Lin et al. *Advances in Difference Equations* (2018) 2018:155
https://doi.org/10.1186/s13662-018-1609-8

Advances in Difference Equations
a SpringerOpen Journal

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Asymptotic properties of a stochastic Lotka–Volterra model with infinite delay and regime switching

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Abstract

We investigate the long-term properties of a stochastic Lotka–Volterra model with infinite delay and Markovian chains on a finite state space. We investigate that the stochastic model admits a unique positive global solution which stays in the way of stochastically ultimate boundedness by constructing Lyapunov functions. Furthermore, the main results that the growth of the solution is slower than time under moderate condition and moment estimation in time average with the power *p* could be controlled are derived, which modified the known ones in recent literatures.

Keywords: Stochastic Lotka–Volterra model; Markovian chains; Infinite delay; Stochastically ultimate boundedness; Moment estimation

1 Model formulation

Gopalsamy proposed a general nonautonomous Lotka–Volterra model with infinite delay for n-interacting species in [1], which was described by an n-dimensional ordinary differential equation

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = x_i(t)\left(b_i + \sum_{j=1}^{n} a_{ij}x_j(t) + \sum_{j=1}^{n} b_{ij}x_j(t-\tau_{ij}) + \sum_{j=1}^{n} c_{ij}\int_{-\infty}^{t} k_{ij}(t-s)x_j(s)\,\mathrm{d}s\right), \tag{1}$$

where $i = 1, 2, \ldots, n$, and the sufficient conditions for the global attractivity of a positive solution of model (1) were obtained therewith. Motivated by model (1), the researchers investigated the modified models [2–26] and obtained some good results regarding n-interacting species in the recent literatures. We have known that the intrinsic growth rates and the carrying capacity of the species often vary when one of the below factors changes, for instance here, the situation of nutrition supply, adequacy of food resources and changes of climate as well. Therefore, the ecosystems governed by the deterministic models would inevitably be affected by the surrounding environmental noises. It is very natural to consider the stochastic ecosystems with the continuous-time Markovian chains $r(t)$ $(t \geq 0)$, which take values in a finite state space $\mathbb{S} = \{1, 2, \ldots, m\}$. Let the Markovian chain $r(t)$ be generated by the generator $\Gamma = (q_{ij})_{m\times m}$

$$\mathrm{P}\{r(t+\Delta t)=j|r(t)=i\} = \begin{cases} q_{ij}\Delta t + \mathrm{o}(\Delta t), & \text{if } i \neq j, \\ 1 + q_{ij}\Delta t + \mathrm{o}(\Delta t), & \text{if } i = j, \end{cases} \tag{2}$$

where $\Delta t > 0$, and $q_{ij} \geq 0$ is the transition rate from state i to state j if $i \neq j$; while $q_{ii} = -\sum_{j\neq i} q_{ij}$.

In this paper, we modify model (1) further and propose a more general nonautonomous Lotka–Volterra model with Markovian switching

$$\begin{aligned} \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = x_i(t)\Biggl(& b_i\bigl(r(t)\bigr) + \sum_{j=1}^{n} a_{ij}\bigl(r(t)\bigr)x_j(t) + \sum_{j=1}^{n} b_{ij}\bigl(r(t)\bigr)x_j(t-\tau_{ij}) \\ & + \sum_{j=1}^{n} c_{ij}\bigl(r(t)\bigr)\int_{-\infty}^{t} k_{ij}(t-s)x_j(s)\,\mathrm{d}s\Biggr), \quad t \geq 0, \end{aligned} \tag{3}$$

where $i = 1, 2, \dots, n$ and $r(t) \in \mathbb{S}$. Assuming that initially, $r(t) = k \in \mathbb{S}$, then (3) obeys the following differential equation:

$$\begin{aligned} \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = x_i(t)\Biggl(& b_i(k) + \sum_{j=1}^{n} a_{ij}(k)x_j(t) + \sum_{j=1}^{n} b_{ij}(k)x_j(t-\tau_{ij}) \\ & + \sum_{j=1}^{n} c_{ij}(k)\int_{-\infty}^{t} k_{ij}(t-s)x_j(s)\,\mathrm{d}s\Biggr), \end{aligned} \tag{4}$$

until $r(t)$ jumps to another state, say $l \in \mathbb{S}$, thus (3) follows the differential equation with state l before $r(t)$ jumps to a new state, that is

$$\begin{aligned} \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = x_i(t)\Biggl(& b_i(l) + \sum_{j=1}^{n} a_{ij}(l)x_j(t) + \sum_{j=1}^{n} b_{ij}(l)x_j(t-\tau_{ij}) \\ & + \sum_{j=1}^{n} c_{ij}(l)\int_{-\infty}^{t} k_{ij}(t-s)x_j(s)\,\mathrm{d}s\Biggr). \end{aligned} \tag{5}$$

Now, we consider the perturbations of intrinsic growth rates and interacting rates between species, that is to say, $b_i(\varsigma)$ and $a_{ij}(\varsigma)$, $b_{ij}(\varsigma)$, $c_{ij}(\varsigma)$ will be disturbed by the white noises in the form of

$$b_i(\varsigma) \to b_i(\varsigma) + \sigma_i(\varsigma)\dot{B}_i(t), \qquad a_{ij}(\varsigma) \to a_{ij}(\varsigma) + \alpha_{ij}(\varsigma)\dot{B}_i(t), \tag{6}$$

$$b_{ij}(\varsigma) \to b_{ij}(\varsigma) + \beta_{ij}(\varsigma)\dot{B}_i(t), \qquad c_{ij}(\varsigma) \to c_{ij}(\varsigma) + \gamma_{ij}(\varsigma)\dot{B}_i(t), \tag{7}$$

where $\dot{B}_i(t)$ are the white noises and $\sigma_i(\varsigma)$ represent the intensities of the white noises in regime $\varsigma \in \mathbb{S}$ for $i = 1, 2, \dots, n$. Next, we are about to investigate the dynamical properties of the following stochastic Lotka–Volterra model under Markovian switching:

$$\begin{aligned}
\mathrm{d}x_i(t) = {} & x_i(t)\Biggl(b_i\bigl(r(t)\bigr) + \sum_{j=1}^{n} a_{ij}\bigl(r(t)\bigr)x_j(t) + \sum_{j=1}^{n} b_{ij}\bigl(r(t)\bigr)x_j(t-\tau_{ij}) \\
& + \sum_{j=1}^{n} c_{ij}\bigl(r(t)\bigr)\int_{-\infty}^{t} k_{ij}(t-s)x_j(s)\,\mathrm{d}s\Biggr)\,\mathrm{d}t \\
& + x_i(t)\Biggl(\sigma_i\bigl(r(t)\bigr) + \sum_{j=1}^{n} \alpha_{ij}\bigl(r(t)\bigr)x_j(t) + \sum_{j=1}^{n} \beta_{ij}\bigl(r(t)\bigr)x_j(t-\tau_{ij}) \\
& + \sum_{j=1}^{n} \gamma_{ij}\bigl(r(t)\bigr)\int_{-\infty}^{t} k_{ij}(t-s)x_j(s)\,\mathrm{d}s\Biggr)\,\mathrm{d}B_i(t), \quad i = 1,2,\dots,n. \qquad (8)
\end{aligned}$$

The initial conditions of model (8) are supposed to be given as follows:

$$\text{(H1)} \quad x_i(\theta) = \varphi_i(\theta) > 0, \qquad \sup_{-\infty<\theta\le 0}\bigl|\varphi_i(\theta)\bigr| < \infty, \tag{9}$$

$$\text{(H2)} \quad \lambda > 0, \qquad \int_0^{\infty} k_{ij}(s)e^{\lambda s}\,\mathrm{d}s = \bar{k}_{ij} < \infty, \tag{10}$$

where φ_i $(i = 1,2,\dots,n)$ are the continuous functions which are defined on the interval $(-\infty,0]$ with its Euclidian norm, and $k_{ij}(s)$ denotes the weight function with the property $\int_0^{\infty} k_{ij}(s)\,\mathrm{d}s \le 1$.

If $\sigma_i(r) = \beta_{ij}(r) = \gamma_{ij}(r) = 0$, then model (8) turns to be a stochastic Lotka–Volterra model with infinite delay, where the stochastically ultimate boundedness and pth $(0 < p \le 2)$ moment in time average of the solution were considered by Wan and Zhou [20]. If $b_{ij}(r) = c_{ij}(r) = 0$, $\sigma_i(r) = \beta_{ij}(r) = \gamma_{ij}(r) = 0$, model (8) becomes a stochastic Lotka–Volterra model with Markovian switching, where the sufficient conditions of the stochastic permanence and extinction were presented by Liu *et al.* [16]. If $b_{ij}(r) = c_{ij}(r) = 0$, $\alpha_i(r) = \beta_{ij}(r) = \gamma_{ij}(r) = 0$, Wu *et al.* [13] discussed the ergodic property of a positive recurrence. Besides, we prefer to mention the related work herewith. For example, Hu and Wang [17] investigated the asymptotic stability in distribution and stochastic boundedness of the solution. Liu and Shen [18] showed the persistence in the mean, extinction, partial permanence, and partial extinction. In addition, Zhu and Yin [19] derived the stochastic boundedness and positive recurrence of the solution.

The framework of this paper will go as follows. We will show that the existence and uniqueness of the positive global solution always holds with probability one for any positive initial value in the next section. Later, the stochastically ultimate boundedness will be derived when constructing a proper function therewith. Consequently, the moment estimation of the solution will be investigated for the stochastic Lotka–Volterra model (8) with Markovian chains in this paper.

2 Existence and uniqueness of global solution

To proceed with our main results in this section, we denote $\mathbb{R}_+ := [0,\infty)$, and we will show that model (8) admits a unique and global solution and the solution will remain in $\mathbb{R}_+^n$ almost surely.

Theorem 2.1 *For any $b_i(r(t)), a_{ij}(r(t)), b_{ij}(r(t)), c_{ij}(r(t)) \in \mathbb{R}_+$ $(i,j = 1,2,\ldots,n)$, model* (8) *admits a unique solution $x(t)$, and the solution remains in $\mathbb{R}^n_+$ with probability one.*

Proof According to Theorem 3.1 in [27], the coefficients of model (8) clearly satisfy the local Lipschitz condition, but do not satisfy the linear growth condition. To show that the solution of model (8) is a global solution, we only need to prove that the explosion time $\tau_e = \infty$ holds almost surely. Let $m_0 > 1$ be sufficiently large such that

$$\frac{1}{m_0} \le \min_{t\ge 0}\bigl|x(t)\bigr| \le \max_{t\ge 0}\bigl|x(t)\bigr| \le m_0, \tag{11}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}}$. We define the stopping time

$$\tau_m = \inf\left\{t \in [0,\tau_e) : x_i(t) \notin \left(\frac{1}{m}, m\right), \text{for some } i = 1,2,\ldots,n\right\}, \quad m \ge m_0. \tag{12}$$

As usual, $\emptyset$ denotes the empty set, we set $\inf\emptyset = \infty$. Clearly, τ_m increases when m tends to infinity. We set

$$\tau_\infty = \lim_{m\to\infty} \tau_m, \tag{13}$$

thus $\tau_\infty \le \tau_e$ by the definition of stopping time. Now, we define two C^2-functions in order to check that $\tau_\infty = \infty$ almost surely:

$$V_1\bigl(x(t)\bigr) = \sum_{i=1}^{n}\bigl(x_i^{0.5}(t) - 1 - 0.5\ln x_i(t)\bigr), \tag{14}$$

$$\begin{aligned} V_2\bigl(x(t)\bigr) = \sum_{i=1}^{n}\sum_{j=1}^{n}\biggl(&\frac{1}{2n}\int_{t-\tau_{ij}}^{t} x_j^2(s)\,\mathrm{d}s + \frac{1}{2n}\int_0^\infty k_{ij}(s)\int_{t-s}^{t} x_j^2(u)\,\mathrm{d}u\,\mathrm{d}s \\ &+ \frac{|\beta(r(\tau_k))|^2}{4(1-l_1)(1-l_2)l_3}\int_{t-\tau_{ij}}^{t} x_j^2(s)\,\mathrm{d}s \\ &+ \frac{|\gamma(r(\tau_k))|^2}{4(1-l_1)(1-l_2)(1-l_3)}\int_0^\infty k_{ij}(s)\int_{t-s}^{t} x_j^2(u)\,\mathrm{d}u\,\mathrm{d}s\biggr), \end{aligned} \tag{15}$$

