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Global solutions and uniform boundedness of attractive/repulsive LV competition systems

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Abstract

In this paper, we study global solutions to the following strongly coupled systems:

$$\begin{cases} u_t = \nabla\cdot(D_1\nabla u - \chi u\nabla v) + (a_1 - b_1u - c_1v)u, & x\in\Omega, t>0, \\ 0 = D_2\Delta v + (a_2 - b_2u - c_2v)v, & x\in\Omega, t>0, \end{cases}$$

over $\Omega\subset\mathbb{R}^N$, $N\geq 2$, subject to homogeneous Neumann boundary conditions and nonnegative initial data. Here D_i, a_i, b_i and c_i, $i=1,2$, are positive constant. It is proved that this system admits global and bounded classical solutions for all $\chi>0$. We also prove the global well-posedness for its repulsive counterpart

$$\begin{cases} u_t = \nabla\cdot(D_1\nabla u + \chi u\nabla v) + (a_1 - b_1u - c_1v)u, & x\in\Omega, t>0, \\ 0 = D_2\Delta v + (a_2 - b_2u - c_2v)v, & x\in\Omega, t>0, \end{cases}$$

provided that $b_1 > \frac{a_2b_2\chi(N-2)}{c_2D_2N}$. Our results extend (Discrete Contin. Dyn. Syst. 35:1239–1284, 2015) to higher dimensions and to its repulsive case.

Keywords: Lotka–Volterra competition system; Global solution; Uniform boundedness

1 Introduction

This paper studies the existence and uniform boundedness of global solutions to the following parabolic–elliptic quasilinear system over a bounded domain Ω in dimension $N\geq 2$:

$$\begin{cases} u_t = \nabla\cdot(D_1\nabla u - \chi u\nabla v) + (a_1 - b_1u - c_1v)u, & x\in\Omega, t>0, \\ 0 = D_2\Delta v + (a_2 - b_2u - c_2v)v, & x\in\Omega, t>0, \\ \frac{\partial u}{\partial \mathbf{n}} = \frac{\partial v}{\partial \mathbf{n}} = 0, & x\in\partial\Omega, t>0, \\ u(x,0) = u_0(x)\geq 0, & x\in\Omega, \end{cases} \tag{1.1}$$

where u and v are functions of space x and time t. D_i, a_i, b_i and c_i, $i=1,2$, are positive constants. $\mathbf{n}$ is the unit outer normal to the boundary $\partial\Omega$. (1.1) is the parabolic–elliptic system of advective competition system studied in [1]. Here $u(x,t)$ and $v(x,t)$ represent the population density of the two competing species at space–time location (x,t). In particular when $\chi > 0$, (1.1) describes the spatial–temporal dynamics of u and v such that the former invades the latter to seek competition subject to Lotka–Volterra kinetics.

It is proved in [1] that, when $N = 1$, (1.1) and its fully parabolic counter-part admit global classical solutions which are uniformly bounded, and when $N \geq 2$, (1.1) admit global and uniformly bounded classical solutions provided that $\chi < 0$ and b_1 is sufficiently large. In this current work, we extend the results in [1] on the global existence and uniform boundedness of classical solutions to (1.1) and our first main result states as follows.

Theorem 1.1 *Let $\Omega \subset \mathbb{R}^N, N \geq 2$ be a bounded domain and $\chi > 0$ be an arbitrary positive constant. Assumes that a_i, b_i, c_i and D_i, $i = 1, 2$, are positive constants. Then, for any nonnegative $u_0 \in C^0(\bar{\Omega})$, there exists one couple (u, v) of nonnegative functions which solve* (1.1) *classically in $\Omega \times (0, \infty)$. Moreover, the solutions are uniformly bounded in the following sense*: $0 < v(x,t) < \frac{a_2}{c_2}, \forall (x,t) \in \Omega \times (0,\infty)$ *and* $\|u(\cdot,t)\|_{L^\infty} \leq C, \forall t \in (0,\infty)$, *for some positive constant C.*

