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# Existence of positive periodic solutions for a class of Gilpin-Ayala ecological models with discrete and distributed time delays

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**Abstract**
In this paper, we deal with a class of Gilpin-Ayala ecological models with discrete and distributed time delays. By employing a fixed point theorem of strict-set-contraction and inequality techniques, some sufficient conditions for the existence of periodic solutions are established. As an application, one example is given to illustrate the validity of our main results.

**MSC:** 34K13; 37N25

**Keywords:** positive periodic solution; Gilpin-Ayala ecological model; discrete and distributed time delays; strict-set-contraction

## 1 Introduction

In this paper, we mainly study the following Gilpin-Ayala-like functional differential system with discrete and distributed time delays:

$$\begin{cases} x_i'(t) = x_i(t)[r_i(t) - F_i(t,x(t),y(t))], & i = 1,2,\ldots,n, \\ y_j'(t) = y_j(t)[-\hat{r}_j(t) + \hat{F}_j(t,x(t),y(t))], & j = 1,2,\ldots,m, \end{cases} \tag{1.1}$$

where $x(t) = (x_1(t),x_2(t),\ldots,x_n(t))$, $y(t) = (y_1(t),y_2(t),\ldots,y_m(t))$,

$$\begin{aligned} F_i\bigl(t,x(t),y(t)\bigr) = {} & \sum_{k=1}^{n} a_{ik}(t)x_k^{\theta_k}\bigl(t-\tau_{ik}(t)\bigr) + \sum_{l=1}^{m} b_{il}(t)y_l^{\vartheta_l}\bigl(t-\sigma_{il}(t)\bigr) \\ & + \sum_{k=1}^{n} c_{ik}(t)\int_{-\infty}^{0} K_{ik}(s)x_k^{\theta_k}(t+s)\,ds + \sum_{l=1}^{m} d_{il}(t)\int_{-\infty}^{0} L_{il}(s)y_l^{\vartheta_l}(t+s)\,ds, \\ \hat{F}_j\bigl(t,x(t),y(t)\bigr) = {} & \sum_{k=1}^{n} \hat{a}_{kj}(t)x_k^{\theta_k}\bigl(t-\hat{\tau}_{kj}(t)\bigr) + \sum_{l=1}^{m} \hat{b}_{lj}(t)y_l^{\vartheta_l}\bigl(t-\hat{\sigma}_{lj}(t)\bigr) \\ & + \sum_{k=1}^{n} \hat{c}_{kj}(t)\int_{-\infty}^{0} \hat{K}_{kj}(s)x_k^{\theta_k}(t+s)\,ds + \sum_{l=1}^{m} \hat{d}_{lj}(t)\int_{-\infty}^{0} \hat{L}_{lj}(s)y_l^{\vartheta_l}(t+s)\,ds, \end{aligned}$$

$r_i, \hat{r}_j, a_{ik}, \hat{a}_{kj}, b_{il}, \hat{b}_{lj}, c_{ik}, \hat{c}_{kj}, d_{il}, \hat{d}_{lj} \in C(\mathbb{R},(0,\infty))$ $(i,k = 1,2,\ldots,n;\ j,l = 1,2,\ldots,m)$ and $\tau_{ik}, \sigma_{il}, \hat{\tau}_{kj}, \hat{\sigma}_{lj} \in C(\mathbb{R},\mathbb{R})$ $(i,k = 1,2,\ldots,n;\ j,l = 1,2,\ldots,m)$ are $\omega$-periodic functions.

$\theta_i > 0$ $(i = 1,2,\ldots,n)$ and $\vartheta_j > 0$ $(j = 1,2,\ldots,m)$ are all constants. $K_{ik}, \hat{K}_{kj}, L_{il}, \hat{L}_{lj} \in C((-\infty,0],(0,\infty))$ with $\int_{-\infty}^0 K_{ik}(s)\,ds = \int_{-\infty}^0 \hat{K}_{kj}(s)\,ds = \int_{-\infty}^0 L_{il}(s)\,ds = \int_{-\infty}^0 \hat{L}_{lj}(s)\,ds = 1$.

The importance of system (1.1) is due to the wide application of functional differential equations in the ecosystem. It is well known that functional differential equations are modeled by mathematical models to describe interactions and changes among species in many ecosystems or biological systems. One of the most famous and important population dynamics models is the Lotka-Volterra predator-prey model proposed by Lotka and Volterra in [1, 2]. This rudimentary and important model of mathematical ecology is expressed in the form of

$$\begin{cases} x'(t) = x(t)(r - ay(t)), \\ y'(t) = y(t)(-d + bx(t)), \end{cases}$$

where $x(t)$ is the density of the prey species at time $t$, $y(t)$ is the density of the predator species at time $t$. $r$ is the intrinsic growth rate of the prey, $a$ is the per-capita rate of predation of the predator, $d$ is the death rate of the predator, $b$ denotes the product of the per-capita rate of predation and the rate of converting the prey into the predator.

The Lotka-Volterra model and its various generalized forms have successfully described the interactions among species in a population dynamics. There have been many papers dealing with its various dynamical properties, and one has seen great progress [3–21]. However, regardless of this fact, the Lotka-Volterra system has a disadvantageous property, that is, the rate of change in the size of each species is a linear function of the sizes of the interacting species. It is worth noticing that Ayala and Gilpin *et al.* [22] conducted experiments on fruit fly dynamics to test the validity of competitions. The model accounting best for the experimental results is given by

$$\begin{cases} x'(t) = r_1 x(t)[1 - (\frac{x(t)}{K_1})^{\theta_1} - a_{12}\frac{y(t)}{K_2}], \\ y'(t) = r_2 y(t)[1 - a_{21}\frac{x(t)}{K_1} - (\frac{y(t)}{K_2})^{\theta_2}], \end{cases}$$

where $r_i$ is the intrinsic rate of growth of species, $K_i$ is the environment carrying capacity of species $i$ in the absence of competition, $\theta_i$ provides a nonlinear measure of interspecific interference, and $a_{ij}$ provides a measure of interspecific interference. Compared with the Lotka-Volterra system, this model called Gilpin-Ayala competition system is somewhat more complicated and accurate. As soon as it was put forward, the Gilpin-Ayala model received extensive attention. Many scholars have studied the dynamics of the Gilpin-Ayala system and its various generalized forms and obtained a lot of good results (see [23–31]).

To the best of our knowledge, there are few papers dealing with the existence of positive periodic solutions of system (1.1) by the theory of strict-set-contraction. Our main purpose of this paper is to establish some new existence conditions of positive periodic solutions for system (1.1) by using a fixed point theorem of strict-set-contraction.

