-m_2} \right) \frac{t^n}{n!} \\
&= \sum_{n=0}^{\infty} \left( \sum_{k=1}^{N} \sum_{l_3=0}^{n} \sum_{l_1=0}^{l_3+k-1} (-1)^{N+l_3+k-l_1} \lambda^{N+n-l_1-1} a_k(N) \frac{(l_3+k)! \binom{n}{l_3} (k)_{n-l_3}}{l_1! (l_3+k-l_1)} D_{l_1,\lambda} \right) \frac{t^n}{n!}. \qquad (2.37)
\end{aligned}$$

By equation (2.37), we finally get the following theorem.

**Theorem 2.2** *For $N = 1, 2, 3, \ldots$, and $n = 0, 1, 2, \ldots$, we have*

$$(N-1)! \sum_{m_2=0}^{n} \sum_{m_1=0}^{m_2} (-N)^{m_1} S_1(m_2, m_1) S_1(n, m_2) \lambda^{n-m_2}$$
$$= \sum_{k=1}^{N} \sum_{l_3=0}^{n} \sum_{l_1=0}^{l_3+k-1} (-1)^{N+l_3+k-l_1} \lambda^{N+n-l_1-1} a_k(N)$$
$$\times \frac{(l_3+k)! \binom{n}{l_3} (k)_{n-l_3}}{l_1!(l_3+k-l_1)} D_{l_1,\lambda}.$$

## 3 Conclusion

Kim and Kim have studied some identities of degenerate Daehee numbers which are derived from the generating function using nonlinear differential equation (see [1]). In this paper, from the viewpoint of the inversion formula to [1], we study the degenerate Daehee number arising from nonlinear differential equation. Therefore we obtain the inversion formula of degenerate Daehee numbers which are related to the some identities of those numbers. In Theorem 2.1, we get the solution of nonlinear differential equation arising from the generating function of the degenerate Daehee number. In Theorem 2.2, we have an explicit expression of the degenerate Daehee number from the result of Theorem 2.1 using the generating function and nonlinear differential equations.

**Acknowledgements**
The authors would like to express their sincere gratitude to the editor, who gave us valuable comments to improve this paper. This paper was supported by Wonkwang University in 2017.

**Competing interests**
The authors declare that they have no competing interests.

**Authors' contributions**
All authors contributed equally to this work. All authors read and approved the final manuscript.

**Author details**
[1]Department of Mathematics, Kwangwoon University, Seoul, 139-701, Republic of Korea. [2]Department of Mathematics Education and RINS, Gyeongsang National University, Gyeongsangnamdo, 52828, Republic of Korea. [3]Division of Mathematics and Informational Statistics, Nanoscale Science and Technology Institute, Wonkwang University, Iksan, 570-749, Republic of Korea.

## Publisher's Note



Received: 13 June 2017 Accepted: 6 July 2017 Published online: 26 July 2017

**References**

1. Kim, T, Kim, DS: Some identities of degenerate Daehee numbers arising from certain differential equations. J. Nonlinear Sci. Appl. **10**, 744-751 (2017)
2. El-Desouky, B, Mustafa, A: New results on higher-order Daehee and Bernoulli numbers and polynomials. Adv. Differ. Equ. **2016**, 32 (2016)
3. Jang, G-W, Kim, T: Revisit of identities for Daehee numbers arising from nonlinear differential equations. Proc. Jangjeon Math. Soc. **20**(2), 163-177 (2017)
4. Kim, BM, Yun, SJ, Park, J-W: On a degenerate λ-*q*-Daeehee polynomials. J. Nonlinear Sci. Appl. **9**(6), 4607-4616 (2016)
5. Kim, DS, Kim, T: Identities arising from higher-order Daehee polynomial bases. Open Math. **13**, 196-208 (2015)
6. Kim, DS, Kim, T: Some identities for Bernoulli numbers of the second kind arising from a nonlinear differential equation. Bull. Korean Math. Soc. **52**, 2001-2010 (2015)
7. Kim, DS, Kim, T, Lee, S-H, Seo, J-J: Higher-order Daehee numbers and polynomials. Int. J. Math. Anal. **8**(5-8), 273-283 (2014)

8. Bayad, A, Kim, T: Higher recurrences for Apostol-Bernoulli numbers. Russ. J. Math. Phys. **19**(1), 1-10 (2012)
9. Bayad, A, Kim, T: Identities for Apostol-type Frobenius-Euler polynomials resulting from the study of a nonlinear operator. Russ. J. Math. Phys. **23**(2), 164-171 (2016)
10. Kim, T: Identities involving Frobenius-Euler polynomials arising from non-linear differential equation. J. Number Theory **132**, 2854-2865 (2012)
11. Kim, T, Dolgy, DV, Kim, DS, Seo, JJ: Differential equations for Changhee polynomials and their applications. J. Nonlinear Sci. Appl. **9**, 2857-2864 (2016)
12. Kim, T, Kim, DS: A note on nonlinear Changhee differential equations. Russ. J. Math. Phys. **23**, 88-92 (2016)
13. Kim, T, Kim, DS, Jang, LC, Kwon, HI: Differential equations associated with Mittag-Leffler polynomials. Glob. J. Pure Appl. Math. **12**(4), 2839-2847 (2016)
14. Kim, T, Kim, DS, Seo, JJ: Differential equations associated with degenerate Bell polynomials. Int. J. Pure Appl. Math. **108**(3), 551-559 (2016)
15. Kim, T, Kim, DS, Seo, J-J, Kwon, HI: Differential equations associated with $\lambda$-Changhee polynomials. J. Nonlinear Sci. Appl. **9**, 3098-3111 (2016)
16. Kim, T, Seo, J-J: Revisit nonlinear differential equations arising from the generating functions of degenerate Bernoulli numbers. Adv. Stud. Contemp. Math. (Kyungshang) **26**(3), 401-406 (2016)
17. Kwon, HI, Kim, T, Seo, J-J: A note on Daehee numbers arising from differential equations. Glob. J. Pure Appl. Math. **12**(3), 2349-2354 (2016)
18. Kwon, JK, Choi, YJ, Jang, MS, Yang, SO, Seong, MS: Some identities involving Changhee polynomials arising from a differential equations. Glob. J. Pure Appl. Math. **12**(6), 4857-4866 (2016)
19. Rim, SH, Jeong, JH, Park, J-W: Some identities involving Euler polynomials arising from a non-linear differential equation. Kyungpook Math. J. **53**, 553-563 (2013)
20. Yardimci, A, Simsek, Y: Identities for Korobov-type polynomials arising from functional equations and *p*-adic integral. J. Nonlinear Sci. Appl. **10**, 2767-2777 (2017)