Equation (1.1) is very similar to the Keller–Segel type chemotaxis system which models the aggregated movement of cellular organisms towards the region high chemical concentration [2]. However, they have quite different kinetics in light that, in chemotaxis models, it is attraction that supports patterns, while here in (1.1) it is repulsion that supports patterns, as suggested by the analysis in [1]. It is well known that large advection rate usually supports blow-ups in chemotaxis system when there is no cellular growth [3]. On the other hand, the logistic growth tends to inhibit solutions from blowing up in finite or infinite time, however, this may not be sufficient when the diffusion is weak or chemotaxis is strong [4–6]. Theorem 1.1 shows that, for the attractive Lotka–Volterra competition system, the solutions are uniformly bounded and blow up in finite or infinite time cannot occur. It is worthwhile to mention that besides the competition model, the advection, which is referred to as the prey–taxis, has been studied in predator–prey models by various authors. See [7–14].

When the advection rate is positive, we have the following repulsive system:

$$\begin{cases} u_t = \nabla \cdot (D_1 \nabla u + \chi u \nabla v) + (a_1 - b_1 u - c_1 v)u, & x \in \Omega, t > 0, \\ 0 = D_2 \Delta v + (a_2 - b_2 u - c_2 v)v, & x \in \Omega, t > 0, \\ \frac{\partial u}{\partial \mathbf{n}} = \frac{\partial v}{\partial \mathbf{n}} = 0, & x \in \partial\Omega, t > 0, \\ u(x,0) = u_0(x) \geq 0, & x \in \Omega, \end{cases} \tag{1.2}$$

where all the constants are assumed to be positive as in (1.1). In [1], it is proved that when $\frac{b_1 D_2}{b_2 \chi}$ is sufficiently large, (1.2) admits global and bounded classical solutions. In this paper we extend the result [1] to the following theorem.

Theorem 1.2 *Let all the conditions in Theorem* 1.1 *hold. Suppose further that* $b_1 > \frac{a_2 b_2 \chi (N-2)}{c_2 D_2 N}$. *Then* (1.2) *admits global and bounded classical solutions and the statements in Theorem* 1.1 *hold true for* (1.2).

Remark 1 We would like to point out that when $N = 2$, Theorem 1.2 holds for any $b_1 > 0$, and then this fact, together with Theorem 1.1, implies that both (1.1) and (1.2) admit global and uniformly bounded classical solutions regardless of the sign and size of the advection rate χ. We note that fully parabolic system of (1.2) was studied by [15] recently, and global existence and boundedness were established provided that the sensitivity function decays super linearly with respect to v. A bifurcation analysis is performed to establish nontrivial patterns.

2 Preliminaries

Our proof of the global existence of (1.1) and (1.2) starts with the local existence and its extensibility criterion due to the classical theory of Amann [16] (Theorem 3.5). Indeed, it is obvious that (1.1) and (1.2) are strictly parabolic systems, therefore both admit locally classical solutions in $\Omega \times T_{\max}$, where $T_{\max} \in (0,\infty]$ is the so called maximal existence time. Moreover, $T_{\max} = \infty$ if $\|u(\cdot,t)\|_{L^\infty} < C$ for each $t \in (0,T_{\max})$, or $T_{\max} < \infty$ and $\lim_{t\to T_{\max}^-} \|u(\cdot,t)\|_{L^\infty} = \infty$. We first collect some important properties of local solutions (u,v) to (1.1) or (1.2) in $\Omega \times (0,T_{\max})$.

Lemma 2.1 *Suppose that $u_0 \ge 0, \not\equiv 0$ in $\bar{\Omega}$. Let (u,v) be a classical nonnegative solution of* (1.1)/(1.2) *in $\Omega \times (0,T_{\max})$. Then there exists a positive constant C_1 dependent on $\|u_0\|_{L^1}$ such that*

$$\int_\Omega u(x,t)\,dx \le C_1, \quad \forall t \in (0,T_{\max}) \tag{2.1}$$

and

$$0 \le v(x,t) \le \frac{a_2}{c_2}, \quad \forall (x,t) \in \Omega \times (0,T_{\max}). \tag{2.2}$$

