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LG–Holling type II diseased predator ecosystem with Lévy noise and white noise

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Abstract

This paper considers an LG–Holling type II diseased predator ecosystem with Lévy noise and white noise. In this prey–predator system, assuming that the predator population is infected by disease and divided into two classes: susceptible predator and infected predator, we show that the system has a unique global positive solution. We investigate the persistence in the mean and extinction for the population, obtain threshold conditions of extinction, persistence in the mean by Itô's formula and a comparison theorem for the stochastic system. In addition, we discuss the uniform boundedness of pth moment with $p > 0$ and reveal the stochastically ultimate boundedness of the system. Finally, some numerical simulations are introduced to illustrate our analytical findings.

Keywords: Leslie–Gower; Prey–predator system; Lévy noise; Persistence in the mean; Ultimate boundedness

1 Introduction

In ecology, prey–predator systems are always the important part, and the most classic model is Lotka–Volterra model. Out of the ordinary, Leslie introduced the following model on the premise that the carrying capacity of predator was proportional to the amount of prey [1, 2]:

$$\begin{cases} \frac{dN_1}{dt} = (r_1 - a_1 N_1 - b_2 N_2)N_1, \\ \frac{dN_2}{dt} = (r_2 - b_2 \frac{N_2}{N_1})N_2, \end{cases} \tag{1}$$

here N_1 and N_2 represented the densities of prey and predator at time t, respectively. r_1 and r_2 were the intrinsic growth rates for prey and predator populations. b_2 was conversion rate. Leslie assumed that the predator population still grew logistically with carrying capacity $\frac{r_2 N_1}{b_2}$.

Leslie assumed that the growth of predator population still satisfied the logistic rule, but the carrying capacity, which was distinguished from the constant in the common model, was determined by the density of prey population. Recently, there have been many experts and researchers studying or improving a Leslie type prey–predator model. Time delay, impulse, food chain, disease, environment pollution, reaction diffusion and so on, have been introduced into the model, see [3–8].

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In [9], Sarwardi introduced a contagious disease among the predators in a predator-prey ecosystem, modeled via the following equations with nonnegative parameters:

$$
\begin{cases}
\frac{dx}{dt} = a_1 x - b_1 x^2 - \frac{c_1 xy}{x+k_1} - \frac{pc_1 xz}{x+k_1}, \\
\frac{dy}{dt} = a_2 y - \frac{c_2 y(y+z)}{x+k_2} - \lambda yz, \\
\frac{dz}{dt} = \lambda yz + a_3 z - \frac{c_3 z(y+z)}{x+k_2}.
\end{cases}
\tag{2}
$$

The system incorporates a Holling type II functional response and a modified version of the Leslie–Gower one. In system (2), the following assumptions are made:

(A1) $x(t)$ represents the density of prey population at time t, and it is assumed that the disease spreads only among the predator, $y(t)$ and $z(t)$ denote the densities of susceptible predator and infected predator at time t, the total predator population is $n(t) = y(t) + z(t)$. The disease transmission rate is λ.

(A2) In the absence of predators, the prey population grows logistically with intrinsic growth rate $a_1 > 0$ and carrying capacity $a_1 b_1^{-1}$. c_1 and pc_1 are the predation rates for susceptible and infected predator. Because of disease infection, the predation ability for the infected predator is a bit less than that for the susceptible predator; therefore, $0 < p < 1$.

(A3) Here a_2, a_3 are the per capita growth rates of each predator population. The parameters k_1, k_2 respectively represent the half saturation constants of prey and predator populations. In addition, the authors introduce intraspecific competition among the predators' sound and infected subpopulations with parameters c_2 and c_3, for which $c_2 > c_3$.

The authors studied the stability of equilibrium point of system (2) and persistence.

Population growth in the natural world is inevitably affected by the environmental perturbations. In the past decades, white noise was introduced into the model to simulate the stochastic perturbation of environment. However, sudden environmental perturbation, such as earthquakes, hurricanes, the flood, etc., cannot be modeled by white noise, which is often used to describe those stable and continuous stochastic interferences. Bao et al. in [10, 11] firstly proposed that these phenomena could be described by Lévy noise. At present, there are many research papers considering Lévy noise [6, 12–17]. In this paper, we use white noise and Lévy noise to simulate the random change of environment. The following system with jumps is considered:

$$
\begin{cases}
dx(t) = \left[a_1 x(t) - b_1 x^2(t) - \frac{c_1 x(t)y(t)}{x(t)+k_1} - \frac{\tilde{p} c_1 x(t)z(t)}{x(t)+k_1}\right] dt \\
\qquad + \sigma_1 x(t)\, dB_1(t) + \int_{\mathbb{Y}} \gamma_1(u) x(t^-) \tilde{N}(dt, du), \\
dy(t) = \left[a_2 y(t) - \frac{c_2 y(t)(y(t)+z(t))}{x(t)+k_2} - \beta y(t)z(t)\right] dt \\
\qquad + \sigma_2 y(t)\, dB_2(t) + \int_{\mathbb{Y}} \gamma_2(u) y(t^-) \tilde{N}(dt, du), \\
dz(t) = \left[\beta y(t)z(t) + a_3 z(t) - \frac{c_3 z(t)(y(t)+z(t))}{x(t)+k_2}\right] dt \\
\qquad + \sigma_3 z(t)\, dB_3(t) + \int_{\mathbb{Y}} \gamma_3(u) z(t^-) \tilde{N}(dt, du),
\end{cases}
\tag{3}
$$

with the initial value $(x_0, y_0, z_0) \in R_+^3$. For convenience, we use $\tilde{p}$ and β instead of p and λ in system (3), respectively. $B_1(t)$, $B_2(t)$ and $B_3(t)$ are mutually independent Brownian motions defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with a filtration $\{\mathcal{F}_t\}_{t \in R_+}$ satisfying the usual conditions. σ_i^2 $(i = 1, 2, 3)$ denotes the intensity of white noise. In system (3), $x(t^-)$, $y(t^-)$ and $z(t^-)$ are the left limits of $x(t)$, $y(t)$ and $z(t)$, respectively. N is a Poisson

counting measure with characteristic measure λ on a measurable subset $\mathbb{Y}$ of $(0, +\infty)$ with $\lambda(\mathbb{Y}) < \infty$, $\tilde{N}(dt, du) = N(dt, du) - \lambda(du)\, dt$, $\gamma_i : \mathbb{Y} \times \Omega \to \mathbb{R}$ is bounded and continuous with respect to λ, and is $\mathcal{B}(\mathbb{Y}) \times \mathcal{F}_t$-measurable, $i = 1, 2, 3$.

For simplicity, we introduce the following notations:

$$w_i = a_i - \frac{\sigma_i^2}{2} - \int_{\mathbb{Y}} \left[\gamma_i(u) - \ln\left(1 + \gamma_i(u)\right)\right]\lambda(du), \quad i = 1, 2, 3.$$

$$R_+^3 = \left\{(\eta_1, \eta_2, \eta_3)^T \in R^3 \mid \eta_i > 0, i = 1, 2, 3\right\}.$$

Assumption 1 There exists a constant c satisfying

$$\int_{\mathbb{Y}} \left[\ln\left(1 + \gamma_i(u)\right)\right]^2 \lambda(du) < c, \quad i = 1, 2, 3.$$

Throughout this paper, we assume that Assumption 1 is true.

2 The existence of global positive solution

Theorem 1 *For any initial value $(x_0, y_0, z_0) \in R_+^3$, system (3) has a unique global solution $(x(t), y(t), z(t)) \in R_+^3$ for $t \geq 0$, and the solution will remain in R_+^3 almost surely.*

Proof At first, let us consider the following system:

$$\begin{cases} du(t) = \left[w_1 - b_1 \exp(u(t)) - \frac{c_1 \exp(v_1(t))}{\exp(u(t)) + k_1} - \frac{\tilde{p}c_1 \exp(v_2(t))}{\exp(u(t)) + k_1}\right] dt \\ \qquad + \sigma_1 \, dB_1(t) + \int_{\mathbb{Y}} \ln(1 + \gamma_1(u))\tilde{N}(dt, du), \\ dv_1(t) = \left[w_2 - \frac{c_2(\exp(v_1(t)) + \exp(v_2(t)))}{\exp(u(t)) + k_2} - \beta \exp(v_2(t))\right] dt \\ \qquad + \sigma_2 \, dB_2(t) + \int_{\mathbb{Y}} \ln(1 + \gamma_2(u))\tilde{N}(dt, du), \\ dv_2(t) = \left[w_3 + \beta \exp(v_1(t)) - \frac{c_3(\exp(v_1(t)) + \exp(v_2(t)))}{\exp(u(t)) + k_2}\right] dt \\ \qquad + \sigma_3 \, dB_3(t) + \int_{\mathbb{Y}} \ln(1 + \gamma_3(u))\tilde{N}(dt, du), \end{cases} \tag{4}$$

on $t \geq 0$ with the initial data $(\ln x_0, \ln y_0, \ln z_0)$. Clearly, the coefficients of system (4) satisfy the local Lipschitz condition, then there is a unique local solution on $[0, \tau_e)$, where τ_e is the explosion time. Therefore, by Itô's formula, $(\exp(u(t)), \exp(v_1(t)), \exp(v_2(t)))$ is the unique positive local solution of system (3) with the initial value $(x_0, y_0, z_0) \in R_+^3$. Whereafter, we will show that $\tau_e = \infty$, namely the solution of system (3) is globally existent.

Consider the stochastic differential equations:

$$\begin{cases} d\Phi_1(t) = \Phi_1(t)[a_1 - b_1\Phi_1(t)]\, dt + \sigma_1\Phi_1(t)\, dB_1(t) + \int_{\mathbb{Y}} \gamma_1(u)\Phi_1(t^-)\tilde{N}(dt, du), \\ d\Phi_2(t) = \Phi_2(t)\left[a_2 - \frac{c_2\Phi_2(t)}{\Phi_1(t) + k_2}\right] dt + \sigma_2\Phi_2(t)\, dB_2(t) + \int_{\mathbb{Y}} \gamma_2(u)\Phi_2(t^-)\tilde{N}(dt, du), \\ d\Phi_3(t) = \Phi_3(t)\left[\beta\Phi_2(t) + a_3 - \frac{c_3\Phi_3(t)}{\Phi_1(t) + k_2}\right] dt \\ \qquad + \sigma_3\Phi_3(t)\, dB_3(t) + \int_{\mathbb{Y}} \gamma_3(u)\Phi_3(t^-)\tilde{N}(dt, du), \end{cases} \tag{5}$$

and

$$\begin{cases} d\Psi_1(t) = \Psi_1(t)\left[a_1 - b_1\Psi_1(t) - \frac{c_1\Phi_2(t)}{k_1} - \frac{\tilde{p}c_1\Phi_3(t)}{k_1}\right] dt \\ \qquad + \sigma_1\Psi_1(t)\, dB_1(t) + \int_{\mathbb{Y}} \gamma_1(u)\Psi_1(t^-)\tilde{N}(dt, du), \\ d\Psi_2(t) = \Psi_2(t)\left[a_2 - \frac{c_2\Psi_2(t)}{k_2} - \frac{c_2\Phi_3(t)}{k_2} - \beta\Phi_3(t)\right] dt \\ \qquad + \sigma_2\Psi_2(t)\, dB_2(t) + \int_{\mathbb{Y}} \gamma_2(u)\Psi_2(t^-)\tilde{N}(dt, du), \\ d\Psi_3(t) = \Psi_3(t)\left[\beta\Psi_2(t) + a_3 - \frac{c_3\Phi_2(t)}{k_2} - \frac{c_3\Psi_3(t)}{k_2}\right] dt \\ \qquad + \sigma_3\Psi_3(t)\, dB_3(t) + \int_{\mathbb{Y}} \gamma_3(u)\Psi_3(t^-)\tilde{N}(dt, du). \end{cases} \tag{6}$$