where $0 < l_i < 1$ $(i = 1,2,3)$, and $|\sigma(r(\tau_k))|$ means the norm of vector $(\sigma_1(r(\tau_k)),\ldots, \sigma_n(r(\tau_k)))$, $|\beta(r(\tau_k))|$ denotes the norm of the matrix $(b_{ij}(r(\tau_k)))_{n\times n}$, so the same to norms $|\alpha(r(\tau_k))|$ and $|\gamma(r(\tau_k))|$. For the sake of simplicity, model (8) could be rewritten in the following form:

$$\mathrm{d}x_i(t) = x_i(t)f_i(t)\,\mathrm{d}t + x_i(t)g_i(t)\,\mathrm{d}B_i(t), \quad i = 1,2,\ldots,n. \tag{16}$$

We define the same differential operator $\mathcal{L}$ associated with equation (8) by

$$\mathcal{L} = \frac{\partial}{\partial t} + \sum_{i=1}^{n} x_i(t)f_i(t)\frac{\partial}{\partial x_i(t)} + \frac{1}{2}\sum_{i=1}^{n} x_i^2(t)g_i^2(t)\frac{\partial^2}{\partial x_i^2(t)}, \tag{17}$$

then the differential operator $\mathcal{L}$ acts on $V_1(x(t))$, generalized Itô's formula gives

$$\begin{aligned} \mathrm{d}V_1\big(x(t)\big) &= 0.5\sum_{i=1}^{n}\big(x_i^{0.5}(t)-1\big)f_i(t)\,\mathrm{d}t + 0.25\sum_{i=1}^{n}\big(-0.5x_i^{0.5}(t)+1\big)g_i^2(t)\,\mathrm{d}t \\ &\quad + 0.5\sum_{i=1}^{n}\big(x_i^{0.5}(t)-1\big)g_i(t)\,\mathrm{d}B_i(t). \end{aligned} \tag{18}$$

The elementary inequality $ab \le \frac{n}{4}a^2 + \frac{1}{n}b^2$ ($a > 0$, $b > 0$) gives that

$$\begin{aligned} &\sum_{i=1}^{n}\big(x_i^{0.5}(t)-1\big)f_i(t) \\ &\quad = \sum_{i=1}^{n}\big(x_i^{0.5}(t)-1\big)\Bigg(b_i\big(r(\tau_k)\big) + \sum_{j=1}^{n}a_{ij}\big(r(\tau_k)\big)x_j(t) \\ &\qquad + \sum_{j=1}^{n}b_{ij}\big(r(\tau_k)\big)x_j(t-\tau_{ij}) + \sum_{j=1}^{n}c_{ij}\big(r(\tau_k)\big)\int_0^\infty k_{ij}(s)x_j(t-s)\,\mathrm{d}s\Bigg) \\ &\quad \le \sum_{i=1}^{n}\big(x_i^{0.5}(t)-1\big)b_i\big(r(\tau_k)\big) + \sum_{i=1}^{n}\sum_{j=1}^{n}\bigg[0.25n\big(x_i^{0.5}(t)-1\big)^2\big(a_{ij}^2\big(r(\tau_k)\big) \\ &\qquad + b_{ij}^2\big(r(\tau_k)\big) + c_{ij}^2\big(r(\tau_k)\big)\big) + \frac{1}{n}x_j^2(t) + \frac{1}{n}x_j^2(t-\tau_{ij}) \\ &\qquad + \frac{1}{n}\bigg(\int_0^\infty k_{ij}(s)x_j(t-s)\,\mathrm{d}s\bigg)^2\bigg]. \end{aligned} \tag{19}$$

The inequalities $(u+v)^2 \le \frac{u^2}{l_i} + \frac{v^2}{1-l_i}$ for $0 < l_i < 1$ and $(\sum_{i=1}^n a_i b_i)^2 \le \sum_{i=1}^n a_i^2 \sum_{i=1}^n b_i^2$ yield that

$$\begin{aligned} g_i^2(t) &\le \frac{1}{l_1}\sigma_i^2\big(r(\tau_k)\big) + \frac{1}{l_2(1-l_1)}\sum_{j=1}^{n}\alpha_{ij}^2\big(r(\tau_k)\big)\sum_{j=1}^{n}x_j^2(t) \\ &\quad + \frac{1}{l_3(1-l_1)(1-l_2)}\sum_{j=1}^{n}\beta_{ij}^2\big(r(\tau_k)\big)\sum_{j=1}^{n}x_j^2(t-\tau_{ij}) \\ &\quad + \frac{1}{(1-l_1)(1-l_2)(1-l_3)}\sum_{j=1}^{n}\gamma_{ij}^2\big(r(\tau_k)\big)\sum_{j=1}^{n}\bigg(\int_0^\infty k_{ij}(s)x_j(t-s)\,\mathrm{d}s\bigg)^2, \end{aligned} \tag{20}$$

which implies that

$$\begin{aligned} &\sum_{i=1}^{n}\big(-0.5x_i^{0.5}(t)+1\big)g_i^2(t) \\ &\quad < \sum_{i=1}^{n}g_i^2(t) \\ &\quad \le \sum_{i=1}^{n}\frac{\sigma_i^2(r(\tau_k))}{l_1} + \sum_{i=1}^{n}\Bigg(\sum_{j=1}^{n}\frac{\alpha_{ij}^2(r(\tau_k))}{l_2(1-l_1)}\sum_{j=1}^{n}x_j^2(t)\Bigg) \end{aligned}$$

$$
\begin{aligned}
&+ \sum_{i=1}^{n}\left(\sum_{j=1}^{n} \frac{\beta_{ij}^2(r(\tau_k))}{(1-l_1)(1-l_2)l_3} \sum_{j=1}^{n} x_j^2(t-\tau_{ij})\right) \\
&+ \sum_{i=1}^{n}\left[\sum_{j=1}^{n} \frac{\gamma_{ij}^2(r(\tau_k))}{(1-l_1)(1-l_2)(1-l_3)} \sum_{j=1}^{n}\left(\int_0^\infty k_{ij}(s)x_j(t-s)\,\mathrm{d}s\right)^2\right] \\
\le{}& \frac{|\sigma(r(\tau_k))|^2}{l_1} + \frac{|\alpha(r(\tau_k))|^2}{(1-l_1)l_2}\left|x(t)\right|^2 + \frac{|\beta(r(\tau_k))|^2}{(1-l_1)(1-l_2)l_3} \sum_{i=1}^{n}\sum_{j=1}^{n} x_j^2(t-\tau_{ij}) \\
&+ \frac{|\gamma(r(\tau_k))|^2}{(1-l_1)(1-l_2)(1-l_3)} \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\int_0^\infty k_{ij}(s)x_j(t-s)\,\mathrm{d}s\right)^2. \qquad (21)
\end{aligned}
$$