## 2 Preliminaries

For convenience, we introduce the notation

$$\delta_i = e^{-\theta_i \int_0^\omega r_i(\tau)\,d\tau}, \qquad \hat{\delta}_j = e^{\vartheta_j \int_0^\omega \hat{r}_j(\tau)\,d\tau}, \qquad f^M = \max_{t\in[0,\omega]}\{f(t)\},$$

$$\Gamma_i = \frac{\theta_i}{1-\delta_i}\int_0^\omega \left[\sum_{k=1}^n \bigl(a_{ik}(s) + c_{ik}(s)\bigr) + \sum_{l=1}^m \bigl(b_{il}(s) + d_{il}(s)\bigr)\right] ds,$$

$$\hat{\Gamma}_j = \frac{\vartheta_j \hat{\delta}_j}{\hat{\delta}_j - 1} \int_0^\omega \left[ \sum_{k=1}^n \big(\hat{a}_{ik}(s) + \hat{c}_{ik}(s)\big) + \sum_{l=1}^m \big(\hat{b}_{il}(s) + \hat{d}_{il}(s)\big) \right] ds,$$

$$\Pi_i = \frac{\theta_i \delta_i^2}{1 - \delta_i} \int_0^\omega \left[ \sum_{k=1}^n \delta_k \big(a_{ik}(s) + c_{ik}(s)\big) + \sum_{l=1}^m \frac{1}{\hat{\delta}_l} \big(b_{il}(s) + d_{il}(s)\big) \right] ds,$$

$$\hat{\Pi}_j = \frac{\vartheta_j}{\hat{\delta}_j(\hat{\delta}_j - 1)} \int_0^\omega \left[ \sum_{k=1}^n \delta_k \big(\hat{a}_{ik}(s) + \hat{c}_{ik}(s)\big) + \sum_{l=1}^m \frac{1}{\hat{\delta}_l} \big(\hat{b}_{il}(s) + \hat{d}_{il}(s)\big) \right] ds,$$

$$\Gamma = \min\left\{ \frac{1}{\Gamma_1}, \dots, \frac{1}{\Gamma_n}, \frac{1}{\hat{\Gamma}_1}, \dots, \frac{1}{\hat{\Gamma}_m} \right\}, \qquad \Pi = \max\left\{ \frac{1}{\Pi_1}, \dots, \frac{1}{\Pi_n}, \frac{1}{\hat{\Pi}_1}, \dots, \frac{1}{\hat{\Pi}_m} \right\},$$

where $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$ and $f(t)$ is a continuous $\omega$-periodic function on $\mathbb{R}$.

Let $x_i^{\theta_i}(t) = u_i(t)$, $y_j^{\vartheta_j}(t) = v_j(t)$, $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$, then system (1.1) changes into the following system:

$$\begin{cases} u_i'(t) = \theta_i u_i(t)[r_i(t) - F_i(t, u(t), v(t))], & i = 1, 2, \dots, n, \\ v_j'(t) = \vartheta_j v_j(t)[-\hat{r}_j(t) + \hat{F}_j(t, u(t), v(t))], & j = 1, 2, \dots, m, \end{cases} \tag{2.1}$$

where $u(t) = (u_1(t), u_2(t), \dots, u_n(t))$, $v(t) = (v_1(t), v_2(t), \dots, v_m(t))$,

$$\begin{aligned} F_i\big(t, u(t), v(t)\big) &= \sum_{k=1}^n a_{ik}(t) u_k\big(t - \tau_{ik}(t)\big) + \sum_{l=1}^m b_{il}(t) v_l\big(t - \sigma_{il}(t)\big) \\ &\quad + \sum_{k=1}^n c_{ik}(t) \int_{-\infty}^0 K_{ik}(s) u_k(t+s)\, ds + \sum_{l=1}^m d_{il}(t) \int_{-\infty}^0 L_{il}(s) v_l(t+s)\, ds, \\ \hat{F}_j\big(t, u(t), v(t)\big) &= \sum_{k=1}^n \hat{a}_{kj}(t) u_k\big(t - \hat{\tau}_{kj}(t)\big) + \sum_{l=1}^m \hat{b}_{lj}(t) v_l\big(t - \hat{\sigma}_{lj}(t)\big) \\ &\quad + \sum_{k=1}^n \hat{c}_{kj}(t) \int_{-\infty}^0 \hat{K}_{kj}(s) u_k(t+s)\, ds + \sum_{l=1}^m \hat{d}_{lj}(t) \int_{-\infty}^0 \hat{L}_{lj}(s) v_l(t+s)\, ds. \end{aligned}$$

Obviously, if $(\bar{u}_1(t), \dots, \bar{u}_n(t), \bar{v}_1(t), \dots, \bar{v}_m(t))$ is a positive $\omega$-periodic solution of system (2.1), then $(\bar{u}_1^{\frac{1}{\theta_1}}(t), \dots, \bar{u}_n^{\frac{1}{\theta_n}}(t), \bar{v}_1^{\frac{1}{\vartheta_1}}(t), \dots, \bar{v}_m^{\frac{1}{\vartheta_m}}(t))$ is a positive $\omega$-periodic solution of system (1.1). Hence, we only need to argue the existence of a positive $\omega$-periodic solutions of system (2.1). To do this, we introduce the following lemma.

**Lemma 2.1** *Let $r \in C(\mathbb{R}, \mathbb{R})$, $a \in \mathbb{R}$ and $y_a \in \mathbb{R}$, the unique solution of the initial value problem*

$$y'(t) = r(t)y(t) + h(t), \qquad y(a) = y_a$$

*is given by*

$$y(t) = y_a e^{\int_a^t r(s)\, ds} + \int_a^t e^{-\int_t^s r(\tau)\, d\tau} h(s)\, ds.$$

The existence of periodic solutions of system (2.1) is equivalent to the existence of periodic solutions of the corresponding integral system. So the following lemma is important in our discussion.

**Lemma 2.2** *$x(t) = (u(t), v(t))^T = (u_1(t), \ldots, u_n(t), v_1(t), \ldots, v_m(t))^T$ is an $\omega$-periodic solution of* (2.1) *is equivalent to $x(t)$ is an $\omega$-periodic solution of the following integral system*:

$$\begin{cases} u_i(t) = \theta_i \int_t^{t+\omega} G_i(t,s)u_i(s)F_i(s,u(s),v(s))\,ds, & i = 1,2,\ldots,n, \\ v_j(t) = \vartheta_j \int_t^{t+\omega} \hat{G}_j(t,s)v_j(s)\hat{F}_j(s,u(s),v(s))\,ds, & j = 1,2,\ldots,m, \end{cases} \tag{2.2}$$

*where*

$$G_i(t,s) = \frac{e^{-\theta_i \int_t^s r_i(\tau)\,d\tau}}{1 - e^{-\theta_i \int_0^\omega r_i(\tau)\,d\tau}}, \quad s \in [t, t+\omega], i = 1,2,\ldots,n, \tag{2.3}$$