According to the comparison theorem for stochastic differential equations in [18], we can get that, for $t \in [0, \tau_e)$,

$$\Psi_1(t) \leq x(t) \leq \Phi_1(t), \qquad \Psi_2(t) \leq y(t) \leq \Phi_2(t), \qquad \Psi_3(t) \leq z(t) \leq \Phi_3(t), \quad \text{a.s.}$$

By virtue of Lemma 4.2 in [10], we can see that $\Phi_i(t)$ and $\Psi_i(t)$ ($i = 1, 2, 3$) will not explode in any finite time, which means that $\Phi_i(t)$ and $\Psi_i(t)$ are existent on $t \geq 0$. Thus, we obtain $\tau_e = +\infty$, the proof of Theorem 1 is completed. □

According to Lemma 4.4 in [10] and equations (5), we know that

$$\limsup_{t \to \infty} \frac{\ln \Phi_i(t)}{\ln t} \leq 1, \quad \text{a.s.}$$

for $i = 1, 2, 3$, when

$$\sup_{t \geq 0} \int_0^t \int_{\mathbb{Y}} \exp(s - t) \big[\gamma_i(u) - \ln\big(1 + \gamma_i(u)\big) \big] \lambda(du) \, ds < \infty.$$

Noting the limit $\lim_{t \to \infty} \frac{\ln t}{t} = 0$, $\limsup_{t \to \infty} \frac{\ln \Phi_i(t)}{t} \leq 0$ is obtained for $i = 1, 2, 3$, a.s. It then follows from the comparison theorem of [18] that

$$\limsup_{t \to \infty} \frac{\ln x(t)}{t} \leq \limsup_{t \to \infty} \frac{\ln \Phi_1(t)}{t} \leq 0, \quad \text{a.s.},$$

$$\limsup_{t \to \infty} \frac{\ln y(t)}{t} \leq \limsup_{t \to \infty} \frac{\ln \Phi_2(t)}{t} \leq 0, \quad \text{a.s.},$$

$$\limsup_{t \to \infty} \frac{\ln z(t)}{t} \leq \limsup_{t \to \infty} \frac{\ln \Phi_3(t)}{t} \leq 0, \quad \text{a.s.}$$

Thus, we have a lemma as follows.

Lemma 1 *Assume that, for any $t \geq 0$ and $i = 1, 2, 3$,*

$$\sup_{t \geq 0} \int_0^t \int_{\mathbb{Y}} \exp(s - t) \big[\gamma_i(u) - \ln\big(1 + \gamma_i(u)\big) \big] \lambda(du) \, ds < \infty.$$

Then, for any initial value $(x_0, y_0, z_0) \in R_+^3$, the solution $(x(t), y(t), z(t))$ of system (3) *satisfies*

$$\limsup_{t \to \infty} \frac{\ln x(t)}{t} \leq 0, \qquad \limsup_{t \to \infty} \frac{\ln y(t)}{t} \leq 0, \qquad \limsup_{t \to \infty} \frac{\ln z(t)}{t} \leq 0, \quad \text{a.s.}$$

3 Persistence in the mean and extinction

Lemma 2 (see Liu et al. [14]) *Suppose that $Z(t) \in C(\Omega \times [0, \infty), R_+)$, and let $\int_{\mathbb{Y}} [\ln(1 + \gamma_i(u))]^2 \lambda(du) < c$, $i = 1, 2$, hold.*

(I) *If there exist two positive constants T and δ_0 such that*

$$\ln Z(t) \leq \delta t - \delta_0 \int_0^t Z(s) \, ds + \alpha B(t) + \sum_{i=1}^2 \delta_i \int_0^t \int_{\mathbb{Y}} \ln\big(1 + \gamma_i(u)\big) \tilde{N}(ds, du), \quad \text{a.s.}$$

for all $t \geq T$, where α, δ_1 and δ_2 are constants, then

$$\begin{cases} \limsup_{t \to \infty} \frac{1}{t} \int_0^t Z(s) \, ds \leq \delta/\delta_0, & a.s., \text{if } \delta \geq 0; \\ \lim_{t \to \infty} Z(t) = 0, & a.s., \text{if } \delta < 0. \end{cases}$$

(II) *If there exist three positive constants T, δ and δ_0 such that*

$$\ln Z(t) \geq \delta t - \delta_0 \int_0^t Z(s) \, ds + \alpha B(t) + \sum_{i=1}^2 \delta_i \int_0^t \int_{\mathbb{Y}} \ln\big(1 + \gamma_i(u)\big) \tilde{N}(ds, du), \quad a.s.$$

for all $t \geq T$, then $\liminf_{t \to \infty} \frac{1}{t} \int_0^t Z(s) \, ds \geq \delta/\delta_0$, a.s.

Definition 1 Assume $m(t)$ is the density of a population at time t:
(1) If $\lim_{t \to \infty} m(t) = 0$, a.s., then species $m(t)$ is said to be extinct.
(2) If $\lim_{t \to \infty} \frac{1}{t} \int_0^t m(s) \, ds > 0$, a.s., then species $m(t)$ is said to be stable in the mean.
(3) If $\liminf_{t \to \infty} \frac{1}{t} \int_0^t m(s) \, ds > 0$, a.s., then species $m(t)$ is said to be strongly persistent in the mean.
(4) If $\limsup_{t \to \infty} \frac{1}{t} \int_0^t m(s) \, ds > 0$, a.s., then species $m(t)$ is said to be weakly persistent in the mean.

Obviously, if species $m(t)$ is stable in the mean, it must be strongly or weakly persistent in the mean.

Making use of Itô's formula to system (3) gives

$$\begin{cases} d \ln x(t) = [w_1 - b_1 x(t) - \frac{c_1 y(t)}{x(t)+k_1} - \frac{\check{p} c_1 z(t)}{x(t)+k_1}] \, dt \\ \qquad + \sigma_1 \, dB_1(t) + \int_{\mathbb{Y}} \ln(1 + \gamma_1(u)) \tilde{N}(dt, du), \\ d \ln y(t) = [w_2 - \frac{c_2(y(t)+z(t))}{x(t)+k_2} - \beta z(t)] \, dt + \sigma_2 \, dB_2(t) + \int_{\mathbb{Y}} \ln(1 + \gamma_2(u)) \tilde{N}(dt, du), \\ d \ln z(t) = [w_3 + \beta y(t) - \frac{c_3(y(t)+z(t))}{x(t)+k_2}] \, dt + \sigma_3 \, dB_3(t) + \int_{\mathbb{Y}} \ln(1 + \gamma_3(u)) \tilde{N}(dt, du). \end{cases} \tag{7}$$

For both sides of system (7), integrating from 0 to t and multiplying by $\frac{1}{t}$, (7) equals

$$\begin{cases} \frac{\ln x(t) - \ln x_0}{t} = w_1 - b_1 \frac{1}{t} \int_0^t x(s) \, ds - \frac{1}{t} \int_0^t \frac{c_1 y(s)}{x(s)+k_1} \, ds - \frac{1}{t} \int_0^t \frac{\check{p} c_1 z(s)}{x(s)+k_1} \, ds + \frac{\sigma_1 B_1(t)}{t} + \frac{M_1(t)}{t}, \\ \frac{\ln y(t) - \ln y_0}{t} = w_2 - \frac{1}{t} \int_0^t \frac{c_2(y(s)+z(s))}{x(s)+k_2} \, ds - \beta \frac{1}{t} \int_0^t z(s) \, ds + \frac{\sigma_2 B_2(t)}{t} + \frac{M_2(t)}{t}, \\ \frac{\ln z(t) - \ln z_0}{t} = w_3 + \beta \frac{1}{t} \int_0^t y(s) \, ds - \frac{1}{t} \int_0^t \frac{c_3(y(s)+z(s))}{x(s)+k_2} \, ds + \frac{\sigma_3 B_3(t)}{t} + \frac{M_3(t)}{t}, \end{cases} \tag{8}$$

where $M_i(t) = \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_i(u)) \tilde{N}(ds, du)$, $i = 1, 2, 3$.

$M_i(t)$ are local martingale, by Proposition 2.4 of [19] and Assumption 1, its quadratic variations are

$$\langle M_i, M_i \rangle(t) = \int_0^t \int_{\mathbb{Y}} \big[\ln\big(1 + \gamma_i(u)\big)\big]^2 \lambda(du) \, ds$$

$$= t \int_{\mathbb{Y}} \big[\ln\big(1 + \gamma_i(u)\big)\big]^2 \lambda(du)$$

$$< ct, \quad i = 1, 2, 3.$$

According to the law of strong large numbers, we have

$$\lim_{t \to \infty} \frac{M_i(t)}{t} = 0, \quad a.s. \ i = 1, 2, 3. \tag{9}$$

Obviously,

$$\lim_{t\to\infty} \frac{B_i(t)}{t} = 0, \quad \text{a.s. } i = 1, 2, 3. \tag{10}$$

In the following, we discuss the persistence in the mean and extinction for prey and predator populations.