By a similar argument, we derive that

$$
\begin{aligned}
\mathrm{d}V_2\bigl(x(t)\bigr) ={}& \Biggl[-\frac{1}{2n}\sum_{i=1}^{n}\sum_{j=1}^{n} x_j^2(t-\tau_{ij}) + \frac{1}{2n}\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^\infty k_{ij}(s)\bigl(x_j^2(t)-x_j^2(t-s)\bigr)\,\mathrm{d}s \\
&+ 0.5\left|x(t)\right|^2 + \frac{|\beta(r(\tau_k))|^2}{4(1-l_1)(1-l_2)l_3}\sum_{i=1}^{n}\sum_{j=1}^{n}\bigl(x_j^2(t)-x_j^2(t-\tau_{ij})\bigr) \\
&+ \frac{|\gamma(r(\tau_k))|^2}{4(1-l_1)(1-l_2)l_3}\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^\infty k_{ij}(s)\bigl(x_j^2(t)-x_j^2(t-\tau_{ij})\bigr)\,\mathrm{d}s\Biggr]\,\mathrm{d}t, \qquad (22)
\end{aligned}
$$

where

$$
\begin{aligned}
\left(\int_0^\infty k_{ij}(s)x_j(t-s)\,\mathrm{d}s\right)^2 &\le \int_0^\infty \bigl(\sqrt{k_{ij}(s)}\bigr)^2\,\mathrm{d}s\cdot\int_0^\infty \bigl(\sqrt{k_{ij}(s)}x_j(t-s)\bigr)^2\,\mathrm{d}s \\
&\le \int_0^\infty k_{ij}(s)x_j^2(t-s)\,\mathrm{d}s, \qquad (23)
\end{aligned}
$$

together with expressions (18), (22), and (23), which gives that

$$
\begin{aligned}
&\mathrm{d}\bigl(V_1\bigl(x(t)\bigr)+V_2\bigl(x(t)\bigr)\bigr) \\
&\le \Biggl(0.5\sum_{i=1}^{n}\bigl(x_i^{0.5}(t)-1\bigr)b_i\bigl(r(\tau_k)\bigr) + 0.125\sum_{i=1}^{n}\sum_{j=1}^{n} n\bigl(x_i^{0.5}(t)-1\bigr)^2\bigl[a_{ij}^2\bigl(r(\tau_k)\bigr) \\
&\quad + b_{ij}^2\bigl(r(\tau_k)\bigr) + c_{ij}^2\bigl(r(\tau_k)\bigr)\bigr] + 1.5\left|x(t)\right|^2 + \frac{1}{4l_1}\left|\sigma\bigl(r(\tau_k)\bigr)\right|^2 \\
&\quad + \frac{1}{4l_2(1-l_1)}\left|\alpha\bigl(r(\tau_k)\bigr)\right|^2\left|x(t)\right|^2 + \frac{n}{4l_3(1-l_1)(1-l_2)}\left|\beta\bigl(r(\tau_k)\bigr)\right|^2\left|x(t)\right|^2 \\
&\quad + \frac{n}{4(1-l_1)(1-l_2)(1-l_3)}\left|\gamma\bigl(r(\tau_k)\bigr)\right|^2\left|x(t)\right|^2\Biggr)\,\mathrm{d}t \\
&\quad + 0.5\sum_{i=1}^{n}\bigl(x_i^{0.5}(t)-1\bigr)g_i(t)\,\mathrm{d}B_i(t) \\
&:= F\bigl(x(t)\bigr)\,\mathrm{d}t + 0.5\sum_{i=1}^{n}\bigl(x_i^{0.5}(t)-1\bigr)g_i(t)\,\mathrm{d}B_i(t), \qquad (24)
\end{aligned}
$$

where $F(x(t))$ is a bounded constant due to the boundedness of all $x(t)$ lying within the interval $(\frac{1}{m}, m)$, that is,

$$\sup_{\frac{1}{m}<x(t)<m} F\bigl(x(t)\bigr) \leq \hat{F} < \infty. \tag{25}$$

Inequality (24) thus becomes

$$\mathrm{d}\bigl(V_1\bigl(x(t)\bigr) + V_2\bigl(x(t)\bigr)\bigr) \leq \hat{F}\,\mathrm{d}t + 0.5\sum_{i=1}^{n}\bigl(x_i^{0.5}(t) - 1\bigr)g_i(t)\,\mathrm{d}B_i(t). \tag{26}$$

Integrating both sides of (26) from 0 to $\tau_m \wedge T$ and taking expectations imply that

$$\begin{aligned}\mathbb{E}V_1\bigl(x(\tau_m \wedge T)\bigr) &< \mathbb{E}\bigl[V_1\bigl(x(\tau_m \wedge T)\bigr) + V_2\bigl(x(\tau_m \wedge T)\bigr)\bigr] \\ &\leq V_1\bigl(x(0)\bigr) + V_2\bigl(x(0)\bigr) + \hat{F}T,\end{aligned} \tag{27}$$

where T is an arbitrary positive constant. For every $w \in \Omega_m = \{\tau_m \leq T\}$, there exists some i such that $x_i(\tau_m, w)$ equals either m or $\frac{1}{m}$, then

$$V_1\bigl(x(\tau_m \wedge T)\bigr) \geq \min\biggl\{\sqrt{m} - 1 - 0.5\ln m, \sqrt{\frac{1}{m}} - 1 + 0.5\ln m\biggr\}, \tag{28}$$