*and*

$$\hat{G}_j(t,s) = \frac{e^{\vartheta_j \int_t^s \hat{r}_j(\tau)\,d\tau}}{e^{\vartheta_j \int_0^\omega \hat{r}_j(\tau)\,d\tau} - 1}, \quad s \in [t, t+\omega], j = 1,2,\ldots,m. \tag{2.4}$$

*Proof* If $(u(t), v(t))$ is an $\omega$-periodic solution of (2.1), by applying Lemma 2.1 and the first equation of (2.1), for $\xi \geq t$, we have

$$u_i(\xi) = u_i(t)e^{\theta_i \int_t^\xi r_i(s)\,ds} - \theta_i \int_t^\xi e^{-\theta_i \int_\xi^s r_i(\tau)\,d\tau} u_i(s)F_i\bigl(s,u(s),v(s)\bigr)\,ds.$$

Let $\xi = t + \omega$ in the above equality and notice that $u_i(t) = u_i(t+\omega)$, $r_i(t+\omega) = r_i(t)$, we have

$$\begin{aligned} u_i(t) = u_i(t+\omega) &= u_i(t)e^{\theta_i \int_t^{t+\omega} r_i(s)\,ds} - \theta_i \int_t^{t+\omega} e^{-\theta_i \int_{t+\omega}^s r_i(\tau)\,d\tau} u_i(s)F_i\bigl(s,u(s),v(s)\bigr)\,ds \\ &= u_i(t)e^{\theta_i \int_0^\omega r_i(s)\,ds} - \theta_i \int_t^{t+\omega} e^{\theta_i \int_0^\omega r_i(\tau)\,d\tau} e^{-\theta_i \int_t^s r_i(\tau)\,d\tau} u_i(s)F_i\bigl(s,u(s),v(s)\bigr)\,ds, \end{aligned}$$

which implies that

$$\begin{aligned} u_i(t) &= \theta_i \int_t^{t+\omega} \frac{e^{-\theta_i \int_t^s r_i(\tau)\,d\tau}}{1 - e^{-\theta_i \int_0^\omega r_i(\tau)\,d\tau}} u_i(s)F_i\bigl(s,u(s),v(s)\bigr)\,ds \\ &= \theta_i \int_t^{t+\omega} G_i(t,s)u_i(s)F_i\bigl(s,u(s),v(s)\bigr)\,ds. \end{aligned}$$

Similarly, we get

$$v_j(t) = \vartheta_j \int_t^{t+\omega} \hat{G}_j(t,s)v_j(s)\hat{F}_j\bigl(s,u(s),v(s)\bigr)\,ds.$$

Thus, we conclude that $(u(t), v(t))$ satisfies (2.2), and vice versa. The proof is complete. □

**Lemma 2.3** *If $r_i(t), \hat{r}_j(t) > 0$, $\forall t \in \mathbb{R}$, and $\theta_i, \vartheta_j > 0$, $i = 1,2,\ldots,n$, $j = 1,2,\ldots,m$, then $G_i(t,s)$ ($i = 1,2,\ldots,n$) and $\hat{G}_j(t,s)$ ($j = 1,2,\ldots,m$) defined by* (2.2) *and* (2.3) *satisfy the following*:

(1) $\frac{\delta_i}{1-\delta_i} \leq G_i(t,s) \leq \frac{1}{1-\delta_i}$, $\forall s \in [t, t+\omega]$, *where* $\delta_i \triangleq e^{-\theta_i \int_0^\omega r_i(\tau)\,d\tau}$, $i = 1,2,\ldots,n$;

(2) $\frac{1}{\hat{\delta}_j - 1} \leq \hat{G}_j(t,s) \leq \frac{\hat{\delta}_j}{\hat{\delta}_j - 1}$, $\forall s \in [t, t+\omega]$, *where* $\hat{\delta}_j \triangleq e^{\vartheta_j \int_0^\omega \hat{r}_j(\tau)\,d\tau}$, $j = 1,2,\ldots,m$;

(3) $G_i(t+\omega, s+\omega) = G_i(t,s)$, $i = 1,2,\ldots,n$, $\hat{G}_j(t+\omega, s+\omega) = \hat{G}_j(t,s)$, $j = 1,2,\ldots,m$.

*Proof* Since $e^{-\theta_i \int_t^s r_i(\tau)\,d\tau}$ is monotone decreasing and $e^{\vartheta_j \int_t^s \hat{r}_j(\tau)\,d\tau}$ is monotone increasing on the variable $s$ in $[t, t+\omega]$, respectively, we have

$$\frac{\delta_i}{1-\delta_i} = \frac{e^{-\theta_i \int_t^{t+\omega} r_i(\tau)\,d\tau}}{1-\delta_i} \le G_i(t,s) \le \frac{e^{-\theta_i \int_t^{t} r_i(\tau)\,d\tau}}{1-\delta_i} = \frac{1}{1-\delta_i},$$
$$\frac{1}{\hat{\delta}_j - 1} = \frac{e^{\vartheta_j \int_t^{t} \hat{r}_j(\tau)\,d\tau}}{\hat{\delta}_j - 1} \le \hat{G}_j(t,s) \le \frac{e^{\vartheta_j \int_t^{t+\omega} \hat{r}_j(\tau)\,d\tau}}{\hat{\delta}_j - 1} = \frac{\hat{\delta}_j}{\hat{\delta}_j - 1}.$$

Thus, assertions (1) and (2) hold. Now we show that assertion (3) holds too. Indeed, by the integration by substitution, we have

$$G_i(t+\omega, s+\omega) = \frac{e^{-\theta_i \int_{t+\omega}^{s+\omega} r_i(\tau)\,d\tau}}{1-e^{-\theta_i \int_0^{\omega} r_i(\tau)\,d\tau}} = \frac{e^{-\theta_i \int_t^{s} r_i(\xi+\omega)\,d\xi}}{1-e^{-\theta_i \int_0^{\omega} r_i(\tau)\,d\tau}} = \frac{e^{-\theta_i \int_t^{s} r_i(\xi)\,d\xi}}{1-e^{-\theta_i \int_0^{\omega} r_i(\tau)\,d\tau}} = G_i(t,s).$$

It is similar to prove that $\hat{G}_j(t+\omega, s+\omega) = \hat{G}_j(t,s)$. The proof of Lemma 2.3 is complete. □

For the sake of obtaining the existence of a periodic solution of system (2.2), we need the following preparations.

Let $X$ be a real Banach space and $K$ be a closed, nonempty subset of $X$. Then $K$ is a cone provided

(i) $k\alpha + l\beta \in K$ for all $\alpha, \beta \in K$ and all $k, l \ge 0$;
(ii) $\alpha, -\alpha \in K$ imply $\alpha = \theta$, here $\theta$ is the zero element of $X$.