Theorem 2

(i) *If $w_i < 0$ ($i = 1, 2, 3$), the prey population $x(t)$ and both the predator populations $y(t)$ and $z(t)$ will be extinct with probability 1.*

(ii) *If $w_1 < 0$, $w_2 < 0$ and $w_3 > 0$, the prey population $x(t)$ and the susceptible predator population $y(t)$ will go to extinction, a.s., the infected predator population $z(t)$ will be stable in the mean, i.e., $\lim_{t\to\infty} \frac{1}{t} \int_0^t z(s)\,ds = \frac{w_3 k_2}{c_3}$.*

(iii) *If $w_1 < 0$, $w_2 > 0$ and $w_3 < 0$, the prey population $x(t)$ will go to extinction, the growth of the predator population is divided into the following two cases:*

   (a) *when $w_3 + \frac{\beta k_2}{c_2} w_2 < 0$, then the susceptible predator $y(t)$ will be stable in the mean, $\lim_{t\to\infty} \frac{1}{t} \int_0^t y(s)\,ds = \frac{w_2 k_2}{c_2}$, and the infected predator $z(t)$ will be extinct, a.s.*

   (b) *when $w_3 + \frac{\beta k_2}{c_2} w_2 > 0$, $w_2 > \frac{(c_2 + k_2\beta)(w_3 c_2 + \beta k_2 w_2)}{c_2 c_3}$ and $w_3 + \beta Q > \frac{c_3 w_2}{c_2}$, where $Q = \frac{k_2}{c_2}\left[w_2 - \frac{(c_2 + k_2\beta)(w_3 c_2 + \beta k_2 w_2)}{c_2 c_3}\right]$, the whole predator population $y(t)$ and $z(t)$ will be strongly persistent in the mean, a.s.*

(iv) *If $w_1 > 0$, $w_2 < 0$ and $w_3 < 0$, then the prey population $x(t)$ will be stable in the mean, namely $\lim_{t\to\infty} \frac{1}{t} \int_0^t x(s)\,ds = \frac{w_1}{b_1}$, a.s. Both the susceptible predator $y(t)$ and the infected predator $z(t)$ will die out, a.s.*

(v) *Assume that, for any $t \geq 0$ and $i = 1, 2, 3$,*

$$\sup_{t\geq 0} \int_0^t \int_{\mathbb{Y}} \exp(s - t)\left[\gamma_i(u) - \ln\left(1 + \gamma_i(u)\right)\right]\lambda(du)\,ds < \infty,$$

then if $w_1 > \frac{(w_2 + w_3)k_2 c_1}{(c_2 + c_3 + k_2\beta)k_1}$, $w_2 > 0$ and $w_3 > 0$, then all the populations will be strongly persistent in the mean, where $\liminf_{t\to\infty}\left[\frac{1}{t}\int_0^t x(s)\,ds\right] \geq \frac{w_1}{b_1} - \frac{(w_2 + w_3)k_2 c_1}{b_1 k_1 (c_2 + c_3 + k_2\beta)}$ and $\liminf_{t\to\infty} \frac{1}{t}\int_0^t [y(s) + z(s)]\,ds \geq \frac{(w_2 + w_3)k_2}{c_2 + c_3 + k_2\beta}$, a.s.

Proof Making use of (8), we get

$$\begin{cases} \frac{1}{t}\ln\frac{x(t)}{x_0} \leq w_1 - b_1\frac{1}{t}\int_0^t x(s)\,ds + \frac{\sigma_1 B_1(t)}{t} + \frac{M_1(t)}{t}, \\ \frac{1}{t}\ln\frac{y(t)}{y_0} \leq w_2 - \frac{1}{t}\int_0^t \frac{c_2 y(s)}{x(s) + k_2}\,ds + \frac{\sigma_2 B_2(t)}{t} + \frac{M_2(t)}{t}, \\ \frac{1}{t}\ln\frac{z(t)}{z_0} \leq w_3 + \beta\frac{1}{t}\int_0^t y(s)\,ds - \frac{1}{t}\int_0^t \frac{c_3 z(s)}{x(s) + k_2}\,ds + \frac{\sigma_3 B_3(t)}{t} + \frac{M_3(t)}{t}. \end{cases} \tag{11}$$

(i) According to (11),

$$\frac{1}{t}\ln\frac{x(t)}{x_0} \leq w_1 - b_1\frac{1}{t}\int_0^t x(s)\,ds + \frac{\sigma_1 B_1(t)}{t} + \frac{M_1(t)}{t},$$

noting (9) and (10), by virtue of Lemma 2, we can get $\lim_{t\to\infty} x(t) = 0$ when $w_1 < 0$. Then, for an arbitrary small constant ε_1 which satisfies $0 < \varepsilon_1 < -\frac{w_3}{\beta}$, there exists a sufficiently

large number T_1 ($T_1 > 0$) such that

$$-\varepsilon_1 \leq x(t) \leq \varepsilon_1 \tag{12}$$

for $t \geq T_1$; therefore,

$$\frac{1}{t} \ln \frac{y(t)}{y_0} \leq w_2 - \frac{1}{t} \int_0^t \frac{c_2 y(s)}{\varepsilon_1 + k_2} \, ds + \frac{\sigma_2 B_2(t)}{t} + \frac{M_2(t)}{t}. \tag{13}$$

Because of $w_2 < 0$, making use of Lemma 2, $\lim_{t\to\infty} y(t) = 0$, a.s. In this case, there is a sufficiently large constant T_2 ($T_2 > T_1$) satisfying $-\varepsilon_1 \leq y(t) \leq \varepsilon_1$, when $t \geq T_2$, a.s. Thus,

$$\begin{aligned}
\frac{1}{t} \ln \frac{z(t)}{z_0} &\leq w_3 + \beta \frac{1}{t} \int_0^t y(s) \, ds - \frac{1}{t} \int_0^t \frac{c_3 z(s)}{x(s) + k_2} \, ds + \frac{\sigma_3 B_3(t)}{t} + \frac{M_3(t)}{t} \\
&\leq w_3 + \beta \frac{1}{t} \int_0^t \varepsilon_1 \, ds - \frac{1}{t} \int_0^t \frac{c_3 z(s)}{\varepsilon_1 + k_2} \, ds + \frac{\sigma_3 B_3(t)}{t} + \frac{M_3(t)}{t} \\
&\leq (w_3 + \beta \varepsilon_1) - \frac{c_3}{\varepsilon_1 + k_2} \frac{1}{t} \int_0^t z(s) \, ds + \frac{\sigma_3 B_3(t)}{t} + \frac{M_3(t)}{t}.
\end{aligned}$$

Notice $0 < \varepsilon_1 < -\frac{w_3}{\beta}$, it follows from Lemma 2 that $\lim_{t\to\infty} z(t) = 0$, a.s.

(ii) Similar to case (i), when $w_1 < 0$, $w_2 < 0$, it easily shows that $\lim_{t\to\infty} x(t) = 0$, $\lim_{t\to\infty} y(t) = 0$, and for an arbitrary small constant $\bar{\varepsilon}_1 > 0$,

$$\frac{1}{t} \ln \frac{z(t)}{z_0} \leq (w_3 + \beta \bar{\varepsilon}_1) - \frac{c_3}{\bar{\varepsilon}_1 + k_2} \frac{1}{t} \int_0^t z(s) \, ds + \frac{\sigma_3 B_3(t)}{t} + \frac{M_3(t)}{t}.$$

Making use of Lemma 2, $w_3 > 0$ and the arbitrariness of $\bar{\varepsilon}_1$, $\limsup_{t\to\infty} \frac{1}{t} \int_0^t z(s) \, ds \leq \frac{w_3 k_2}{c_3}$, a.s. is obviously verified. In contrast,

$$\frac{1}{t} \ln \frac{z(t)}{z_0} \geq w_3 - \frac{c_3}{k_2} \frac{1}{t} \int_0^t y(s) \, ds - \frac{c_3}{k_2} \frac{1}{t} \int_0^t z(s) \, ds + \frac{\sigma_3 B_3(t)}{t} + \frac{M_3(t)}{t}.$$

Noting that $\lim_{t\to\infty} y(t) = 0$, a.s. and Lemma 2, we have

$$\liminf_{t\to\infty} \frac{1}{t} \int_0^t z(s) \, ds \geq \frac{w_3 k_2}{c_3}, \quad \text{a.s.,}$$

then we obtain $\lim_{t\to\infty} \frac{1}{t} \int_0^t z(s) \, ds = \frac{w_3 k_2}{c_3}$, a.s.

(iii) It is easy to show that $\lim_{t\to\infty} x(t) = 0$, a.s. when $w_1 < 0$. On the basis of (13) and Lemma 2, for $t > T_1$, $\limsup_{t\to\infty} \frac{1}{t} \int_0^t y(s) \, ds \leq \frac{w_2(\varepsilon_1 + k_2)}{c_2}$, a.s. under the condition of $w_2 > 0$. Considering the arbitrariness of ε_1, $\limsup_{t\to\infty} \frac{1}{t} \int_0^t y(s) \, ds \leq \frac{w_2 k_2}{c_2}$, a.s. Therefore, for an arbitrary constant $\varepsilon_3 > 0$ ($0 < \varepsilon_3 < \frac{1}{\beta} |w_3 + \frac{\beta k_2}{c_2} w_2|$), there exists a number T_3 ($T_3 > T_1$) such that

$$\frac{1}{t} \int_0^t y(s) \, ds \leq \frac{w_2 k_2}{c_2} + \varepsilon_3, \quad \text{a.s.}$$

for $t > T_3$. Making use of (12), then for $t > T_3$, we have

$$\frac{1}{t} \ln \frac{z(t)}{z_0} \leq w_3 + \beta \frac{1}{t} \int_0^t y(s) \, ds - \frac{1}{t} \int_0^t \frac{c_3 z(s)}{\varepsilon_1 + k_2} \, ds + \frac{\sigma_3 B_3(t)}{t} + \frac{M_3(t)}{t}$$

$$\leq w_3 + \beta \left(\frac{w_2 k_2}{c_2} + \varepsilon_3 \right) - \frac{c_3}{\varepsilon_1 + k_2} \frac{1}{t} \int_0^t z(s) \, ds + \frac{\sigma_3 B_3(t)}{t} + \frac{M_3(t)}{t}.$$

(a) If $w_3 + \frac{\beta k_2}{c_2} w_2 < 0$, then

$$\lim_{t \to \infty} z(t) = 0, \quad \text{a.s.} \tag{14}$$

For the susceptible predator population,

$$\frac{1}{t} \ln \frac{y(t)}{y_0} \geq w_2 - \frac{1}{t} \int_0^t \frac{c_2 y(s)}{k_2} \, ds - \frac{1}{t} \int_0^t \frac{c_2 z(s)}{k_2} \, ds - \beta \frac{1}{t} \int_0^t z(s) \, ds + \frac{\sigma_2 B_2(t)}{t} + \frac{M_2(t)}{t}.$$

Because of (9), (10), (14) and Lemma 2, we have $\liminf_{t \to \infty} \frac{1}{t} \int_0^t y(s) \, ds \geq \frac{w_2 k_2}{c_2}$, a.s. Then $\lim_{t \to \infty} \frac{1}{t} \int_0^t y(s) \, ds = \frac{w_2 k_2}{c_2}$, a.s.

(b) If $w_3 + \frac{\beta k_2}{c_2} w_2 > 0$, by virtue of Lemma 2 and the arbitrariness of ε_1 and ε_3, we have

$$\limsup_{t \to \infty} \frac{1}{t} \int_0^t z(s) \, ds \leq \frac{(w_3 c_2 + \beta k_2 w_2) k_2}{c_2 c_3}, \quad \text{a.s.} \tag{15}$$

Then

$$\frac{1}{t} \ln \frac{y(t)}{y_0} \geq w_2 - \frac{c_2}{k_2} \frac{1}{t} \int_0^t y(s) \, ds - \left(\frac{c_2}{k_2} + \beta \right) \frac{1}{t} \int_0^t z(s) \, ds + \frac{\sigma_2 B_2(t)}{t} + \frac{M_2(t)}{t}.$$

Notice (15), for sufficiently large t, $\frac{1}{t} \int_0^t z(s) \, ds \leq \frac{(w_3 c_2 + \beta k_2 w_2) k_2}{c_2 c_3} + \varepsilon_4$, a.s., where ε_4 is an arbitrary positive number. Hence, by (9), (10), Lemma 2 and the arbitrariness of ε_4, we have

$$\liminf_{t \to \infty} \frac{1}{t} \int_0^t y(s) \, ds \geq \frac{k_2}{c_2} \left[w_2 - \frac{(c_2 + k_2 \beta)(w_3 c_2 + \beta k_2 w_2)}{c_2 c_3} \right],$$

when $w_2 > \frac{(c_2 + k_2 \beta)(w_3 c_2 + \beta k_2 w_2)}{c_2 c_3}$. Here we define $Q = \frac{k_2}{c_2} [w_2 - \frac{(c_2 + k_2 \beta)(w_3 c_2 + \beta k_2 w_2)}{c_2 c_3}]$.