which leads to

$$\begin{aligned}&\mathbb{P}\{\tau_m \leq T\}\min\biggl\{\sqrt{m} - 1 - 0.5\ln m, \sqrt{\frac{1}{m}} - 1 + 0.5\ln m\biggr\} \\ &\quad\leq \mathbb{E}\bigl[1_{\Omega_m}(\omega)V_1\bigl(x(\tau_m \wedge T)\bigr)\bigr] \leq V_1\bigl(x(0)\bigr) + V_2\bigl(x(0)\bigr) + \hat{F}T,\end{aligned} \tag{29}$$

where $1_{\Omega_m}(\omega)$ is the indicator function of Ω_m. We therefore get that

$$\lim_{m\to\infty} \mathbb{P}\{\tau_m \leq T\} = 0, \tag{30}$$

when m tends to infinity. From the arbitrariness of a positive number T, the following assertion holds:

$$\mathbb{P}\{\tau_\infty = \infty\} = 1. \tag{31}$$

The proof is complete. □

3 Stochastically ultimate boundedness

The moment estimation of the solution to model (8) will be investigated as the first part of this section. Then the stochastically ultimate boundedness of the solution follows later in Theorem 3.2.

Lemma 3.1 *If condition* (H2) *holds, then there exists positive constants* p, λ *such that*

$$\limsup_{t\to\infty} \mathbb{E}\bigl|x(t)\bigr|^p \leq \frac{\bar{Q}}{\lambda} n^{\frac{p}{2}}, \tag{32}$$

where $\bar{Q}$ *is a positive constant and* $0 < p < 1$.

Proof Let us define a C^2-function

$$V_3\bigl(x(t)\bigr) = \sum_{i=1}^{n} x_i^p(t). \tag{33}$$

For any given $\lambda > 0$, applying Itô's formula to $e^{\lambda t}V_3(x(t))$ and taking expectation, we derive that

$$e^{\lambda t}\mathbb{E}\bigl(V_3\bigl(x(t)\bigr)\bigr) = \mathbb{E}V_3\bigl(x(0)\bigr) + \mathbb{E}\int_0^t e^{\lambda s}\bigl[\mathcal{L}V_3\bigl(x(s)\bigr) + \lambda V_3\bigl(x(s)\bigr)\bigr]\,\mathrm{d}s, \tag{34}$$

where

$$\begin{aligned}
\mathcal{L}V_3\bigl(x(t)\bigr) &= p\sum_{i=1}^{n} x_i^p(t)f_i(t) + \frac{p(p-1)}{2}\sum_{i=1}^{n} x_i^p(t)g_i^2(t) \\
&\leq \sum_{i=1}^{n} p x_i^p(t) b_i\bigl(r(\tau_k)\bigr) \\
&\quad + \frac{np^2}{4}\sum_{i=1}^{n}\sum_{j=1}^{n} x_i^{2p}(t)\bigl[a_{ij}^2\bigl(r(\tau_k)\bigr) + b_{ij}^2\bigl(r(\tau_k)\bigr) + c_{ij}^2\bigl(r(\tau_k)\bigr)\bigr] \\
&\quad + \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\biggl(\int_0^\infty k_{ij}(s)x_j(t-s)\,\mathrm{d}s\biggr)^2 + \bigl|x(t)\bigr|^2 + \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} x_j^2(t-\tau_{ij}) \\
&\quad - \frac{p(1-p)}{2}\sum_{i=1}^{n} x_i^p(t)\biggl[\sigma_i^2\bigl(r(\tau_k)\bigr) + \alpha_{ii}^2\bigl(r(\tau_k)\bigr)x_i^2 + \beta_{ii}^2\bigl(r(\tau_k)\bigr)x_i^2(t-\tau_{ii}) \\
&\quad + \gamma_{ii}^2\bigl(r(\tau_k)\bigr)\biggl(\int_0^\infty k_{ii}(s)x_i(t-s)\,\mathrm{d}s\biggr)^2\biggr].
\end{aligned} \tag{35}$$

Again we define

$$\begin{aligned}
V_4\bigl(x(t)\bigr) &= \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\tau_{ij}}^{t} e^{\lambda(s+\tau_{ij})}x_j^2(s)\,\mathrm{d}s \\
&\quad + \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^\infty k_{ij}(s)\int_{t-s}^{t} e^{\lambda(u+s)}x_j^2(u)\,\mathrm{d}u\,\mathrm{d}s,
\end{aligned} \tag{36}$$

according to the same argument, we then derive that

$$\begin{aligned}
\mathrm{d}V_4\bigl(x(t)\bigr) \leq \biggl(& e^{\lambda(t+\tau)}\bigl|x(t)\bigr|^2 - \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} e^{\lambda t}x_j^2(t-\tau_{ij}) \\
&+ e^{\lambda t}\bar{k}\bigl|x(t)\bigr|^2 - \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} e^{\lambda t}\int_0^\infty k_{ij}(s)x_j^2(t-s)\,\mathrm{d}s\biggr)\,\mathrm{d}t,
\end{aligned} \tag{37}$$

where

$$\tau = \max_{1\leq i,j\leq n}\tau_{ij}, \qquad \bar{k} = \max_{1\leq i,j\leq n}\bar{k}_{ij}. \tag{38}$$