Let $E$ be a Banach space and $K$ be a cone in $E$. The semi-order induced by the cone $K$ is denoted by $\le$. That is, $x \le y$ if and only if $y - x \in K$. In addition, for a bounded subset $A \subset E$, let $\alpha_E(A)$ denote the (Kuratowski) measure of non-compactness defined by

$$\begin{aligned}\alpha_E(A) = \inf\bigl\{&\delta > 0 : A \text{ admits a finite cover by subsets of } A_i \subset A \\ &\text{such that } \operatorname{diam}(A_i) \le \delta\bigr\},\end{aligned}$$

where $\operatorname{diam}(A_i)$ denotes the diameter of the set $A_i$.

Let $E$, $F$ be two Banach spaces and $D \subset E$. A continuous and bounded map $\Phi : \overline{\Omega} \to F$ is called $k$-set contractive if, for any bounded set $S \subset D$, we have

$$\alpha_F\bigl(\Phi(S)\bigr) \le k\alpha_E\bigl(\Phi(S)\bigr).$$

$\Phi$ is called strict-set-contractive if it is $k$-set-contractive for some $0 \le k < 1$. Particularly, completely continuous operators are 0-set-contractive.

The following lemma is useful for the proof of our main results of this paper.

**Lemma 2.4** ([32, 33]) *Let $K$ be a cone in the real Banach space $X$ and $K_{r,R} = \{x \in K : r \le \|x\| \le R\}$ with $R > r > 0$. Suppose that $\Phi : K_{r,R} \to K$ is strict-set-contractive such that one of the following two conditions is satisfied*:

(i) $\Phi x \not\le x$, $\forall x \in K$, $\|x\| = r$ *and* $\Phi x \not\ge x$, $\forall x \in K$, $\|x\| = R$.
(ii) $\Phi x \not\ge x$, $\forall x \in K$, $\|x\| = r$ *and* $\Phi x \not\le x$, $\forall x \in K$, $\|x\| = R$.

*Then $\Phi$ has at least one fixed point in $K_{r,R}$.*

Let $C(\mathbb{R},\mathbb{R}^{n+m})$ be a set of the continuous function $x:\mathbb{R}\to\mathbb{R}^{n+m}$. Define $X=\{x: x\in C(\mathbb{R},\mathbb{R}^{n+m}), x(t+\omega)=x(t)\}$ endowed with the norm defined by $\|x\|=\max_{1\le i\le n+m}|x_i|_0$, where $|x_i|_0=\sup_{t\in[0,\omega]_{\mathbb{T}}}\{|x_i(t)|\}$, $i=1,2,\ldots,n+m$. Then $X$ is a Banach space. In view of Lemma 2.3, we define the cone $K$ in $X$ as

$$K=\left\{x=(u_1,\ldots,u_n,v_1,\ldots,v_m)\in X: u_i(t)\ge\delta_i|u_i|_0, v_j(t)\ge\frac{1}{\hat{\delta}_j}|v_j|_0, t\in[0,\omega]\right\}.$$

Let the map $\Phi$ be defined by

$$(\Phi x)(t)=\big((\Phi_1x)(t),\ldots,(\Phi_nx)(t),(\Psi_1x)(t),\ldots,(\Psi_mx)(t)\big)^T, \tag{2.5}$$

where $x\in K$, $t\in\mathbb{R}$,

$$\begin{aligned}
(\Phi_ix)(t)&=\theta_i\int_t^{t+\omega}G_i(t,s)u_i(s)F_i\big(s,u(s),v(s)\big)\,ds,\quad i=1,2,\ldots,n,\\
(\Psi_jx)(t)&=\vartheta_j\int_t^{t+\omega}\hat{G}_j(t,s)v_j(s)\hat{F}_j\big(s,u(s),v(s)\big)\,ds,\quad j=1,2,\ldots,m,
\end{aligned}$$

and $G_i(t,s)$ $(i=1,2,\ldots,n)$, $\hat{G}_j(t,s)$ $(j=1,2,\ldots,m)$ defined by (2.2) and (2.3), respectively.

**Lemma 2.5** *$\Phi:K\to K$ defined by* (2.5) *is well defined, that is,* $\Phi(K)\subset K$.

*Proof* For any $x\in K$, it is clear that $\Phi x\in C(\mathbb{R},\mathbb{R}^{n+m})$. In view of Lemma 2.3 and (2.5), we obtain

$$\begin{aligned}
(\Phi_ix)(t+\omega)&=\theta_i\int_{t+\omega}^{t+2\omega}G_i(t+\omega,s)u_i(s)F_i\big(s,u(s),v(s)\big)\,ds\\
&=\theta_i\int_t^{t+\omega}G_i(t+\omega,\tau+\omega)u_i(\tau+\omega)F_i\big(\tau+\omega,u(\tau+\omega),v(\tau+\omega)\big)\,d\tau\\
&=\theta_i\int_t^{t+\omega}G_i(t+\omega,\tau+\omega)u_i(\tau+\omega)\Bigg[\sum_{k=1}^n a_{ik}(\tau+\omega)u_k\big(\tau+\omega-\tau_{ik}(\tau+\omega)\big)\\
&\quad+\sum_{l=1}^m b_{il}(\tau+\omega)v_l\big(\tau+\omega-\sigma_{il}(\tau+\omega)\big)\\
&\quad+\sum_{k=1}^n c_{ik}(\tau+\omega)\int_{-\infty}^0 K_{ik}(s)u_k(\tau+\omega+s)\,ds\\
&\quad+\sum_{l=1}^m d_{il}(\tau+\omega)\int_{-\infty}^0 L_{il}(s)v_l(\tau+\omega+s)\,ds\Bigg]\,d\tau\\
&=\theta_i\int_t^{t+\omega}G_i(t,\tau)u_i(\tau)\Bigg[\sum_{k=1}^n a_{ik}(\tau)u_k\big(\tau-\tau_{ik}(\tau)\big)+\sum_{l=1}^m b_{il}(\tau)v_l\big(\tau-\sigma_{il}(\tau)\big)\\
&\quad+\sum_{k=1}^n c_{ik}(\tau)\int_{-\infty}^0 K_{ik}(s)u_k(\tau+s)\,ds+\sum_{l=1}^m d_{il}(\tau)\int_{-\infty}^0 L_{il}(s)v_l(\tau+s)\,ds\Bigg]\,d\tau\\
&=\theta_i\int_t^{t+\omega}G_i(t,\tau)u_i(\tau)F_i\big(\tau,u(\tau),v(\tau)\big)\,d\tau=(\Phi_ix)(t),
\end{aligned}$$