In this case,

$$\frac{1}{t} \ln \frac{z(t)}{z_0} \geq w_3 + \beta \frac{1}{t} \int_0^t y(s) \, ds - \frac{c_3}{k_2} \frac{1}{t} \int_0^t y(s) \, ds - \frac{c_3}{k_2} \frac{1}{t} \int_0^t z(s) \, ds + \frac{\sigma_3 B_3(t)}{t} + \frac{M_3(t)}{t}.$$

For sufficiently large t, we have

$$\frac{1}{t} \ln \frac{z(t)}{z_0} \geq \left[w_3 + \beta Q - \frac{c_3}{k_2} \frac{w_2 k_2}{c_2} \right] - \frac{c_3}{k_2} \frac{1}{t} \int_0^t z(s) \, ds + \frac{\sigma_3 B_3(t)}{t} + \frac{M_3(t)}{t}.$$

If $w_3 + \beta Q > \frac{c_3 w_2}{c_2}$, then by Lemma 2, we get

$$\liminf_{t \to \infty} \frac{1}{t} \int_0^t z(s) \, ds \geq \frac{[w_3 + \beta Q - \frac{c_3 w_2}{c_2}] k_2}{c_3}, \quad \text{a.s.}$$

(iv) According to (11), we have

$$\frac{1}{t}\ln\frac{x(t)}{x_0} \le w_1 - b_1\frac{1}{t}\int_0^t x(s)\,ds + \frac{\sigma_1 B_1(t)}{t} + \frac{M_1(t)}{t}.$$

Making use of Lemma 2, (9) and (10), we have

$$\limsup_{t\to\infty}\frac{1}{t}\int_0^t x(s)\,ds \le \frac{w_1}{b_1}, \quad \text{a.s.} \tag{16}$$

and

$$\frac{1}{t}\ln\frac{y(t)}{y_0} \le w_2 + \frac{\sigma_2 B_2(t)}{t} + \frac{M_2(t)}{t}.$$

Because of $w_2 < 0$ and (9), (10), we get $\limsup_{t\to\infty}\frac{\ln y(t)}{t} < 0$, a.s., namely

$$\lim_{t\to\infty} y(t) = 0, \quad \text{a.s.} \tag{17}$$

Then

$$\frac{1}{t}\ln\frac{z(t)}{z_0} \le w_3 + \beta\frac{1}{t}\int_0^t y(s)\,ds + \frac{\sigma_3 B_3(t)}{t} + \frac{M_3(t)}{t}.$$

Considering (9), (10), (17) and $w_3 < 0$, we have $\limsup_{t\to\infty}\frac{\ln z(t)}{t} < 0$; therefore,

$$\lim_{t\to\infty} z(t) = 0, \quad \text{a.s.} \tag{18}$$

At this time, for sufficiently large t,

$$\frac{1}{t}\ln\frac{x(t)}{x_0} \ge w_1 - b_1\frac{1}{t}\int_0^t x(s)\,ds - \frac{c_1}{k_1}\frac{1}{t}\int_0^t y(s)\,ds - \frac{\tilde{p}c_1}{k_1}\frac{1}{t}\int_0^t z(s)\,ds + \frac{\sigma_1 B_1(t)}{t} + \frac{M_1(t)}{t}.$$

By (17), (18) and Lemma 2,

$$\liminf_{t\to\infty}\frac{1}{t}\int_0^t x(s)\,ds \ge \frac{w_1}{b_1}, \quad \text{a.s.} \tag{19}$$

Combining (19) with (16), we have $\lim_{t\to\infty}\frac{1}{t}\int_0^t x(s)\,ds = \frac{w_1}{b_1}$, a.s.

(v) It is easy to have $\limsup_{t\to\infty}\frac{1}{t}\int_0^t x(s)\,ds \le \frac{w_1}{b_1}$, a.s. when $w_1 > 0$ and

$$\frac{1}{t}\ln\frac{y(t)}{y_0} \ge w_2 - \frac{c_2}{k_2}\frac{1}{t}\int_0^t y(s)\,ds - \left(\frac{c_2}{k_2} + \beta\right)\frac{1}{t}\int_0^t z(s)\,ds + \frac{\sigma_2 B_2(t)}{t} + \frac{M_2(t)}{t},$$

$$\frac{1}{t}\ln\frac{z(t)}{z_0} \ge w_3 - \frac{c_3}{k_2}\frac{1}{t}\int_0^t y(s)\,ds - \frac{c_3}{k_2}\frac{1}{t}\int_0^t z(s)\,ds + \frac{\sigma_3 B_3(t)}{t} + \frac{M_3(t)}{t}.$$

Then

$$\frac{1}{t}\ln\frac{y(t)}{y_0} + \frac{1}{t}\ln\frac{z(t)}{z_0} \ge (w_2 + w_3) - \frac{c_2 + c_3}{k_2}\frac{1}{t}\int_0^t y(s)\,ds - \frac{c_2 + c_3 + k_2\beta}{k_2}\frac{1}{t}\int_0^t z(s)\,ds$$

$$+ \sum_{i=2}^3\left[\frac{\sigma_i B_i(t)}{t} + \frac{M_i(t)}{t}\right],$$

namely

$$
\frac{1}{t} \ln\big(y(t)z(t)\big) - \frac{1}{t} \ln(y_0 z_0) \geq (w_2 + w_3) - \frac{c_2 + c_3 + k_2 \beta}{k_2} \frac{1}{t} \int_0^t \big[y(s) + z(s)\big] ds
$$
$$
+ \sum_{i=2}^3 \left[\frac{\sigma_i B_i(t)}{t} + \frac{M_i(t)}{t} \right].
$$

By virtue of the formula $(a + b)^2 \geq 2ab$ for $a, b \in R_+$, we can get $2\ln(a + b) \geq \ln 2 + \ln(ab)$, hence

$$
\frac{2 \ln(y(t) + z(t))}{t} - \frac{\ln 2}{t} - \frac{\ln(y_0 z_0)}{t} \geq (w_2 + w_3) - \frac{(c_2 + c_3 + k_2 \beta)}{k_2} \frac{1}{t} \int_0^t \big[y(s) + z(s)\big] ds
$$
$$
+ \sum_{i=2}^3 \left[\frac{\sigma_i B_i(t)}{t} + \frac{M_i(t)}{t} \right],
$$

then

$$
\frac{\ln(y(t) + z(t))}{t} - \frac{\ln 2}{2t} - \frac{\ln(y_0 z_0)}{2t} \geq \frac{(w_2 + w_3)}{2} - \frac{(c_2 + c_3 + k_2 \beta)}{2k_2} \frac{1}{t} \int_0^t \big[y(s) + z(s)\big] ds
$$
$$
+ \frac{1}{2} \sum_{i=2}^3 \left[\frac{\sigma_i B_i(t)}{t} + \frac{M_i(t)}{t} \right].
$$

By Lemma 2, we get

$$
\liminf_{t \to \infty} \frac{1}{t} \int_0^t \big[y(s) + z(s)\big] ds \geq \frac{(w_2 + w_3)k_2}{c_2 + c_3 + k_2 \beta}, \quad \text{a.s.}
$$

In this case, considering $0 < \tilde{p} < 1$, we have

$$
\frac{1}{t} \ln \frac{x(t)}{x_0} \geq w_1 - b_1 \frac{1}{t} \int_0^t x(s) ds - \frac{c_1}{k_1} \frac{1}{t} \int_0^t \big[y(s) + z(s)\big] ds + \frac{\sigma_1 B_1(t)}{t} + \frac{M_1(t)}{t}.
$$

Taking the inferior limit, by virtue of Lemma 1 and (9), (10), we get

$$
0 \geq \limsup_{t \to \infty} \frac{1}{t} \ln \frac{x(t)}{x_0} \geq \liminf_{t \to \infty} \frac{1}{t} \ln \frac{x(t)}{x_0}
$$
$$
\geq w_1 - b_1 \liminf_{t \to \infty} \left[\frac{1}{t} \int_0^t x(s) ds \right] - \frac{c_1}{k_1} \liminf_{t \to \infty} \left[\frac{1}{t} \int_0^t \big[y(s) + z(s)\big] ds \right]
$$
$$
+ \liminf_{t \to \infty} \left[\frac{\sigma_1 B_1(t)}{t} + \frac{M_1(t)}{t} \right].
$$

Hence,

$$
\liminf_{t \to \infty} \left[\frac{1}{t} \int_0^t x(s) ds \right] \geq \frac{w_1}{b_1} - \frac{(w_2 + w_3)k_2 c_1}{b_1 k_1 (c_2 + c_3 + k_2 \beta)}, \quad \text{a.s.} \qquad \square
$$

4 Stochastically ultimate boundedness

From the biological point of view, the nonexplosion and positiveness property of solution in a population dynamical system is not enough. In this section we will show that the pth moment of solution (3) is bounded.

Definition 2 The solution $X(t)$ of system (3) is called stochastically ultimate bounded if, for any $\epsilon \in (0, 1)$, there is a constant $H := H(\epsilon)$ such that, for any $X(0) \in R_+^3$,

$$\limsup_{t \to \infty} P\{|X(t)| \leq H\} \geq 1 - \epsilon.$$

Theorem 3 *Assume* $(x(t), y(t), z(t))^T$ *to be a solution to system* (3) *with the initial value* $(x_0, y_0, z_0) \in R_+^3$. *If* $k_2 > \frac{(1+a_1)^2}{4b_1}$ *for all* $p > 0$, *there exist constants* $\bar{K}_i(p)$ $(i = 1, 2, 3)$ *such that*

$$\limsup_{t \to \infty} E[x^p(t)] \leq \bar{K}_1(p), \qquad \limsup_{t \to \infty} E[y^p(t)] \leq \bar{K}_2(p), \qquad \limsup_{t \to \infty} E[z^p(t)] \leq \bar{K}_3(p).$$