Further, we combine (23), (35), and (37) to get that

$$\begin{aligned}
&\mathrm{d}\big(e^{\lambda t}V_3\big(x(t)\big)+V_4\big(x(t)\big)\big)\\
&\quad=\mathrm{d}V_4\big(x(t)\big)+\lambda e^{\lambda t}V_3\big(x(t)\big)\,\mathrm{d}t+e^{\lambda t}\mathcal{L}V_3\big(x(t)\big)\,\mathrm{d}t+e^{\lambda t}\sum_{i=1}^{n}px_i^p(t)g_i(t)\,\mathrm{d}B_i(t)\\
&\quad\le e^{\lambda t}\Bigg\{\lambda V_3\big(x(t)\big)+\big(e^{\lambda\tau}+\bar{k}+1\big)\big|x(t)\big|^2+\sum_{i=1}^{n}px_i^p(t)b_i\big(r(\tau_k)\big)\\
&\qquad+\frac{np^2}{4}\sum_{i=1}^{n}\sum_{j=1}^{n}x_i^{2p}(t)\big[a_{ij}^2\big(r(\tau_k)\big)+b_{ij}^2\big(r(\tau_k)\big)+c_{ij}^2\big(r(\tau_k)\big)\big]\\
&\qquad-\frac{p(1-p)}{2}\sum_{i=1}^{n}x_i^p(t)\bigg[\sigma_i^2\big(r(\tau_k)\big)+\alpha_{ii}^2\big(r(\tau_k)\big)x_i^2(t)\\
&\qquad+\beta_{ii}^2\big(r(\tau_k)\big)x_i^2(t-\tau_{ii})+\gamma_{ii}^2\big(r(\tau_k)\big)\\
&\qquad\times\bigg(\int_0^\infty k_{ij}(s)x_i(t-s)\,\mathrm{d}s\bigg)^2\bigg]\Bigg\}\,\mathrm{d}t+e^{\lambda t}\sum_{i=1}^{n}px_i^p(t)g_i(t)\,\mathrm{d}B_i(t)\\
&\quad<e^{\lambda t}Q\big(x(t)\big)\,\mathrm{d}t+e^{\lambda t}\sum_{i=1}^{n}px_i^p(t)g_i(t)\,\mathrm{d}B_i(t),
\end{aligned}\tag{39}$$

where the term with $p(1-p)$ is positive, and the boundedness $\bar{Q}=\sup_{x(t)\in\mathbb{R}_+^n}Q(x(t))<\infty$ is derived due to the boundedness of $x(t)\in\mathbb{R}_+^n$. Integrating both sides of (39) and taking expectation from 0 to t therefore give

$$\begin{aligned}
e^{\lambda t}\mathbb{E}V_3\big(x(t)\big)&\le V_3\big(x(0)\big)+V_4\big(x(0)\big)+\mathbb{E}\int_0^t e^{\lambda s}Q\big(x(s)\big)\,\mathrm{d}s\\
&\le V_3\big(x(0)\big)+V_4\big(x(0)\big)+\frac{\bar{Q}e^{\lambda t}}{\lambda},
\end{aligned}\tag{40}$$

which implies that

$$\limsup_{t\to\infty}\mathbb{E}V_3\big(x(t)\big)\le\frac{\bar{Q}}{\lambda}.\tag{41}$$

The inequality $|x(t)|^2\le n\max_i x_i^2(t)$ admits the following inequality:

$$\big|x(t)\big|^p\le n^{\frac{p}{2}}\max_i x_i^p(t)\le n^{\frac{p}{2}}\sum_{i=1}^{n}x_i^p(t)=n^{\frac{p}{2}}V_3\big(x(t)\big),\tag{42}$$

which yields that

$$\limsup_{t\to\infty}\mathbb{E}\big|x(t)\big|^p\le\frac{\bar{Q}}{\lambda}n^{\frac{p}{2}}.\tag{43}$$

The proof is complete. □

Theorem 3.2 *If condition* (H2) *holds, then the following property is valid for any arbitrary positive constant* $\varepsilon \in (0,1)$:

$$\limsup_{t\to\infty} \mathbb{P}\{|x(t)| \le \zeta\} \ge 1-\varepsilon, \tag{44}$$

that is, the solution $x(t)$ *of model* (8) *is stochastically ultimately bounded.*

Proof We take $p=\frac{1}{2}$ in (43), then there exists a positive constant $\frac{\bar{Q}}{\lambda}n^{\frac{1}{4}}$ such that

$$\limsup_{t\to\infty} \mathbb{E}|x(t)|^{\frac{1}{2}} \le \frac{\bar{Q}}{\lambda}n^{\frac{1}{4}}. \tag{45}$$

For any positive constant ε, let

$$\zeta = \frac{\bar{Q}^2}{\lambda^2\varepsilon^2}n^{\frac{1}{2}}, \tag{46}$$

the Chebyshev's inequality gives

$$\limsup_{t\to\infty} \mathbb{P}\{|x(t)| \ge \zeta\} \le \limsup_{t\to\infty} \frac{\mathbb{E}|x(t)|^{\frac{1}{2}}}{\zeta^{\frac{1}{2}}} \le \varepsilon, \tag{47}$$

which yields that

$$\limsup_{t\to\infty} \mathbb{P}\{|x(t)| < \zeta\} \ge 1-\varepsilon. \tag{48}$$

The proof is complete. □

4 Moment estimation of the solution

We are about to discuss other properties of the solution to model (8) in this section, for instance, the absolute value of the solution will grow slower than time t when t goes to infinity. Moreover, the pth moment in time average of the solution to model (8) is discussed in Theorem 4.2.

Theorem 4.1 *If condition* (H2) *holds, then the solution of model* (8) *has the property that*

$$\limsup_{t\to\infty} \frac{\ln|x(t)|}{\ln t} \le 1 \quad a.s. \tag{49}$$

Proof In order to prove this theorem, we need to define two C^2-functions:

$$V_5(x(t)) = \sum_{i=1}^n x_i(t), \tag{50}$$

$$\begin{aligned} V_6(x(t)) &= \sum_{i=1}^n\sum_{j=1}^n |b_{ij}(r(\tau_k))| \int_{t-\tau_{ij}}^t e^{\lambda(s+\tau_{ij})}x_j(s)\,\mathrm{d}s \\ &\quad + \sum_{i=1}^n\sum_{j=1}^n |c_{ij}(r(\tau_k))| \int_0^\infty k_{ij}(s)\int_{t-s}^t e^{\lambda(u+s)}x_j(u)\,\mathrm{d}u\,\mathrm{d}s. \end{aligned} \tag{51}$$