that is, $(\Phi_i x)(t+\omega) = (\Phi_i x)(t)$, $\forall t \in \mathbb{R}$, $i = 1,2,\ldots,n$. Similarly, we have $(\Psi_j x)(t+\omega) = (\Psi_j x)(t)$, $\forall t \in \mathbb{R}$, $j = 1,2,\ldots,m$. So $\Phi x \in X$. For any $x \in K$, we have

$$|\Phi_i x|_0 \le \frac{\theta_i}{1-\delta_i}\int_0^{\omega} u_i(s)F_i\bigl(s,u(s),v(s)\bigr)\,ds, \quad i = 1,2,\ldots,n,$$

$$|\Psi_j x|_0 \le \frac{\hat{\delta}_j\vartheta_j}{\hat{\delta}_j - 1}\int_0^{\omega} v_j(s)\hat{F}_j\bigl(s,u(s),v(s)\bigr)\,ds, \quad j = 1,2,\ldots,m,$$

and

$$\begin{aligned}(\Phi_i x)(t) &\ge \frac{\delta_i\theta_i}{1-\delta_i}\int_t^{t+\omega} u_i(s)F_i\bigl(s,u(s),v(s)\bigr)\,ds \\ &= \frac{\delta_i\theta_i}{1-\delta_i}\int_0^{\omega} u_i(s)F_i\bigl(s,u(s),v(s)\bigr)\,ds \ge \delta_i|\Phi_i x|_0, \\ (\Psi_j x)(t) &\ge \frac{\vartheta_j}{\hat{\delta}_j - 1}\int_t^{t+\omega} v_j(s)\hat{F}_j\bigl(s,u(s),v(s)\bigr)\,ds \\ &= \frac{\vartheta_j}{\hat{\delta}_j - 1}\int_0^{\omega} v_j(s)\hat{F}_j\bigl(s,u(s),v(s)\bigr)\,ds \ge \frac{1}{\hat{\delta}_j}|\Psi_j x|_0.\end{aligned}$$

So $\Phi x \in K$. This completes the proof of Lemma 2.5. □

**Lemma 2.6** *$\Phi : K \to K$ defined by* (2.5) *is completely continuous.*

*Proof* It is easy to see that $\Phi$ is continuous and bounded. Now we show that $\Phi$ maps bounded sets into relatively compact sets. Let $\Omega \subset K$ be an arbitrary open bounded set in $K$, then there exists a number $R > 0$ such that $\|x\| < R$ for any $x = (u_1,\ldots,u_n, v_1,\ldots,v_m)^T \in \Omega$. We prove that $\overline{\Phi(\Omega)}$ is compact. In fact, for any $x \in \Omega$ and $t \in [0,\omega]$, we have

$$\begin{aligned}\bigl|(\Phi_i x)(t)\bigr| &= \theta_i\int_t^{t+\omega} G_i(t,s)u_i(s)F_i\bigl(s,u(s),v(s)\bigr)\,ds \\ &\le \frac{\theta_i}{1-\delta_i}\int_0^{\omega} u_i(s)F_i\bigl(s,u(s),v(s)\bigr)\,ds \\ &= \frac{\theta_i}{1-\delta_i}\int_0^{\omega} u_i(s)\Biggl[\sum_{k=1}^{n} a_{ik}(s)u_k\bigl(s-\tau_{ik}(s)\bigr) + \sum_{l=1}^{m} b_{il}(s)v_l\bigl(s-\sigma_{il}(s)\bigr) \\ &\quad + \sum_{k=1}^{n} c_{ik}(s)\int_{-\infty}^{0} K_{ik}(\tau)u_k(\tau+s)\,d\tau + \sum_{l=1}^{m} d_{il}(s)\int_{-\infty}^{0} L_{il}(\tau)v_l(\tau+s)\,d\tau\Biggr]\,ds \\ &\le \frac{\theta_i\omega|u_i|_0}{1-\delta_i}\Biggl[\sum_{k=1}^{n} a_{ik}^M|u_k|_0 + \sum_{l=1}^{m} b_{il}^M|v_l|_0 \\ &\quad + \sum_{k=1}^{n} c_{ik}^M\int_{-\infty}^{0} K_{ik}(\tau)|u_k|_0\,d\tau + \sum_{l=1}^{m} d_{il}^M\int_{-\infty}^{0} L_{il}(\tau)|v_l|_0\,d\tau\Biggr] \\ &\le \frac{\theta_i\omega\|x\|}{1-\delta_i}\Biggl[\sum_{k=1}^{n} a_{ik}^M\|x\| + \sum_{l=1}^{m} b_{il}^M\|x\|\end{aligned}$$

$$+\sum_{k=1}^{n} c_{ik}^{M} \int_{-\infty}^{0} K_{ik}(\tau)\|x\|\,d\tau + \sum_{l=1}^{m} d_{il}^{M} \int_{-\infty}^{0} L_{il}(\tau)\|x\|\,d\tau \Bigg]$$
$$< \frac{\theta_i \omega R^2}{1-\delta_i}\left[\sum_{k=1}^{n}\left(a_{ik}^{M}+c_{ik}^{M}\right)+\sum_{l=1}^{m}\left(b_{il}^{M}+d_{il}^{M}\right)\right] \triangleq A_i, \quad i=1,2,\ldots,n,$$

and

$$\left|(\Phi_i x)'(t)\right| = \theta_i\left|r_i(t)(\Phi_i x)(t) - u_i(t)F_i\bigl(t,u(t),v(t)\bigr)\right|$$
$$\leq \theta_i r_i^M A_i + R^2\left[\sum_{k=1}^{n}\left(a_{ik}^{M}+c_{ik}^{M}\right)+\sum_{l=1}^{m}\left(b_{il}^{M}+d_{il}^{M}\right)\right] \triangleq B_i, \quad i=1,2,\ldots,n.$$

Similarly, for any $x \in \Omega$ and $t \in [0,\omega]$, we have

$$\left|(\Psi_j x)(t)\right| < \frac{\hat{\delta}_j \vartheta_j \omega R^2}{\hat{\delta}_j - 1}\left[\sum_{k=1}^{n}\left(\hat{a}_{ik}^{M}+\hat{c}_{ik}^{M}\right)+\sum_{l=1}^{m}\left(\hat{b}_{il}^{M}+\hat{d}_{il}^{M}\right)\right] \triangleq \hat{A}_j, \quad j=1,2,\ldots,m,$$

and

$$\left|(\Psi_j x)'(t)\right| \leq \vartheta_j \hat{r}_j^M \hat{A}_j + R^2\left[\sum_{k=1}^{n}\left(\hat{a}_{ik}^{M}+\hat{c}_{ik}^{M}\right)+\sum_{l=1}^{m}\left(\hat{b}_{il}^{M}+\hat{d}_{il}^{M}\right)\right] \triangleq \hat{B}_j, \quad j=1,2,\ldots,m.$$

Hence,

$$\left\|(\Phi x)\right\| \leq \max\{A_1,\ldots,A_n,\hat{A}_1,\ldots,\hat{A}_m\}, \qquad \left\|(\Phi x)'\right\| \leq \max\{B_1,\ldots,B_n,\hat{B}_1,\ldots,\hat{B}_m\}.$$

It follows from Lemma 2.4 in [34] that $\Phi(\bar{\Omega})$ is relatively compact in $X$. The proof of Lemma 2.6 is complete. □

## 3 Main results

In this section, we shall give our main results.