Proof Making use of Itô's formula and then taking expectations yield

$$d(e^t x^p(t)) = e^t x^p(t)\left\{1 + p\left[a_1 - b_1 x - \frac{c_1 y}{x + k_1} - \frac{\tilde{p}c_1 z}{x + k_1}\right] + \frac{p(p-1)}{2}\sigma_1^2 \right.$$
$$+ \int_{\mathbb{Y}} \left[(1 + \gamma_1(u))^p - 1 - p\gamma_1(u)\right]\lambda(du)\bigg\} dt + p\sigma_1 e^t x^p(t)\, dB_1(t)$$
$$+ \int_{\mathbb{Y}} e^t x^p(t)\left[(1 + \gamma_1(u))^p - 1\right]\tilde{N}(dt, du),$$

$$E(e^t x^p(t)) = x^p(0) + pE\int_0^t e^s x^p(s)\left\{\frac{1}{p} + a_1 + \frac{(p-1)}{2}\sigma_1^2 \right.$$
$$+ \frac{1}{p}\int_{\mathbb{Y}}\left[(1 + \gamma_1(u))^p - 1 - p\gamma_1(u)\right]\lambda(du)$$
$$- b_1 x(s) - \frac{c_1 y(s)}{x(s) + k_1} - \frac{\tilde{p}c_1 z(s)}{x(s) + k_1}\bigg\} ds$$
$$\leq x^p(0) + pE\int_0^t e^s x^p(s)\left\{\frac{1}{p} + a_1 + \frac{(p-1)}{2}\sigma_1^2 \right.$$
$$+ \frac{1}{p}\int_{\mathbb{Y}}\left[(1 + \gamma_1(u))^p - 1 - p\gamma_1(u)\right]\lambda(du) - b_1 x(s)\bigg\} ds.$$

Note for a function $f(v) = v^p(a - bv)$, $v \in [0, +\infty)$, $p > 0$, $a > 0$, $b > 0$, $f(v)$ has a maximum value at $\frac{ap}{(p+1)b}$, namely

$$f(v) \leq f\left(\frac{ap}{(p+1)b}\right) = \frac{a^{p+1}p^p}{(p+1)^{p+1}b^p}. \tag{20}$$

Therefore,

$$E(e^t x^p(t)) \leq x^p(0) + E\int_0^t e^s \bar{K}_1(p)\, ds \leq x^p(0) + \bar{K}_1(p)(e^t - 1),$$

where

$$\bar{K}_1(p) = \frac{[1 + a_1 p + \frac{p(p-1)}{2}\sigma_1^2 + \int_{\mathbb{Y}}[(1 + \gamma_1(u))^p - 1 - p\gamma_1(u)]\lambda(du)]^{p+1}}{(p+1)^{p+1}b_1^p}.$$

Let $t \to +\infty$, we have

$$\limsup_{t \to \infty} E[x^p(t)] \leq \bar{K}_1(p), \quad \text{a.s.} \tag{21}$$

Similarly, we can compute

$$
\begin{aligned}
d\left(e^t y^p(t)\right) = e^t y^p(t) &\left\{ 1 + p\left[a_2 - \frac{c_2(y+z)}{x+k_2} - \beta z\right] + \frac{p(p-1)}{2}\sigma_2^2 \right. \\
&\left. + \int_{\mathbb{Y}} \left[(1 + \gamma_2(u))^p - p\gamma_2(u) - 1\right]\lambda(du) \right\} dt + p\sigma_2 e^t y^p(t)\, dB_2(t) \\
&+ \int_{\mathbb{Y}} e^t y^p(t)\left[(1 + \gamma_2(u))^p - 1\right]\tilde{N}(dt, du).
\end{aligned}
$$

Thus, taking expectations leads to

$$
\begin{aligned}
E\left(e^t y^p(t)\right) = y^p(0) + E\int_0^t e^s y^p(s) &\left[1 + pa_2 + \frac{p(p-1)}{2}\sigma_2^2 \right. \\
&+ \int_{\mathbb{Y}} \left[(1 + \gamma_2(u))^p - p\gamma_2(u) - 1\right]\lambda(du) \\
&\left. - \frac{c_2 p(y(s) + z(s))}{x(s) + k_2} - p\beta z(s) \right] ds \\
\le y^p(0) + E\int_0^t e^s y^p(s) &\left[1 + pa_2 + \frac{p(p-1)}{2}\sigma_2^2 \right. \\
&+ \int_{\mathbb{Y}} \left[(1 + \gamma_2(u))^p - p\gamma_2(u) - 1\right]\lambda(du) \\
&\left. - \frac{c_2 p y(s)}{k_2} + \frac{c_2 p x(s) y(s)}{k_2(x(s) + k_2)} \right] ds.
\end{aligned}
$$

By virtue of (21), for arbitrary small $\varepsilon_0 > 0$ ($\varepsilon_0 < k_2 - \bar{K}_1(1)$), there exists a positive constant T_0 satisfying

$$
E\left(x^p(t)\right) \le \bar{K}_1(p) + \varepsilon_0
$$

for $t > T_0$. Therefore,

$$
\begin{aligned}
E\left(e^t y^p(t)\right) &\le y^p(0) + \int_0^t e^s E\left\{ y^p(s)\left[1 + pa_2 + \frac{p(p-1)}{2}\sigma_2^2 + \int_{\mathbb{Y}} \left[(1 + \gamma_2(u))^p \right. \right. \right. \\
&\qquad \left. \left. \left. - p\gamma_2(u) - 1\right]\lambda(du) - \frac{c_2 p y(s)}{k_2} \right] \right\} ds + \frac{c_2 p}{k_2^2}\int_0^t e^s E\left(x(s)\right)E\left(y^{p+1}(s)\right) ds \\
&\le y^p(0) + \int_0^t e^s E\left\{ y^p(s)\left[1 + pa_2 + \frac{p(p-1)}{2}\sigma_2^2 + \int_{\mathbb{Y}} \left[(1 + \gamma_2(u))^p \right. \right. \right. \\
&\qquad \left. \left. \left. - p\gamma_2(u) - 1\right]\lambda(du) - \frac{c_2 p y(s)}{k_2} \right] \right\} ds + \frac{c_2 p}{k_2^2}\left(\bar{K}_1(1) + \varepsilon_0\right)\int_0^t e^s E\left(y^{p+1}(s)\right) ds \\
&\le y^p(0) + \int_0^t e^s E\left\{ y^p(s)\left[1 + pa_2 + \frac{p(p-1)}{2}\sigma_2^2 + \int_{\mathbb{Y}} \left[(1 + \gamma_2(u))^p \right. \right. \right. \\
&\qquad \left. \left. \left. - p\gamma_2(u) - 1\right]\lambda(du) - \frac{c_2 p y(s)}{k_2} + \frac{c_2 p}{k_2^2}\left(\bar{K}_1(1) + \varepsilon_0\right)y(s) \right] \right\} ds.
\end{aligned}
$$

If $k_2 > \bar{K}_1(1)$, then $k_2 > \bar{K}_1(1) + \varepsilon_0$. According to (20), the following result is available:

$$E\big(e^t y^p(t)\big) \le y^p(0) + \int_0^t e^s K_2(p)\, ds = y^p(0) + K_2(p)\big(e^t - 1\big).$$

Here

$$K_2(p) = \frac{[1 + pa_2 + \frac{p(p-1)}{2}\sigma_2^2 + \int_{\mathbb{Y}}[(1+\gamma_2(u))^p - p\gamma_2(u) - 1]\lambda(du)]^{p+1}}{(p+1)^{p+1}}$$

$$\cdot \frac{k_2^{2p}}{c_2^p(k_2 - \bar{K}_1(1) - \varepsilon_0)^p},$$

where $\bar{K}_1(1) = \frac{(1+a_1)^2}{4b_1}$. Similarly, $\limsup_{t\to\infty} E[y^p(t)] \le K_2(p)$, here because of the arbitrariness of ε_0, define

$$\bar{K}_2(p) = \frac{[1 + pa_2 + \frac{p(p-1)}{2}\sigma_2^2 + \int_{\mathbb{Y}}[(1+\gamma_2(u))^p - p\gamma_2(u) - 1]\lambda(du)]^{p+1}}{(p+1)^{p+1}} \cdot \frac{k_2^{2p}}{c_2^p(k_2 - \bar{K}_1(1))^p},$$

thus,

$$\limsup_{t\to\infty} E\big[y^p(t)\big] \le \bar{K}_2(p). \tag{22}$$

And

$$d\big(e^t z^p(t)\big) = e^t z^p(t)\left\{ 1 + p\left[\beta y + a_3 - \frac{c_3(y+z)}{x+k_2} \right] + \frac{p(p-1)}{2}\sigma_3^2 \right.$$

$$\left. + \int_{\mathbb{Y}}\big[(1+\gamma_3(u))^p - p\gamma_3(u) - 1\big]\lambda(du) \right\} dt + p\sigma_3 e^t z^p(t)\, dB_3(t)$$

$$+ \int_{\mathbb{Y}} e^t z^p(t)\big[(1+\gamma_3(u))^p - 1\big]\tilde{N}(dt, du),$$

$$E\big(e^t z^p(t)\big) = z^p(0) + E\int_0^t e^s z^p(s)\left[1 + pa_3 + \frac{p(p-1)}{2}\sigma_3^2 \right.$$

$$+ \int_{\mathbb{Y}}\big[(1+\gamma_3(u))^p - p\gamma_3(u) - 1\big]\lambda(du)$$

$$\left. + p\beta y(s) - \frac{c_3 p(y(s) + z(s))}{x(s) + k_2} \right] ds$$

$$\le z^p(0) + E\int_0^t e^s z^p(s)\left[1 + pa_3 + \frac{p(p-1)}{2}\sigma_3^2 \right.$$

$$+ \int_{\mathbb{Y}}\big[(1+\gamma_3(u))^p - p\gamma_3(u) - 1\big]\lambda(du)$$

$$\left. + p\beta y(s) - \frac{c_3 pz(s)}{x(s) + k_2} \right] ds$$

$$\le z^p(0) + E\int_0^t p\beta e^s z^p(s) y(s)\, ds + E\int_0^t e^s z^p(s)\left[1 + pa_3 + \frac{p(p-1)}{2}\sigma_3^2 \right.$$

$$\left. + \int_{\mathbb{Y}}\big[(1+\gamma_3(u))^p - p\gamma_3(u) - 1\big]\lambda(du) - \frac{c_3 pz(s)}{k_2} + \frac{c_3 px(s)z(s)}{k_2(x(s) + k_2)} \right] ds.$$