Proof First of all, we see that (2.2) is immediate from standard maximum principles for elliptic equations. To verify (2.1), we integrate the u-equation in (1.1) over Ω to get

$$\frac{d}{dt}\int_\Omega u = a_1\int_\Omega u - b_1\int_\Omega u^2 - c_1\int_\Omega uv \le a_1\int_\Omega u - b_1\int_\Omega u^2. \tag{2.3}$$

Since $(a_1+1)\int_\Omega u \le b_1\int_\Omega u^2 + \frac{(a_1+1)^2}{4b_1}|\Omega|$ by Young's inequality, we find that

$$\frac{d}{dt}\int_\Omega u + \int_\Omega u \le \frac{(a_1+1)^2}{4b_1}|\Omega|,$$

then we conclude from Grönwall's lemma that

$$\int_\Omega u \le \max\left\{\int_\Omega u_0, \frac{4b_1}{(a_1+1)^2|\Omega|}\right\},$$

from which (2.1) follows. Here we have not used the sign of χ; hence the arguments carry over for (1.2). □

3 Proof of Theorem 1.1

In order to prove Theorem 1.1, we shall see that it suffices to show the uniform boundedness of $\|u(\cdot,t)\|_{L^\infty}$ for $t \in (0,\infty)$. To this end, we first prove the boundedness of $\|u(\cdot,t)\|_{L^p}$ for each finite p, then we can send p to ∞ by the Moser–Alikakos iteration [17].

Lemma 3.1 *For each $p \in [2, \infty)$, there exists a positive constant C_p which depends on $\|u_0\|_{L^p}$ and the system parameters such that*

$$\int_\Omega u^p(x,t) \le C_p, \quad \forall t \in (0, T_{\max}). \tag{3.1}$$

Proof Testing the *u-equation* in (1.1) by u^{p-1} and then integrating it over Ω by parts, we obtain

$$\begin{aligned}
&\frac{1}{p}\frac{d}{dt}\int_\Omega u^p \\
&\quad = \int_\Omega D_1 u^{p-1}\Delta u - \chi \int_\Omega u^{p-1}\nabla\cdot(u\nabla v) + \int_\Omega (a_1 - b_1 u - c_1 v)u^p \\
&\quad = -(p-1)D_1\int_\Omega u^{p-2}|\nabla u|^2 + \frac{p-1}{p}\chi\int_\Omega \nabla u^p\cdot\nabla v + \int_\Omega (a_1 - b_1 u - c_1 v)u^p \\
&\quad = -\frac{4(p-1)D_1}{p^2}\int_\Omega \bigl|\nabla u^{\frac{p}{2}}\bigr|^2 - \frac{p-1}{p}\chi\int_\Omega u^p\Delta v + \int_\Omega (a_1 - b_1 u - c_1 v)u^p.
\end{aligned} \tag{3.2}$$

From the v-equation in (1.1) we have

$$\Delta v = \frac{b_2}{D_2}uv - \left(\frac{a_2}{D_2} - \frac{c_2}{D_2}v\right)v. \tag{3.3}$$

By substituting (3.3) into (3.2), we derive

$$\begin{aligned}
&\frac{1}{p}\frac{d}{dt}\int_\Omega u^p + \frac{4(p-1)D_1}{p^2}\int_\Omega \bigl|\nabla u^{\frac{p}{2}}\bigr|^2 \\
&\quad = -\frac{p-1}{p}\chi\int_\Omega u^p\left(\frac{b_2}{D_2}uv - \left(\frac{a_2}{D_2} - \frac{c_2}{D_2}v\right)v\right) + \int_\Omega (a_1 - b_1 u - c_1 v)u^p \\
&\quad = -\frac{(p-1)b_2\chi}{pD_2}\int_\Omega vu^{p+1} + \frac{p-1}{p}\chi\int_\Omega\left(\frac{a_2}{D_2} - \frac{c_2}{D_2}v\right)vu^p \\
&\qquad + \int_\Omega (a_1 - c_1 v)u^p - \int_\Omega b_1 u^{p+1} \\
&\quad = -\int_\Omega\left(b_1 + \frac{(p-1)b_2\chi}{pD_2}v\right)u^{p+1} + \int_\Omega\left(a_1 - c_1 v + \frac{p-1}{p}\chi\left(\frac{a_2}{D_2} - \frac{c_2}{D_2}v\right)v\right)u^p \\
&\quad \le -\tilde{b}_1\int_\Omega u^{p+1} + \tilde{a}_1\int_\Omega u^p,
\end{aligned} \tag{3.4}$$