**Theorem 3.1** *If* $\Gamma < 1$, *then system* (1.1) *has at least one positive* $\omega$*-periodic solution*.

*Proof* Take $0 < r < \Gamma$ and $R > \Pi$. Noting that $0 < \delta_i < 1$ and $\hat{\delta}_j > 1$, we have $\Gamma_i > \Pi_i$ and $\hat{\Gamma}_j > \hat{\Pi}_j$. Then we obtain $0 < r < \Gamma < \Pi < R$. It follows from Lemmas 2.5-2.6 and $\Gamma < 1$ that $\Phi$ is strict-set-contractive on $K_{r,R}$. By Lemma 2.2, it is easy to see that if there exists $x^* \in K$ such that $\Phi x^* = x^*$, then $x^*$ is one positive $\omega$-periodic solution of system (2.1). Now, we shall prove that condition (ii) of Lemma 2.4 holds.

First, we prove that $\Phi x \ngeq x$, $\forall x \in K$, $\|x\| = r$. Otherwise, there exists $x \in K$, $\|x\| = r$ such that $\Phi x \neq x$. So, $\|x\| > 0$ and $\Phi x - x \in K$, which implies that

$$(\Phi_i x)(t) - u_i(t) \geq \delta_i |\Phi_i x - u_i|_0 \geq 0, \quad \forall t \in [0,\omega], i = 1,2,\ldots,n, \tag{3.1}$$

and

$$(\Psi_j x)(t) - v_j(t) \geq \frac{1}{\hat{\delta}_j} |\Psi_j x - v_j|_0 \geq 0, \quad \forall t \in [0,\omega], j = 1,2,\ldots,m. \tag{3.2}$$

Moreover, for $t \in [0,\omega]$, we have

$$\begin{aligned}
\big|(\Phi_i x)(t)\big| &= \theta_i \int_t^{t+\omega} G_i(t,s)u_i(s)F_i\big(s,u(s),v(s)\big)\,ds \le \frac{\theta_i}{1-\delta_i}\int_0^{\omega} u_i(s)F_i\big(s,u(s),v(s)\big)\,ds \\
&= \frac{\theta_i}{1-\delta_i}\int_0^{\omega} u_i(s)\Bigg[\sum_{k=1}^{n} a_{ik}(s)u_k\big(s-\tau_{ik}(s)\big) + \sum_{l=1}^{m} b_{il}(s)v_l\big(s-\sigma_{il}(s)\big) \\
&\quad + \sum_{k=1}^{n} c_{ik}(s)\int_{-\infty}^{0} K_{ik}(\tau)u_k(\tau+s)\,d\tau + \sum_{l=1}^{m} d_{il}(s)\int_{-\infty}^{0} L_{il}(\tau)v_l(\tau+s)\,d\tau\Bigg]\,ds \\
&\le \frac{\theta_i|u_i|_0}{1-\delta_i}\int_0^{\omega}\Bigg[\sum_{k=1}^{n} a_{ik}(s)|u_k|_0 + \sum_{l=1}^{m} b_{il}(s)|v_l|_0 \\
&\quad + \sum_{k=1}^{n} c_{ik}(s)\int_{-\infty}^{0} K_{ik}(\tau)|u_k|_0\,d\tau + \sum_{l=1}^{m} d_{il}(s)\int_{-\infty}^{0} L_{il}(\tau)|v_l|_0\,d\tau\Bigg]\,ds \\
&\le \frac{\theta_i\|x\|^2}{1-\delta_i}\int_0^{\omega}\Bigg[\sum_{k=1}^{n}\big(a_{ik}(s)+c_{ik}(s)\big) + \sum_{l=1}^{m}\big(b_{il}(s)+d_{il}(s)\big)\Bigg]\,ds \\
&= \Gamma_i\|x\|^2 < \frac{\Gamma_i}{\Gamma}\|x\| \le \|x\| = r, \quad i=1,2,\ldots,n.
\end{aligned} \tag{3.3}$$

Similarly, for $t \in [0,\omega]$, we have

$$\begin{aligned}
\big|(\Psi_j x)(t)\big| &= \vartheta_j \int_t^{t+\omega} \hat{G}_j(t,s)v_j(s)\hat{F}_j\big(s,u(s),v(s)\big)\,ds \le \frac{\vartheta_j\hat{\delta}_j}{\hat{\delta}_j-1}\int_0^{\omega} v_j(s)\hat{F}_j\big(s,u(s),v(s)\big)\,ds \\
&= \frac{\vartheta_j\hat{\delta}_j}{\hat{\delta}_j-1}\int_0^{\omega} v_j(s)\Bigg[\sum_{k=1}^{n} \hat{a}_{ik}(s)u_k\big(s-\tau_{ik}(s)\big) + \sum_{l=1}^{m} \hat{b}_{il}(s)v_l\big(s-\sigma_{il}(s)\big) \\
&\quad + \sum_{k=1}^{n} \hat{c}_{ik}(s)\int_{-\infty}^{0} \hat{K}_{ik}(\tau)u_k(\tau+s)\,d\tau + \sum_{l=1}^{m} \hat{d}_{il}(s)\int_{-\infty}^{0} \hat{L}_{il}(\tau)v_l(\tau+s)\,d\tau\Bigg]\,ds \\
&\le \frac{\vartheta_j\hat{\delta}_j|v_j|_0}{\hat{\delta}_j-1}\int_0^{\omega}\Bigg[\sum_{k=1}^{n} \hat{a}_{ik}(s)|u_k|_0 + \sum_{l=1}^{m} \hat{b}_{il}(s)|v_l|_0 \\
&\quad + \sum_{k=1}^{n} \hat{c}_{ik}(s)\int_{-\infty}^{0} \hat{K}_{ik}(\tau)|u_k|_0\,d\tau + \sum_{l=1}^{m} \hat{d}_{il}(s)\int_{-\infty}^{0} \hat{L}_{il}(\tau)|v_l|_0\,d\tau\Bigg]\,ds \\
&\le \frac{\vartheta_j\hat{\delta}_j\|x\|^2}{\hat{\delta}_j-1}\int_0^{\omega}\Bigg[\sum_{k=1}^{n}\big(\hat{a}_{ik}(s)+\hat{c}_{ik}(s)\big) + \sum_{l=1}^{m}\big(\hat{b}_{il}(s)+\hat{d}_{il}(s)\big)\Bigg]\,ds \\
&= \hat{\Gamma}_j\|x\|^2 < \frac{\hat{\Gamma}_j}{\Gamma}\|x\| \le \|x\| = r, \quad j=1,2,\ldots,m.
\end{aligned} \tag{3.4}$$