On the basis of Young's inequality, we have

$$
y(t)z^p(t) \leq \frac{y^{p+1}(t)}{p+1} + \frac{p}{p+1}\left(z^p(t)\right)^{\frac{p+1}{p}}
$$

$$
\leq \frac{y^{p+1}(t)}{p+1} + \frac{p}{p+1}z^{p+1}(t).
$$

Notice $\frac{p}{p+1} < 1$, we have $y(t)z^p(t) \leq \frac{y^{p+1}(t)}{p+1} + z^{p+1}(t)$, then

$$
E\left(e^t z^p(t)\right) \leq z^p(0) + p\beta E \int_0^t e^s z^{p+1}(s)\,ds + p\beta E \int_0^t e^s \frac{1}{p+1}y^{p+1}(s)\,ds
$$

$$
+ E\int_0^t e^s z^p(s)\left[1 + pa_3 + \frac{p(p-1)}{2}\sigma_3^2\right.
$$

$$
+ \int_{\mathbb{Y}}\left[\left(1 + \gamma_3(u)\right)^p - p\gamma_3(u) - 1\right]\lambda(du) - \frac{c_3 pz(s)}{k_2} + \left.\frac{c_3 px(s)z(s)}{k_2^2}\right]ds
$$

$$
\leq z^p(0) + \frac{p\beta}{p+1}E \int_0^t e^s y^{p+1}(s)\,ds + E\int_0^t e^s z^p(s)\left[1 + pa_3 + \frac{p(p-1)}{2}\sigma_3^2\right.
$$

$$
+ \int_{\mathbb{Y}}\left[\left(1 + \gamma_3(u)\right)^p - p\gamma_3(u) - 1\right]\lambda(du) - \left.\left(\frac{c_3 p}{k_2} - p\beta\right)z(s)\right]ds
$$

$$
+ \frac{c_3 p}{k_2^2}\int_0^t e^s E(x(s))E\left(z^{p+1}(s)\right)ds.
$$

Considering (22), we can see that there exist positive constants $\vec{K}_2(p)$ and $\vec{T}_2$ $(\vec{T}_2 > T_0)$, for $t > \vec{T}_2$, one can obtain that $E[y^p(t)] \leq \vec{K}_2(p)$. Then, for $t > \vec{T}_2$,

$$
E\left(e^t z^p(t)\right) \leq z^p(0) + \frac{p\beta}{p+1}\vec{K}_2(p+1)\left[e^t - 1\right] + \int_0^t e^s E\left\{z^p(s)\left[1 + pa_3 + \frac{p(p-1)}{2}\sigma_3^2\right.\right.
$$

$$
+ \int_{\mathbb{Y}}\left[\left(1 + \gamma_3(u)\right)^p - p\gamma_3(u) - 1\right]\lambda(du)
$$

$$
\left.\left.- \left(\frac{c_3 p}{k_2} - p\beta - \frac{c_3 p}{k_2^2}\left(\bar{K}_1(1) + \varepsilon_0\right)\right)z(s)\right]\right\}ds
$$

$$
\leq z^p(0) + \frac{p\beta}{p+1}\vec{K}_2(p+1)\left[e^t - 1\right] + \int_0^t e^s K_3(p)\,ds
$$

$$
\leq z^p(0) + \frac{p\beta}{p+1}\vec{K}_2(p+1)\left[e^t - 1\right] + K_3(p)\left[e^t - 1\right],
$$

where

$$
K_3(p) = \frac{\left[1 + pa_3 + \frac{p(p-1)}{2}\sigma_3^2 + \int_{\mathbb{Y}}\left[(1 + \gamma_3(u))^p - p\gamma_3(u) - 1\right]\lambda(du)\right]^{p+1}}{(p+1)^{p+1}}
$$

$$
\cdot \frac{p^p}{\left[\frac{c_3 p}{k_2} - p\beta - \frac{c_3 p}{k_2^2}(\bar{K}_1(1) + \varepsilon_0)\right]^p}.
$$

Take the superior limit results in

$$
\limsup_{t \to \infty} E\left[z^p(t)\right] \leq \frac{p\beta}{p+1}\vec{K}_2(p+1) + K_3(p).
$$

Notice the arbitrariness of ε_0, let

$$
\begin{aligned}
\bar{K}_3(p) = {} & \frac{p\beta}{p+1}\bar{K}_2(p+1) + \frac{[1 + pa_3 + \frac{p(p-1)}{2}\sigma_3^2 + \int_{\mathbb{Y}}[(1+\gamma_3(u))^p - p\gamma_3(u) - 1]\lambda(du)]^{p+1}}{(p+1)^{p+1}} \\
& \cdot \frac{p^p}{[\frac{c_3 p}{k_2} - p\beta - \frac{c_3 p}{k_2^2}\bar{K}_1(1)]^p},
\end{aligned}
$$

clearly,

$$
\limsup_{t\to\infty} E\big[z^p(t)\big] \le \bar{K}_3(p),
$$

thus the proof of Theorem 3 is completed. □

By virtue of the formula of Lemma 2.1 in [10], we have

$$
n^{(1-\frac{p}{2})\wedge 0}|x|^p \le \sum_{i=1}^n x_i^p \le n^{(1-\frac{p}{2})\vee 0}|x|^p
$$

for $\forall p > 0$, $x \in R_+^n$, where $R_+^n := \{x \in R^n, x_i > 0, 1 \le i \le n\}$.

For solution $X(t) = (x(t), y(t), z(t))^T \in R_+^3$ and $p > 0$, we have

$$
3^{(1-\frac{p}{2})\wedge 0}\big|X(t)\big|^p \le x^p(t) + y^p(t) + z^p(t).
$$

By Theorem 3,

$$
\limsup_{t\to\infty} E\big[X(t)\big]^p \le \left(\frac{1}{3}\right)^{(1-\frac{p}{2})\wedge 0} \limsup_{t\to\infty} E\big[x^p(t) + y^p(t) + z^p(t)\big] \le \left(\frac{1}{3}\right)^{(1-\frac{p}{2})\wedge 0} \sum_{i=1}^3 \bar{K}_i(p).
$$

As an application of Theorem 3, together with the Chebyshev inequality, we can also establish the following corollary.

Corollary 1 *Under conditions of Theorem* 3, *the solution of system* (3) *is stochastically ultimate bounded.*

5 Numerical simulation

In this section, we give some examples and numerical simulations to illustrate our analytical findings. These numerical simulations are given by the Euler scheme of [20].

For system (3), we choose the initial value $(x_0, y_0, z_0) = (0.2, 0.1, 0.2)$, and

$$
\begin{aligned}
& b_1 = 0.3, && c_1 = 0.4, && \tilde{p} = 0.1, && c_2 = 0.6, && k_1 = 0.5, \\
& k_2 = 0.5, && \beta = 0.4, && c_3 = 0.5, && \sigma_i = 0.5 \quad (i = 1, 2, 3),
\end{aligned}
\tag{23}
$$

$\mathbb{Y} = (0, +\infty)$, $\lambda(\mathbb{Y}) = 1$. In the following, we change the values of part parameters to observe the asymptotic behavior of solution for system (3).

Figure 1 Solutions of system (3) for $a_1 = 0.02$, $a_2 = 0.03$, $a_3 = 0.02$, $\gamma_1(u) = \gamma_3(u) = 0.2$, $\gamma_2(u) = 0.4$, the other parameters are the same as in (23), with step size $\Delta t = 0.1 > 0$
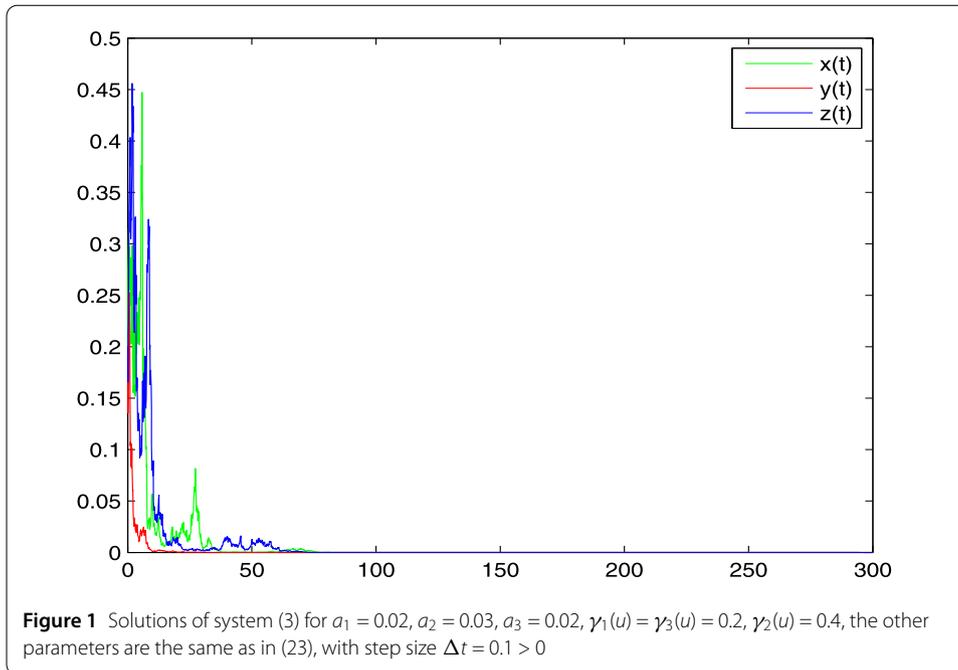
(1) In Figure 1, let $a_1 = 0.02$, $a_2 = 0.03$, $a_3 = 0.02$, $\gamma_1(u) = \gamma_3(u) = 0.2$, $\gamma_2(u) = 0.4$, the other parameters are the same as above, then

$$w_1 = a_1 - \frac{\sigma_1^2}{2} - \int_{\mathbb{Y}} \left[\gamma_1(u) - \ln\left(1 + \gamma_1(u)\right) \right] \lambda(du) = -0.1227 < 0,$$

$$w_2 = a_2 - \frac{\sigma_2^2}{2} - \int_{\mathbb{Y}} \left[\gamma_2(u) - \ln\left(1 + \gamma_2(u)\right) \right] \lambda(du) = -0.1585 < 0,$$

$$w_3 = a_3 - \frac{\sigma_3^2}{2} - \int_{\mathbb{Y}} \left[\gamma_3(u) - \ln\left(1 + \gamma_3(u)\right) \right] \lambda(du) = -0.1227 < 0.$$

Thus, in accordance with case (i) in Theorem 2, all the species will go to extinction, Figure 1 confirms it.

(2) In Figure 2, let $a_1 = 0.02$, $a_2 = 0.03$, $a_3 = 0.5$, $\gamma_1(u) = \gamma_2(u) = 0.2$, $\gamma_3(u) = 0.4$, the other parameters remain the same as in (23), then

$$w_1 = -0.1227 < 0, \qquad w_2 = -0.1127 < 0, \qquad w_3 = 0.3115 > 0.$$

By the case (ii) of Theorem 2, the prey population and the susceptible predator population will be extinct, the infected predator population will be stable in the mean. Figure 2 shows it.