where $\tilde{a}_1$ and $\tilde{b}_1$ are both positive constants. Solving this differential inequality by Grönwall's lemma gives rise to (3.1). □

Proof of Theorem 1.1 Choosing $p = N + 1$ in (3.1), we conclude from (2.2) and the elliptic regularity argument that $\|v\|_{W^{2,N+1}} \le C$ for some positive constant C independent of t, then we conclude from the embedding (for ∇v) $W^{1,N+1}(\Omega) \hookrightarrow L^\infty(\Omega)$ or Morrey's inequality (e.g., p. 280 of [18])that $\|\nabla v\|_{L^\infty} < C$.

For simplicity of notations and without loss of generality, we assume that $\|\nabla v\|_{L^\infty} < 1$. Indeed, in the coming analysis, we can set $\tilde{\chi} := \chi\|\nabla v\|_{L^\infty}$, and by skipping the tilde we can proceed the same way as we shall do. Now, similar to (3.2), for each $p > 2$ we obtain

$$\begin{aligned}\frac{1}{p}\frac{d}{dt}\int_\Omega u^p &\le -\frac{4(p-1)D_1}{p^2}\int_\Omega \left|\nabla u^{\frac{p}{2}}\right|^2 + \frac{2(p-1)\chi}{p}\int_\Omega u^{\frac{p}{2}}\left|\nabla u^{\frac{p}{2}}\right| + a_1\int_\Omega u^p \\ &\le -\frac{2D_1(p-1)}{p^2}\int_\Omega |\nabla u^{\frac{p}{2}}|^2 + \left(\frac{(p-1)\chi^2}{2D_1} + a_1\right)\int_\Omega u^p, \end{aligned} \tag{3.5}$$

where the last inequality follows from Young's inequality. Now the rest of the arguments follow from the standard Moser–Alikakos L^p-iteration and we shall sketch the main parts. We recall from Gagliardo–Nirenberg–Sobolev inequality that, for any $u \in H^1$ and any $\epsilon > 0$

$$\left\|u^{\frac{p}{2}}\right\|^2_{L^2(\Omega)} \le \epsilon\left\|\nabla u^{\frac{p}{2}}\right\|^2_{L^2(\Omega)} + C\left\|u^{\frac{p}{2}}\right\|^2_{L^1(\Omega)},$$

and where C is a positive constant dependent on Ω and ϵ. Let $\epsilon = \frac{2D_1^2(p-1)}{p^2((p-1)\chi^2+2a_1D_1)}$, then $\frac{D_1(p-1)}{p^2\epsilon} = \frac{(p-1)\chi^2}{2D_1} + a_1$ and we have from (3.5) that for p large there exists C_2 (independent of p) such that

$$\frac{d}{dt}\int_\Omega u^p \le -\left(\frac{p(p-1)\chi^2}{2D_1} + pa_1\right)\int_\Omega u^p + \frac{2p(p-1)C_2\chi^2(1+\frac{p\chi}{D_1})}{D_1}\left(\int_\Omega u^{\frac{p}{2}}\right)^2.$$

Solving this inequality with $\kappa = \frac{p(p-1)\chi^2}{2D_1} + pa_1$, we obtain

$$\int_\Omega u^p \le e^{-\kappa t}\int_\Omega u_0^p + 4C_2\left(1+\frac{p\chi}{D_1}\right)\sup_{t\in(0,T)}\left(\int_\Omega u^{\frac{p}{2}}\right)^2.$$