From (3.1)-(3.4), we get $\|x\| \le \|\Phi x\| < r = \|x\|$, which is a contradiction. Next, we prove that $\Phi x \nleq x$, $\forall x \in K$, $\|x\| = R$ also holds. Indeed, we only need to prove that $\Phi x \nless x$, $\forall x \in K$, $\|x\| = R$. For the sake of contradiction, suppose that there exists $x \in K$ and $\|x\| = R$ such that $\Phi x < x$. Thus $x - \Phi x \in K \setminus \{\theta = (0,0,\ldots,0)^T\}$. Furthermore, for any $t \in [0,\omega]$, we have

$$u_i(t) - (\Phi x)(t) \ge \delta_i|u_i - \Phi_i x|_0 \ge 0, \quad i=1,2,\ldots,n, \tag{3.5}$$

and

$$v_j(t) - (\Psi x)(t) \geq \frac{1}{\hat{\delta}_j} |v_j - \Psi_j x|_0 \geq 0, \quad j = 1, 2, \ldots, m. \tag{3.6}$$

For any $t \in [0, \omega]$, we have

$$\begin{aligned}
\left|(\Phi_i x)(t)\right| &= \theta_i \int_t^{t+\omega} G_i(t,s) u_i(s) F_i\bigl(s, u(s), v(s)\bigr)\, ds \geq \frac{\theta_i \delta_i}{1-\delta_i} \int_0^\omega u_i(s) F_i\bigl(s, u(s), v(s)\bigr)\, ds \\
&= \frac{\theta_i \delta_i}{1-\delta_i} \int_0^\omega u_i(s) \Biggl[ \sum_{k=1}^n a_{ik}(s) u_k\bigl(s - \tau_{ik}(s)\bigr) + \sum_{l=1}^m b_{il}(s) v_l\bigl(s - \sigma_{il}(s)\bigr) \\
&\quad + \sum_{k=1}^n c_{ik}(s) \int_{-\infty}^0 K_{ik}(\tau) u_k(\tau + s)\, d\tau + \sum_{l=1}^m d_{il}(s) \int_{-\infty}^0 L_{il}(\tau) v_l(\tau + s)\, d\tau \Biggr] ds \\
&\geq \frac{\theta_i \delta_i^2 |u_i|_0}{1-\delta_i} \int_0^\omega \Biggl[ \sum_{k=1}^n a_{ik}(s) \delta_k |u_k|_0 + \sum_{l=1}^m b_{il}(s) \frac{1}{\hat{\delta}_l} |v_l|_0 \\
&\quad + \sum_{k=1}^n c_{ik}(s) \int_{-\infty}^0 K_{ik}(\tau) \delta_k |u_k|_0\, d\tau + \sum_{l=1}^m d_{il}(s) \int_{-\infty}^0 L_{il}(\tau) \frac{1}{\hat{\delta}_l} |v_l|_0\, d\tau \Biggr] ds \\
&\geq \frac{\theta_i \delta_i^2 \|x\|^2}{1-\delta_i} \int_0^\omega \Biggl[ \sum_{k=1}^n \delta_k \bigl(a_{ik}(s) + c_{ik}(s)\bigr) + \sum_{l=1}^m \frac{1}{\hat{\delta}_l} \bigl(b_{il}(s) + d_{il}(s)\bigr) \Biggr] ds \\
&= \Pi_i \|x\|^2 > \frac{\Pi_i}{\Pi} \|x\| \geq \|x\| = R, \quad i = 1, 2, \ldots, n.
\end{aligned} \tag{3.7}$$

Similarly, for $t \in [0, \omega]$, we have

$$\begin{aligned}
\left|(\Psi_j x)(t)\right| &= \vartheta_j \int_t^{t+\omega} \hat{G}_j(t,s) v_j(s) \hat{F}_j\bigl(s, u(s), v(s)\bigr)\, ds \geq \frac{\vartheta_j}{\hat{\delta}_j - 1} \int_0^\omega v_j(s) \hat{F}_j\bigl(s, u(s), v(s)\bigr)\, ds \\
&= \frac{\vartheta_j}{\hat{\delta}_j - 1} \int_0^\omega v_j(s) \Biggl[ \sum_{k=1}^n \hat{a}_{ik}(s) u_k\bigl(s - \tau_{ik}(s)\bigr) + \sum_{l=1}^m \hat{b}_{il}(s) v_l\bigl(s - \sigma_{il}(s)\bigr) \\
&\quad + \sum_{k=1}^n \hat{c}_{ik}(s) \int_{-\infty}^0 \hat{K}_{ik}(\tau) u_k(\tau + s)\, d\tau + \sum_{l=1}^m \hat{d}_{il}(s) \int_{-\infty}^0 \hat{L}_{il}(\tau) v_l(\tau + s)\, d\tau \Biggr] ds \\
&\geq \frac{\vartheta_j |v_j|_0}{\hat{\delta}_j(\hat{\delta}_j - 1)} \int_0^\omega \Biggl[ \sum_{k=1}^n \hat{a}_{ik}(s) \delta_k |u_k|_0 + \sum_{l=1}^m \hat{b}_{il}(s) \frac{1}{\hat{\delta}_l} |v_l|_0 \\
&\quad + \sum_{k=1}^n \hat{c}_{ik}(s) \int_{-\infty}^0 \hat{K}_{ik}(\tau) \delta_k |u_k|_0\, d\tau + \sum_{l=1}^m \hat{d}_{il}(s) \int_{-\infty}^0 \hat{L}_{il}(\tau) \frac{1}{\hat{\delta}_l} |v_l|_0\, d\tau \Biggr] ds \\
&\geq \frac{\vartheta_j \|x\|^2}{\hat{\delta}_j(\hat{\delta}_j - 1)} \int_0^\omega \Biggl[ \sum_{k=1}^n \delta_k \bigl(\hat{a}_{ik}(s) + \hat{c}_{ik}(s)\bigr) + \sum_{l=1}^m \frac{1}{\hat{\delta}_l} \bigl(\hat{b}_{il}(s) + \hat{d}_{il}(s)\bigr) \Biggr] ds \\
&= \hat{\Pi}_j \|x\|^2 > \frac{\hat{\Pi}_j}{\Pi} \|x\| \geq \|x\| = R, \quad j = 1, 2, \ldots, m.
\end{aligned} \tag{3.8}$$