Generalized Itô's formula yields

$$\mathrm{d}V_5\big(x(t)\big) = \sum_{i=1}^{n} x_i(t)f_i(t)\,\mathrm{d}t + \sum_{i=1}^{n} x_i(t)g_i(t)\,\mathrm{d}B_i(t), \tag{52}$$

$$\begin{aligned}\mathrm{d}V_6\big(x(t)\big) = {} & \Bigg(\sum_{i=1}^{n}\sum_{j=1}^{n}\big|b_{ij}\big(r(\tau_k)\big)\big|e^{\lambda(t+\tau_{ij})}x_j(t) \\ & - \sum_{i=1}^{n}\sum_{j=1}^{n}\big|b_{ij}\big(r(\tau_k)\big)\big|e^{\lambda t}x_j(t-\tau_{ij}) \\ & + \sum_{i=1}^{n}\sum_{j=1}^{n}\big|c_{ij}\big(r(\tau_k)\big)\big|\int_0^\infty k_{ij}(s)e^{\lambda(t+s)}x_j(s)\,\mathrm{d}s \\ & - \sum_{i=1}^{n}\sum_{j=1}^{n}\big|c_{ij}\big(r(\tau_k)\big)\big|\int_0^\infty k_{ij}(s)e^{\lambda t}x_j(t-s)\,\mathrm{d}s\Bigg)\,\mathrm{d}t.\end{aligned} \tag{53}$$

For any given $\lambda > 0$, expressions (52) and (53) imply that

$$\begin{aligned}&\mathrm{d}\big(e^{\lambda t}\ln V_5\big(x(t)\big) + V_6\big(x(t)\big)\big) \\ &\quad = \mathrm{d}V_6\big(x(t)\big) + e^{\lambda t}\left(\lambda \ln V_5\big(x(t)\big) + \sum_{i=1}^{n} x_i(t)f_i(t)V_5^{-1}\big(x(t)\big) - \frac{|x^{\mathrm{T}}(t)g(t)|^2}{2V_5^2(x(t))}\right)\mathrm{d}t \\ &\qquad + \frac{e^{\lambda t}|x^{\mathrm{T}}(t)g(t)|}{V_5(x(t))}\,\mathrm{d}B(t).\end{aligned} \tag{54}$$

We denote

$$M^*(t) = \int_0^t \frac{e^{\lambda s}|x^{\mathrm{T}}(t)g(t)|}{V_5(x(t))}\,\mathrm{d}B(s), \tag{55}$$

then the strong law of large numbers (see Theorem 3.4 in [27]) for a local martingale $M^*(t)$ yields that

$$\lim_{t\to\infty}\frac{M^*(t)}{t} = 0 \quad \text{a.s.} \tag{56}$$

and

$$\big\langle M^*(t), M^*(t)\big\rangle = \int_0^t \frac{e^{2\lambda s}|x^{\mathrm{T}}(t)g(t)|^2}{V_5^2(x(t))}\,\mathrm{d}s. \tag{57}$$

The exponential martingale inequality (see Theorem 7.4 in [27]) shows that when choosing

$$T = \nu, \qquad \alpha = \frac{\eta}{e^{\lambda k}}, \qquad \beta = \frac{\nu e^{\lambda k \ln k}}{\eta}, \tag{58}$$

and letting $\nu > 1$, $0 < \eta < 1$, we get

$$\mathbb{P}\left\{\sup_{0\le t\le \nu}\left(M^*(t) - \frac{\eta}{2e^{\lambda k}}\big\langle M^*(t), M^*(t)\big\rangle\right) > \frac{\nu e^{\lambda k}}{\eta}\ln k\right\} \le \frac{1}{k^\nu}. \tag{59}$$

By the Borel–Cantelli lemma, for almost all $w \in \Omega$, there exists a random integer $v_0 = v_0(w)$ such that

$$M^*(t) \le \frac{\eta}{2e^{\lambda k}} \langle M^*(t), M^*(t) \rangle + \frac{ve^{\lambda k}}{\eta} \ln k, \quad v \ge v_0(\omega). \tag{60}$$

Therefore, we have

$$\begin{aligned}
&\mathrm{d}\bigl(e^{\lambda t} \ln V_5\bigl(x(t)\bigr) + V_6\bigl(x(t)\bigr)\bigr) \\
&\quad \le e^{\lambda t} \Biggl[\sum_{i=1}^{n} \sum_{j=1}^{n} \bigl| b_{ij}\bigl(r(\tau_k)\bigr)\bigr| e^{\lambda \tau_{ij}} x_j(t) - \frac{|x^{\mathrm{T}}(t) g(t)|^2}{2V_5^2(x(t))} \\
&\qquad + \sum_{i=1}^{n} \sum_{j=1}^{n} \bigl| c_{ij}\bigl(r(\tau_k)\bigr)\bigr| \int_0^{\infty} k_{ij}(s) e^{\lambda s} x_j(s)\, \mathrm{d}s \\
&\qquad + \lambda \ln V_5\bigl(x(t)\bigr) + \sum_{i=1}^{n} \bigl| b_i\bigl(r(\tau_k)\bigr)\bigr| + \sum_{i=1}^{n} \sum_{j=1}^{n} \bigl| a_{ij}\bigl(r(\tau_k)\bigr)\bigr| x_j(t) \Biggr] \mathrm{d}t \\
&\qquad + \frac{e^{\lambda t} |x^{\mathrm{T}}(t) g(t)|}{V_5(x(t))} \mathrm{d}B(t) \\
&\quad \le e^{\lambda t} \biggl[\sqrt{n} \bigl| x(t) \bigr| (a + b + c) + \lambda \ln V_5\bigl(x(t)\bigr) + \bar{b} - \frac{|x^{\mathrm{T}}(t) g(t)|^2}{2V_5^2(x(t))} \biggr] \mathrm{d}t \\
&\qquad + \frac{e^{\lambda t} |x^{\mathrm{T}}(t) g(t)|}{V_5(x(t))} \mathrm{d}B(t),
\end{aligned} \tag{61}$$

where

$$a = \max_j \sum_{i=1}^{n} \bigl| a_{ij}\bigl(r(\tau_k)\bigr)\bigr|, \qquad b = \max_j \sum_{i=1}^{n} \bigl| b_{ij}\bigl(r(\tau_k)\bigr)\bigr| e^{\lambda \tau_{ij}}, \tag{62}$$

$$c = \max_j \sum_{i=1}^{n} \bigl| c_{ij}\bigl(r(\tau_k)\bigr)\bigr| \bar{k}_{ij}, \qquad \bar{b} = \sum_{i=1}^{n} \bigl| b_i\bigl(r(\tau_k)\bigr)\bigr|. \tag{63}$$