Let $M(p) = \max\{\|u_0\|_{L^\infty}, \sup_{t\in(0,T)}\|u(\cdot,t)\|_{L^p}\}$, it follows that for p large there exists C_3 independent of p

$$M(p) \le \left(C_3 + \frac{C_3 p\chi}{D_1}\right)^{\frac{1}{p}} M(p/2).$$

Choosing $p = 2^i$ and then sending $i \to \infty$, we can apply a Moser–Alikakos iteration [16] to obtain the boundedness of $M(\infty)$, therefore $\|u(\cdot,t)\|_{L^\infty}$ is uniformly bounded in $(0,T)$ for each $T \in (0,\infty)$. Therefore we must have $T_{\max} = \infty$ and the solution (u,v) is globally bounded. Finally, one can apply standard elliptic and parabolic regularity theory to show that the solution is classical. □

4 Proof of Theorem 1.2

We proceed to prove Theorem 1.2. First we present the following result parallel to Lemma 3.1.

Lemma 4.1 *Let* (u,v) *be a solution of* (1.2) *in* $\Omega \times (0,T_{\max})$. *Let* $p^* > 1$ *be given by*

$$p^* = \begin{cases} \frac{\frac{a_2b_2\chi}{c_2D_2}}{\frac{a_2b_2\chi}{c_2D_2} - b_1} & \text{if } b_1 < \frac{a_2b_2\chi}{c_2D_2}, \\ \infty & \text{if } b_1 \ge \frac{a_2b_2\chi}{c_2D_2}. \end{cases}$$

Then, for each $p \in (1, p^*)$, there exists a positive constant C_p dependent on $\|u_0\|_{L^p(\Omega)}$ such that

$$\int_\Omega u^p(x,t) \le C_p, \quad \forall t \in (0, T_{\max}). \tag{4.1}$$

Proof Similar to (3.2) we test the u-equation in (1.2) by u^{p-1} and then integrate it over Ω by parts to have

$$\begin{aligned} &\frac{1}{p}\frac{d}{dt}\int_\Omega u^p + \frac{4(p-1)D_1}{p^2}\int_\Omega \bigl|\nabla u^{\frac{p}{2}}\bigr|^2 \\ &\quad \le -\int_\Omega \biggl(b_1 - \frac{(p-1)b_2\chi}{pD_2}v\biggr)u^{p+1} + \int_\Omega \biggl(a_1 + \frac{(p-1)c_2\chi}{pD_2}v^2\biggr)u^p \\ &\quad \le -\biggl(b_1 - \frac{(p-1)a_2b_2\chi}{pc_2D_2}\biggr)\int_\Omega u^{p+1} + \biggl(a_1 + \frac{(p-1)a_2^2\chi}{pc_2D_2}\biggr)\int_\Omega u^p. \end{aligned} \tag{4.2}$$

It is obvious that $\mu := b_1 - \frac{(p-1)a_2b_2\chi}{pc_2D_2}$ is positive thanks to the condition on p. We can use Young's inequality and Hölder's inequality to find that for some positive constant C_4

$$\begin{aligned} &\frac{1}{p}\frac{d}{dt}\int_\Omega u^p + \frac{4(p-1)D_1}{p^2}\int_\Omega \bigl|\nabla u^{\frac{p}{2}}\bigr|^2 \\ &\quad \le -\frac{\mu}{2}\int_\Omega u^{p+1} + C_4 \le -\frac{\mu}{2|\Omega|^{\frac{1}{p}}}\biggl(\int_\Omega u^p\biggr)^{\frac{p+1}{p}} + C_4, \end{aligned} \tag{4.3}$$

from which (4.1) follows thanks to the Grönwall lemma. □

Lemma 4.2 *Let (u, v) be a nonnegative local solution of (1.2) in $\Omega \times (0, T_{\max})$. Suppose that the following condition holds*:

$$b_1 > \frac{a_2b_2\chi(N-2)}{c_2D_2N}. \tag{4.4}$$

Then, for each $p \in [1, \infty)$, there exists a positive constant C_p dependent on $\|u_0\|_{L^p(\Omega)}$ such that

$$\int_\Omega u^p(x,t) \le C_p, \quad \forall t \in (0, T_{\max}). \tag{4.5}$$

Proof Thanks to (4.4) there always exists $p_0 \in (\frac{N}{2}, p^*)$ such that $b_1 > \frac{(p_0-1)a_2b_2\chi}{p_0c_2D_2}$, which, in the light of Lemma 4.1, implies that

$$\int_\Omega u^{p_0}(x,t) \le C_{p_0}, \quad \forall t \in (0, \infty). \tag{4.6}$$