From (3.5)-(3.8), we obtain $\|x\| > \|\Phi x\| \geq R$, which is a contradiction. Therefore, condition (ii) of Lemma 2.4 holds. By Lemma 2.4, we see that $\Phi$ has at least one positive nonzero fixed point in $K_{r,R}$. Therefore, system (2.1) has at least one positive $\omega$-periodic solution $(u_1^*(t),\ldots,u_n^*(t),v_1^*(t),\ldots,v_m^*(t))$. Thus, system (1.1) has at least one positive $\omega$-periodic solution $((u_1^*(t))^{\frac{1}{\theta_1}},\ldots,(u_n^*(t))^{\frac{1}{\theta_n}},(v_1^*(t))^{\frac{1}{\vartheta_1}},\ldots,(v_m^*(t))^{\frac{1}{\vartheta_m}})$. The proof of Theorem 3.1 is complete. □

System (1.1) contains the following $n$-species Gilpin-Ayala competitive population dynamics model:

$$x_i'(t)=x_i(t)\left[r_i(t)-\sum_{k=1}^n a_{ik}(t)x_k^{\theta_k}\bigl(t-\tau_{ik}(t)\bigr)-\sum_{k=1}^n c_{ik}(t)\int_{-\infty}^0 K_{ik}(s)x_k^{\theta_k}(t+s)\,ds\right], \tag{3.9}$$

where $i=1,2,\ldots,n$, $r_i,a_{ik},c_{ik}\in C(\mathbb{R},(0,\infty))$ $(i,k=1,2,\ldots,n)$ and $\tau_{ik}\in C(\mathbb{R},\mathbb{R})$ $(i,k=1,2,\ldots,n)$ are $\omega$-periodic functions. $\theta_k>0$ $(k=1,2,\ldots,n)$ is a constant. $K_{ik}\in C((-\infty,0],(0,\infty))$ with $\int_{-\infty}^0 K_{ik}(s)\,ds=1$. There exists a positive integer $p$ such that $t_{i,k+p}=t_k+\omega$, $k\in\mathbb{Z}$. Without loss of generality, we also assume that $[0,\omega)\cap\{t_k:k\in\mathbb{Z}\}=\{t_1,t_2,\ldots,t_p\}$.

Similar to the previous arguments, we conclude the existence of a positive $\omega$-periodic solution for system (3.9) as follows.

**Theorem 3.2** *If* $\Lambda=\min\{\frac{1}{\Lambda_1},\frac{1}{\Lambda_2},\ldots,\frac{1}{\Lambda_n}\}<1$, *where* $\Lambda_i=\frac{\theta_i}{1-\delta_i}\sum_{k=1}^n\int_0^\omega[a_{ik}(s)+c_{ik}(s)]\,ds$, *then system* (3.9) *has at least one positive $\omega$-periodic solution.*

## 4 Illustrative example

Consider the following two-species Gilpin-Ayala population model:

$$\begin{cases} x'(t)=x(t)[r(t)-a(t)x^\theta(t-\tau(t))-b(t)y^\vartheta(t-\sigma(t)) \\ \qquad -c(t)\int_{-\infty}^0 K(s)x^\theta(t+s)\,ds-d(t)\int_{-\infty}^0 L(s)y^\vartheta(t+s)\,ds], \\ y'(t)=y(t)[-\hat{r}(t)+\hat{a}(t)x^\theta(t-\hat{\tau}(t))+\hat{b}(t)y^\vartheta(t-\hat{\sigma}(t)) \\ \qquad +\hat{c}(t)\int_{-\infty}^0 \hat{K}(s)x^\theta(t+s)\,ds+\hat{d}(t)\int_{-\infty}^0 \hat{L}(s)y^\vartheta(t+s)\,ds], \end{cases} \tag{4.1}$$

where $\theta=\frac{1}{2}$, $\vartheta=2$, $r(t)=\frac{(2+\cos t)\ln 2}{2\pi}$, $\hat{r}(t)=\frac{(2+\sin t)\ln 2}{8\pi}$, $a(t)=\frac{3+\cos 2t}{9\pi}$, $b(t)=\frac{2-\sin 3t}{9\pi}$, $c(t)=\frac{1+\pi|\sin t|}{9\pi}$, $d(t)=\frac{2+\cos 5t}{24\pi}$, $\hat{a}(t)=\frac{3-\cos 2t}{9\pi}$, $\hat{b}(t)=\frac{2+\sin 4t}{24\pi}$, $\hat{c}(t)=\frac{1+\pi|\sin t|}{24\pi}$, $\hat{d}(t)=\frac{2+\cos 3t}{24\pi}$, $\tau(t)=\frac{|\sin t|}{2}$, $\sigma(t)=\frac{1+\sin t}{3}$, $\hat{\tau}(t)=\frac{|\cos 2t|}{3}$, $\hat{\sigma}(t)=\frac{2-|\cos t|}{4}$, $K(s)=\hat{K}(s)=e^s$, $L(s)=\hat{L}(s)=\sqrt{\frac{2}{\pi}}e^{-\frac{s^2}{2}}$.

Obviously, $r(t)$, $\hat{r}(t)$, $a(t)$, $b(t)$, $c(t)$, $d(t)$, $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$, $\hat{d}(t)$, $\tau(t)$, $\sigma(t)$, $\hat{\tau}(t)$ and $\hat{\sigma}(t)$ are all positive $2\pi$-periodic functions. By a simple calculation, we have

$$\int_{-\infty}^0 K(s)\,ds=\int_{-\infty}^0 \hat{K}(s)\,ds=\int_{-\infty}^0 L(s)\,ds=\int_{-\infty}^0 \hat{L}(s)\,ds=1,$$

$$\delta=e^{-\theta\int_0^{2\pi} r(s)\,ds}=\frac{1}{2},\qquad \hat{\delta}=e^{\vartheta\int_0^{2\pi}\hat{r}(s)\,ds}=2,$$

$$\Gamma_1=\frac{\theta}{1-\delta}\int_0^{2\pi}\bigl[a(s)+b(s)+c(s)+d(s)\bigr]\,ds=2,$$

$$\hat{\Gamma}_1=\frac{\vartheta\hat{\delta}}{\hat{\delta}-1}\int_0^{2\pi}\bigl[\hat{a}(s)+\hat{b}(s)+\hat{c}(s)+\hat{d}(s)\bigr]\,ds=3,$$

$$\Gamma=\min\left\{\frac{1}{\Gamma_1},\frac{1}{\hat{\Gamma}_1}\right\}=\min\left\{\frac{1}{2},\frac{1}{3}\right\}=\frac{1}{3}<1.$$

Thus, all the assumptions of Theorem 3.1 are satisfied. Hence, system (4.1) has at least one positive $2\pi$-periodic solution.

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The authors declare to have no competing interests.

**Authors' contributions**
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