For all $0 \le t \le \tau_k \le k$ and $k \ge k_0(w)$, it follows $e^t \le e^k$ and $V_5^2(x(t)) \le n|x(t)|^2$, we then get

$$\begin{aligned}
e^{\lambda t} \ln V_5\bigl(x(t)\bigr) &\le e^{\lambda t} \ln V_5\bigl(x(t)\bigr) + V_6\bigl(x(t)\bigr) \\
&= \ln V_5\bigl(x(0)\bigr) + V_6\bigl(x(0)\bigr) + \int_0^t \mathrm{d}\bigl(e^{\lambda s} \ln V_5\bigl(x(s)\bigr) + V_6\bigl(x(s)\bigr)\bigr) \\
&\le C_0 + \int_0^t e^{\lambda s} \bigl[\bigl((a + b + c)\sqrt{n} + \lambda \ln \sqrt{n} \bigr) \bigl| x(s) \bigr| + \bar{b} \bigr] \mathrm{d}s \\
&\quad - \int_0^t (1 - \eta) \frac{e^{\lambda s} |x^{\mathrm{T}}(s) g(x(s))|^2}{2n|x(s)|^2} \mathrm{d}s + \frac{ve^{\lambda k}}{\eta} \ln k \\
&\le C_0 + \int_0^t e^{\lambda s} \bigl[\bigl((a + b + c)\sqrt{n} + \lambda \ln \sqrt{n} \bigr) \bigl| x(s) \bigr| + \bar{b} \bigr] \mathrm{d}s + \frac{ve^{\lambda k}}{\eta} \ln k \\
&:= C_0 + \int_0^t e^{\lambda s} U\bigl(x(s)\bigr) \mathrm{d}s + \frac{ve^{\lambda k}}{\eta} \ln k \\
&< C_0 + C_1 \lambda^{-1} \bigl(e^{\lambda t} - 1\bigr) + \frac{ve^{\lambda k}}{\eta} \ln k
\end{aligned} \tag{64}$$

due to the boundedness of x in $\mathbb{R}_+^n$. For $0 \leq k-1 \leq t \leq k$ and $k \geq k_0(w)$, expression (64) thus gives

$$\frac{\ln V_5(x(t))}{\ln t} \leq \frac{C_0 + \eta^{-1}\nu e^{\lambda k}\ln k + \lambda^{-1}C_1(e^{\lambda t}-1)}{e^{\lambda t}\ln(k-1)}, \tag{65}$$

taking superior limit on both sides of (64) as t tends to infinity, which then becomes

$$\limsup_{t\to\infty}\frac{\ln V_5(x(t))}{\ln t} \leq \frac{\nu e^{\lambda}}{\eta} \quad \text{a.s.} \tag{66}$$

Letting $\nu \to 1$, $\eta \to 1$, and $\lambda \to 0$ yields

$$\limsup_{t\to\infty}\frac{\ln V_5(x(t))}{\ln t} \leq 1 \quad \text{a.s.} \tag{67}$$

Noting that $|x(t)|^2 \leq n\max_i x_i^2(t)$, which leads to

$$\frac{V_5^2(x(t))}{n} \leq \bigl|x(t)\bigr|^2 \leq nV_5^2\bigl(x(t)\bigr), \tag{68}$$

then we have that

$$\limsup_{t\to\infty}\frac{\ln|x(t)|}{\ln t} \leq 1 \quad \text{a.s.} \tag{69}$$

The proof is complete. □

Theorem 4.2 *If condition* (H1) *is valid, for any positive constant p, then there exists a positive constant C such that the following property holds*:

$$\limsup_{T\to\infty}\frac{1}{T}\int_0^T \mathbb{E}\bigl|x(t)\bigr|^p\,\mathrm{d}t \leq C. \tag{70}$$

Proof Let

$$H\bigl(x(t)\bigr) = F\bigl(x(t)\bigr) + \bigl|x(t)\bigr|^p, \tag{71}$$

where $F(x(t))$ is the same as in (24). Then there exists a positive constant C such that

$$\sup_{x(t)\in\mathbb{R}_+^n} H\bigl(x(t)\bigr) \leq C. \tag{72}$$

Integrating both sides of (18) from 0 to T and taking expectation give

$$V_1\bigl(x(T)\bigr) + V_2\bigl(x(T)\bigr) - V_1\bigl(x(0)\bigr) - V_2\bigl(x(0)\bigr) \leq \int_0^T F\bigl(x(t)\bigr)\,\mathrm{d}t, \tag{73}$$

the relationship $|x(t)|^p = H(x(t)) - F(x(t))$ implies that

$$\int_0^T \mathbb{E}\bigl|x(t)\bigr|^p\,\mathrm{d}t \leq CT + V_1\bigl(x(0)\bigr) + V_2\bigl(x(0)\bigr) - V_1\bigl(x(T)\bigr) - V_2\bigl(x(T)\bigr). \tag{74}$$

Letting $T \to \infty$ yields that

$$\limsup_{t\to\infty} \frac{1}{T}\int_0^T \mathbb{E}\big|x(t)\big|^p \,\mathrm{d}t \le C. \tag{75}$$

The proof is complete. □

5 Example and conclusion

We take the weight function $k_{ij}(t) = e^{-(\lambda+1)t}$ in (8), then (H2) is always valid because $\int_0^\infty k_{ij}(s)e^{\lambda s}\,\mathrm{d}s = 1 < \infty$. Therefore the existence and uniqueness of the positive solution is derived with probability one as time approaches infinity. Moreover, the pth moment of the solution is controlled by a bounded constant as presented in Lemma 3.1, and the solution is stochastically ultimately bounded as proved in Theorem 3.2. In addition, the absolute value of the solution will not explode in a long run, and the pth moment in the mean always keeps a constant no matter how large the time scale is.

Acknowledgements
The authors would like to thank the anonymous referees for their helpful comments and suggestions. This research work is supported by the National Natural Science Foundation of China (No. 11201075) and the Natural Science Foundation of Fujian Province of China (No. 2016J01015).

Competing interests
The authors declare that they have no competing interests in the manuscript.

Authors' contributions
All authors conceived of the study and carried out the proof. All authors of this paper read and approved the final manuscript.

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Publisher's Note

Received: 16 November 2017 Accepted: 20 April 2018 Published online: 02 May 2018

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