(3) In Figure 3, we assume $a_1 = 0.06$, $a_2 = 0.6$, $a_3 = 0.2$, $\sigma_3 = 0.8$, $\beta = k_2 = 0.2$, $\gamma_1(u) = 0.4$, $\gamma_2(u) = \gamma_3(u) = 0.2$, the other parameters are the same as in (23), at this time

$$w_1 = -0.1285 < 0, \qquad w_2 = 0.4573 > 0,$$

$$w_3 = -0.1377 < 0, \qquad w_3 + \frac{\beta k_2}{c_2} w_2 = -0.1072 < 0.$$

Figure 2 Solutions of system (3) for $a_1 = 0.02$, $a_2 = 0.03$, $a_3 = 0.5$, $\gamma_1(u) = \gamma_2(u) = 0.2$, $\gamma_3(u) = 0.4$, the other parameters remain the same as in (23), with step size $\Delta t = 0.1 > 0$

Figure 3 Solutions of system (3) for $a_1 = 0.06$, $a_2 = 0.6$, $a_3 = 0.2$, $\sigma_3 = 0.8$, $\beta = k_2 = 0.2$, $\gamma_1(u) = 0.4$, $\gamma_2(u) = \gamma_3(u) = 0.2$, the other parameters are the same as in (23), with step size $\Delta t = 0.1 > 0$
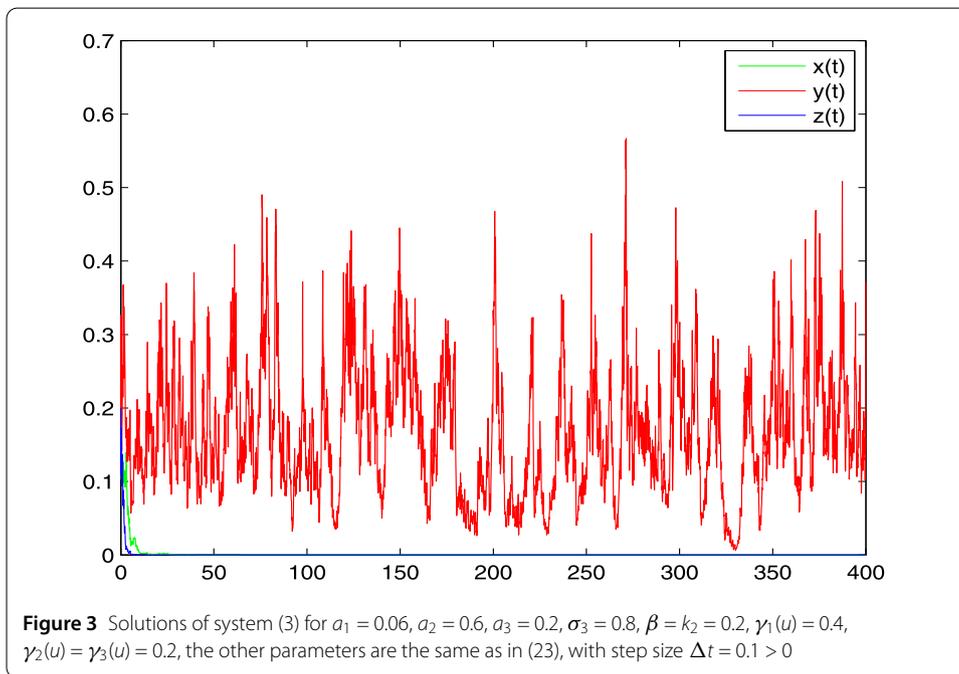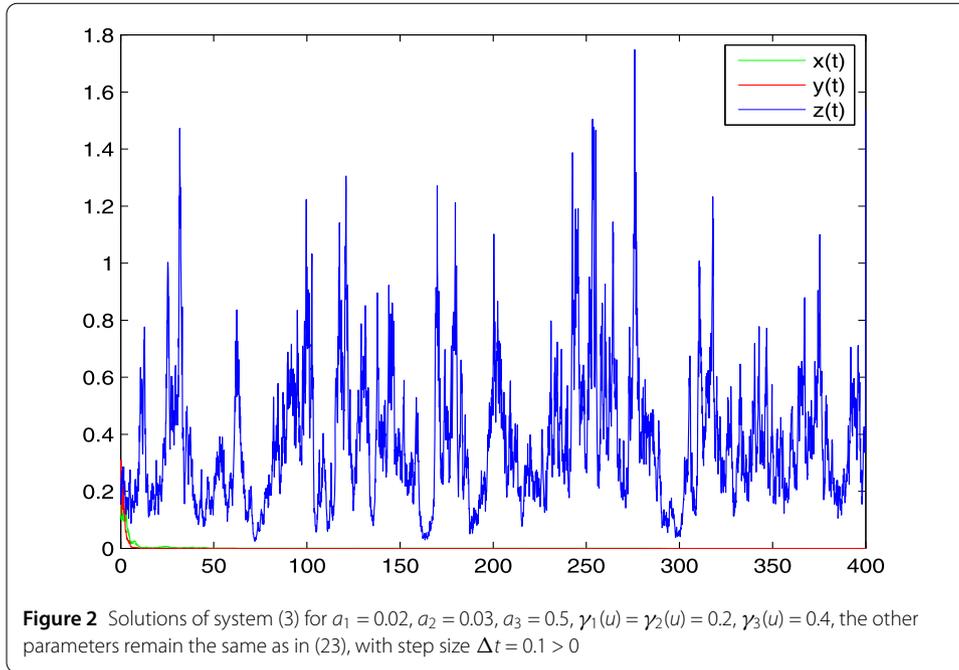
By the condition of (iii)(a), the prey population $x(t)$ and the infected population $z(t)$ will go to extinction, the susceptible predator population $y(t)$ will be stable in the mean.

(4) In Figure 4, we choose $a_1 = 0.3$, $a_2 = 0.03$, $a_3 = 0.02$, $\gamma_1(u) = 0.2$, $\gamma_2(u) = \gamma_3(u) = 0.4$, the other parameters are the same as in (23), then

$$w_1 = 0.1573 > 0, \qquad w_2 = -0.1585 < 0, \qquad w_3 = -0.1685 < 0.$$

Figure 4 Solutions of system (3) for $a_1 = 0.3$, $a_2 = 0.03$, $a_3 = 0.02$, $\gamma_1(u) = 0.2$, $\gamma_2(u) = \gamma_3(u) = 0.4$, the other parameters are the same as in (23), with step size $\Delta t = 0.1 > 0$

Figure 5 Solutions of system (3) for $a_1 = 0.75$, $a_2 = 0.7$, $a_3 = 0.3$, $c_2 = 0.3$, $c_3 = 0.2$, $k_2 = 0.6$, $\beta = 0.4$, $\sigma_3 = 0.7$, $\gamma_1(u) = 0.4$, $\gamma_2(u) = \gamma_3(u) = 0.2$, the other parameters are the same as in (23), with step size $\Delta t = 0.1 > 0$
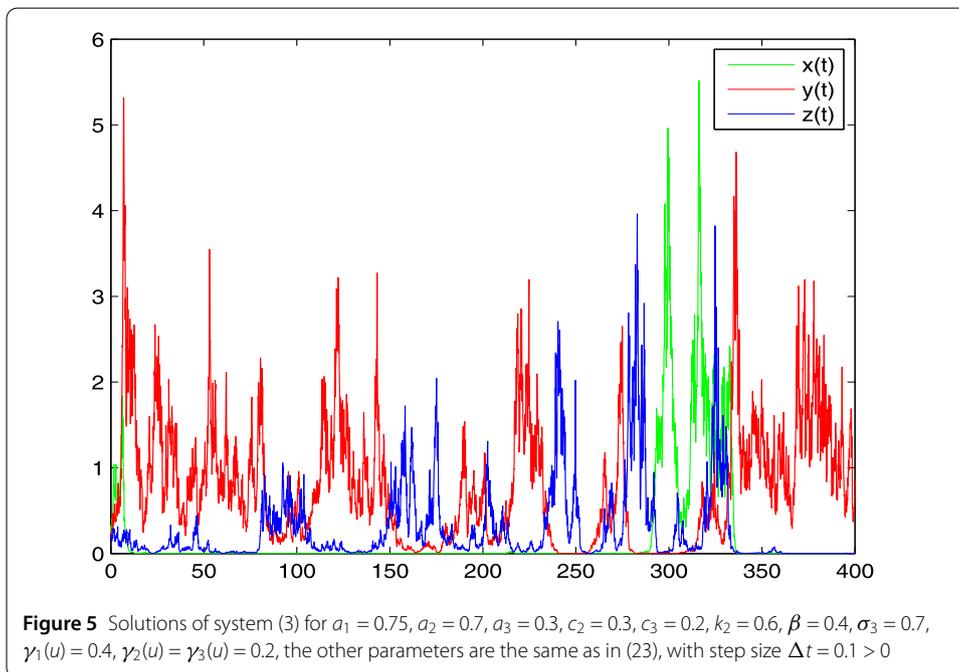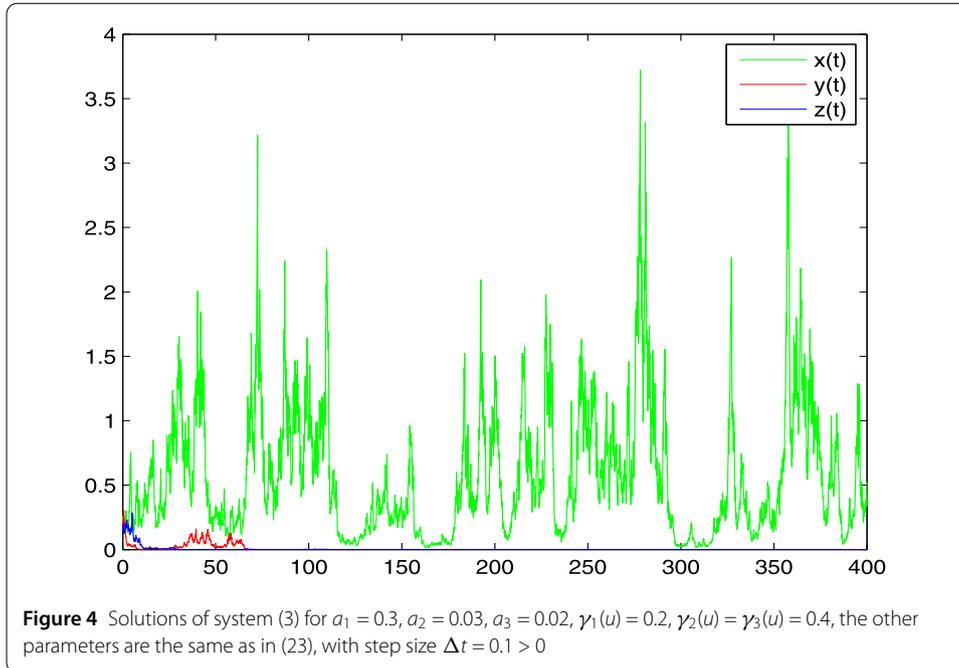
On the basis of (iv) in Theorem 2, the prey population will be stable in the mean, the whole predator population will die out, which is consistent with Figure 4.