Now each for $p > p_0$, similar to (3.5) we obtain

$$\frac{1}{p}\frac{d}{dt}\int_\Omega u^p + \frac{4(p-1)D_1}{p^2}\int_\Omega \bigl|\nabla u^{\frac{p}{2}}\bigr|^2 \le C_5\biggl(\int_\Omega u^{p+1} + 1\biggr) \tag{4.7}$$

from (4.2) where C_5 is a positive constant independent of p. To further estimate (4.7), we use Gagliardo–Nirenberg interpolation inequality and Young's inequality to estimate that

$$\begin{aligned}\int_\Omega u^{p+1} &= \left\|u^{\frac{p}{2}}\right\|_{L^{\frac{2(p+1)}{p}}(\Omega)}^{\frac{2(p+1)}{p}} \\ &\le C_6\left\|\nabla u^{\frac{p}{2}}\right\|_{L^2(\Omega)}^{\frac{2(p+1)}{p}h}\cdot\left\|u^{\frac{p}{2}}\right\|_{L^{\frac{2p_0}{p}}(\Omega)}^{\frac{2(p+1)}{p}(1-h)} + C_6\left\|u^{\frac{p}{2}}\right\|_{L^{\frac{2p_0}{p}}(\Omega)}^2 \\ &\le C_7\left\|\nabla u^{\frac{p}{2}}\right\|_{L^2(\Omega)}^{\frac{2(p+1)}{p}h} + C_7, \end{aligned} \tag{4.8}$$

where

$$h = \frac{\frac{p}{2(p+1)} - \frac{p}{2p_0}}{\frac{1}{2} - \frac{1}{N} - \frac{p}{2p_0}} \in (0,1) \tag{4.9}$$

and we have applied the fact that $\|u(\cdot,t)\|_{L^{p_0}}$ is bounded thanks to (4.6). Moreover, because of $p > p_0 > \frac{N}{2}$, we note that

$$\frac{2(p+1)}{p}h = \frac{1 - \frac{p+1}{2p_0}}{\frac{1}{2} - \frac{1}{N} - \frac{p}{2p_0}} < 2$$

and this implies that

$$\int_\Omega u^{p+1} \le \epsilon\left\|\nabla u^{\frac{p}{2}}\right\|_{L^2(\Omega)}^2 + C_\epsilon.$$

Finally, we conclude from (4.7)–(4.9) that

$$\frac{1}{p}\frac{d}{dt}\int_\Omega u^p + \frac{2(p-1)D_1}{p^2}\int_\Omega\left|\nabla u^{\frac{p}{2}}\right|^2 \le -C_8\left(\int_\Omega u^p\right)^{\frac{p+1}{p}} + C_9. \tag{4.10}$$

This gives rise to (4.5) and the proof of Lemma 4.2 completes. □

Proof of Theorem 1.2 By choosing $p > N + 1$ fixed in (4.5), we have $\|\nabla v(\cdot,t)\|_{L^\infty} < C$ for all $t \in (0,\infty)$. Then by the same arguments as in (3.5) we can show the uniform boundedness of $\|u(\cdot,t)\|_{L^\infty}$ for all $t \in (0,\infty)$. Therefore $T_{\max} = \infty$ and the local solution (u,v) is global. Moreover, one can apply the standard regularity theory to show that both u and v are classical in $\bar{\Omega} \times (0,\infty)$. □

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Competing interests
The author declares that they have no competing interests.

Authors' contributions
All authors read and approved the final manuscript.

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