(5) In Figure 5, let $a_1 = 0.75$, $a_2 = 0.7$, $a_3 = 0.3$, $c_2 = 0.3$, $c_3 = 0.2$, $k_2 = 0.6$, $\beta = 0.4$, $\sigma_3 = 0.7$, $\gamma_1(u) = 0.4$, $\gamma_2(u) = \gamma_3(u) = 0.2$, the other parameters are the same as in (23), then

$$w_1 = 0.5615 > 0, \qquad w_2 = 0.5573 > 0, \qquad w_3 = 0.0373 > 0,$$

Figure 6 Solutions of deterministic system (2), all the parameters are the same as in Figure 1

Figure 7 Solutions of deterministic system (2), all the parameters are the same as in Figure 4
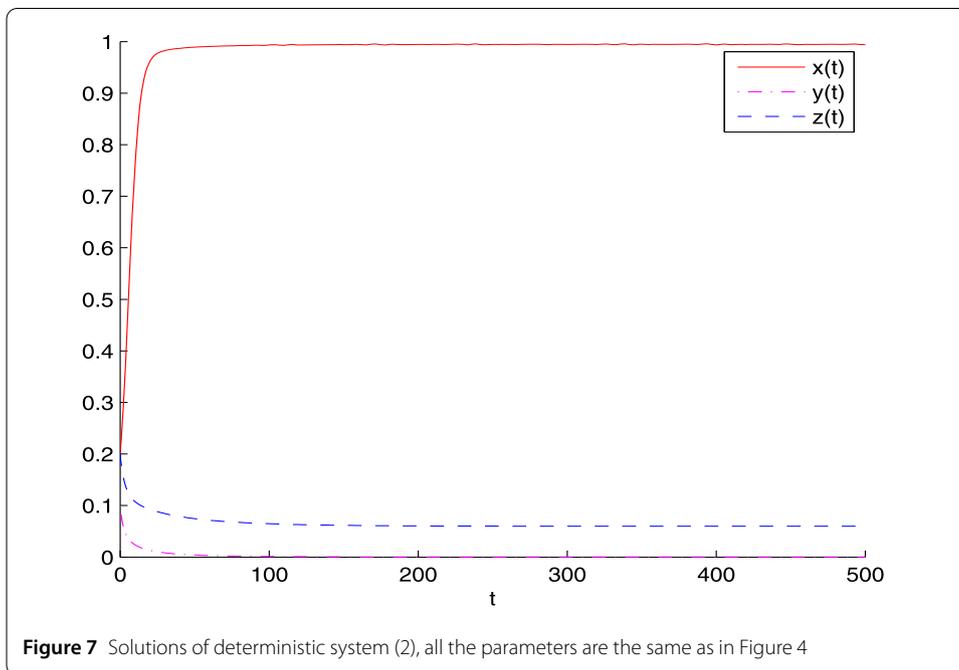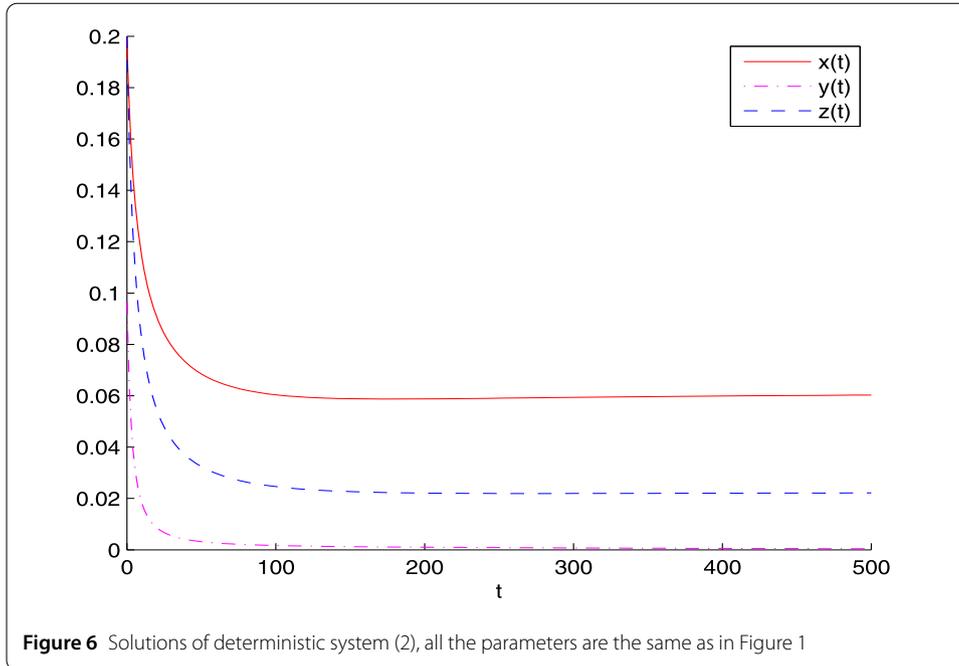
and $\frac{(w_2+w_3)k_2c_1}{(c_2+c_3+k_2\beta)k_1} = 0.3857$, $w_1 > \frac{(w_2+w_3)k_2c_1}{(c_2+c_3+k_2\beta)k_1}$. By virtue of (v) in Theorem 2, all the populations will be strongly persistent in the mean.

In the following, we discuss the solution of deterministic system (2), assume that all the parameters are the same as in Figures 1, 4, 5. Figure 6 shows that the susceptible predator population will be extinct, but the prey and infected predator populations will be stable, which is different from Figure 1.

Figure 7 illustrates that the prey population and the infected predator population will be stable, and the susceptible predator population will go to extinction. Compare Figure 7

Figure 8 Solutions of deterministic system (2), all the parameters are the same as in Figure 5
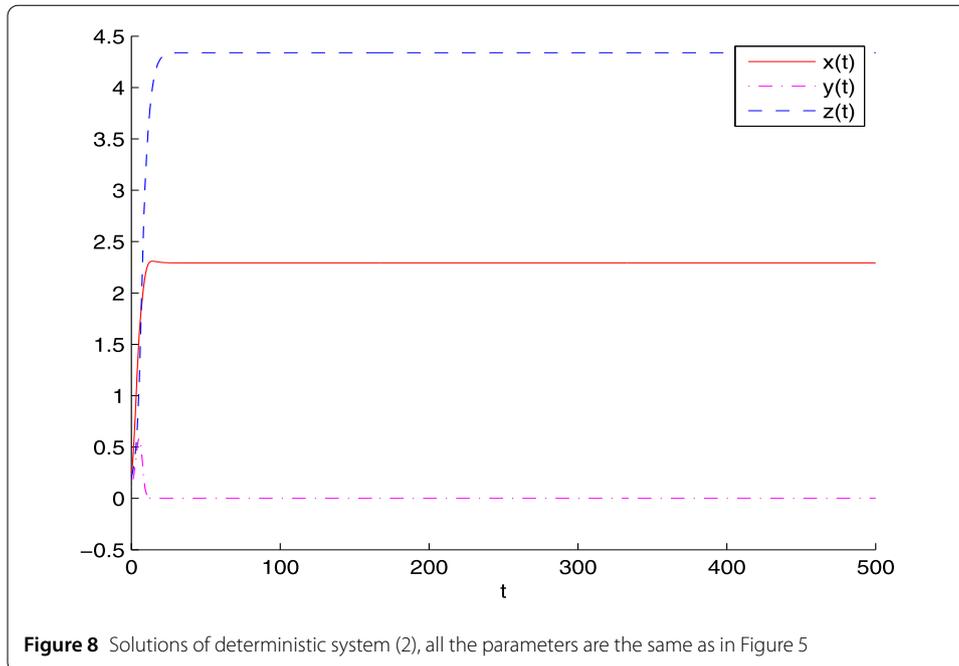
with Figure 4, the behavior of solution for system (2) is distinct from that for stochastic system (3). Figure 8 confirms it again, we give no more explanations.

These results show that, for a stochastic system, not only the growth rate but also the Lévy noise and Gauss white noise play an important role in the persistence of population. Sometimes, the stochastic perturbation of environment can make species extinct.

6 Conclusions

In this paper, we discuss an LG–Holling type II diseased predator ecosystem with Lévy noise and Gauss white noise. We show that the system admits a unique global positive solution; we also investigate persistence in the mean and extinction of all the populations, uniformly finite pth moment with $p > 0$ and stochastic ultimate boundedness. The threshold conditions of extinction or persistence in the mean for prey and predator populations illustrate that w_i is the key value, which means that a_i is advantageous to the population, white noise and Lévy noise will go against the persistence of population. From the numerical simulation, we find that Lévy noise and Gauss white noise have an important effect on persistence for a stochastic system; occasionally, it may be a determinant factor. Thus, we must consider random changes of environment in the mathematical model.

Competing interests
The author declares that no competing interests exist.

Authors' contributions
The author has read and approved the final manuscript.

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References
 1. Leslie, P.H.: Some further notes on the use of matrices in population mathematics. Biometrika **35**, 213–245 (1948)
 2. Leslie, P.H.: A stochastic model for studying the properties of certain biological systems numerical methods. Biometrika **45**, 16–31 (1958)
 3. Pielou, E.C.: Mathematical Ecology. Wiley, New York (1977)
 4. Korobeinikov, A.: A Lyapunov function for Leslie–Gower predator–prey models. Appl. Math. Lett. **14**, 697–699 (2001)
 5. Zhou, X., Cui, J., Shi, X., Song, X.: A modified Leslie–Gower predator–prey model with prey infection. J. Appl. Math. Comput. **33**, 471–487 (2010)
 6. Liu, M., Wang, K.: Dynamics of a Leslie–Gower Holling-type II predator-prey system with Lévy jumps. Nonlinear Anal. **85**, 204–213 (2013)
 7. Flores, J.D., Gonzalez-Olivares, E.: A modified Leslie–Gower predator–prey model with ratio-dependent functional response and alternative food for the predator. Math. Methods Appl. Sci. **40**, 2313–2328 (2017)
 8. Xu, Y., Liu, M., Yang, Y.: Analysis of a stochastic two-predators one-prey system with modified Leslie–Gower and Holling-type II schemes. J. Math. Anal. Appl. **7**, 713–727 (2017)
 9. Sarwardi, S., Haque, M., Venturino, E.: Global stability and persistence in LG–Holling type II diseased predator ecosystems. J. Biol. Phys. **37**, 91–106 (2011)
10. Bao, J., Mao, X., Yin, G., Yuan, C.: Competitive Lotka–Volterra population dynamics with jumps. Nonlinear Anal. **74**, 6601–6616 (2011)
11. Bao, J., Yuan, C.: Stochastic population dynamics driven by Lévy noise. J. Math. Anal. Appl. **391**, 363–375 (2012)
12. Liu, M., Deng, M., Do, B.: Analysis of a stochastic logistic model with diffusion. Appl. Math. Comput. **266**, 169–182 (2015)
13. Liu, Q., Chen, Q.: Asymptotic behavior of a stochastic non-autonomous predator-prey system with jumps. Appl. Math. Comput. **271**, 418–428 (2015)
14. Liu, M., Wang, K.: Stochastic Lotka–Volterra systems with Levy noise. J. Math. Anal. Appl. **410**, 750–763 (2014)
15. Zhao, Y., Yuan, S.: Optimal harvesting policy of a stochastic two-species competitive model with Levy noise in a polluted environment. Physica A **477**, 20–33 (2017)
16. Liu, L., Meng, X., Zhang, T.: Optimal control strategy for an impulsive stochastic competition system with time delays and jumps. Physica A **477**, 99–113 (2017)
17. Liu, Q., Jiang, D., Shi, N., Hayat, T., Alsaedi, A.: Stochastic mutualism model with Levy jumps. Commun. Nonlinear Sci. Numer. Simul. **43**, 78–90 (2017)
18. Peng, S., Zhu, X.: Necessary and sufficient condition for comparison theorem of 1-dimensional stochastic differential equations. Stoch. Process. Appl. **116**, 370–380 (2006)
19. Kunita, H.: Itô's stochastic calculus: its surprising power for applications. Stoch. Process. Appl. **120**, 622–652 (2010)
20. Protter, P., Talay, D.: The Euler scheme for Lévy driven stochastic differential equations. Ann. Probab. **25**, 393–423 (1997)